Conformal description of inflation and primordial B-modes

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Outline

Predictions of inflation

- Nearly scale-invariant power spectrum

$$P_{\mathscr{R}}(k) = \left\langle |\mathscr{R}(k)|^2 \right\rangle \propto k^{n_{\mathscr{R}}-4} \text{ with } n_{\mathscr{R}} \approx 1$$

CMB observations: $n_{\mathscr{R}} = 0.960 \pm 0.007$ at 1σ

- Nearly Gaussian fluctuations

$$f_{\rm NL} = \frac{5}{12} \lim_{k_3 \to 0} \frac{\langle \mathscr{R}(k_1)\mathscr{R}(k_2)\mathscr{R}(k_3)\rangle}{P_{\mathscr{R}}(k_1)P_{\mathscr{R}}(k_2) + 2 \text{ perm}} \ll 1$$

CMB observations: $f_{\rm NL} = 2.7 \pm 5.8$ at 1σ

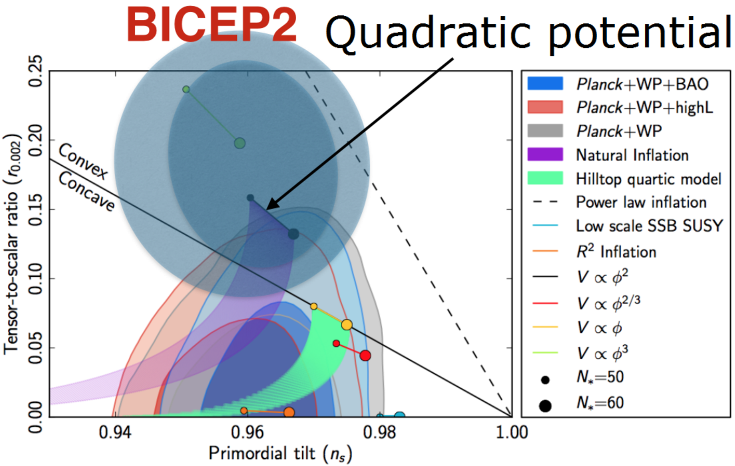
- Nearly scale-invariant gravitational waves

$$P_T(k) = \sum_{s=+,\times} \left\langle |h_s(k)|^2 \right\rangle \propto k^{n_T-3} \text{ with } n_T = -2\epsilon$$

Model dependent: $r \equiv P_T/P_{\mathscr{R}} = \mathscr{O}(0.1) \sim \mathscr{O}(10^{-5})$

This is the situation until 16 Mar, 2014...

Status of inflation models after BICEP2



Starobinsky $R+\alpha R^2$ is not favored, but chaotic $m^2\phi^2$ model is back?

Recovering Starobinsky model

$$\mathscr{L} = \sqrt{-g}\left[\frac{R}{12}\left(\chi^2-\phi^2\right)+\frac{1}{2}\partial^\mu\chi\partial_\mu\chi-\frac{1}{2}\partial^\mu\phi\partial_\mu\phi-\frac{\lambda}{4}\phi^2\left(\chi-\phi\right)^2\right]$$

Invariant under local conformal transformations

$$g_{\mu\nu}\to e^{-2\sigma(x)}g_{\mu\nu}\,,\quad \chi\to e^{\sigma(x)}\chi\,,\quad \phi\to e^{\sigma(x)}\phi$$

Gauge fixing: choose $\sigma(x)$ in such a way that $\chi^2-\phi^2=6$

$$\chi=\sqrt{6}\cosh\left(\frac{\varphi}{\sqrt{6}}\right)\,,\quad \phi=\sqrt{6}\sinh\left(\frac{\varphi}{\sqrt{6}}\right)$$

$$\mathscr{L} = \sqrt{-g}\left[\frac{R}{2}-\frac{1}{2}\partial^\mu\varphi\partial_\mu\varphi-\frac{9}{4}\lambda e^{-4\varphi/\sqrt{6}}\left(1-e^{2\varphi/\sqrt{6}}\right)^2\right]$$

Starobinsky model $R+\alpha R^2$ in Einstein frame with $\alpha=1/(18\lambda)$

$SO(1,1)$ symmetry: T-model

$$\mathscr{L} = \sqrt{-g}\left\{\frac{R}{12}\left(\chi^2-\phi^2\right)+\frac{1}{2}\left[\left(\partial_\mu\chi\right)^2-\left(\partial_\mu\phi\right)^2\right]-\frac{1}{36}F(\phi/\chi)\left(\chi^2-\phi^2\right)^2\right\}$$

Claims are:

- $SO(1,1)$ symmetry is preserved except for $F(\phi/\chi)$
- With $z\equiv\phi/\chi=\tanh\left(\varphi/\sqrt{6}\right)$, $SO(1,1)$ is restored as $z\to 1$
- Inflation happens there, and ends as $SO(1,1)$ is broken
- With a specific form $F(z)=\lambda z^{2p}$, after gauge fixing, $V(z)=F(z)$
- Predictions are similar to Starobinsky model

Conformal description

But we can do more than Starobinsky-like models w/o $SO(1,1)$

- Conformal symm = gauge symm (Padilla et al. 2013, Hertzberg 2014)
- No need to stick to (seemingly) conformal symm
- More than Starobinsky-like models can be embedded

What we do

- General relation between slow-roll parameters
- Specific form of potential to support SR
- Large class of models with 2 non-trivial examples

Slow-roll conditions

Keep conformal inv manifest but not $SO(1,1)$ preservation

$$V(\phi,\chi) = V_0\chi^4 f(\phi/\chi) \underset{\chi^2-\phi^2=6}{\longrightarrow} V(z) = 36V_0\frac{f(z)}{\left(1-z^2\right)^2}$$

Two 2nd order poles, $z=-1$ and $z=1$, which may spoil SR

$$\frac{V_\varphi}{V} = \frac{1}{\sqrt{6}}\left[4z + \left(1-z^2\right)\frac{f'}{f}\right] \equiv \frac{g(z)}{\sqrt{6}}$$

$$\to \epsilon \equiv \frac{1}{2}\left(\frac{V_\varphi}{V}\right)^2 = \frac{g^2(z)}{12}$$

$$\eta \equiv \frac{\dot\epsilon}{H\epsilon} = -\frac{1-z^2}{3}g'(z)$$

Unless f'/f has a 1st order pole at $z=1$ with a residue 2, $\epsilon = \mathcal{O}(1)$

Non-analytic case at $z = 1$

$g(1 - z \equiv \xi)$ is not an analytic function at $\xi = 0$ and $g'(\xi)$ is divergent

$$g(\xi) = -2\lambda\xi\log\xi \to V(\phi,\chi) \sim \chi^4 \left(\phi/\chi - 1\right)^{2+\lambda(\phi/\chi-1)}$$

- Slow-roll parameter

$$\epsilon = \frac{\lambda^2}{3}\xi^2(\log\xi)^2 \to \xi = \frac{-\sqrt{3\epsilon}}{\lambda W_{-1}(-\sqrt{3\epsilon}/\lambda)} = \exp\left[W_{-1}\left(-\frac{\sqrt{3\epsilon}}{\lambda}\right)\right]$$

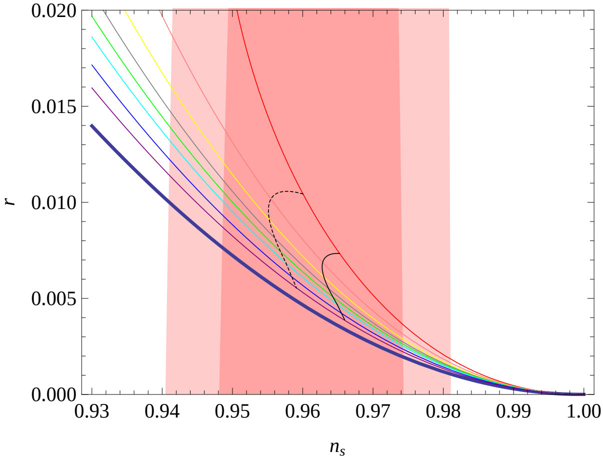
- Spectral index and tensor-to-scalar ratio

$$n_s = 1 - \sqrt{\frac{r}{3}}\left[1 + \frac{1}{W_{-1}(-\sqrt{3r}/\lambda)}\right]\left[1 - \frac{\sqrt{3r}}{8\lambda W_{-1}(-\sqrt{3r}/\lambda)}\right] - \frac{r}{8}$$

- Number of e-folds

$$N = \frac{3}{2\lambda}\mathrm{li}\left(\frac{1}{\xi}\right) \to \xi = \left[\mathrm{li}^{-1}\left(\frac{2\lambda N}{3}\right)\right]^{-1}$$

Prediction of the model



In general, r is very small

Chaotic inflation embedded in conformal description

- Slow-roll parameter ansatz: $\eta = \mu\epsilon$ with $\mu = \mathcal{O}(1)$

$$g(z) = \frac{4}{\mu \tanh^{-1} z} = \frac{8}{\mu}\left[\log\left(\frac{1+z}{1-z}\right)\right]^{-1} \to V(\varphi) = V_0 36^{1-1/\mu}\varphi^{4/\mu}$$

We can embed chaotic inflation with a power-law potential

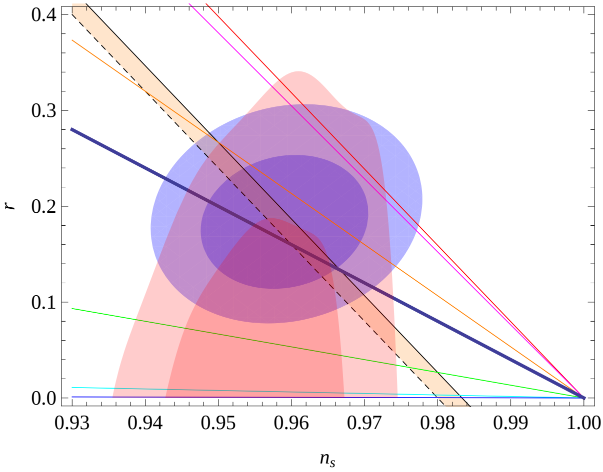
- Number of e-folds

$$N = \frac{3}{16}\mu\log^2\left(\frac{1+z}{1-z}\right) \to z = \tanh\left(2\sqrt{\frac{N}{3\mu}}\right)$$

SR parameters in terms of N

$$\epsilon = \frac{1}{\mu N}, \quad \eta = \frac{1}{N} = \mu\epsilon$$

Predictions of the model



At the center of BICEP2 data

Conclusions

- BICEP2 raised serious questions on Starobinsky-like models
- Conformal description with $SO(1,1)$ symmetry
  - Originally to generalize Starobinsky model (e.g. T-model)
  - More general than previously studied
  - Chaotic inflation with large r can be described
- True(?) conformal symm description would be possible