Gravity from Cosmology

Pedro G. Ferreira
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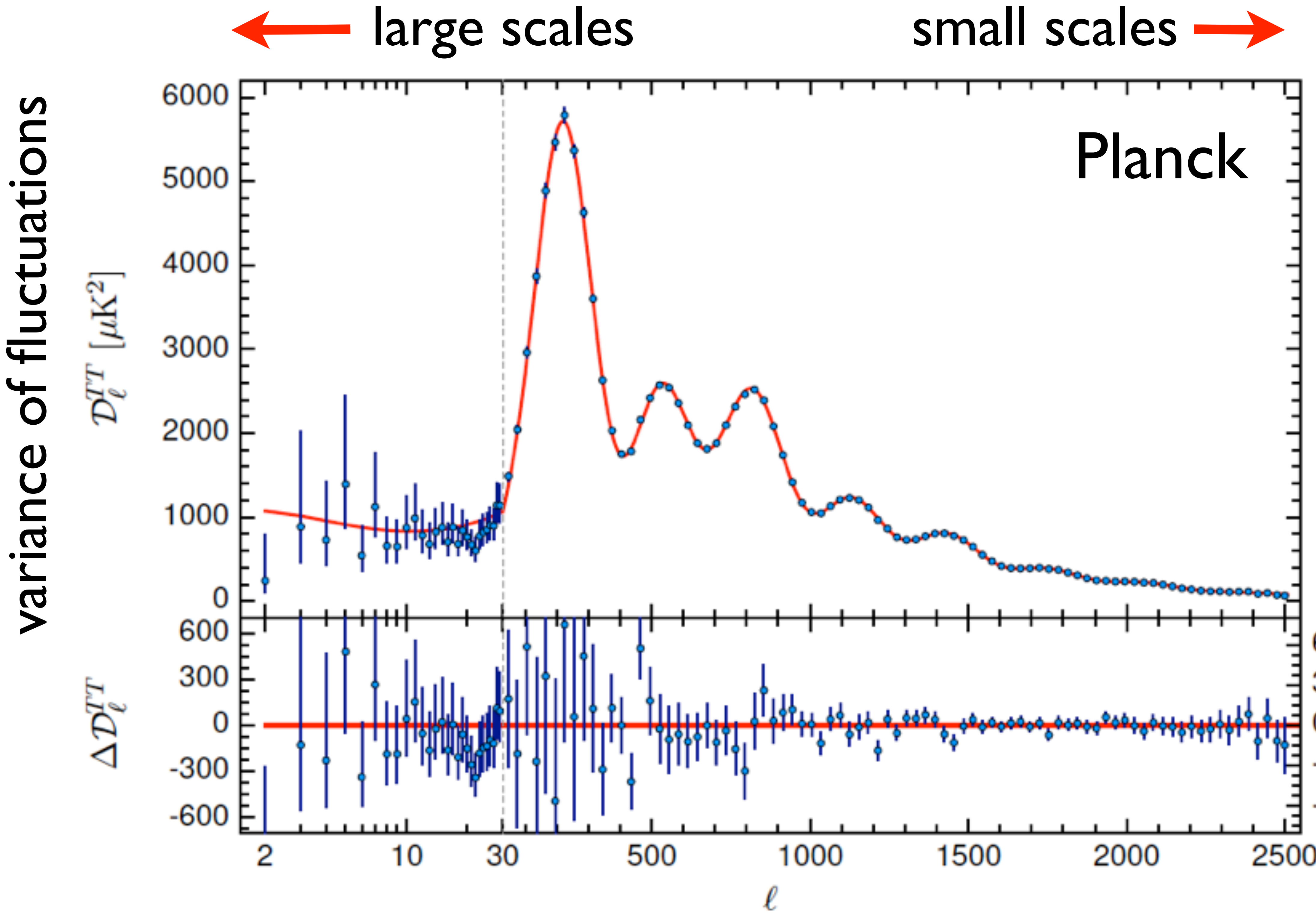
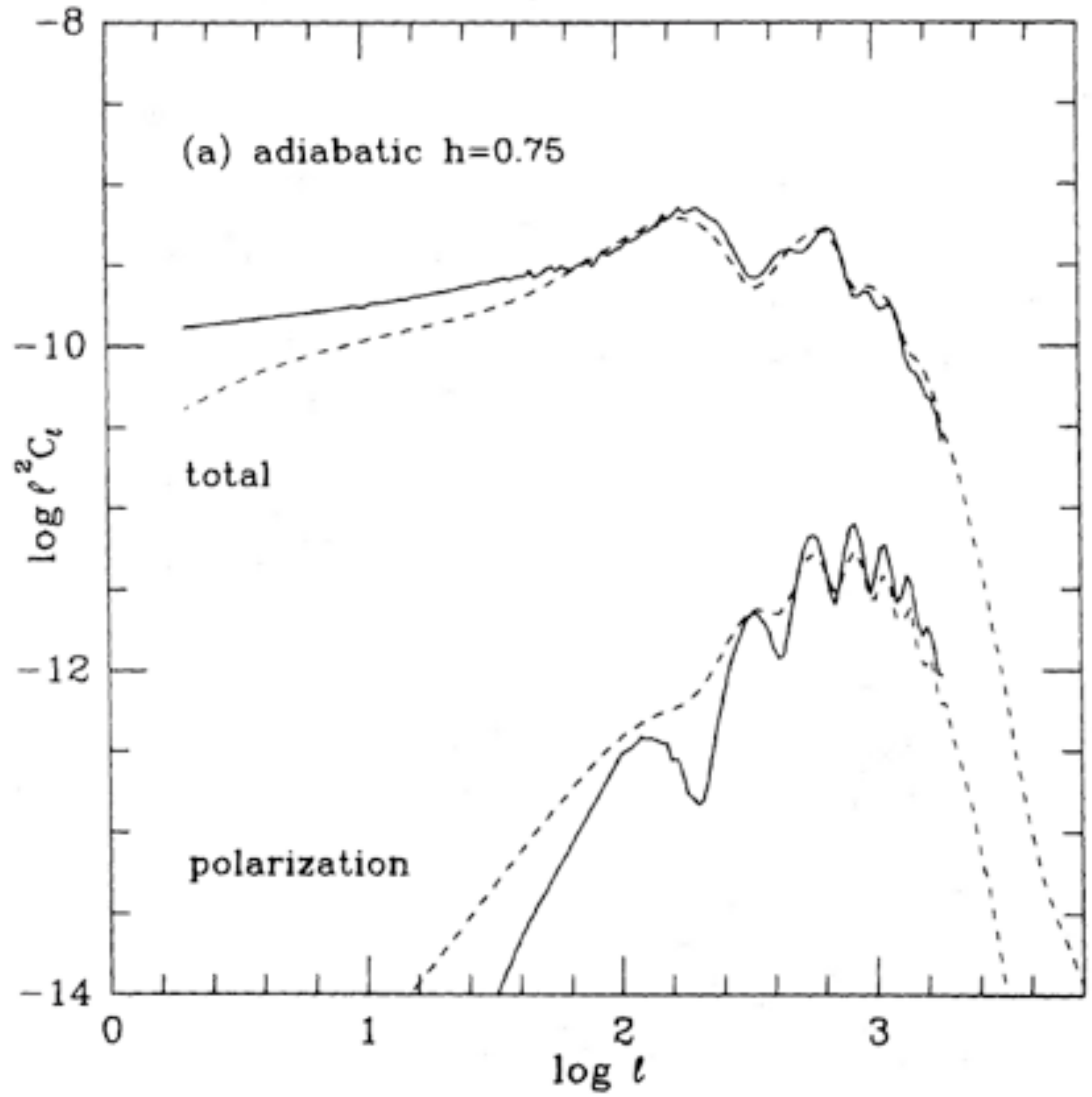
The state of General Relativity in 1957

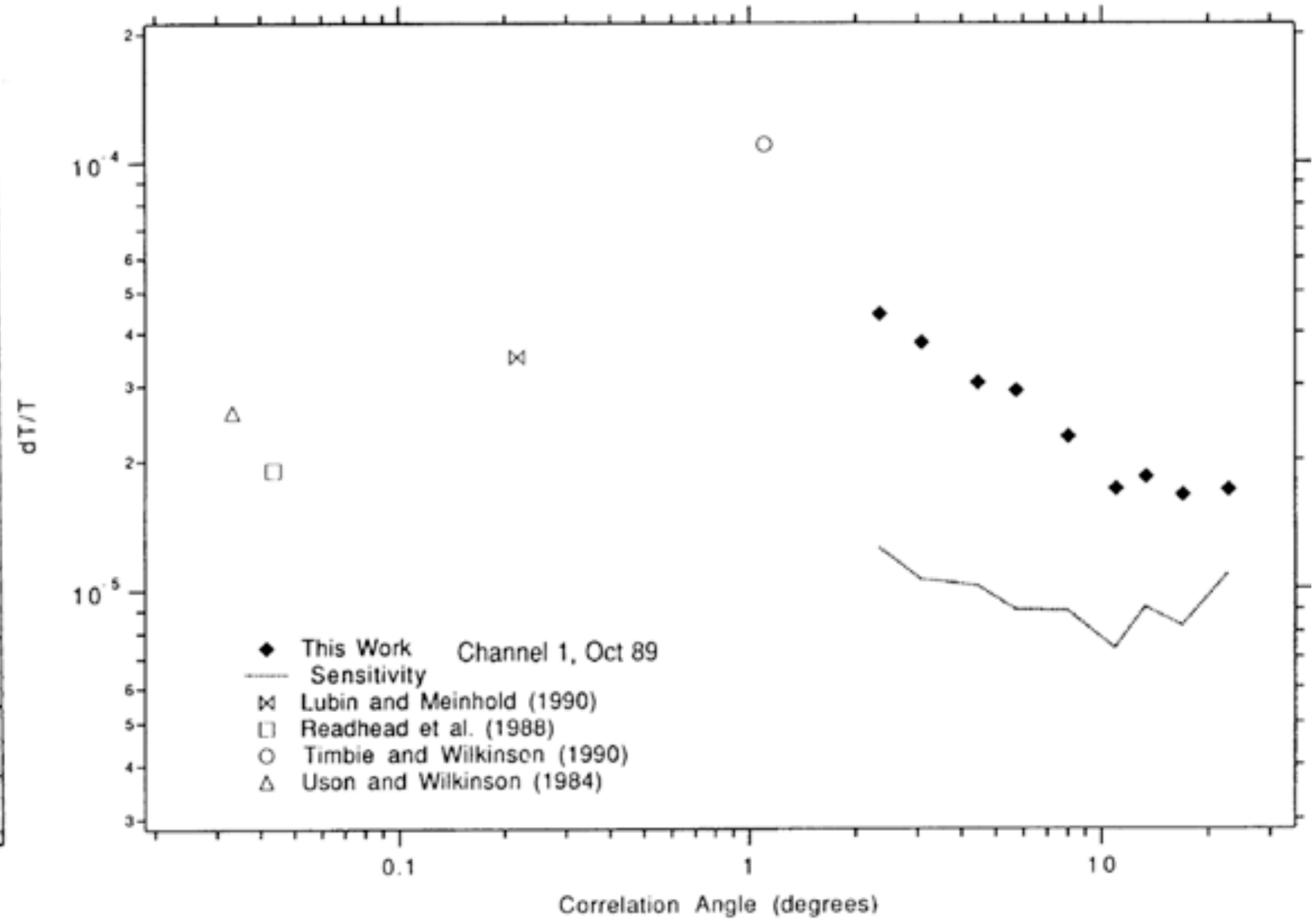
"There exists. . . one serious difficulty, and that is the lack of experiments. Furthermore, we are not going to get any experiments, so we have to take the viewpoint of how to deal with the problems where no experiments are available. ... the best viewpoint is to pretend that there are experiments and calculate. In this field we are not pushed by experiments but pulled by imagination."

R. Feynman, Chapel Hill workshop on gravitation (1957)

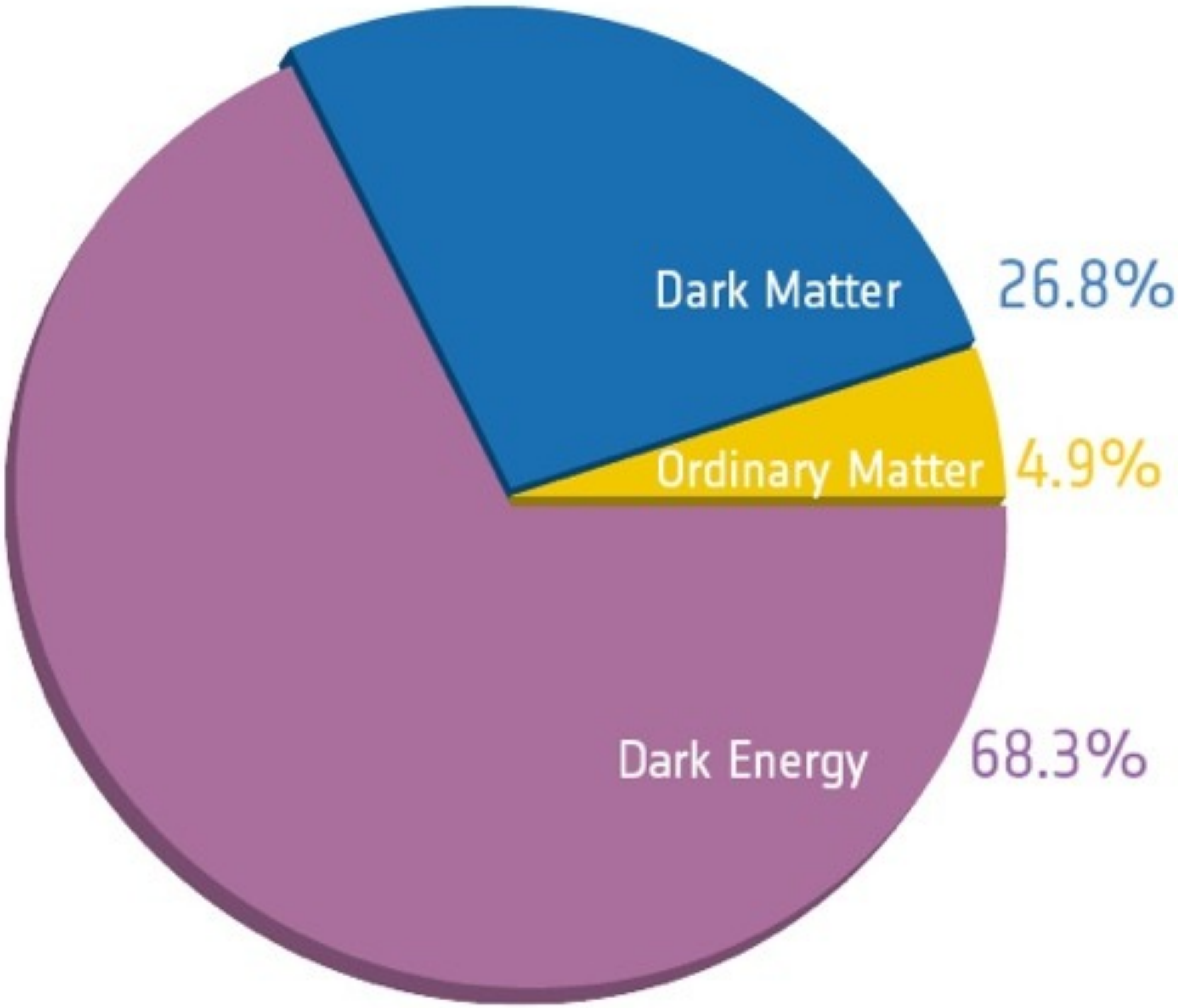

large scales
small scales
variance of fluctuations
Planck
$\mathcal{D}_\ell^{TT}$ [μK^2]
$\Delta\mathcal{D}_\ell^{TT}$
6000
5000
4000
3000
2000
1000
0
600
300
0
-300
-600
60
30
0
-30
-60
2
10
30
500
1000
1500
2000
2500
ℓ



Bond & Efstathiou 1987

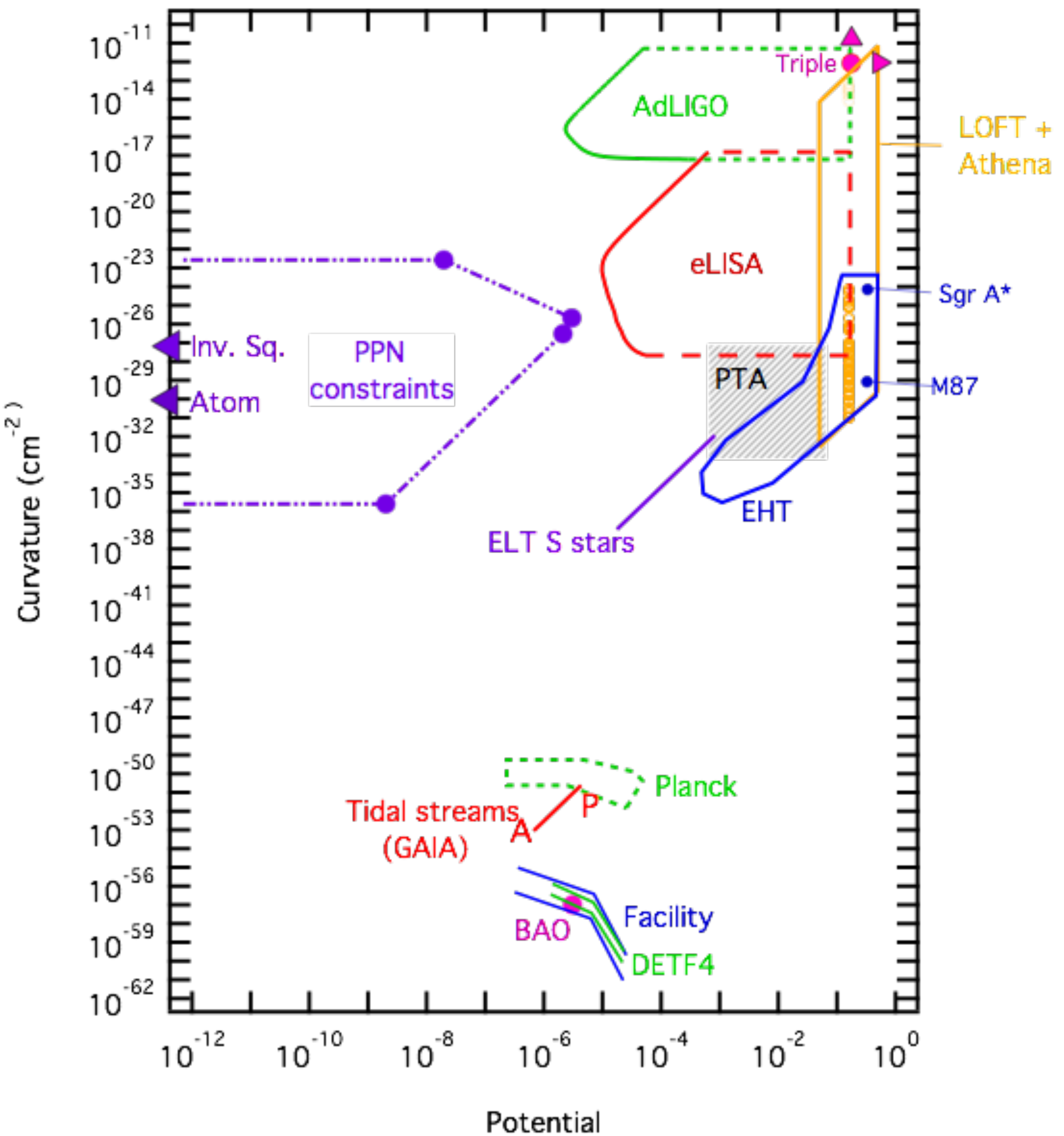


Meyer, Cheng and Page 1991


Dark Matter
26.8%
Ordinary Matter
4.9%
Dark Energy
68.3%

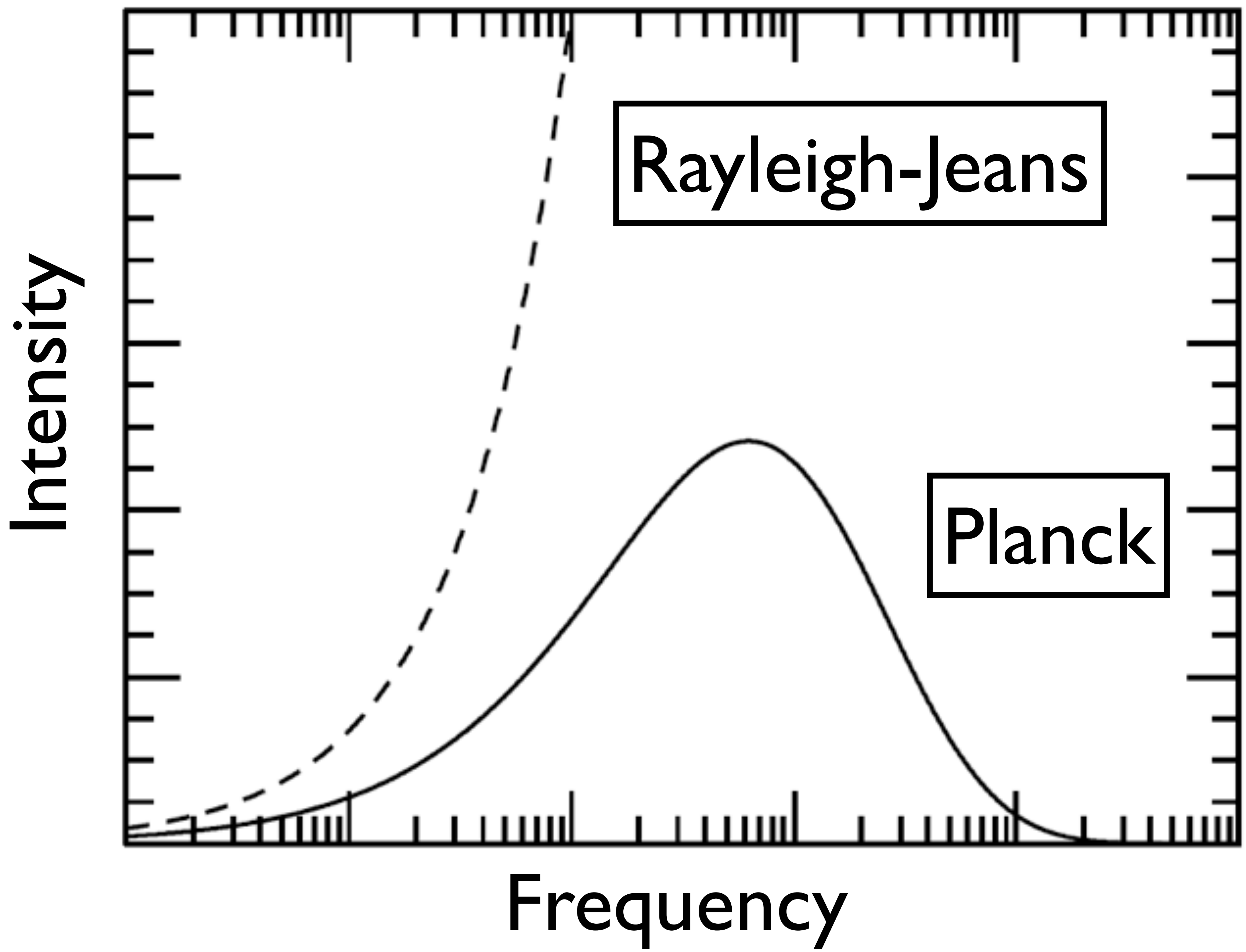
"The elegant logic of general relativity theory, and its precision tests, recommend GR as the first choice for a working model for cosmology. But the Hubble length is fifteen orders of magnitude larger than the length scale of the precision tests, at the astronomical unit and smaller, a spectacular extrapolation."

Jim Peebles, IAU 2000



(Baker, Psaltis & Skordis 2014)

Big puzzles ...



... small inconsistencies.

- Lamb shift
- Wu parity violation experiment
- Fitch-Cronin CP violation
- Precession of perihelion of Mercury

The Theory

Einstein Gravity

Curvature

$$\frac{1}{16\pi G}\int d^4x\sqrt{-g}[R(g)-2\Lambda]+\int d^4x\sqrt{-g}\mathcal{L}(g,\mathrm{matter})$$

Metric of space-time

Lovelock's theorem (1971) :*"The only second-order, local gravitational field equations derivable from an action containing solely the 4D metric tensor (plus related tensors) are the Einstein field equations with a cosmological constant."*

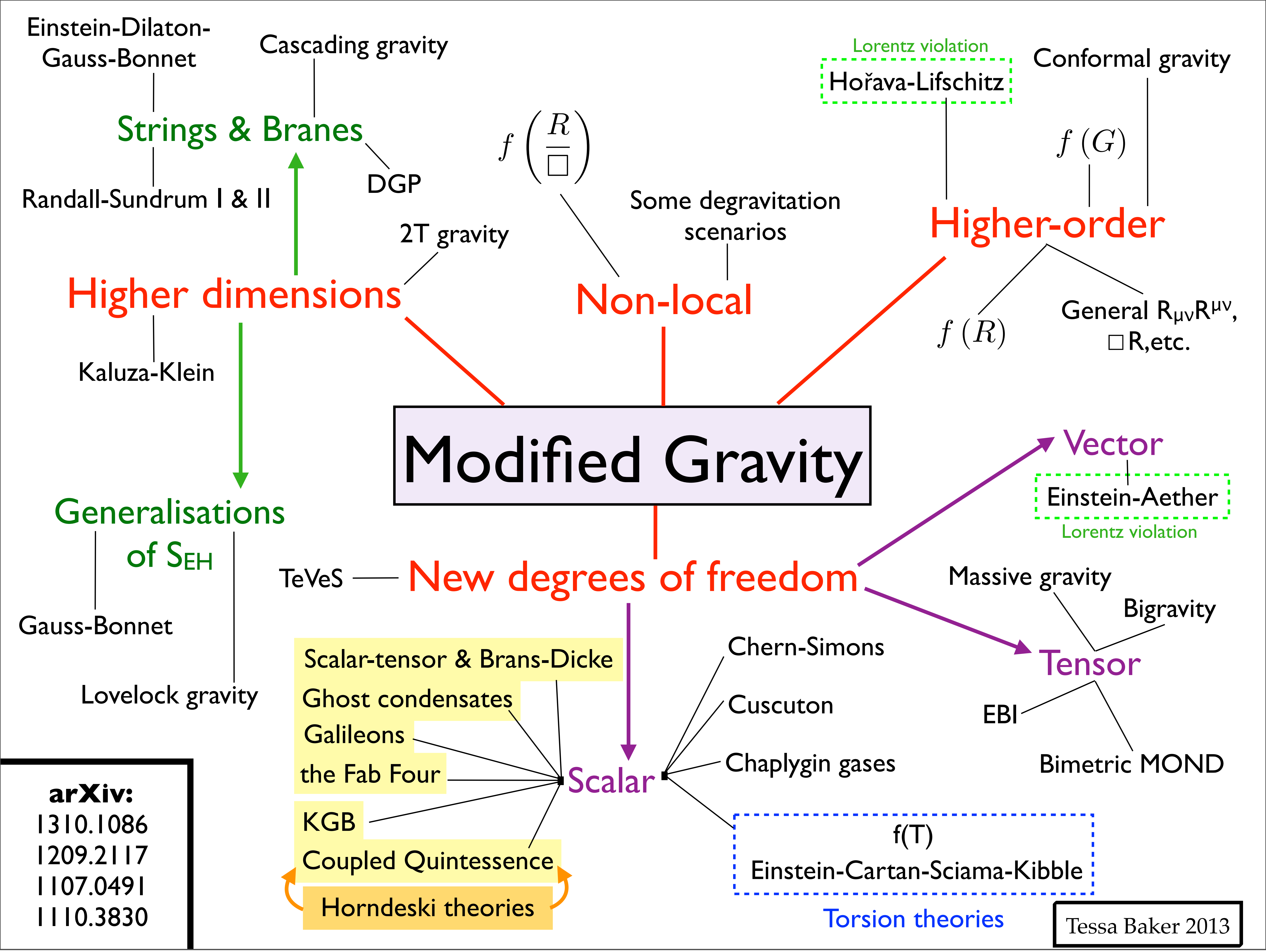
See also Hojman, Kuchar & Teitelboim (1976)

Jordan-Brans-Dicke Theory

One free parameter

$$S = \int d^4x\sqrt{-g}\left[\phi R - \frac{\omega_{\mathrm{BD}}}{\phi}\nabla_\mu\phi\nabla^\mu\phi + V + \mathscr{L}_M[g_{\mu\nu}, \varphi]\right\}$$

Cassini (Bertotti et al 2003) $\omega_{\mathrm{BD}} > 40,000$



Modified Gravity
Higher dimensions
Non-local
Higher-order
New degrees of freedom
Strings & Branes
Einstein-Dilaton-Gauss-Bonnet
Cascading gravity
Randall-Sundrum I & II
DGP
2T gravity
Kaluza-Klein
Generalisations of S_{EH}
Gauss-Bonnet
Lovelock gravity
$f\left(\frac{R}{\Box}\right)$
Some degravitation scenarios
Lorentz violation
Hořava-Lifschitz
Conformal gravity
$f(G)$
$f(R)$
General $R_{\mu\nu}R^{\mu\nu}$, $\Box R$, etc.
Vector
Einstein-Aether
Lorentz violation
Massive gravity
Bigravity
Tensor
EBI
Bimetric MOND
TeVeS
Scalar-tensor & Brans-Dicke
Ghost condensates
Galileons
the Fab Four
KGB
Coupled Quintessence
Horndeski theories
Scalar
Chern-Simons
Cuscuton
Chaplygin gases
f(T)
Einstein-Cartan-Sciama-Kibble
Torsion theories
arXiv:
1310.1086
1209.2117
1107.0491
1110.3830
Tessa Baker 2013

Extra degrees of freedom

metric → add ϕ , A^{μ} ,$f_{\alpha\beta}$ etc.

4D → e.g. in 5 dimensions:

$$g_{AB} = \begin{pmatrix} g_{\alpha\beta} & A_{\alpha} \\ A_{\beta} & \phi \end{pmatrix}$$

2nd order → e.g. if $\int d^4x\sqrt{-g}f(R)$ define $\phi = \frac{df}{dR}$.

Local → e.g. $\phi = \frac{R}{\Box}$.

All transform differently under diffeomorphisms

Background $ds^2 = g^{(0)}_{\mu\nu} dx^\mu dx^\nu = -dt^2 + a^2(t)(d\vec{x})^2$

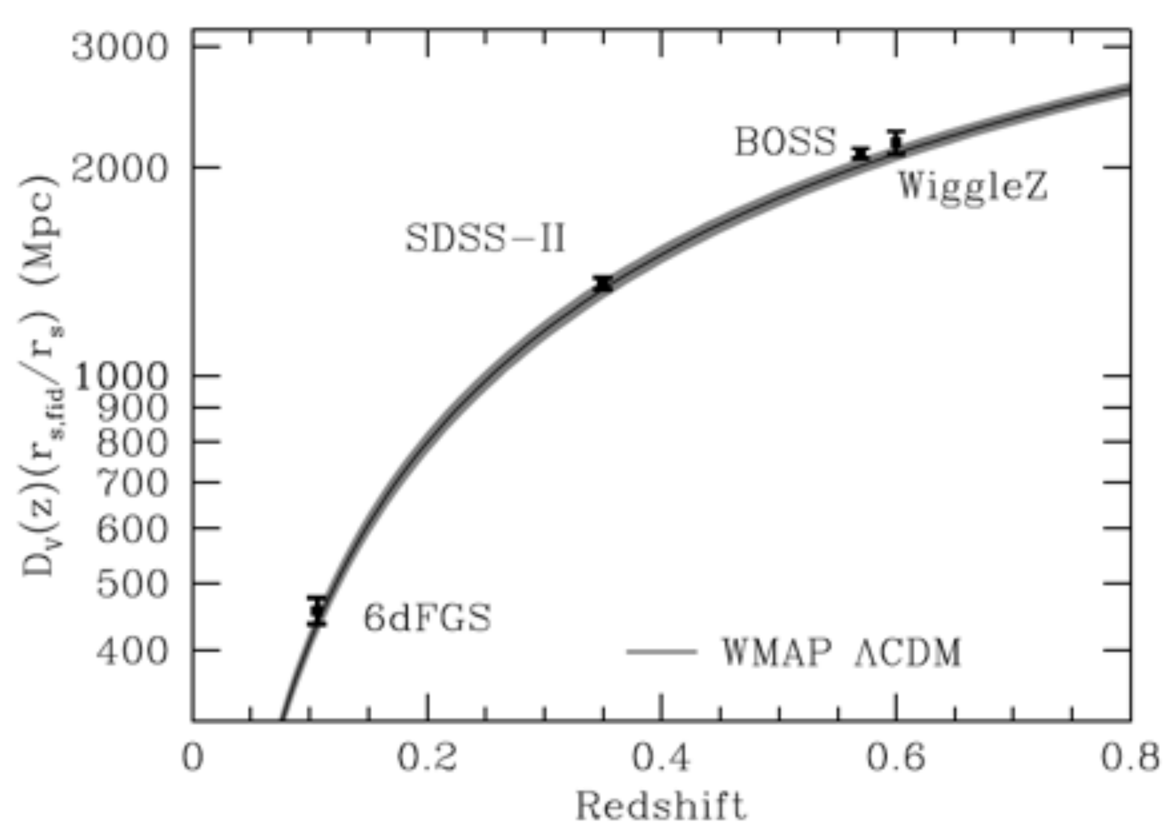
$$G_{\alpha\beta} = 8\pi G T_{\alpha\beta} + \underline{U_{\alpha\beta}}$$

where $U_{\alpha\beta}(a, \dot{a}, \rho_M, P_M, \phi, \cdots)$ $\quad P_X = \underline{w}\rho_X$

Homogeneity and isotropy

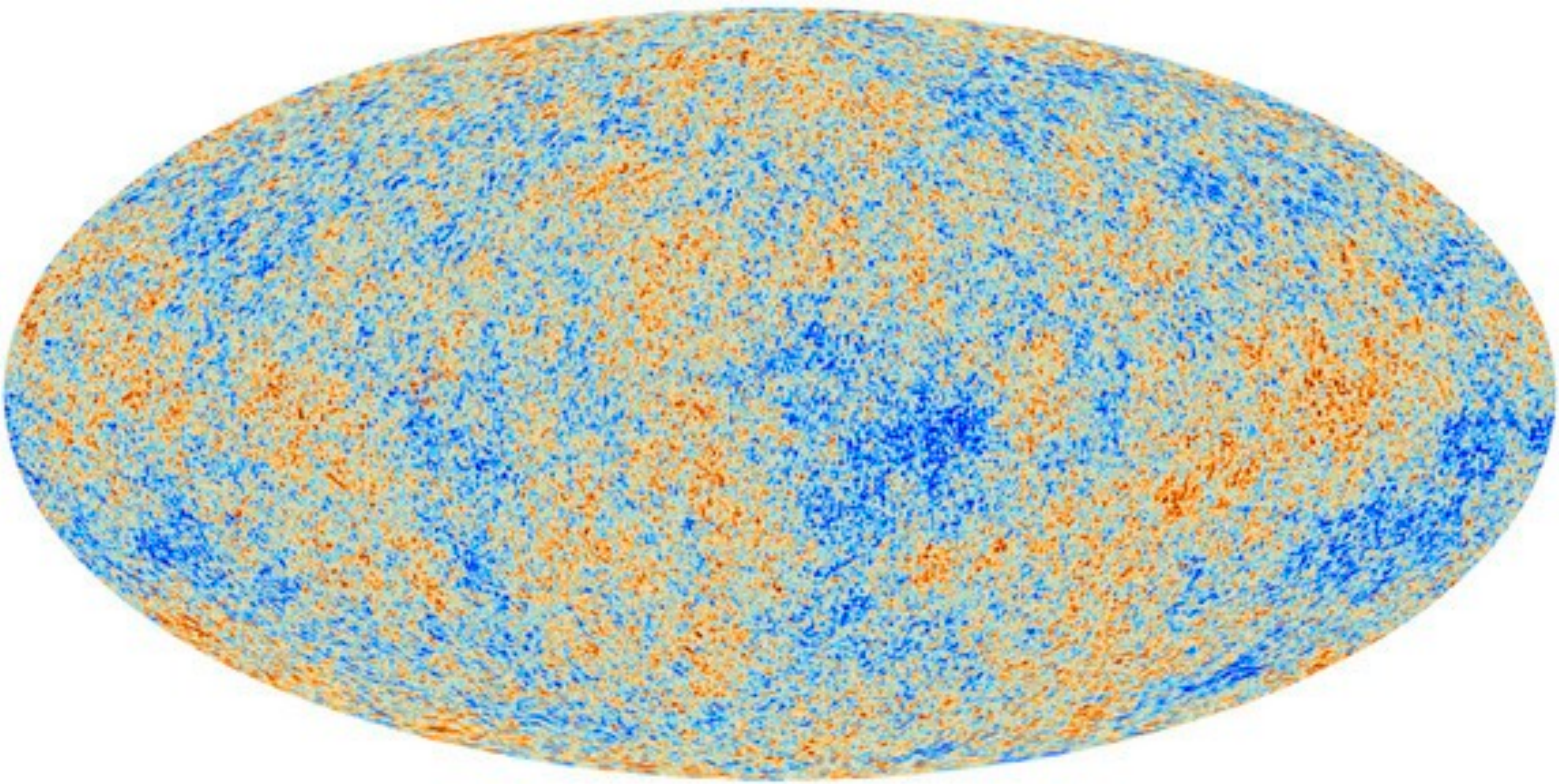
$$U_{\alpha\beta} = 8\pi G \begin{pmatrix} \rho_X & 0 \\ 0 & a^2 P_X \delta_{ij} \end{pmatrix}$$

BOSS, Anderson et al 2013.



Bianchi identities

$$\nabla^\alpha [8\pi G T_{\alpha\beta} + U_{\alpha\beta}] = 0$$



$$\frac{\delta T}{T} \sim 10^{-5}$$ → linear perturbation theory

Linear Perturbations

$$\begin{aligned} g_{\alpha\beta} &= g^{(0)}_{\alpha\beta} + h_{\alpha\beta} \\ \rho_M &= \bar{\rho}_M(1+\delta_M) \\ \phi &= \phi_0 + \delta\phi \end{aligned}$$

Construct most general quadratic action which has:

- upto 2nd order in time derivatives
- $h_{\alpha\beta} \to h_{\alpha\beta} + \nabla_\alpha \xi_\beta + \nabla_\beta \xi_\alpha$ where $x^\alpha \to x^\alpha + \xi^\alpha$
- inherits symmetries of the background

Linear Perturbations

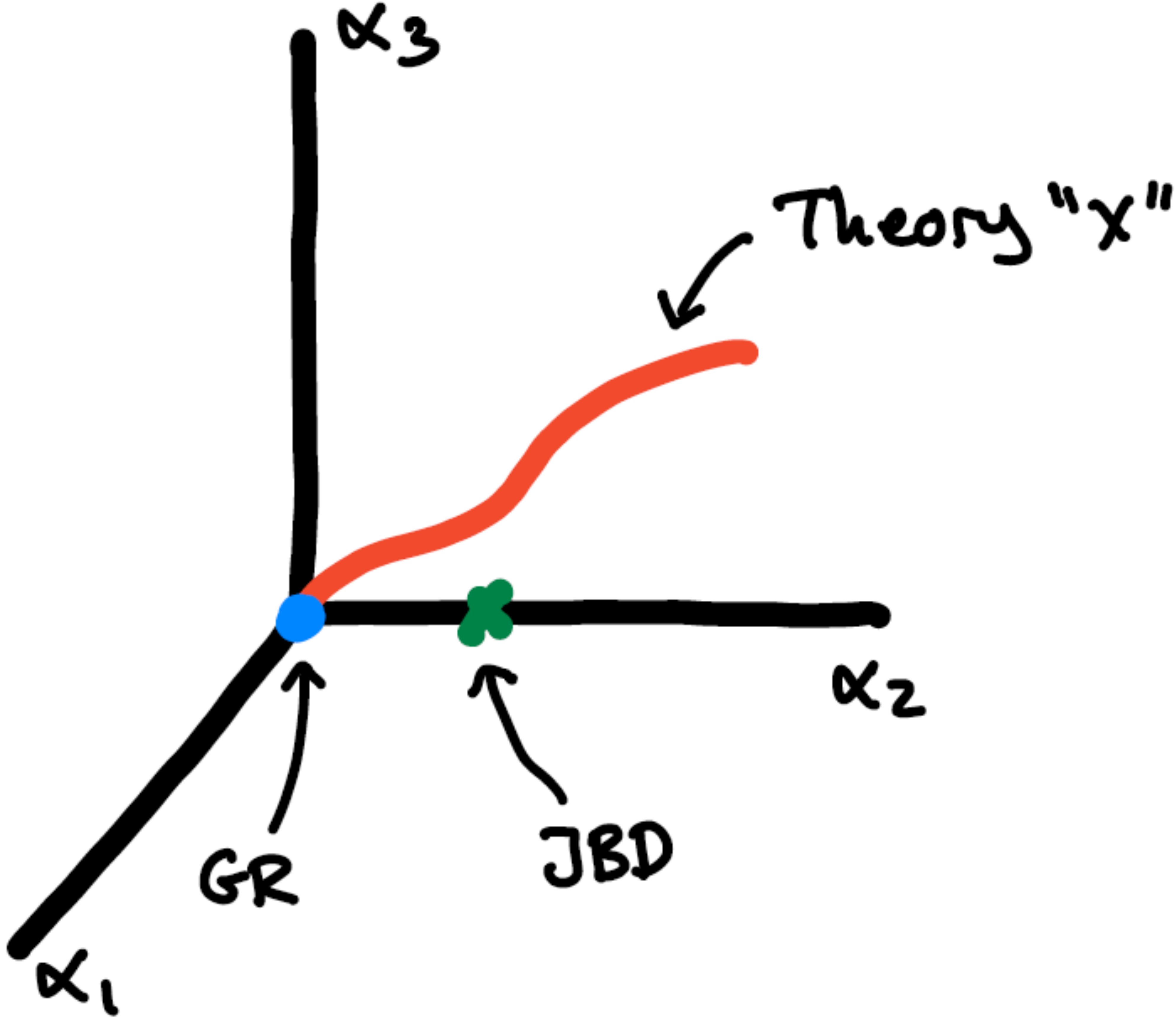
$$S = \int d^4x\sqrt{-g^{(0)}}\,[\alpha_1\nabla^\mu h^{\alpha\beta}\nabla_\mu h_{\alpha\beta} + \alpha_2\nabla^\mu h^{\alpha\beta}\nabla_\alpha h_{\mu\beta}$$
$$+\alpha_3\nabla^\mu h\nabla^\alpha h_{\mu\alpha} + \alpha_4\nabla^\mu h\nabla_\mu h + \alpha_5\nabla^\mu h_{\mu\alpha}\nabla^\alpha\delta\phi + \cdots]$$

Properties:

- $(\alpha_1, \alpha_2, \cdots)$ are functions of t
- $\alpha_X(t)$ depend on transf. props of extra fields
- clear mapping theory $\longleftrightarrow$ $\alpha_X(t)$
- clear physical interpretation of each $\alpha_X(t)$

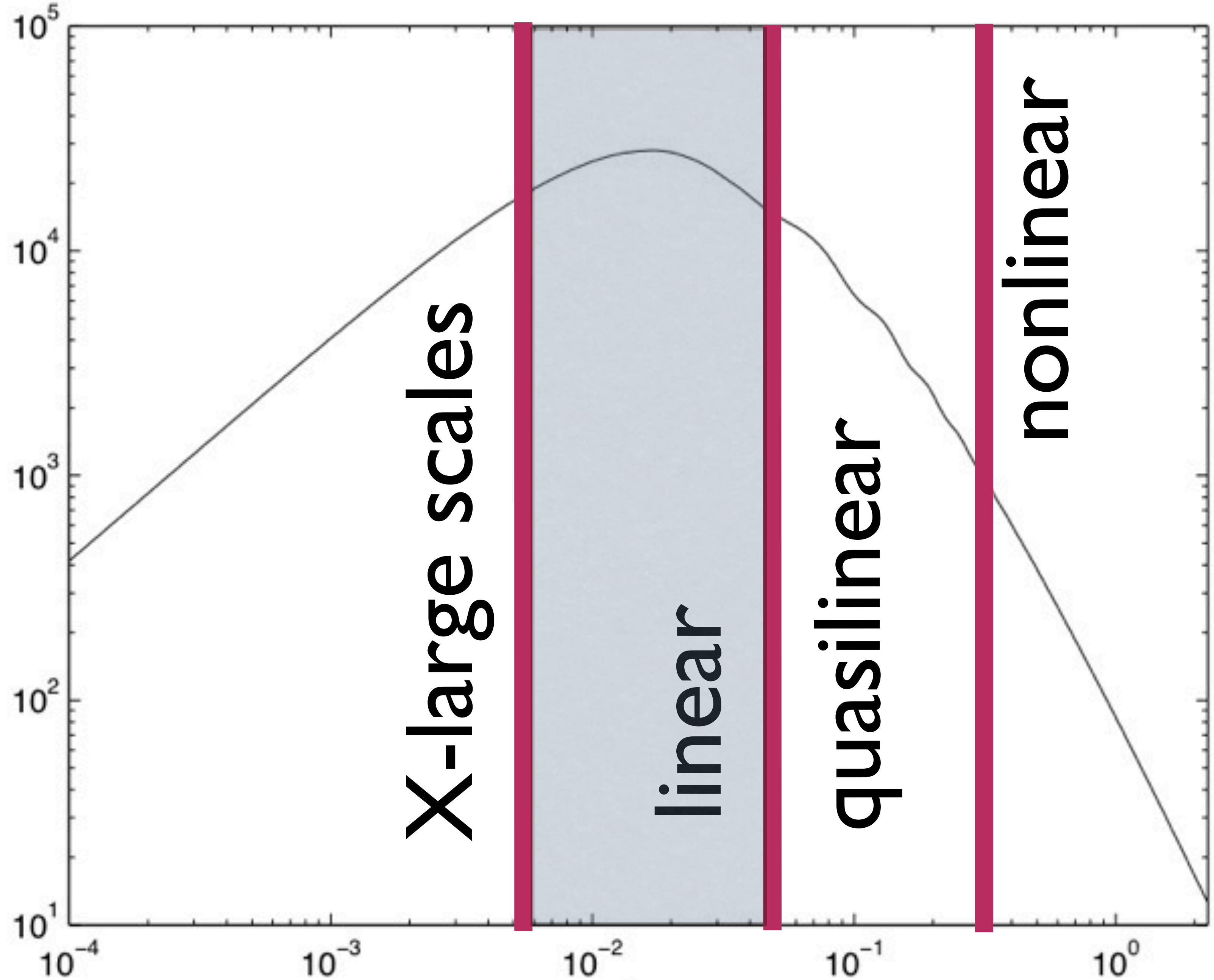
Examples:

- Scalar-tensor (Horndeski): five $\alpha_X(t)$
- Vector-tensor (Einstein-Aether, Proca): nine $\alpha_X(t)$
- Tensor-tensor (Bigravity, massive gravity): three $\alpha_X(t)$



α_3
Theory "x"
α_2
GR
JBD
α_1

“amplitude of clustering”



“inverse length scale”

$$N(k) \propto k^3$$

More statistical power →

The Data

A preferred length scale- the horizon

$$\mathcal{H}^{-1} \equiv \left(\frac{\dot{a}}{a}\right)^{-1} \propto \tau \simeq 3000h^{-1}\text{Mpc}$$

Most surveys $\ll \tau$ so that $k\tau \gg 1$

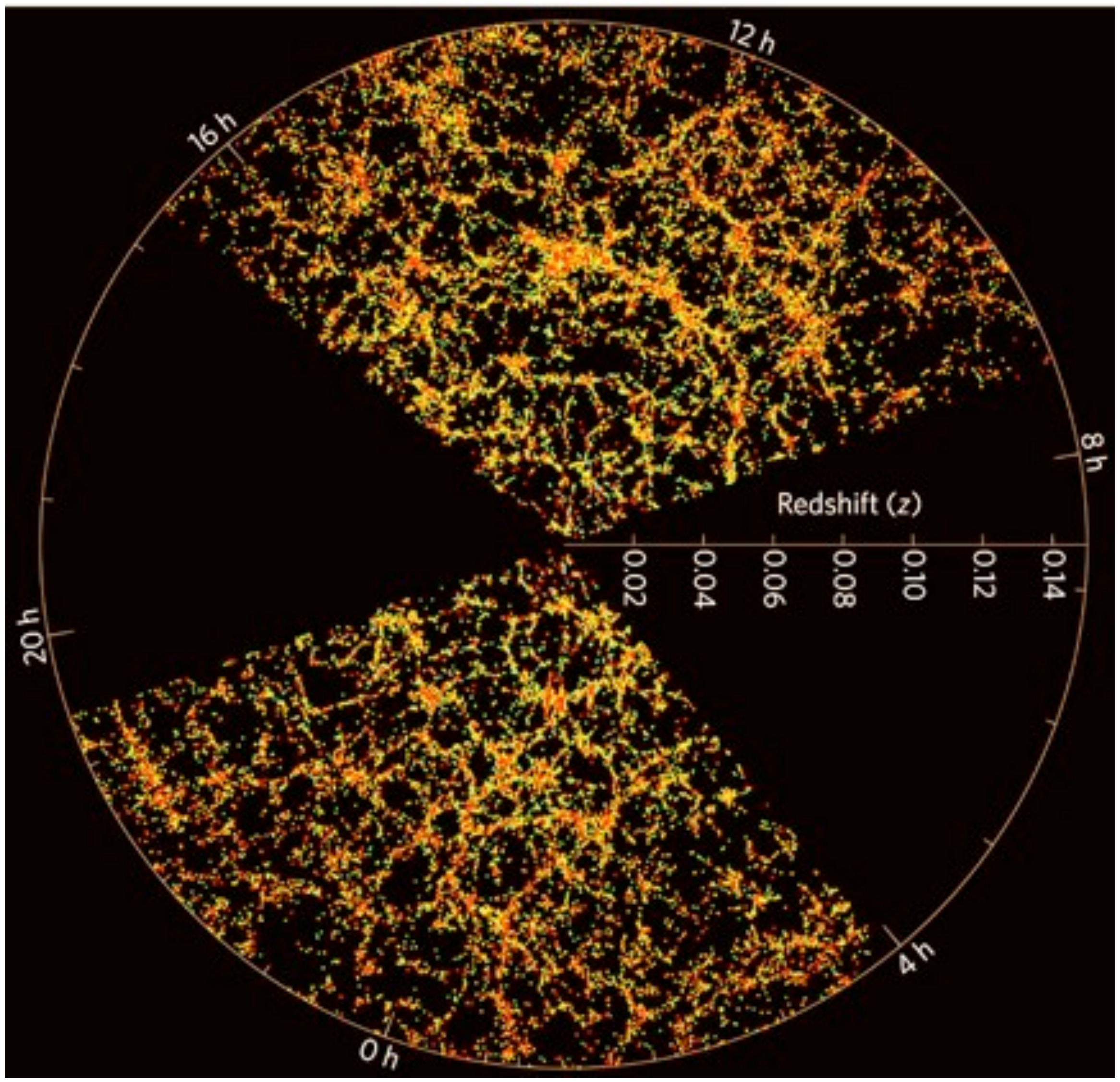
Newtonian potentials: $h_{\alpha\beta} = 2\begin{pmatrix} \Phi & 0 \\ 0 & a^2\Psi\delta_{ij} \end{pmatrix}$

Einstein equations: $-k^2\Phi = 4\pi G\underline{\mu} a^2\rho\Delta$

$$\underline{\gamma}\Psi = \Phi$$

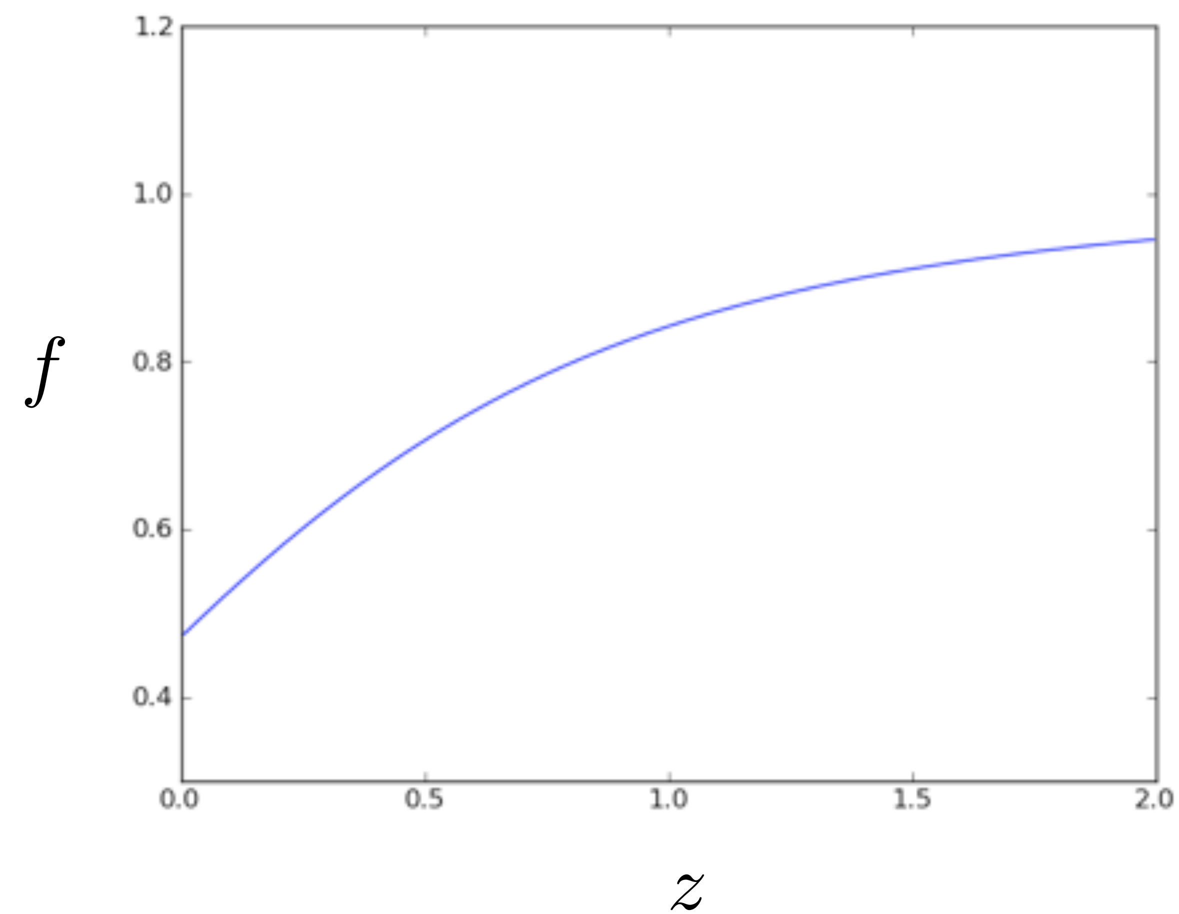
(μ, γ) are rational functions of $\alpha_X(t)$ and k^2

We measure matter and light.



Growth rate

$$f(k,a) = \frac{d\ln\delta_M(k,a)}{d\ln a}$$



f satisfies a simple ODE

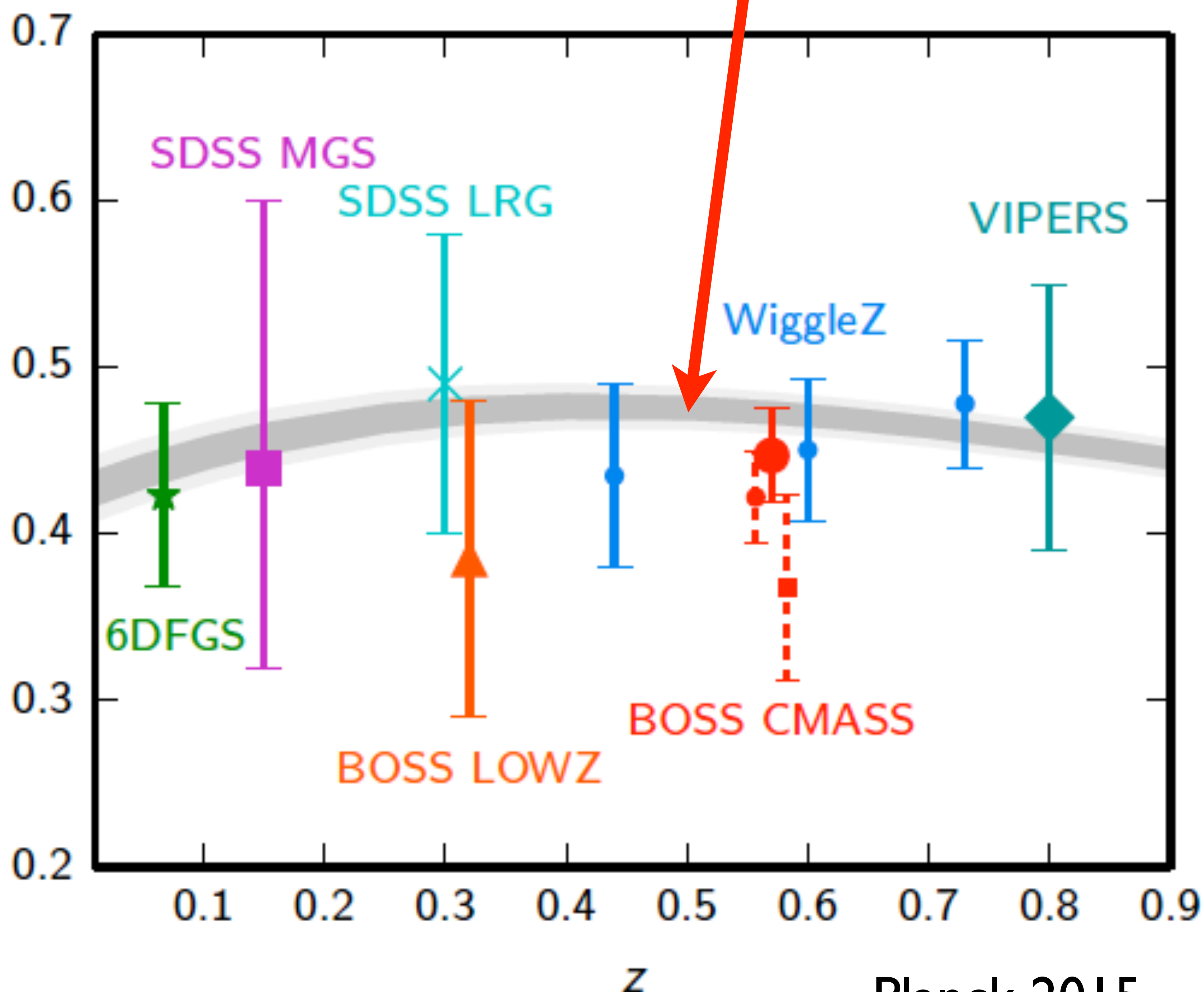
$$\frac{df}{d\ln a} + qf + f^2 = \frac{3}{2}\Omega_M\xi$$

with $q = \frac{1}{2}[1 - 3w(1-\Omega_M)]$ and $\xi = \frac{\mu}{\gamma}$

Planck “prediction”

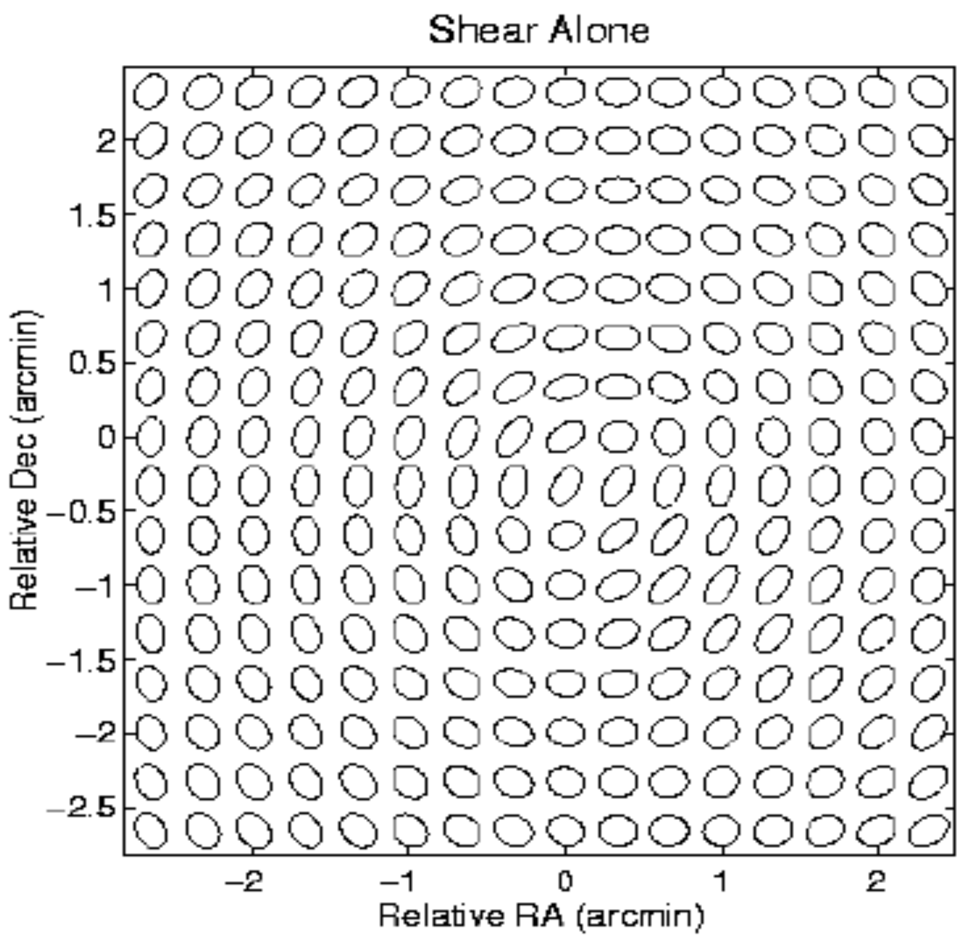
“growth rate of structure”

$$f\sigma_8 \propto \frac{d\delta}{d\ln a}$$



Planck 2015

Weak Lensing



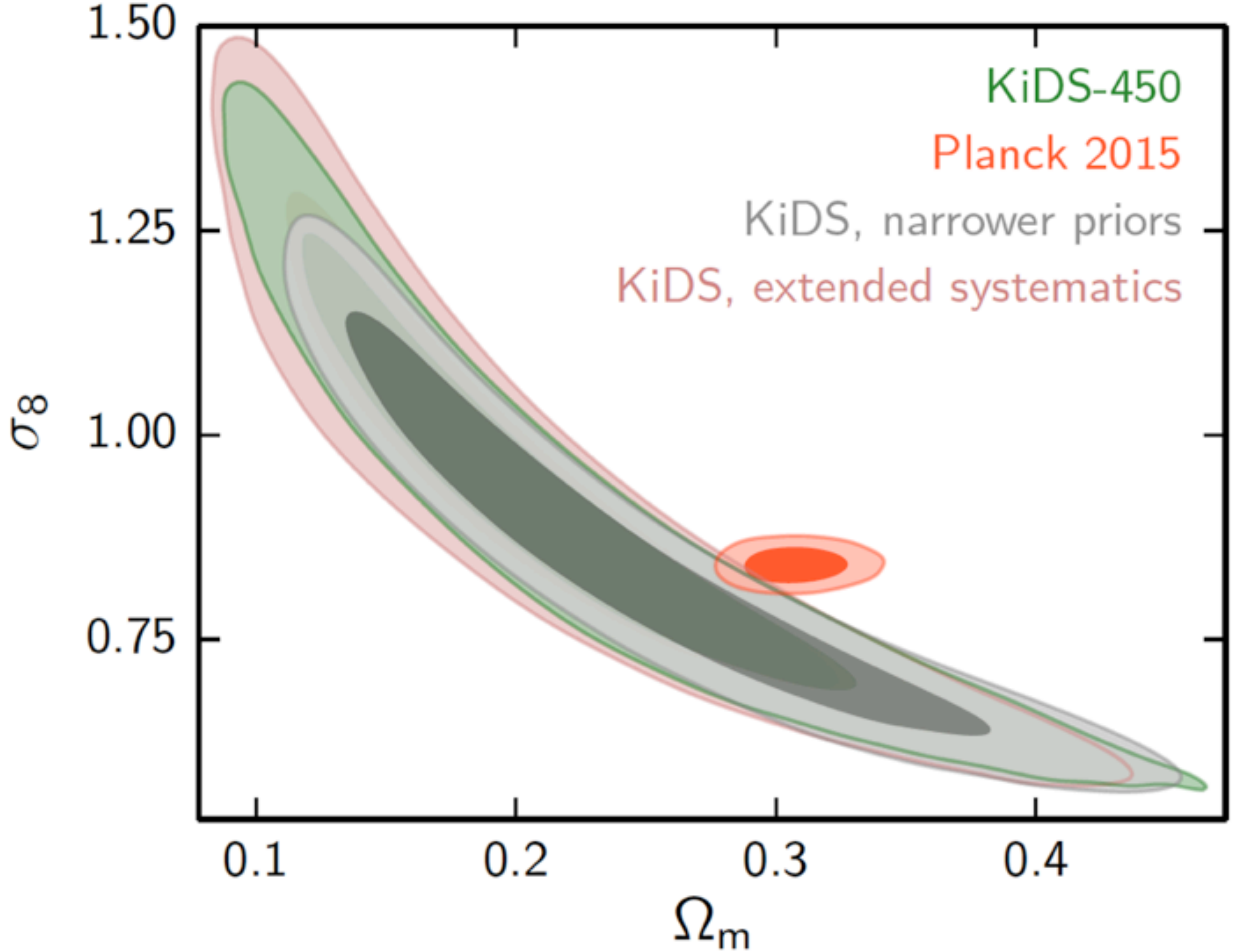
$$\text{shear} \simeq \int_0^{\chi} \nabla_{\perp}^2[\Phi + \Psi](\chi') \left[\chi' \left(1 - \frac{\chi'}{\chi}\right)\right] d\chi'$$

$$\text{shear} \sim \Sigma \equiv \mu \left(1 + \frac{1}{\gamma}\right)$$

Sarah Bridle lectures (2003)

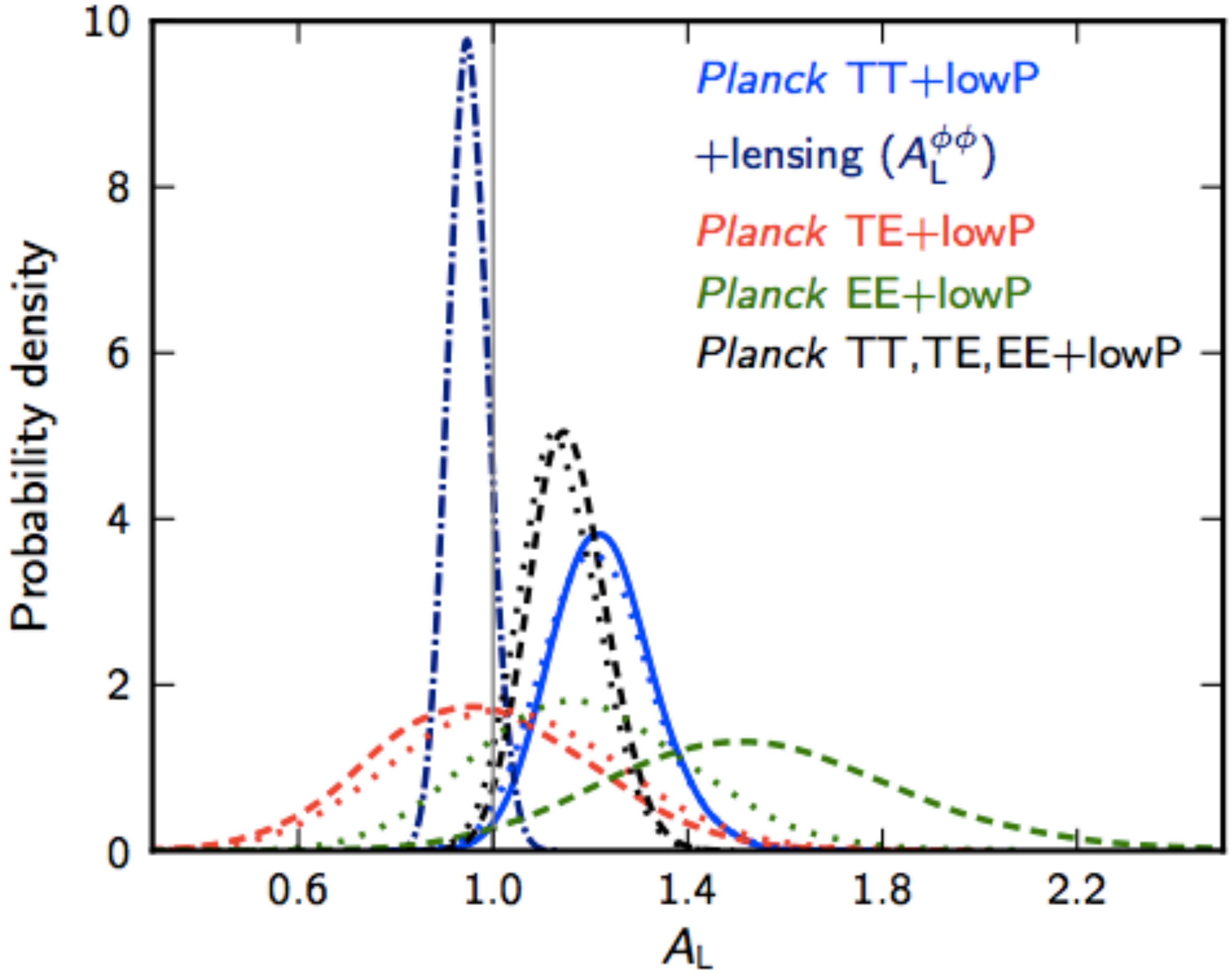
Joudaki et al 2016

“amplitude of clustering at 8 h^{-1} Mpc”

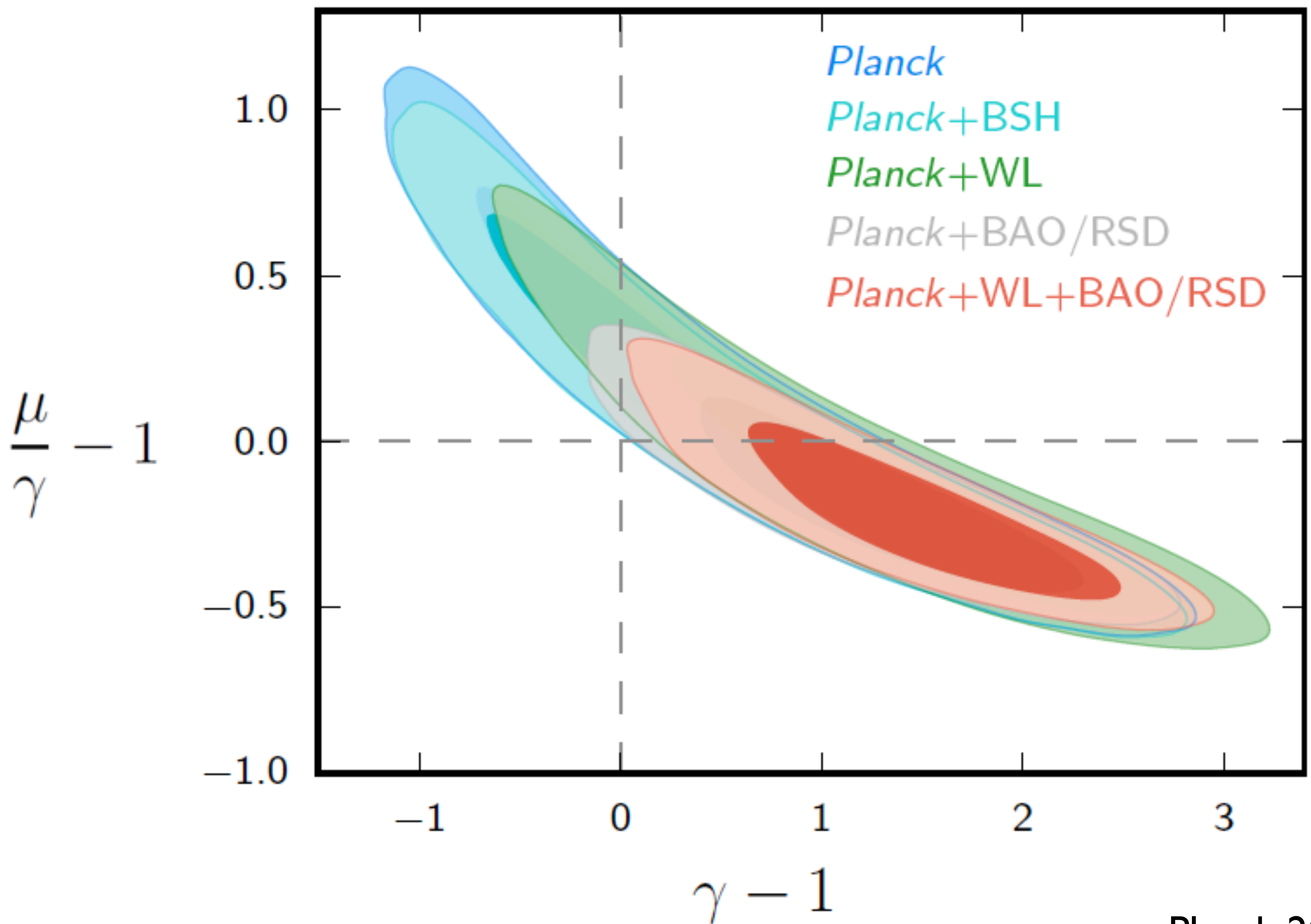


“matter density”

Lensing of CMB

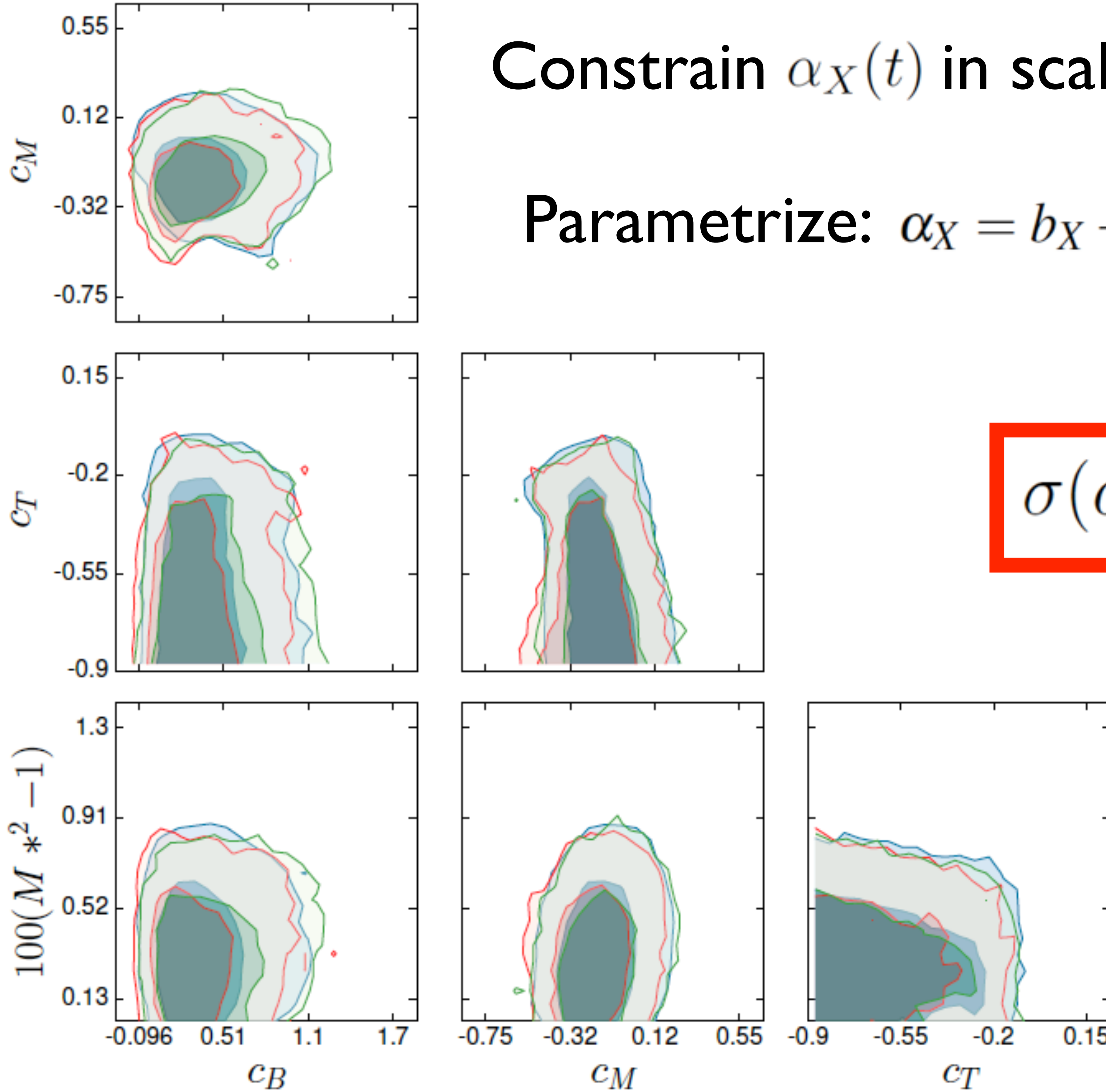


Planck 2015


Planck
Planck+BSH
Planck+WL
Planck+BAO/RSD
Planck+WL+BAO/RSD
1.0
0.5
0.0
−0.5
−1.0
$\frac{\mu}{\gamma} - 1$
−1
0
1
2
3
$\gamma - 1$

Planck 2015

Constrain $\alpha_X(t)$ in scalar-tensor theories



Parametrize: $\alpha_X = b_X + c_X \dfrac{\Omega_{\rm DE}(z)}{\Omega_{\rm DE}(z=0)}$

$$\sigma(\alpha_X) \sim 0.5$$

Bellini et al 2016

Jordan-Brans-Dicke Theory

One free parameter

$$S = \int d^4x\sqrt{-g}\left[\phi R - \frac{\omega_{\mathrm{BD}}}{\phi}\nabla_\mu\phi\nabla^\mu\phi + V + \mathscr{L}_M[g_{\mu\nu},\varphi]\right\}$$

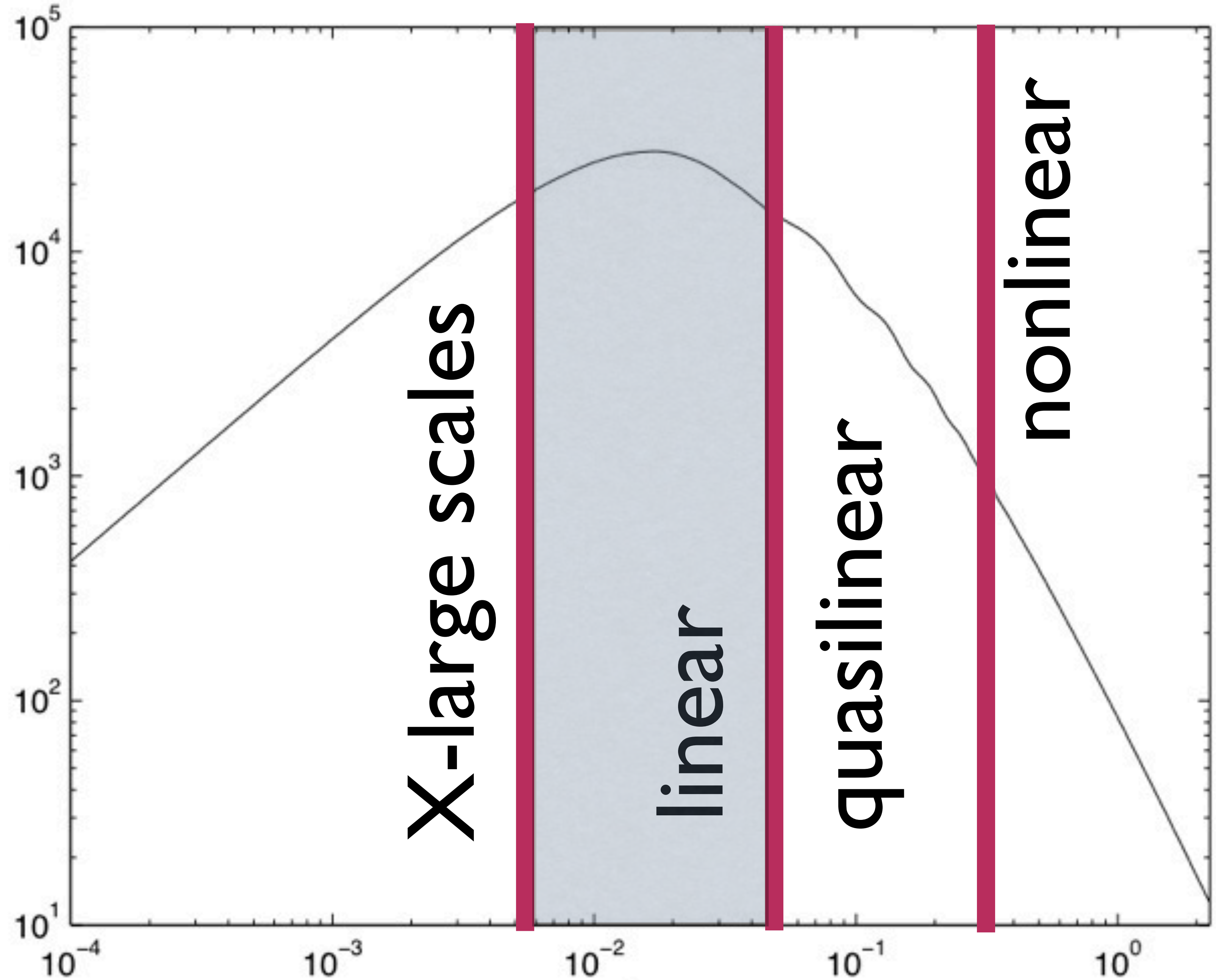
Cassini (Bertotti et al 2003) $\omega_{\mathrm{BD}} > 40,000$

Planck (Avilez & Skordis 2015) $\omega_{\mathrm{BD}} > 1,000$

The Challenge

Systematics: non-linear physics

“amplitude of clustering”



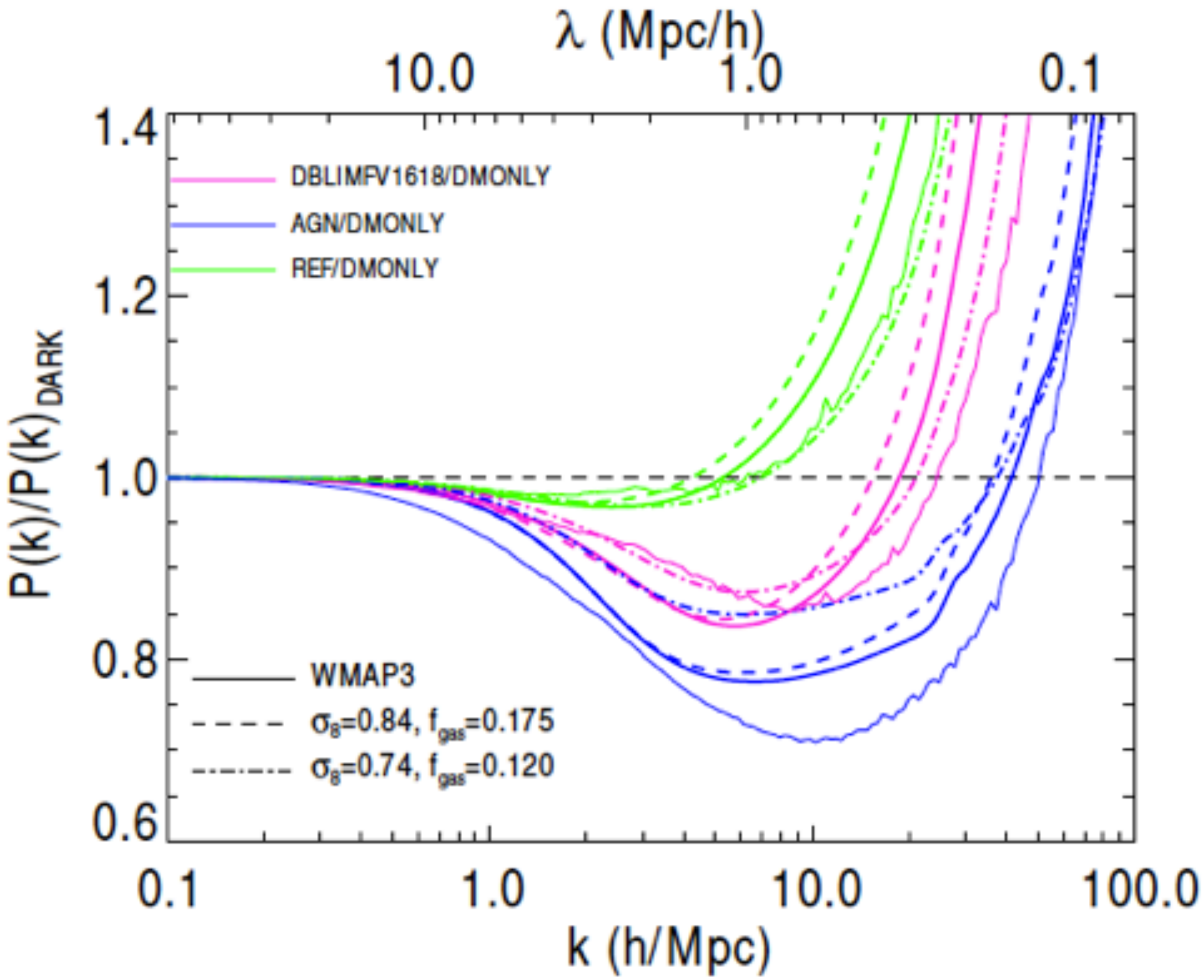
“inverse length scale”

$$N(k) \propto k^3$$

More statistical power →

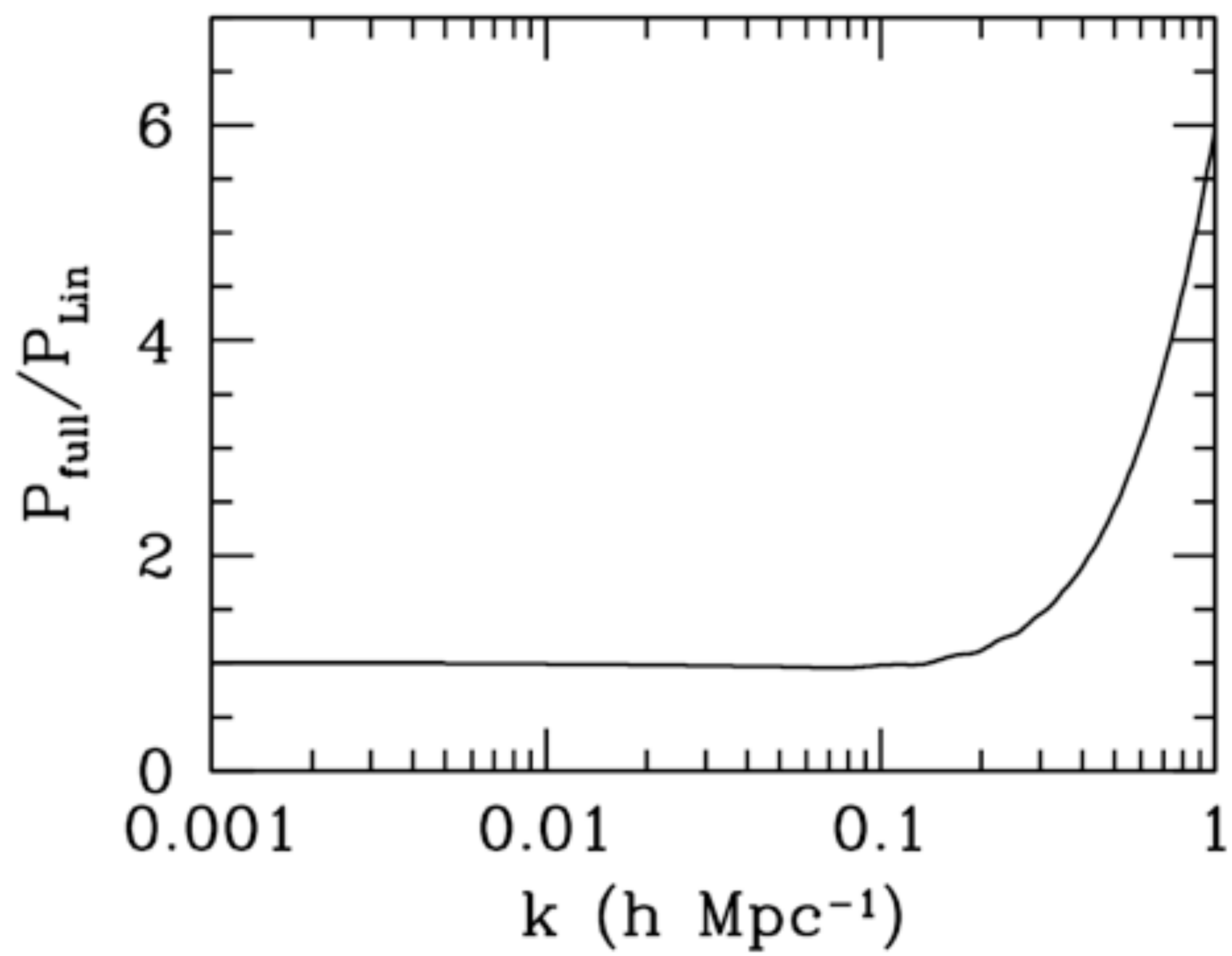
Systematics: non-linear physics

baryonic feedback

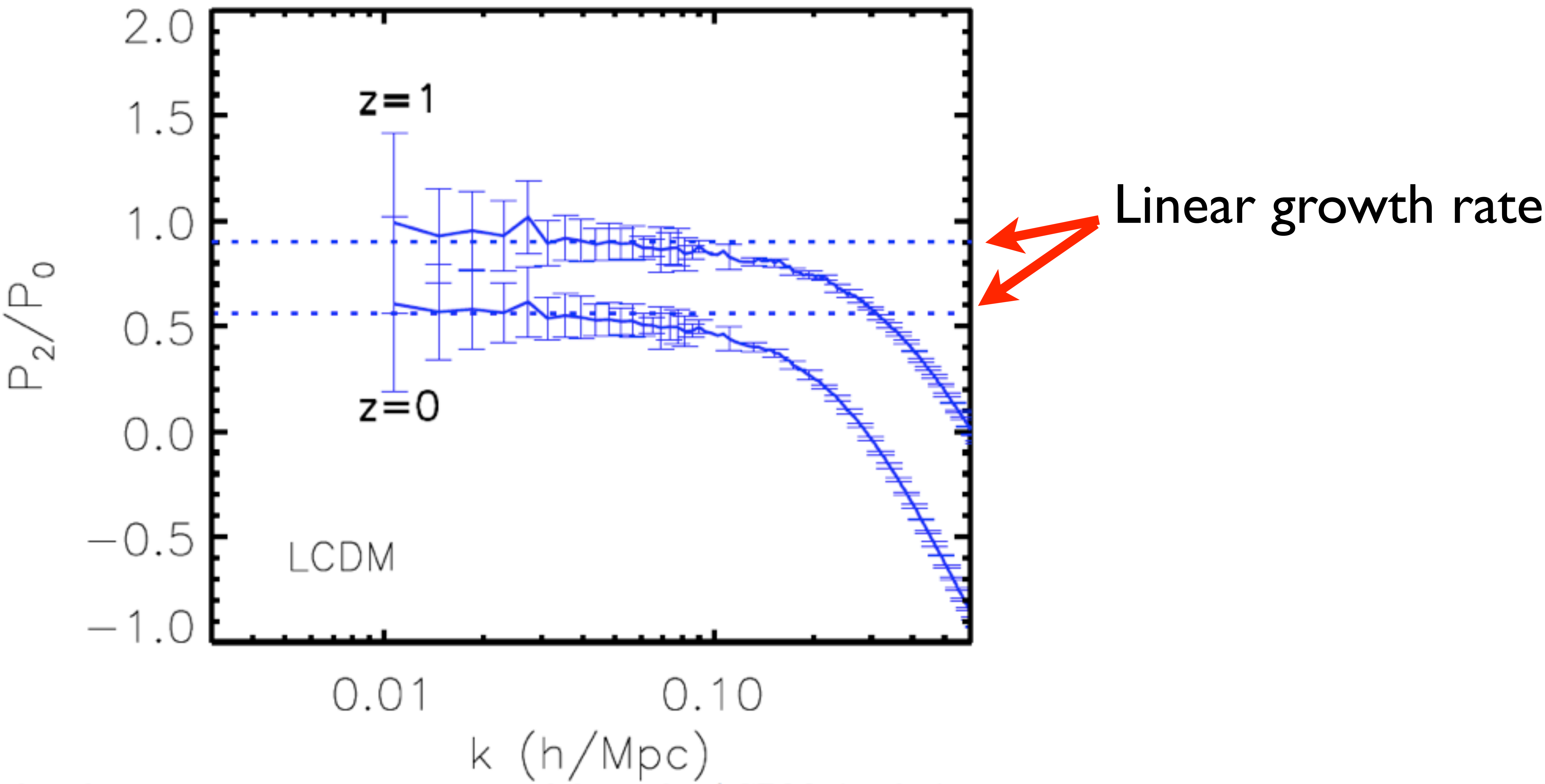


Sembloni et al 2012

non-linear growth



Systematics: non-linear physics



Jennings, Baugh & Pascoli 2015

Systematics: screening

Newtonian potential

$$\Phi = -\frac{GM}{r}$$

$$\ddot{\vec{r}} = -\nabla[\Phi + \phi]$$

Fifth force

$$\phi = -\frac{\tilde{G}M}{r}e^{-mr}$$

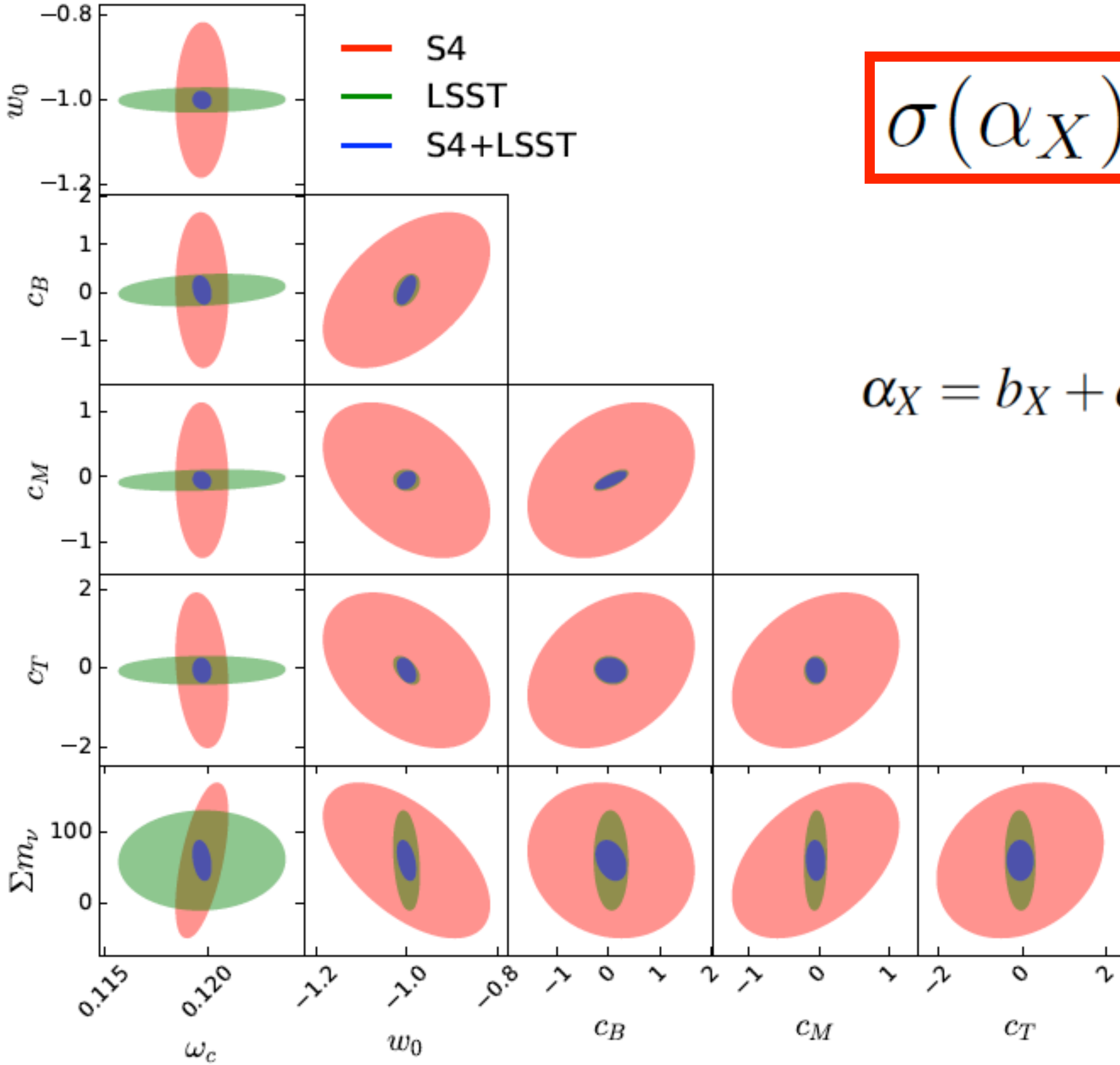
Chameleon: $m = m(\rho)$ $\quad m \to \infty$ when $\rho \gg \rho_S$

Vainshtein: $\tilde{G} \to 0$ when $r \ll r_V$

The Future

The Future is now

Data Type	Now	Soon	Future
Photo-z:LSS (weak lensing)	DES, RCS, KIDS	HSC	LSST, Euclid, SKA, WFIRST
Spectro-z (BAO, RSD, ...)	BOSS	DESI,PFS,HETDEX, Weave	Euclid, SKA
SN Ia	HST, Pan-STARRS, SCP, SDSS, SNLS	DES, J-PAS	JWST,LSST
CMB/ISW	WMAP, Planck	AdvACT	Simons Array, Stage IV, LiteBird
sub-mm, small scale lensing, SZ	ACT, SPT,Planck, ACTPol,SPTPol,	PolarBear,Spider, Vista	CCAT, SKA
X-Ray clusters	ROSAT, XMM, Chandra	XMM, XCS, eRosita	
HI Tomography	GBT	Meerkat, Baobab, Chime, Kat 7	SKA



$$\sigma(\alpha_X) \sim 0.1$$

$$\alpha_X = b_X + c_X \frac{\Omega_{\rm DE}(z)}{\Omega_{\rm DE}(z=0)}$$

Alonso et al 2016

Jordan-Brans-Dicke Theory

One free parameter

$$S = \int d^4x\sqrt{-g}\left[\phi R - \frac{\omega_{\rm BD}}{\phi}\nabla_\mu\phi\nabla^\mu\phi + V + \mathscr{L}_M[g_{\mu\nu},\varphi]\right\}$$

Cassini (Bertotti et al 2003) $\omega_{\rm BD} > 40,000$

Planck (Avilez & Skordis 2015) $\omega_{\rm BD} > 1,000$

LSST+SKA+S4 (Alonso et al 2016) $\omega_{\rm BD} > 20,000$

"An important contribution of the general theory of relativity to cosmology has been to keep out theologians by a straightforward application of tensor analysis."

E. Schucking

D. Alonso, L. Amendola, M. Amin, T. Baker, R.Bean, E. Bellini, C. Blake, P. Bull, P. Brax, S. Daniels, A. Davies, D. Leonard, G. Gubitosi, P. G. Ferreira, J. Gleyzes, W. Hu, L. Hui, C, Heymans, S. Joudaki, K. Koyama, M. Kunz, M. Lagos, D. Langlois, E. Linder, L. Lombrisier, D. Mota, A. Narimani, J. Noller, J. Peacock, F. Piazza, D. Pogosian, D. Sapone, D. Scott, I. Sawicki, A. Silvestri, F. Simpson, A. Taylor, F. Vernizzi, H. Winther, J. Zuntz, ...