YUKAWA INSTITUTE FOR THEORETICAL PHYSICS

Cosmological Quests for the Next Decade
KASI, 21 April, 2017

Inflationary massive gravity and its observational signatures

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C. Lin & MS, PLB 752, 84 (2016) [arXiv:1504.01373]
G. Domenech, T. Hiramatsu, C. Lin, MS, M. Shiraishi, Y. Wang, arXiv:1701.05554
S. Kuroyanagi, C. Lin, MS, S. Tsujikawa, in preparation.

Introduction

Inflation: the origin of Big Bang

Brout, Englert & Gunzig '77, Starobinsky '79, Guth '81, Sato '81, …

- Inflation is a quasi-exponential expansion of the Universe at its very early stage; perhaps at $t \sim 10^{-36}$ sec.

- It was meant to solve the initial condition (singularity, horizon & flatness, etc.) problems in Big-Bang Cosmology:
- if any of them can be said to be solved depends on precise definitions of the problems.

- Quantum vacuum fluctuations during inflation turn out to play the most important role. They give the initial condition for all the structures in the Universe.

- Cosmic gravitational wave background is also generated.

In summary, the picture that emerges is in complete accord with the kinematic generalities of causal cosmology presented in Section 2. For $y < y_0$, one has $p < 0$ $(p \simeq -\sigma)$. For $y > y_0$, p becomes positive and λ undergoes an inflection. The situation is summarized in Figs. 1 and 2.

The Creation of the Universe as a Quantum Phenomenon

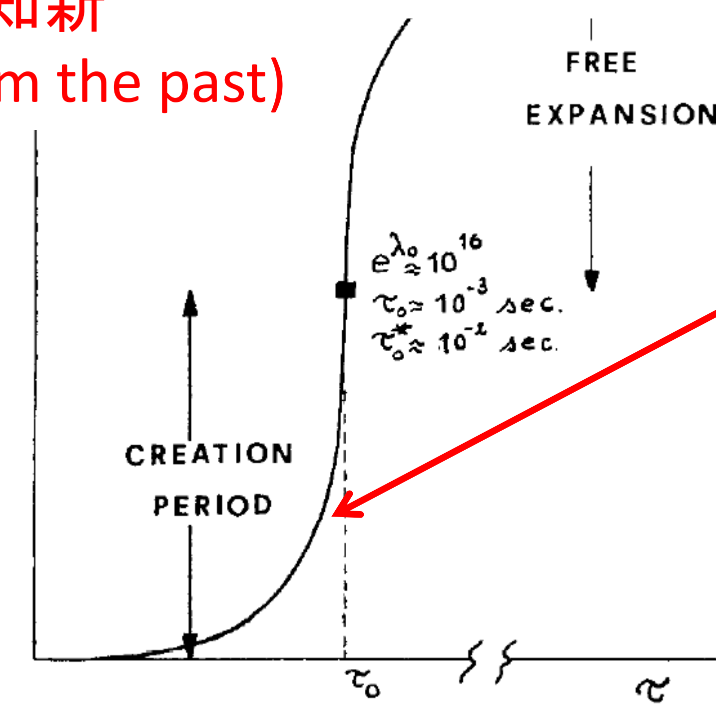
R. Brout, F. Englert, and E. Gunzig

Faculté des Sciences, Université Libre de Bruxelles, Bruxelles, Belgium

Received July 7, 1977

温故知新

(learning from the past)



$$ds^2 = -dt^2 + a^2(t) dH^2_{(3)};$$

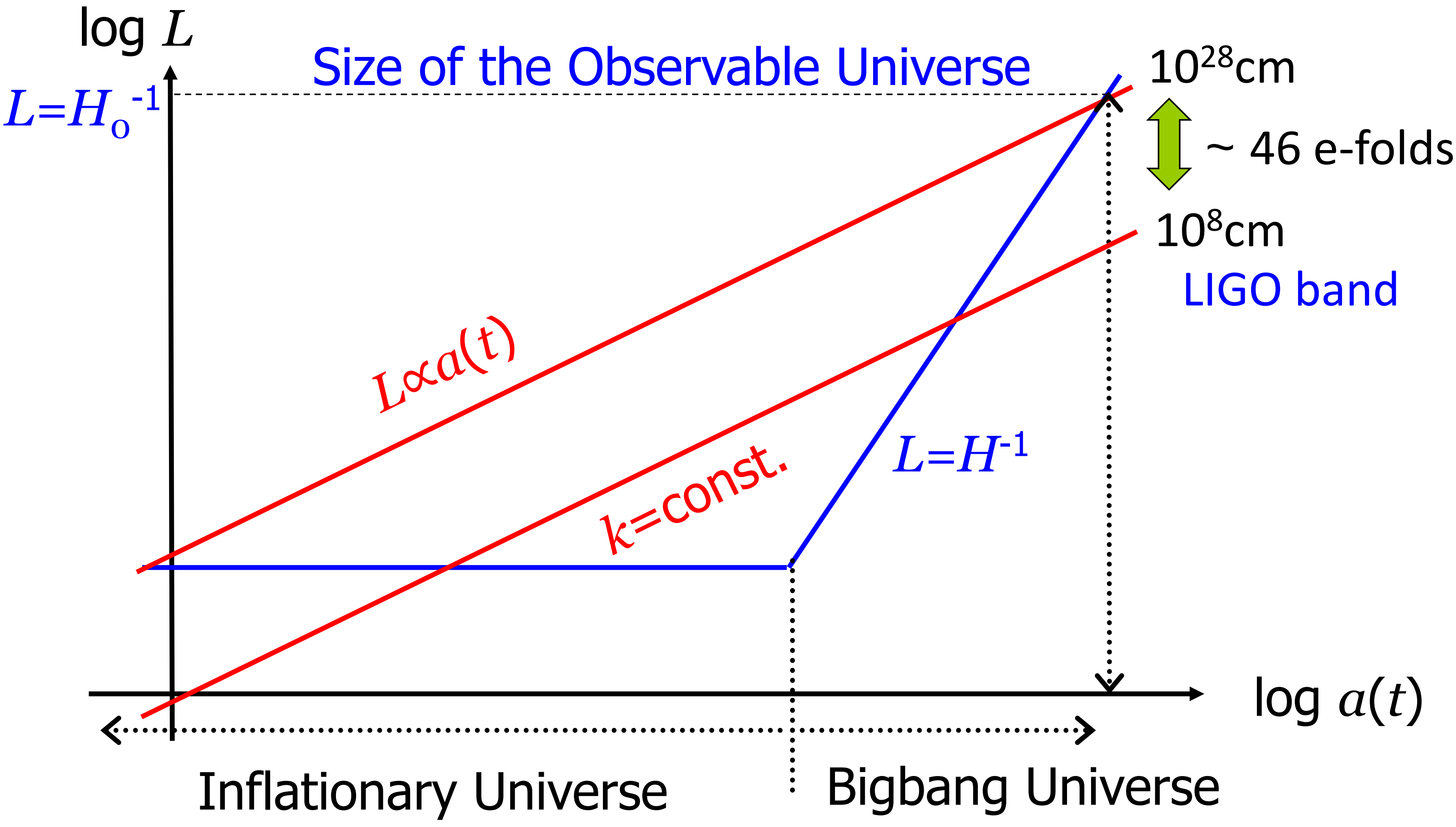
$$a(t) \simeq H^{-1} \sinh Ht$$

Creation of Open Universe!

Now in the context of String Landscape

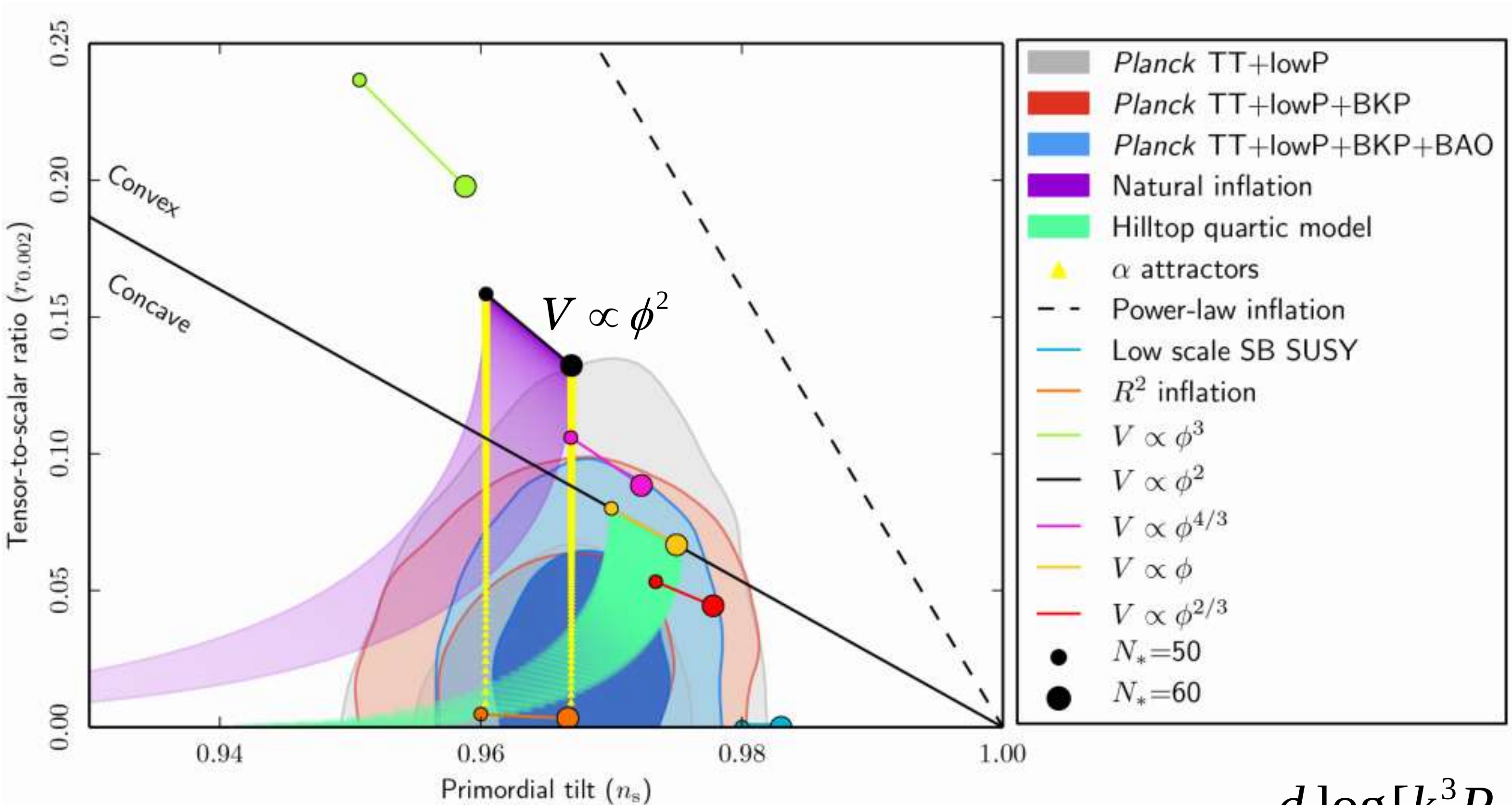
Fig. 1. λ as a function of kinematical time τ for $\delta = 0$. Time scales are calculated for $m = 1$ GeV.

length scales of the inflationary universe



Planck constraints on inflation

Planck 2015 XX



$$n_s - 1 \equiv \frac{d \log[k^3 P_S(k)]}{d \log k}$$

- scalar spectral index: $n_s \sim 0.96$
- tensor-to-scalar ratio: $r < 0.1$
- simplest $V \propto \phi^2$ model is almost excluded

$$r \equiv \frac{P_T(k)}{P_S(k)}$$

Current status

- scalar spectral index: $n_s<1$ at ~ 5 σ
- tensor/scalar ratio: $r < 0.1$ implies $E_{inflation} < 10^{16}$ GeV
- simple, canonical models are on verge of extinction ($m^2\phi^2$ model excluded at > 2 σ)
- R^2 (Starobinsky) model seems to fit best. But why? (large R^2 correction but negligible higher order terms)
- f_{NL}^{local} <O(1) suggests (effectively) single-field slow-roll (but non-slow-roll models with f_{NL}^{local} =O(1) not excluded)

↓

some element of non-canonicality is needed

Massive Gravity?

The idea of massive gravity

- Gauge theory:

Higgs VEV spontaneously breaks gauge symmetry

⇨ massive gauge field

- Gravity:

Spontaneous broken general covariance

⇨ massive gravitons

dRGT gravity

Boulware-Deser (BD) ghost must be removed

spin 2 = 2+2+1 (+1) dof
(tensor+vector+scalar)

This assumes however Poincare symmetry on flat background.
If ∃ no background, covariance should NOT be violated.

Spontaneous broken local Lorentz invariance
= existence of a preferred frame

⇨ massive tensor modes!
(helicity 2)

Dubovsky's model

Dubovsky 2004

- 4 scalar (Stuckelberg) fields: $\varphi^a = (\varphi^0, \ \varphi^i)$

$$ds^2 = \eta_{ab} e_\mu^{(a)} e_\nu^{(b)} dx^\mu dx^\nu : \quad e_\mu^{(a)} \to \bar{e}_\mu^{(a)} = \Lambda^a{}_b(x) e_\nu^{(b)}, \quad \Lambda^a{}_b \in SO(3,1)$$

- VEV spontaneously breaks local SO(3,1) symmetry

$$e_\mu^{(a)} = \frac{\partial \varphi^a}{\partial x^\mu} : \quad \varphi^a = \delta_\mu^a x^\mu \to e_\mu^{(a)} = \delta_\mu^a$$

- required symmetry (Poincare symmetry is not imposed)

$$\varphi^i \to \Lambda_j^i \varphi^j, \quad \varphi^i \to \varphi^i + \xi^i(\varphi^0); \quad \Lambda_j^i \in \text{global } SO(3)$$

- action: $S = \frac{M_P^2}{2} \int d^4x \left[R + m_g^2 f(X, Z^{ij}) \right]$ 2+1 dof (tensor+scalar)

$$X = g^{\mu\nu} \partial_\mu \varphi^0 \partial_\nu \varphi^0 = g^{00} = N^{-2}$$ ← Lapse fcn.

$$Z^{ij} = g^{\mu\nu} \partial_\mu \varphi^i \partial_\nu \varphi^j - \frac{g^{\mu\alpha} \partial_\mu \varphi^0 \partial_\alpha \varphi^i g^{\nu\alpha} \partial_\nu \varphi^0 \partial_\beta \varphi^j}{X} = h^{ij}$$ ← 3-metric

only φ^0 becomes dynamical

Inflationary massive gravity: minimal model

- Identify φ^0 with inflaton: $\phi = \varphi^0$

$$S = \int d^4x \left[\frac{M_P^2}{2} R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) - \frac{9}{8} M_P^2 m_g^2(\phi) \frac{\delta Z^{ij} \delta Z^{ij}}{Z^2} \right]$$

$$\delta Z^{ij} = Z^{ij} - 3 \frac{Z^{ik} Z^{kj}}{Z}; \quad Z = Z^{ii}$$

assumption: $\begin{cases} m_g^2 \ll H^2 & \text{during inflation} \\ m_g^2 = \frac{1}{2} \lambda \phi^2 & \text{during reheating} \end{cases}$

- Symmetry:

$\varphi^i \to \Lambda^i_j \varphi^j, \quad \varphi^i \to \lambda \varphi^i; \quad \Lambda^i_j \in$ global $SO(3)$, $\lambda = const.$

These symmetries guarantees φ^i to be non-dynamical.

$\because \quad \varphi^i = x^i + \delta\phi^i : \quad \delta\phi^i = w^{ij}(t) x^j + v(t) x^i + O(1), \; w^{ij} = -w^{ji}$

at leading order in gradient expansion

notes on non-dynamical modes

- helicity 1 and traceless helicity 0 modes (=3 NG bosons) at leading order on spatially flat slicing (~ decoupling limit)

$$S = \frac{9}{4} m_g^2 M_P^2 \int d^4x \left[\left(\partial_i \pi^j\right)^2 + \frac{1}{3}\left(\partial_i \pi^i\right)^2 \right] + O(M_P^{-2}) \sim \Lambda^4 k^2 \left(\pi^i\right)^2$$

$$\varphi^i = x^i + \pi^i \qquad \Lambda^4 \equiv m_g^2 M_P^2 : \text{ cutoff scale}$$

- become dynamical at higher orders

$$\delta S \sim \Lambda^2 \int d^4x \sqrt{-g}\, g^{\mu\nu} \partial_\mu \delta Z^{ij} \partial_\nu \delta Z^{ij} + \cdots \sim \Lambda^2 k^2 (\dot{\pi}^i)^2 + \cdots$$

rescaling: $k\Lambda\pi^i \to \pi^i$

$$S \sim \int d^4x \left[(\dot{\pi}^i)^2 + \Lambda^2 (\pi^i)^2 + \cdots \right] \qquad \Lambda^2 \gg H^2$$

massive enough: can be integrated out

massive tensor perturbation

- tensor 2nd order action

$$S_T^{(2)} = \frac{M_P^2}{8}\int d^3x\,dt\,a^3\left[(\dot{\gamma}_{ij})^2 - \left(\frac{k^2}{a^2} + m_g^2(\phi)\right)\gamma_{ij}^2\right]$$

- quantization

$$\gamma_{ij} = \sum_{s=\pm}\frac{d^3k}{(2\pi)^{3/2}}\left[a(\boldsymbol{k},s)e_{ij}(\boldsymbol{k},s)\gamma_k(t)e^{i\boldsymbol{k}\cdot\boldsymbol{x}} + h.c.\right]$$

$\gamma_k(t)$: positive frequency fcn.

$a(\boldsymbol{k},s)$: annihilation operator

$e_{ij}(\boldsymbol{k},s)$: polarization tensor

$$e_{ij}(\boldsymbol{k},s)e^{ij}(\boldsymbol{k},s') = \delta_{ss'}$$

- eom

$$\ddot{\gamma}_k + 3H\dot{\gamma}_k + \left(m_g^2 + \frac{k^2}{a^2}\right)\gamma_k = 0 \; : \quad \gamma_k\dot{\gamma}_k^* - \dot{\gamma}_k\gamma_k^* = \frac{i}{a^3}$$

KG normalization

tensor spectrum

- during inflation $0 < m_g^2 \ll H^2$

$$P_T(k) = \frac{2H^2}{\pi^2 M_P^2}\left(\frac{k}{k_f}\right)^{2m_g^2/3H^2}$$ at the end of inflation $k_f = a(t_f)H(t_f)$

spectral index: $n_T \approx -2\varepsilon + \frac{2m_g^2}{3H^2}; \ \varepsilon = -\frac{\dot{H}}{H^2}$

if $\frac{m_g^2}{3H^2} > \varepsilon$, blue-tilted!

- Lyth bound

$\Delta\phi \simeq 15 M_P r^{1/2}$: distance traveled by ϕ during inflation

$r = \frac{P_T(k)}{P_S(k)} \simeq 16\varepsilon$ for standard slow-roll inflation

if $r > 0.001$, ϕ travels more than Planck distance

beyond validity of QFT?

Observational Signatures?

resonant GW amplification

Lin & MS '15

$$m_g^2 = \frac{1}{2}\lambda\phi^2;\ \phi = \phi_f\left(\frac{a_f}{a}\right)^{3/2}\sin(Mt+\theta)e^{-\Gamma Mt}$$ after inflation

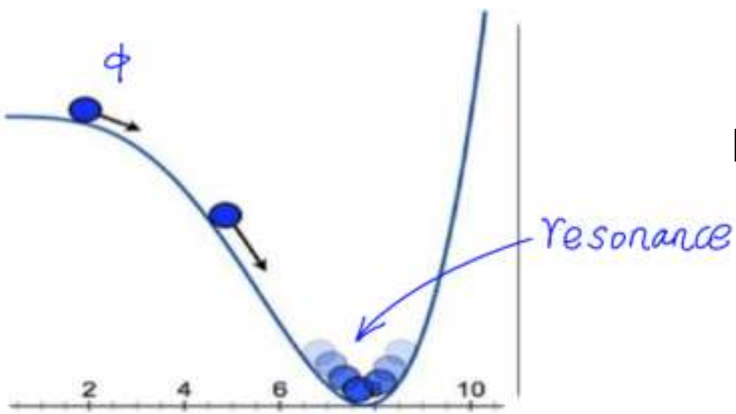
M : inflaton mass

ΓM : decay (reheating) rate

- Mathiew-like eq. for $k/a << H < m_g$

$$\ddot{\gamma}_k + 3H\dot{\gamma}_k + \left(m_g^2(\phi) + \frac{k^2}{a^2}\right)\gamma_k = 0$$

($\frac{k^2}{a^2} \to$ 0)

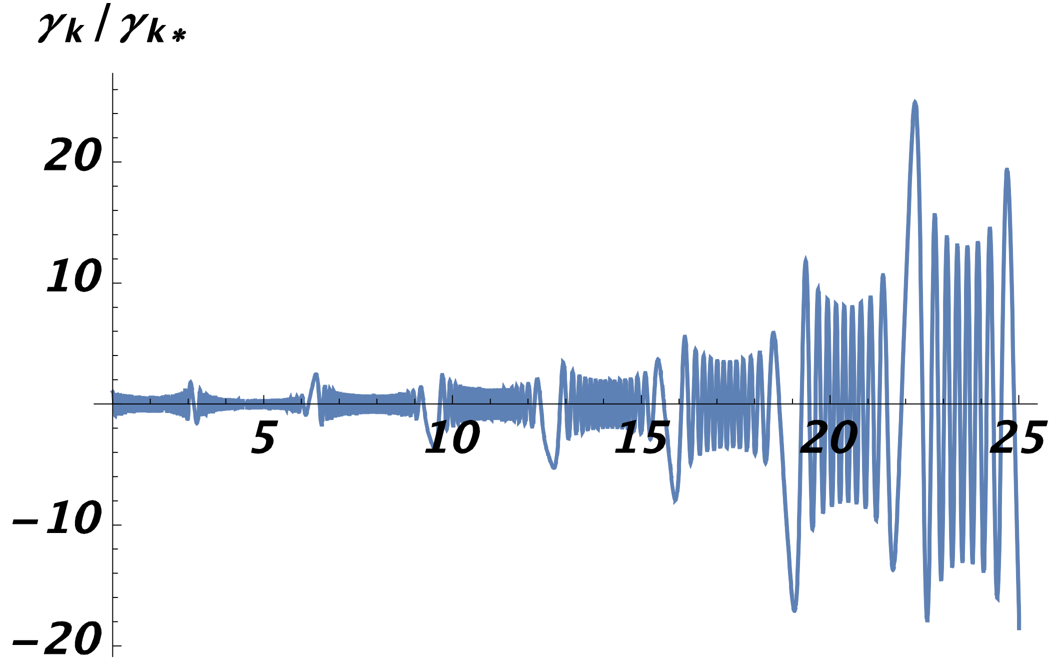


$$\left[\frac{d^2}{dx^2} + \frac{2}{x}\frac{d}{dx} + \frac{\xi e^{-\Gamma x}}{x^2}\sin^2(x)\right]\gamma_k = 0$$

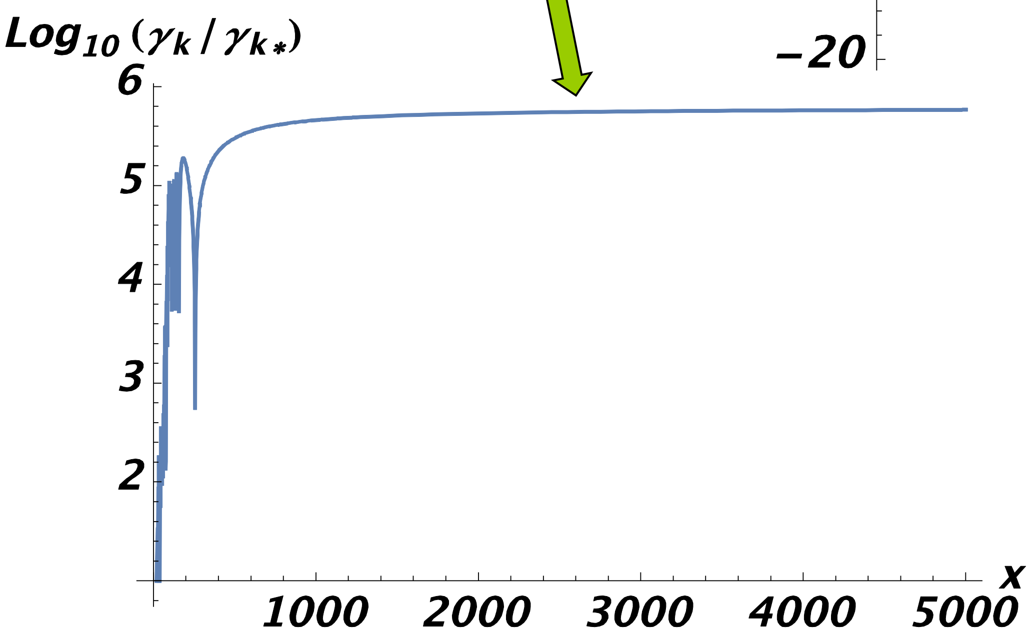
$$x = Mt,\ \xi = \frac{\lambda\phi_f^2}{3\pi M^2}$$

broad parametric resonance for $\frac{\xi e^{-\Gamma M x}}{x^2} \gg 1$

broad parametric resonance

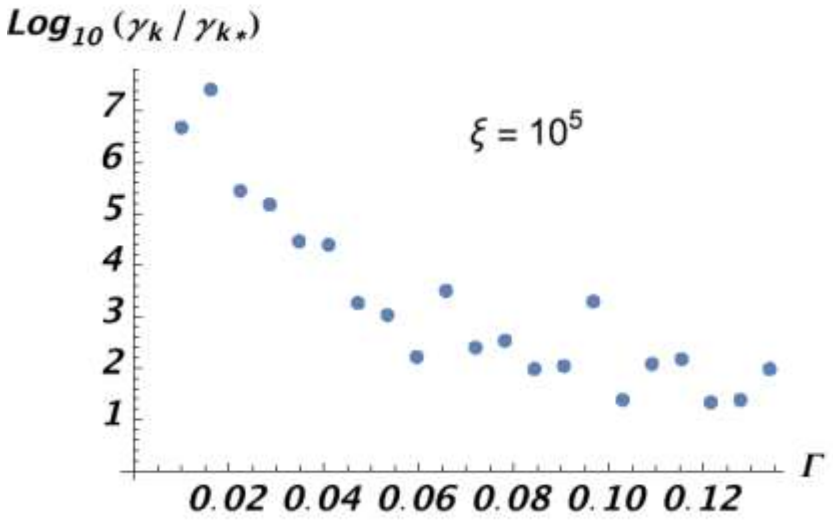


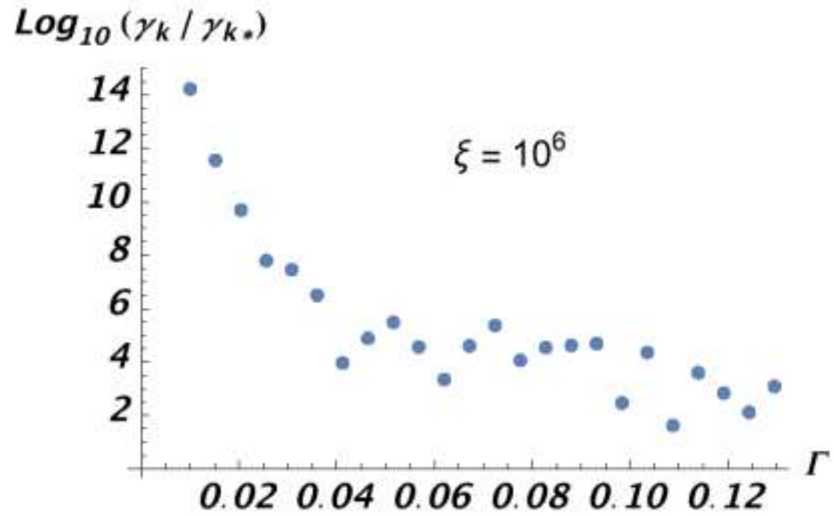
amplified by
a factor ~ 10^6!

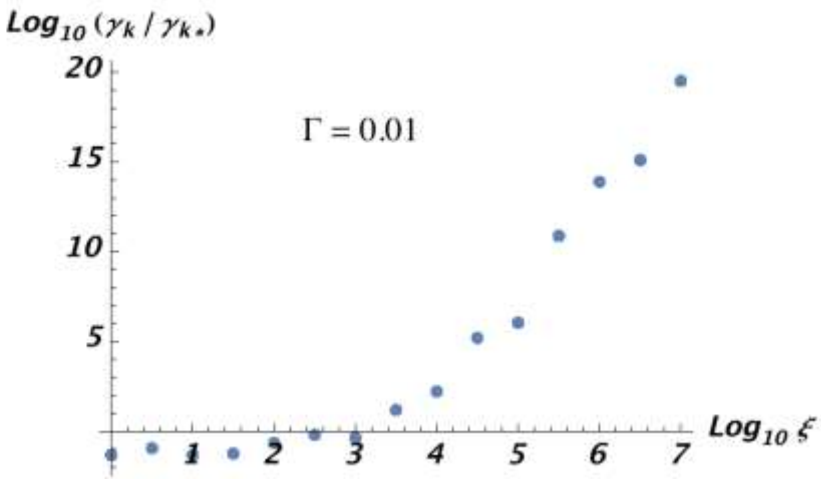


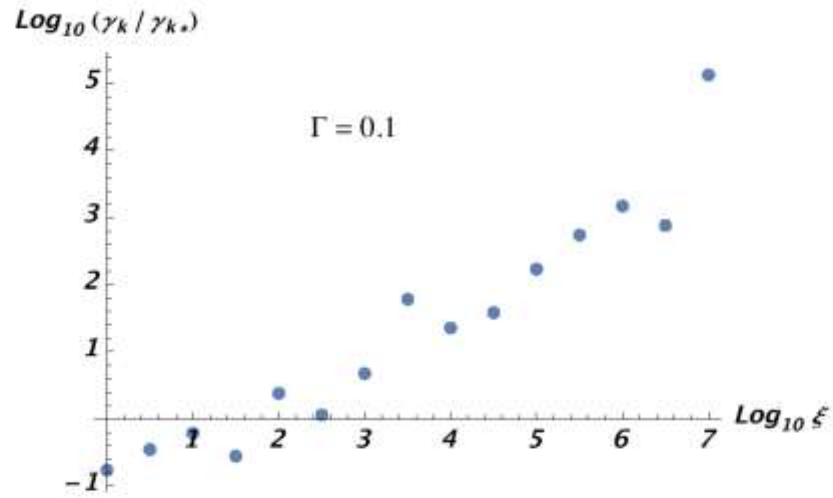
$$\xi = \frac{\lambda \phi_f^2}{3\pi M^2} = 10^6,\ \Gamma = 0.05$$

parameter dependence









evading Lyth bound

- tensor perturbation can be exponentially amplified by broad parametric resonance:

$$P_T(k) = A P_{T,0}(k);\; A \gg 1$$

- scalar (curvature) perturbation remains the same:

$$P_S(k) = P_{S,0}(k)$$

$$r = \frac{P_T(k)}{P_S(k)} = 16\varepsilon A$$

Lyth bound is modified as $\Delta\phi \simeq 15 M_P \sqrt{\frac{r}{A}}$

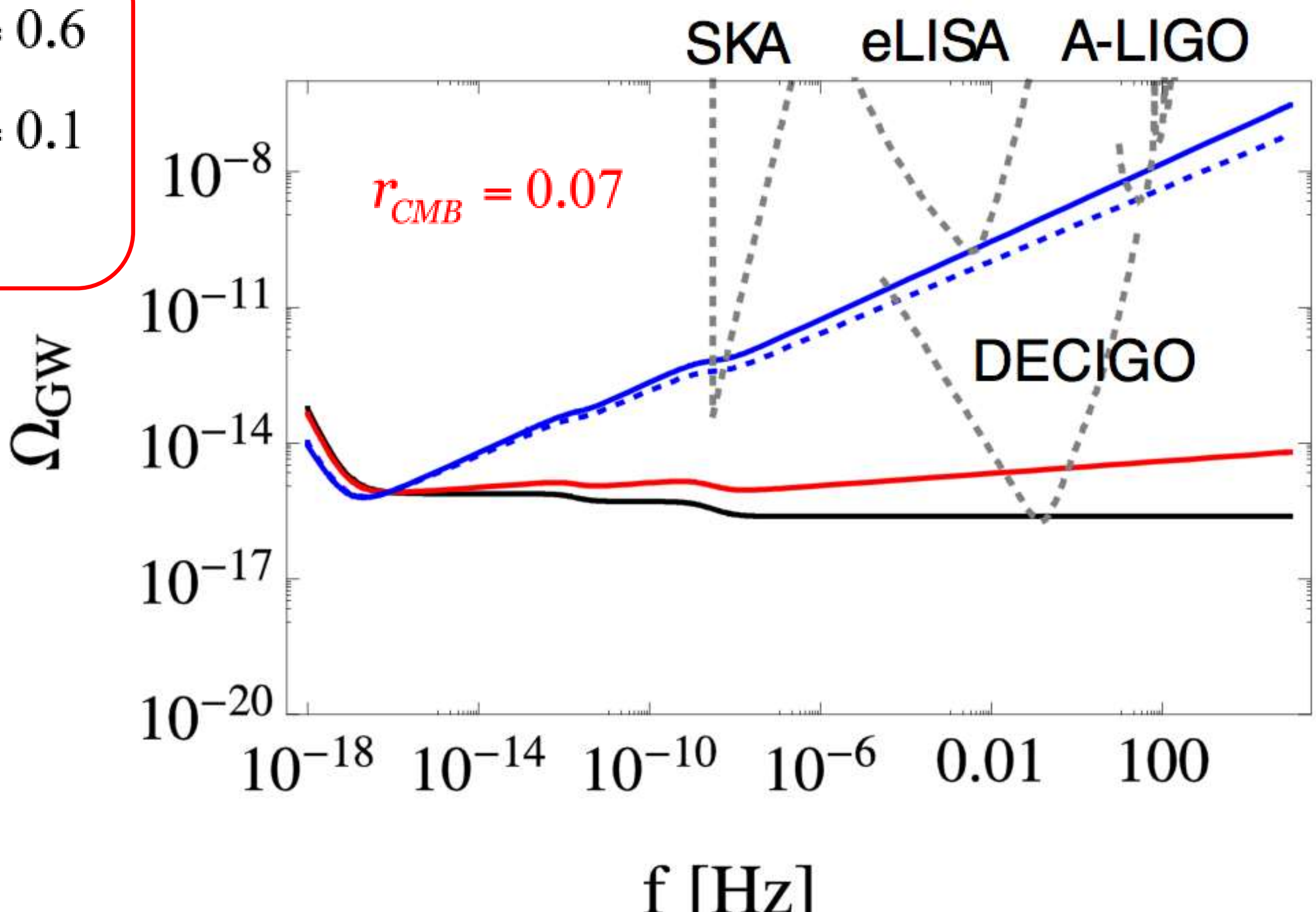
tensor perturbation can be large enough to be detected without invalidating low-energy EFT

strongly blue-tilted GW spectrum?

Kuroyanagi, Lin, MS & Tsujikawa ’17 (in prep)

- : $m_g^2 / H^2 = 0.6$
- : $m_g^2 / H^2 = 0.1$
- : $n_T = 0$

$$n_T \approx \frac{2m_g^2}{3H^2}$$



non-Gaussianity?

Domenech, Hiramatsu, Lin, MS, Shiraishi & Wang '17

- 3rd order Hamiltonian in $\delta\phi=0$ spatially isotropic gauge:

$$H_{\rm int}=-L_{\rm int}\supset\lambda_{SST}M_P^2\varepsilon\int d^3x\,a\,\gamma_{ij}\partial_i\mathcal{R}_c\partial_i\mathcal{R}_c+\cdots$$

$$\lambda_{SST}=\frac{m_g^2}{4\varepsilon H^2}+\lambda$$

$$\propto\delta Z^{ij}\nabla^\mu\varphi^i\nabla^\nu\varphi^j\partial_\mu\phi\partial_\nu\phi$$

coupling term that could appear in low-energy EFT from (unknown) UV physics

- 3-pt fcn:

$$\left\langle\gamma^s_{\boldsymbol{k}_1}\mathcal{R}_{\boldsymbol{k}_2}\mathcal{R}_{\boldsymbol{k}_3}\right\rangle=(2\pi)^3\delta^{(3)}\left(\sum\boldsymbol{k}_i\right)\frac{\lambda_{SST}H^4}{4\varepsilon M_P^4}\frac{e^{-s}_{ij}k_{2i}k_{3j}}{\prod k_i^3}\left(k_t-\frac{\sum_{i<j}k_ik_j}{k_t}-\frac{k_1k_2k_3}{k_t^2}\right)$$

$$k_t=k_1+k_2+k_3$$

dominates CMB 3-pt fcn if λ_{SST} >> 1

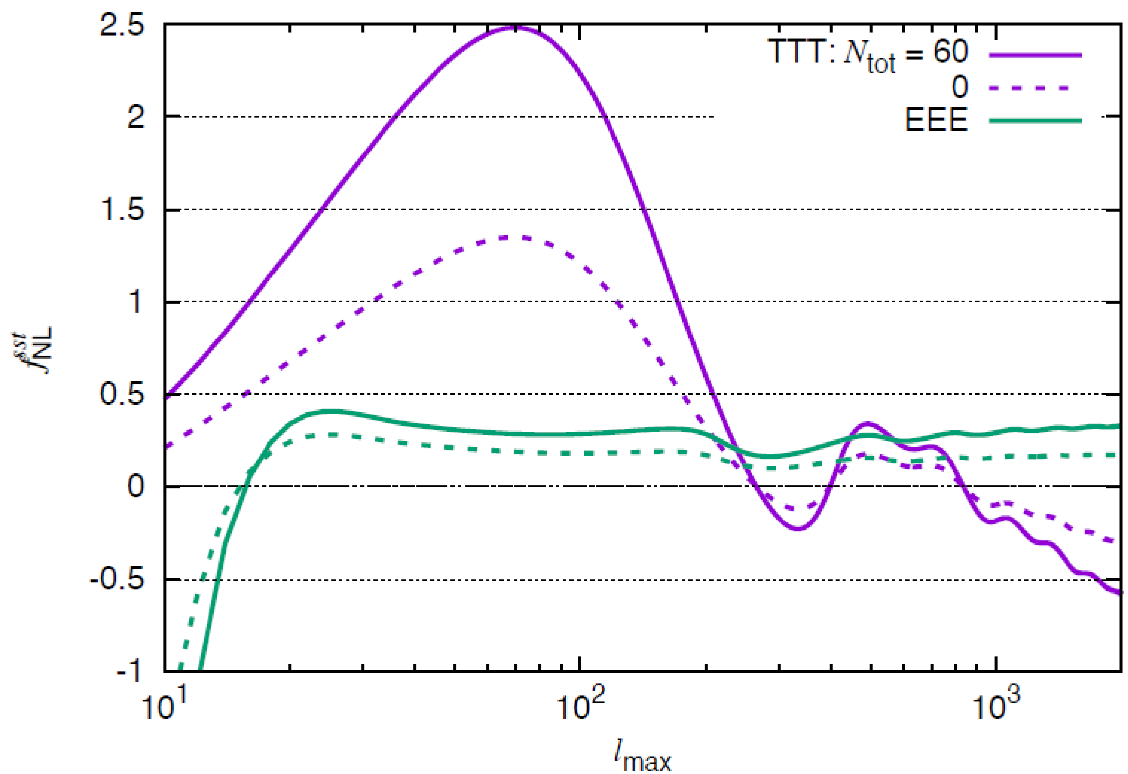
scale-dependent non-Gaussianity

$$\langle \delta T \delta T \delta T \rangle \propto \langle \gamma_{ij} \partial_i \mathcal{R}_c \partial_j \mathcal{R}_c \rangle$$

$\gamma_{ij} \propto a^{-1}$ after horizon re-entry

small scale modes re-enters horizon earlier

➡ large $\boldsymbol{\ell}$ multipoles are suppressed



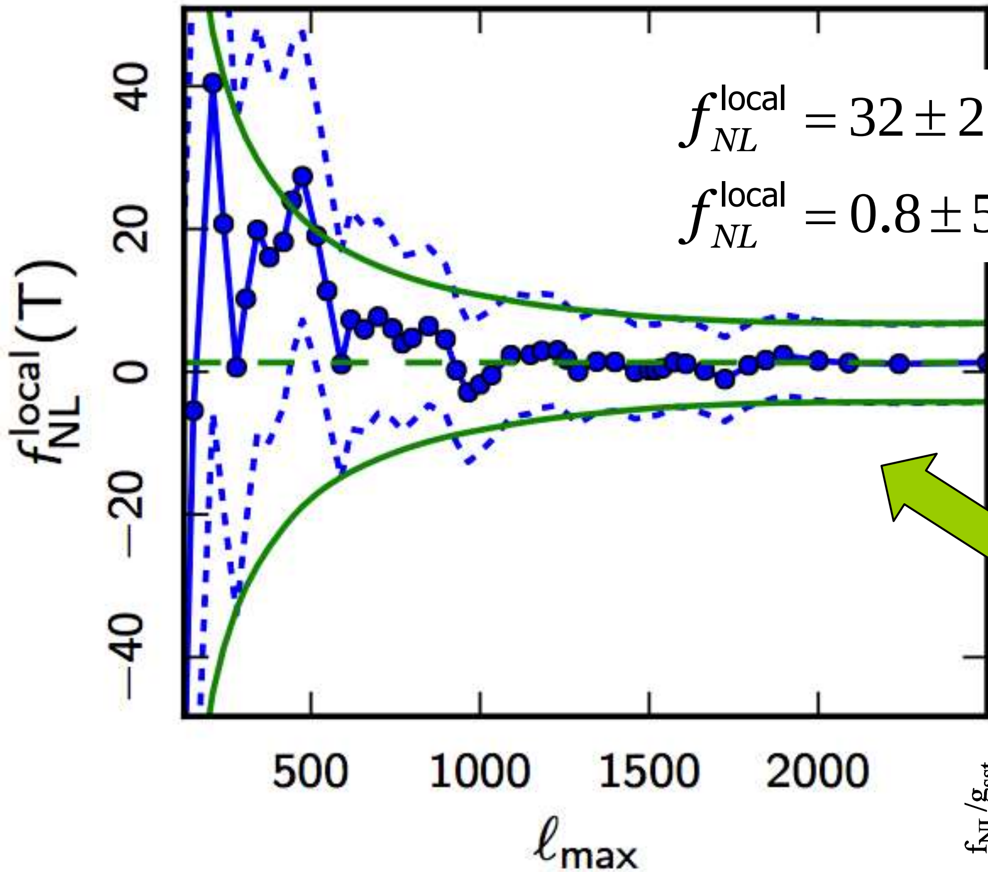
$\ell_{\max}$ dependence of $f_{NL}^{\rm local}$

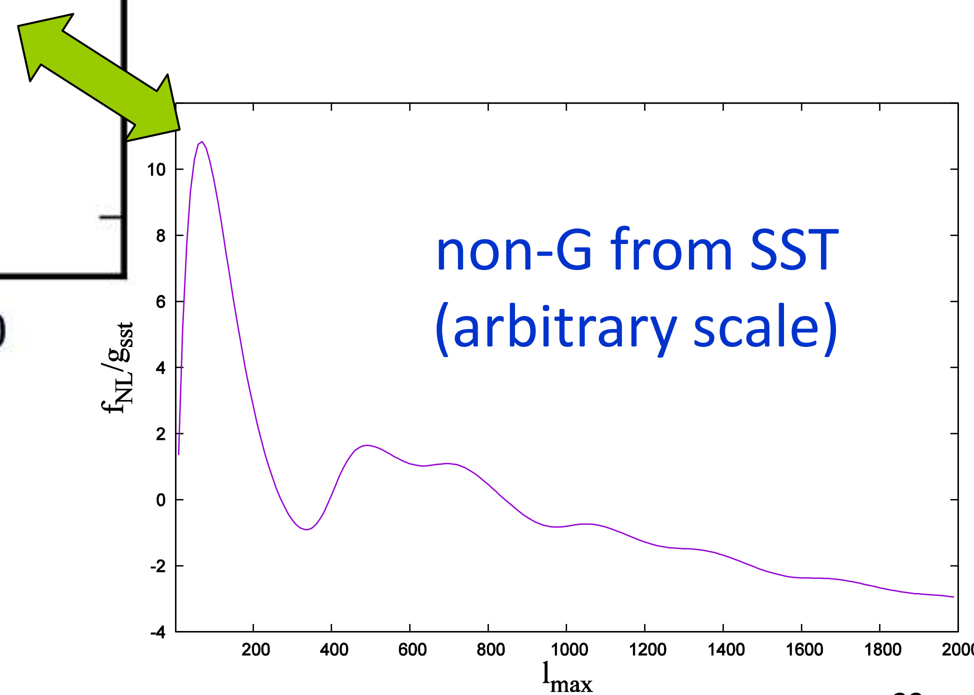
$\lambda_{SST} = 100, \ \varepsilon = 0.01$

WMAP 2010/Planck 2015

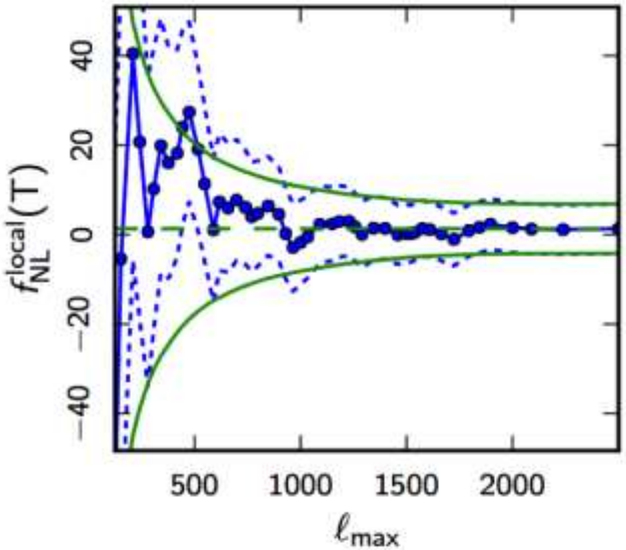
$$f_{NL}^{\text{local}} = 32 \pm 21 \quad \text{for } \ell_{\max} \simeq 500 \quad \text{WMAP 2010}$$

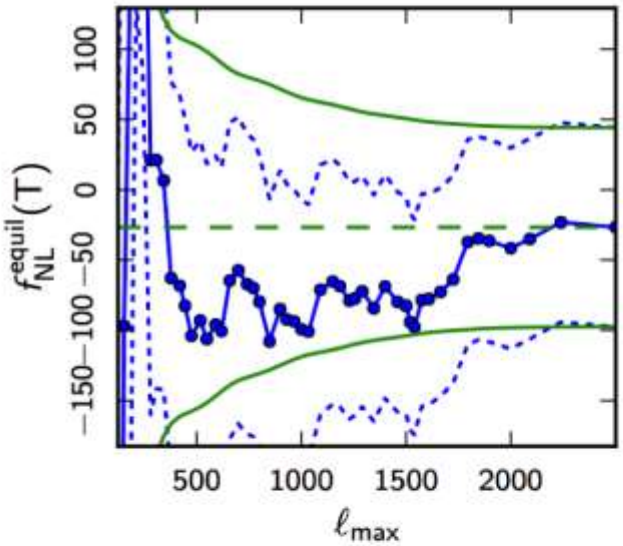
$$f_{NL}^{\text{local}} = 0.8 \pm 5.0 \quad \text{for } \ell_{\max} \simeq 2500 \quad \text{Planck 2015}$$



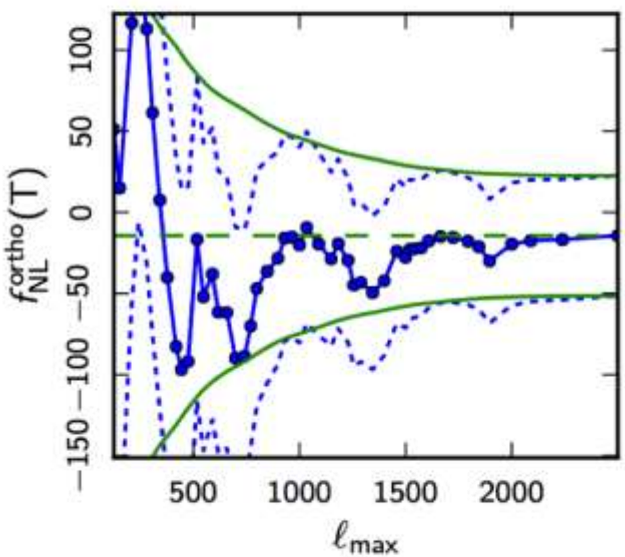


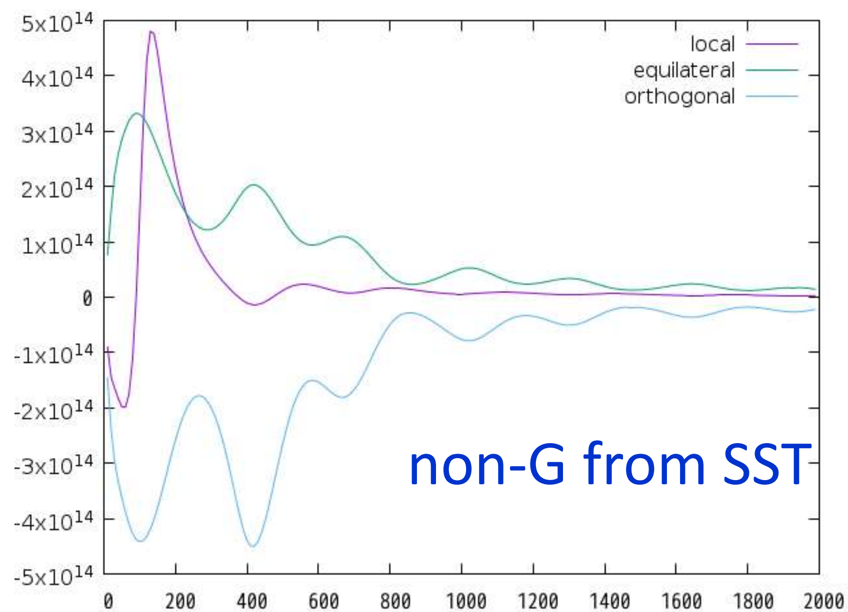
more from Planck 2015/other shapes





a hint of non-G due to SST but not yet conclusive





Summary

- Inflation is a natural platform for modified gravity

  Inflation = scalar-tensor theory

- GW (tensor mode) can become massive during inflation without encountering BD ghost problem

  symmetry: $\varphi^i \to \Lambda^i{}_j \varphi^j, \ \varphi^i \to \lambda \varphi^i; \ \Lambda^i{}_j \in SO(3), \ \lambda = const.$

- GW can be parametrically amplified during reheating

  evading Lyth bound even if $r > 0.001$

- GW spectrum may be blue-tilted

  primordial GW may be detectable by LIGO/Virgo/KAGRA…

- 3rd order interaction can give rise to sizable scale-dependent non-Gaussianity

  ∃ already a hint in WMAP/Planck data ⇨ needs further tests!