Cosmology in vector-tensor theories

Shinji Tsujikawa

(Tokyo University of Science)

Collaboration with

A. De Felice, L. Heisenberg, R. Kase, S. Mukohyama, Y. Zhang

There have been many attempts for constructing dark energy models in the framework of scalar-tensor theories.

Many of them belong to the so-called Horndeski theories.

$$S = \int d^4x \sqrt{-g}\, L$$

Most general scalar-tensor theories with second-order equations of motion

$$
\begin{aligned}
L = {} & G_2(\phi, X) + G_3(\phi, X)\Box\phi + G_4(\phi, X)R - 2G_{4,X}(\phi, X)\left[(\Box\phi)^2 - \phi^{;\mu\nu}\phi_{;\mu\nu}\right] \\
& + G_5(\phi, X)G_{\mu\nu}\phi^{;\mu\nu} + \frac{1}{3}G_{5,X}(\phi, X)[(\Box\phi)^3 - 3(\Box\phi)\phi_{;\mu\nu}\phi^{;\mu\nu} + 2\phi_{;\mu\nu}\phi^{;\mu\sigma}\phi^{;\nu}{}_{;\sigma}]
\end{aligned}
$$

Single scalar field ϕ with $X = g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$

Horndeski derived this action at the age of 25 (1973).

R and $G_{\mu\nu}$ are the 4-dimensional Ricci scalar and the Einstein tensors, respectively.

- General Relativity corresponds to $G_4 = M_{\rm pl}^2/2$.

- Horndeski theories accommodate a wide variety of gravitational theories like Brans-Dicke theory, $f(R)$ gravity, and covariant Galileons.

What happens for a vector field instead of a scalar field ?

(i) Maxwell field (massless)

Lagrangian: $\mathcal{L}_F = -\dfrac{1}{4}F_{\mu\nu}F^{\mu\nu}$

There are two transverse polarizations (electric and magnetic fields).

(ii) Proca field (massive)

Lagrangian: $\mathcal{L}_F = -\dfrac{1}{4}F_{\mu\nu}F^{\mu\nu} - \dfrac{1}{2}m^2 A_\mu A^\mu$

Introduction of the mass m of the vector field A_μ allows the propagation in the longitudinal direction due to the breaking of $U(1)$ gauge invariance.

2 transverse and 1 longitudinal = 3 DOFs

The Electric and Magnetic Fields

Longitudinal propagation
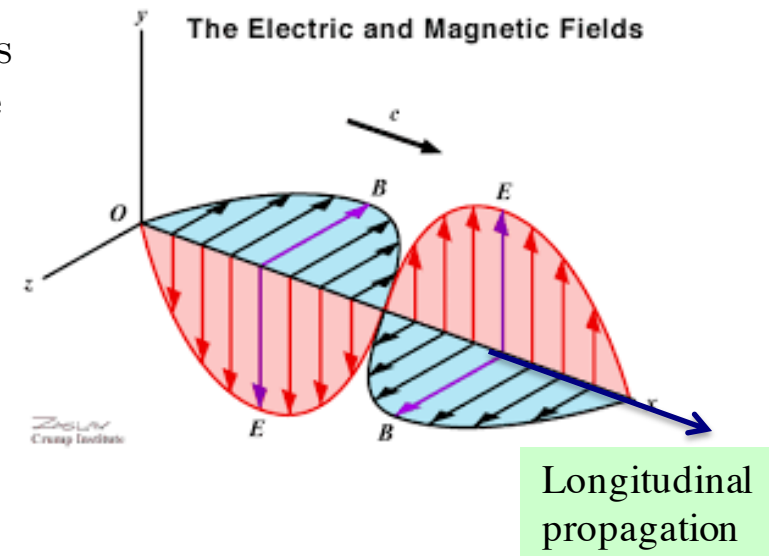
Generalized Proca theories

On general curved backgrounds, it is possible to extend the massive Proca theories to those containing three DOFs (besides two tensor polarizations).

Heisenberg Lagrangian (2014)   See also Tasinato (2014)

$$\mathcal{L}_2 = G_2(X, F, Y),$$
$$\mathcal{L}_3 = G_3(X)\nabla_\mu A^\mu,$$
$$\mathcal{L}_4 = G_4(X)R + G_{4,X}(X)\left[(\nabla_\mu A^\mu)^2 - \nabla_\rho A_\sigma \nabla^\sigma A^\rho\right],$$
$$\mathcal{L}_5 = G_5(X)G_{\mu\nu}\nabla^\mu A^\nu - \frac{1}{6}G_{5,X}(X)[(\nabla_\mu A^\mu)^3 - 3\nabla_\mu A^\mu \nabla_\rho A_\sigma \nabla^\sigma A^\rho + 2\nabla_\rho A_\sigma \nabla^\gamma A^\rho \nabla^\sigma A_\gamma]$$
$$- g_5(X)\tilde{F}^{\alpha\mu}\tilde{F}^\beta{}_\mu \nabla_\alpha A_\beta,$$
$$\mathcal{L}_6 = G_6(X)L^{\mu\nu\alpha\beta}\nabla_\mu A_\nu \nabla_\alpha A_\beta + \frac{1}{2}G_{6,X}(X)\tilde{F}^{\alpha\beta}\tilde{F}^{\mu\nu}\nabla_\alpha A_\mu \nabla_\beta A_\nu,$$

$\}$ Intrinsic vector mode

where
$$X = -\frac{1}{2}A_\mu A^\mu, \qquad F = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}, \qquad Y = A^\mu A^\nu F_\mu{}^\alpha F_{\nu\alpha}$$

$$L^{\mu\nu\alpha\beta} = \frac{1}{4}\epsilon^{\mu\nu\rho\sigma}\epsilon^{\alpha\beta\gamma\delta}R_{\rho\sigma\gamma\delta}, \qquad \tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}$$

The non-minimal derivatives couplings like $G_4(X)R$ are required to keep the equations of motion up to second order.

Taking the scalar limit $A^\mu \to \nabla^\mu\pi$, the above Lagrangian recovers a sub-class of Horndeski theories (with $\mathcal{L}_6$ vanishing).
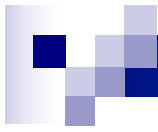
Cosmology in generalized Proca theories

Can we realize a viable cosmology with the late-time acceleration?

Vector field: $A^\mu = (\phi(t), 0, 0, 0)$ (which does not break spatial isotropy)

Variation of the Heisenberg action with respect to $g_{\mu\nu}$ on the flat FLRW background leads to

$$G_2 - G_{2,X}\phi^2 - 3G_{3,X}H\phi^3 + 6G_4H^2 - 6(2G_{4,X} + G_{4,XX}\phi^2)H^2\phi^2 + G_{5,XX}H^3\phi^5 + 5G_{5,X}H^3\phi^3 = \rho_M \, ,$$

$$G_2 - \dot\phi\phi^2 G_{3,X} + 2G_4\left(3H^2 + 2\dot H\right) - 2G_{4,X}\phi\left(3H^2\phi + 2H\dot\phi + 2\dot H\phi\right) - 4G_{4,XX}H\dot\phi\phi^3$$

$$+ G_{5,XX}H^2\dot\phi\phi^4 + G_{5,X}H\phi^2(2\dot H\phi + 2H^2\phi + 3H\dot\phi) = -P_M \, .$$

The matter density ρ_M and the pressure P_M obey the continuity equation

$$\dot\rho_M + 3H(\rho_M + P_M) = 0$$

Variation of the action with respect to A^μ leads to

$$\phi\left(G_{2,X} + 3G_{3,X}H\phi + 6G_{4,X}H^2 + 6G_{4,XX}H^2\phi^2 - 3G_{5,X}H^3\phi - G_{5,XX}H^3\phi^3\right) = 0 \, .$$

The branch $\phi \neq 0$ gives the solution where ϕ depends on H alone, which allows the existence of de Sitter solutions with constant ϕ and H.

Vector Galileons

The Lagrangian of vector Galileons which recover the Galilean symmetry in the scalar limit $(A_\mu \to \partial_\mu \pi)$ on the flat space-time is given by

$$G_2(X) = b_2 X\,, \qquad G_3(X) = b_3 X\,, \qquad G_4(X) = \frac{M_{\rm pl}^2}{2} + b_4 X^2\,, \qquad G_5(X) = b_5 X^2.$$

We substitute these functions into the vector-field equation:

$$G_{2,X} + 3G_{3,X} H\phi + 6G_{4,X} H^2 + 6G_{4,XX} H^2\phi^2 - 3G_{5,X} H^3\phi - G_{5,XX} H^3\phi^3 = 0$$

Taking note that $X = \phi^2/2$, the background EOM admits the solution
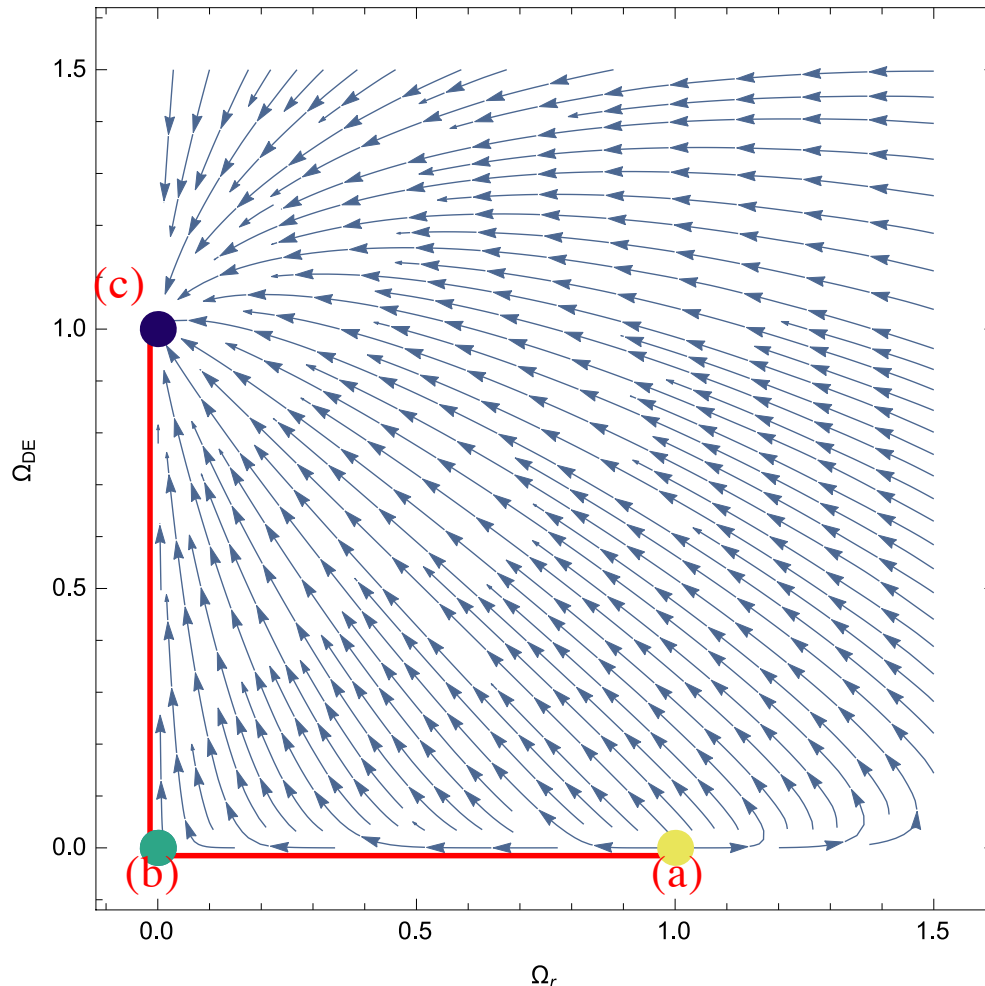
$$\phi H = \text{constant.}$$

The temporal component ϕ is small in the early cosmological epoch, but it grows with the decrease of H.

The solution finally approaches the de Sitter attractor characterized by

$$\phi = \text{constant,} \qquad H = \text{constant.}$$

Phase-space trajectories for vector Galileons

The de Sitter fixed point (c) is always stable against homogeneous perturbations, so it corresponds to the late-time attractor.

The dark energy equation of state w_{DE} is -2 during the matter era.

This case is excluded from the joint data analysis of SN Ia, CMB, and BAO.

(a) Radiation point: $w_{\text{DE}} = -7/3$
(b) Matter point: $w_{\text{DE}} = -2$
(c) De Sitter point: $w_{\text{DE}} = -1$

Generalizations of vector Galileons

Let us consider the case in which ϕ is related with H according to

$$\phi^p \propto H^{-1} \qquad (p > 0)$$

This solution can be realized for

$$G_2(X) = b_2 X^{p_2} , \qquad G_3(X) = b_3 X^{p_3} , \qquad G_4(X) = \frac{M_{\rm pl}^2}{2} + b_4 X^{p_4} , \qquad G_5(X) = b_5 X^{p_5} ,$$

where

$$p_3 = \frac{1}{2}(p + 2p_2 - 1) , \qquad p_4 = p + p_2 , \qquad p_5 = \frac{1}{2}(3p + 2p_2 - 1) .$$

The vector Galileon corresponds to $p_2 = p = 1$.

The dark energy and radiation density parameters obey

$$\Omega'_{\rm DE} = \frac{(1 + s)\Omega_{\rm DE}(3 + \Omega_r - 3\Omega_{\rm DE})}{1 + s\,\Omega_{\rm DE}} ,$$

$$\Omega'_r = -\frac{\Omega_r[1 - \Omega_r + (3 + 4s)\Omega_{\rm DE}]}{1 + s\,\Omega_{\rm DE}} ,$$

where $s \equiv \dfrac{p_2}{p} .$

There are 3 fixed points:

(a) $(\Omega_{\rm DE}, \Omega_r) = (0, 1)$
(b) $(\Omega_{\rm DE}, \Omega_r) = (0, 0)$
(c) $(\Omega_{\rm DE}, \Omega_r) = (1, 0)$

Dark energy equation of state

$$w_{\mathrm{DE}} = -\frac{3(1+s) + s\,\Omega_r}{3(1 + s\,\Omega_{\mathrm{DE}})}.$$
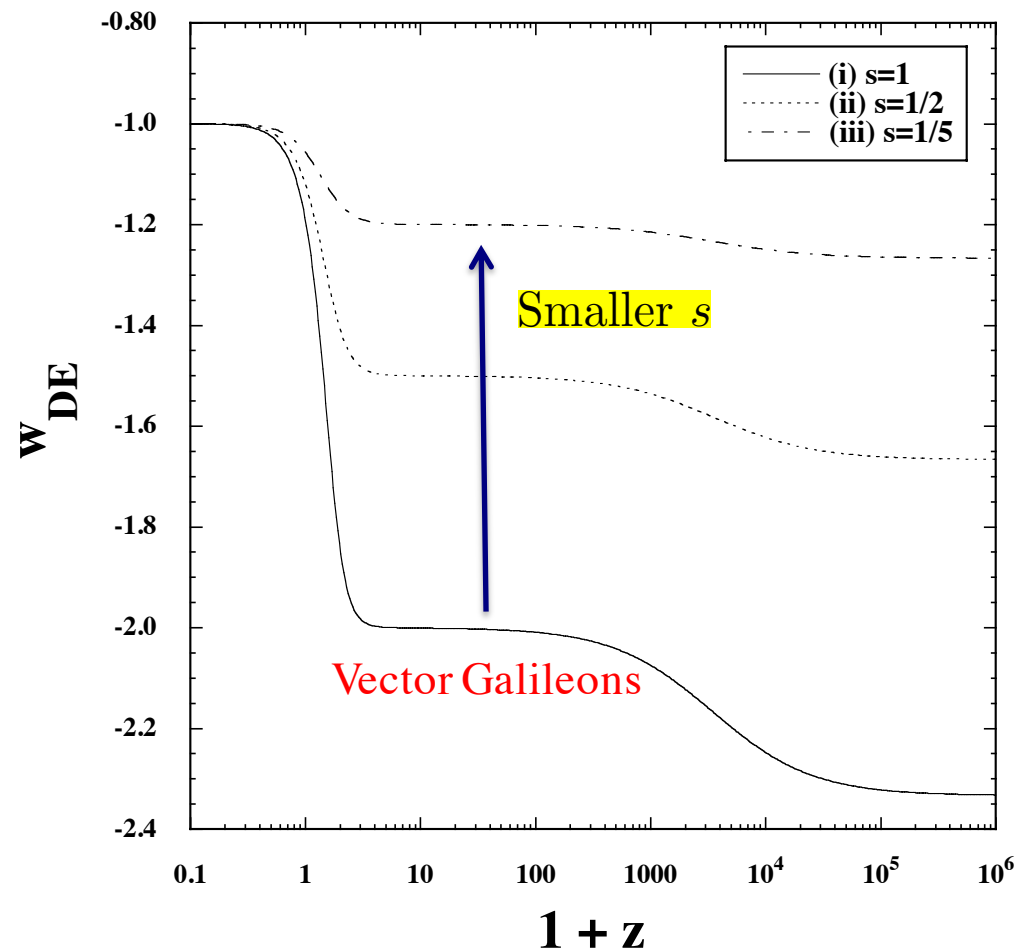
$\Longrightarrow$

(a) $w_{\mathrm{DE}} = -1 - 4s/3$ in the radiation era,
(b) $w_{\mathrm{DE}} = -1 - s$ in the matter era,
(c) $w_{\mathrm{DE}} = -1$ in the de Sitter era

The vector Galileon corresponds to the case $p_2 = p = 1$, i.e., $s = 1$.

For smaller $s = p_2/p$ close to 0, $w_{\mathrm{DE}} = -1 - s$ approaches -1.

For larger p the field ϕ evolves more slowly as $\phi \propto H^{-1/p}$, so w_{DE} approaches -1.
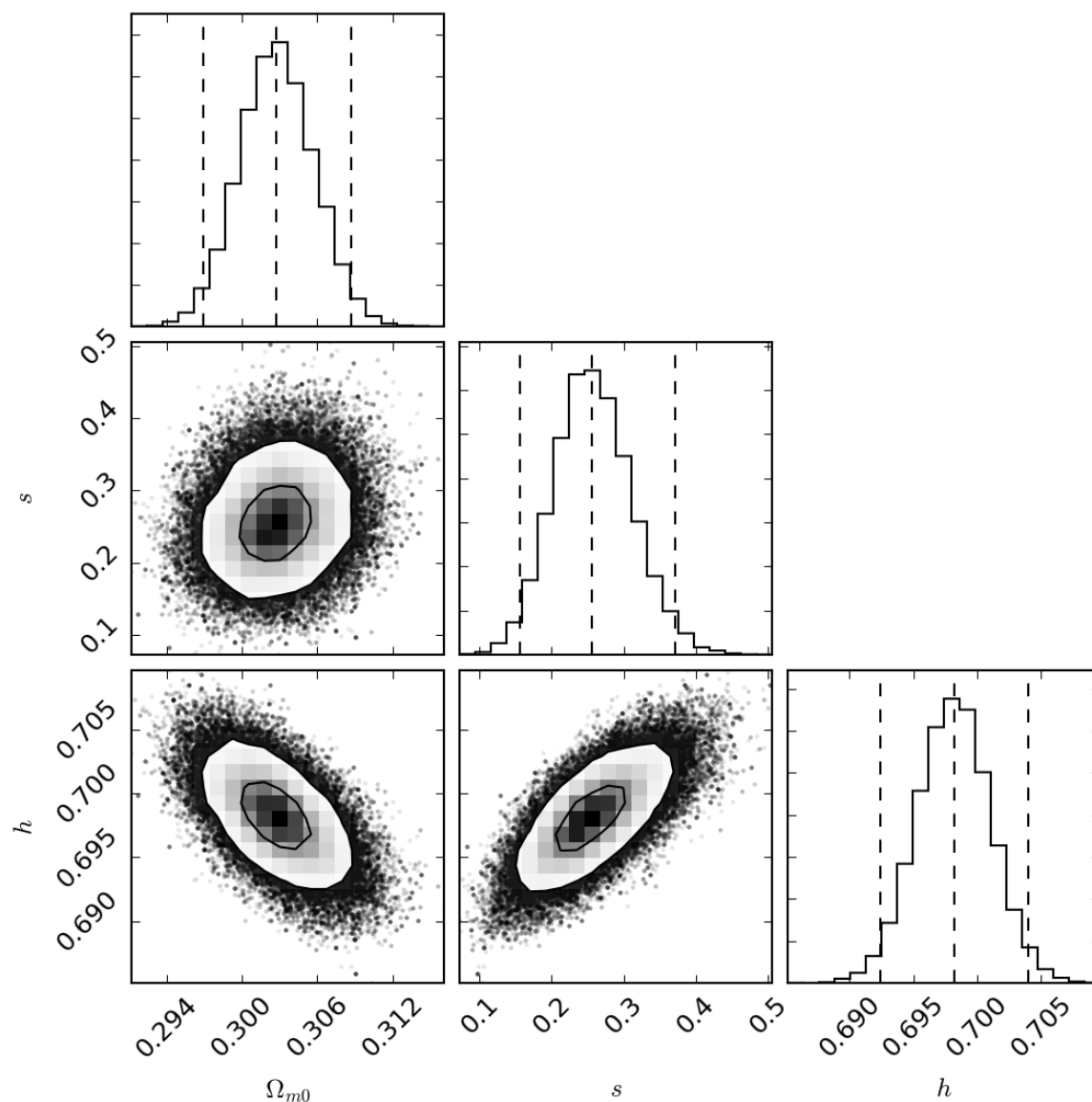
Latest observational constraints

A. De Felice, L. Heisenberg, ST, 1703.09573.

The joint data analysis of SN Ia, CMB shift parameter, BAO, and H0 give the bound

$$\Omega_{m0} = 0.3027^{+0.0060}_{-0.0057},$$
$$h = 0.6981^{+0.0059}_{-0.0057},$$
$$s = 0.254^{+0.118}_{-0.097}.$$

(95 %CL)

The model fits the data better than the LCDM at the background level.

Cosmological perturbations in generalized Proca theories

We need to study perturbations on the flat FLRW background to study

> (i) Conditions for avoiding ghosts and instabilities,
> (ii) Observational signatures for the matter distribution in the Universe.

In doing so, let us consider the perturbed metric in flat gauge:

$$ds^2 = -(1 + 2\alpha)\, dt^2 + 2\left(\partial_i \chi + V_i\right) dt\, dx^i + a^2(t)\left(\delta_{ij} + h_{ij}\right) dx^i dx^j \,,$$

where α, χ are scalar perturbations, V_i and h_{ij} are the vector and tensor perturbations, respectively, obeying

$$\partial^i V_i = 0 \,,$$
$$\partial^i h_{ij} = 0 \,, \qquad h_i{}^i = 0 \,.$$

We also consider the perturbations of the vector field, as

$$A^0 = \phi(t) + \delta\phi \,,$$
$$A^i = \frac{1}{a^2} \delta^{ij} \left(\partial_j \chi_V + E_j\right)$$

where $\delta\phi$ and χ_V are scalar perturbations, while E_j is the vector perturbation obeying $\partial^j E_j = 0$.

Theoretical consistency and observational signatures

● There are 6 theoretically consistent conditions associated with tensor, vector, and scalar perturbations:

No ghosts: $\quad q_t > 0, \ q_v > 0, \ q_s > 0$

No instabilities: $\quad c_t^2 > 0, \ c_v^2 > 0, \ c_s^2 > 0$

See arXiv:1603.05806 for details.

There exists a wide range of parameter space consistent with these conditions.

● The effective gravitational coupling associated with the growth of large-scale structures can be smaller than the Newton constant.

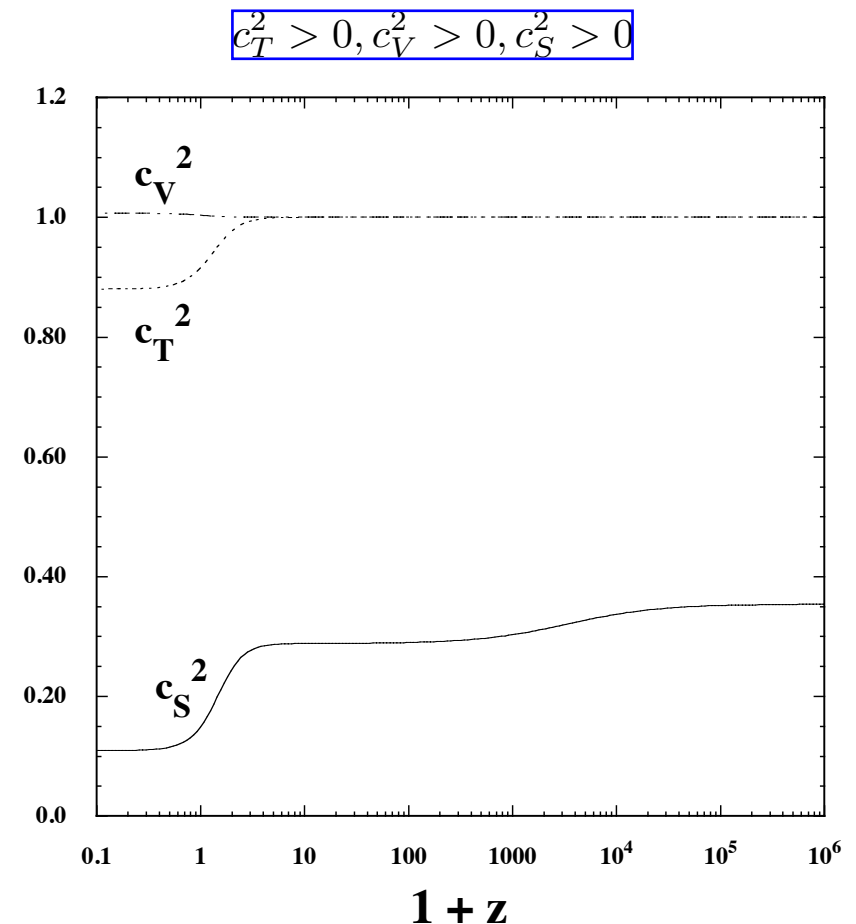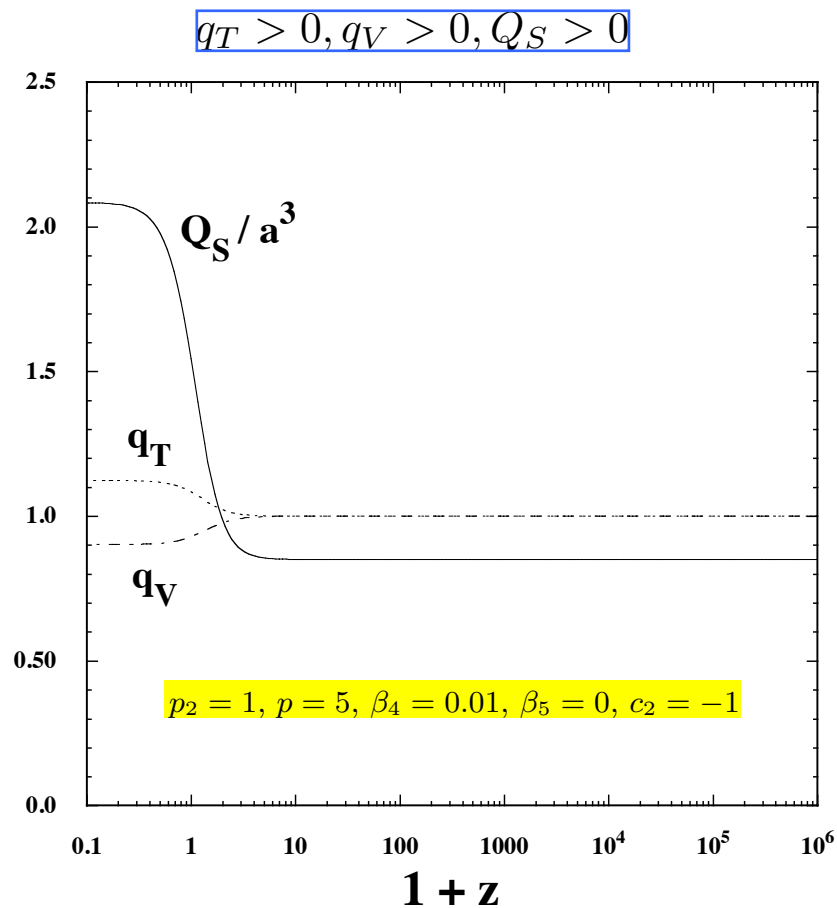The existence of the intrinsic vector mode can lead to

$$G_{\text{eff}} < G$$

See arXiv:1605.05066 for details.

A model consistent with no-ghost and stability conditions

$$G_2(X) = b_2 X, \qquad G_3(X) = b_3 X^{p_3}, \qquad G_4(X) = \frac{M_{\mathrm{pl}}^2}{2} + b_4 X^{p_4}, \qquad G_5(X) = 0.$$

Provided that $0 < \beta_4 < 1/[6(2p+1)]$, there exists the parameter space in which all the theoretically consistent conditions are satisfied.

$q_T > 0, q_V > 0, Q_S > 0$

$c_T^2 > 0, c_V^2 > 0, c_S^2 > 0$

$p_2 = 1,\ p = 5,\ \beta_4 = 0.01,\ \beta_5 = 0,\ c_2 = -1$

Cosmic growth in generalized Proca theories

Under the quasi-static approximation on sub-horizon scales, the matter perturbation obeys

$$\ddot{\delta}_M + 2H\dot{\delta}_M - 4\pi G_{\text{eff}} \rho_M \delta_M \simeq 0$$

where the effective gravitational coupling is

$$G_{\text{eff}} = \frac{\xi_2 + \xi_3}{\xi_1}$$

$$\xi_1 = 4\pi \phi^2 \left(w_2 + 2Hq_T\right)^2,$$

$$\xi_2 = \left[H\left(w_2 + 2Hq_T\right) - \dot{w}_1 + 2\dot{w}_2 + \rho_M\right]\phi^2 - \frac{w_2^2}{q_V},$$

$$\xi_3 = \frac{1}{8H^2\phi^2 q_S^3 q_T c_S^2}\left[2\phi^2\left\{q_S[w_2\dot{w}_1 - (w_2 - 2Hq_T)\dot{w}_2] + \rho_M w_2[3w_2(w_2 + 2Hq_T) - q_S]\right\}\right.$$
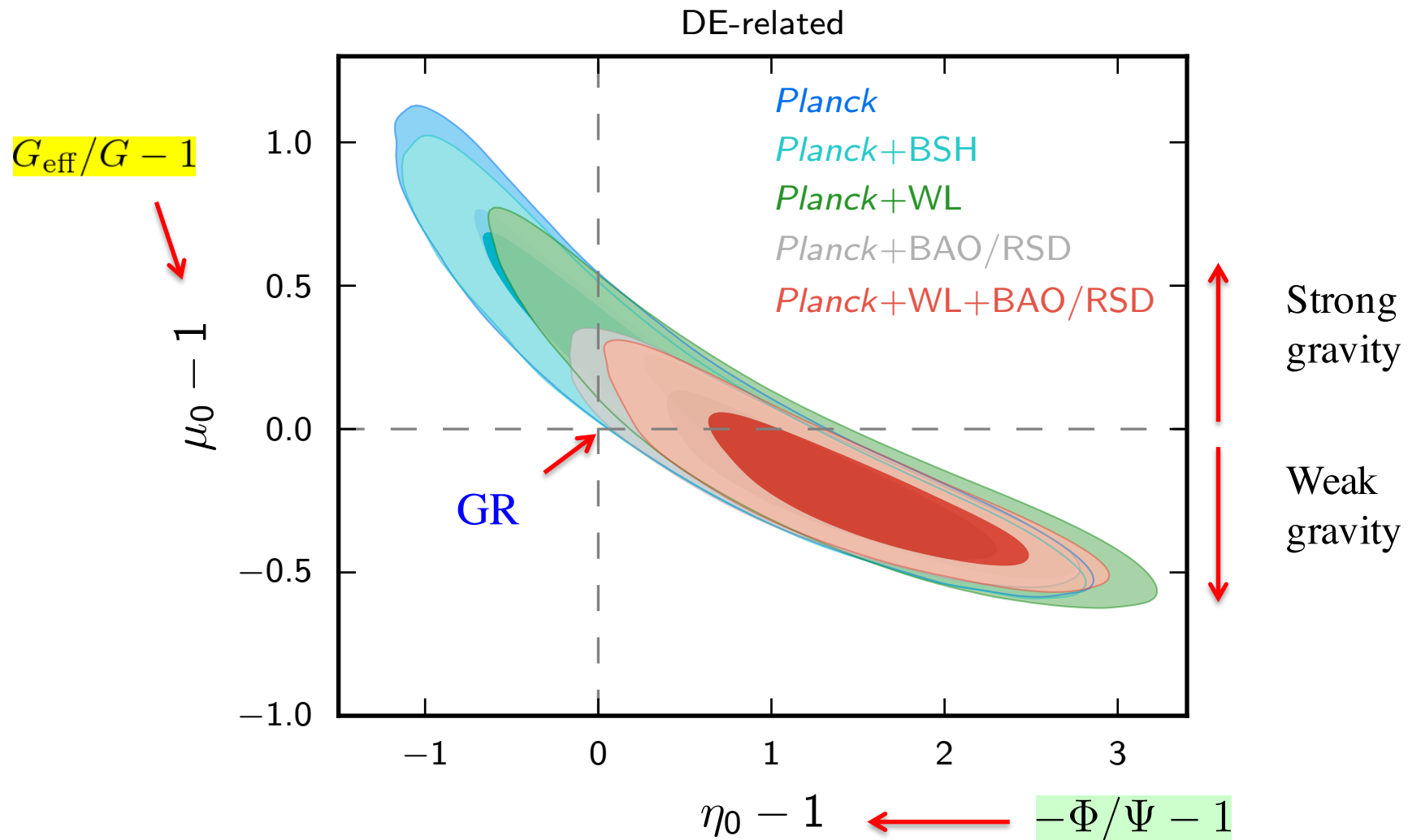$$\left. - \frac{q_S}{q_V}w_2\left\{w_6\phi(w_2 + 2Hq_T) - w_2(w_2 - 2Hq_T)\right\}\right]^2.$$

ξ_3 is positive under the no-ghost and stability conditions (which enhances the gravitational attraction).

For smaller q_V close to 0, there is a tendency that G_{eff} decreases.

Planck constraints on the effective gravitational coupling and the gravitational slip parameter

Ade et al (2015)

G_{eff}/G and Φ/Ψ are assumed to be constant.

DE-related

$G_{\mathrm{eff}}/G - 1$

$\mu_0 - 1$

GR

Planck
Planck+BSH
Planck+WL
Planck+BAO/RSD
Planck+WL+BAO/RSD

Strong gravity

Weak gravity

$\eta_0 - 1$ ← $-\Phi/\Psi - 1$

Weak gravity in generalized Proca theories

G_{eff} is modified through the intrinsic vector mode through the quantity q_V.
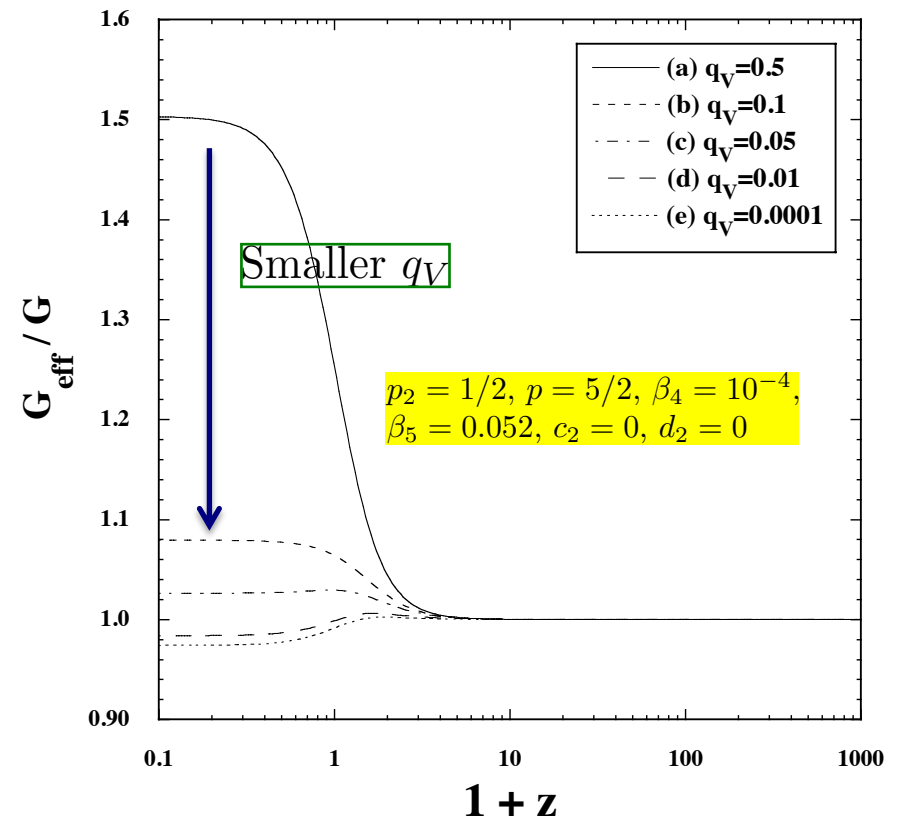
For a massive vector field with $G_2 = F + m^2 X$ we have

$$q_V = 1 - 4g_5 H\phi + 2G_6 H^2 + 2G_{6,X} H^2 \phi^2$$

Effect of the intrinsic vector mode

For smaller q_V approaching 0, the effect of the vector field tends to reduce the gravitational attraction.

It is possible to see signatures of the intrinsic vector mode in redshift-space distortion measurements.

Smaller q_V

$p_2 = 1/2,\ p = 5/2,\ \beta_4 = 10^{-4},$
$\beta_5 = 0.052,\ c_2 = 0,\ d_2 = 0$

Legend:
(a) q_V=0.5
(b) q_V=0.1
(c) q_V=0.05
(d) q_V=0.01
(e) q_V=0.0001

Observational signatures in red-shift space distortions (RSD)

From the RSD measurement we can constrain the growth rate of matter perturbations: $f = \dot{\delta}_m/(H\delta_m)$.
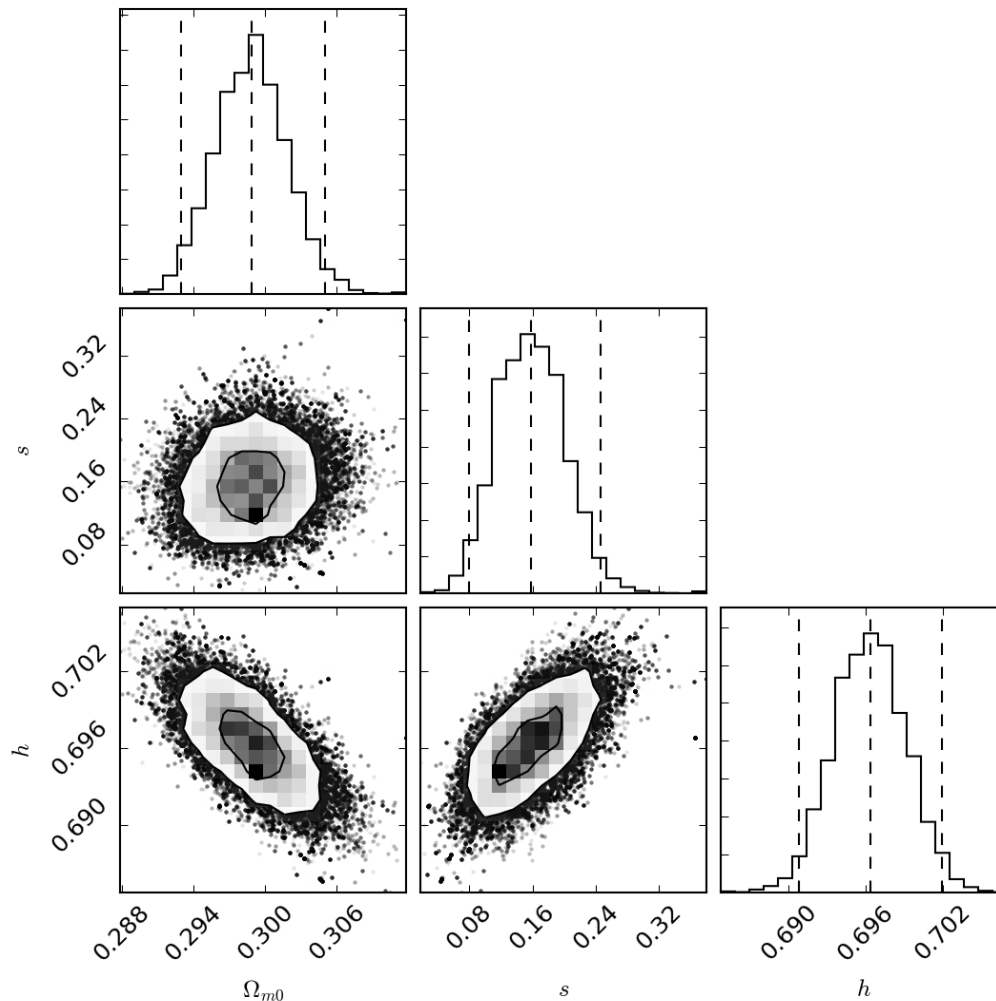
Planck best-fit value

$\sigma_8 = 0.82$
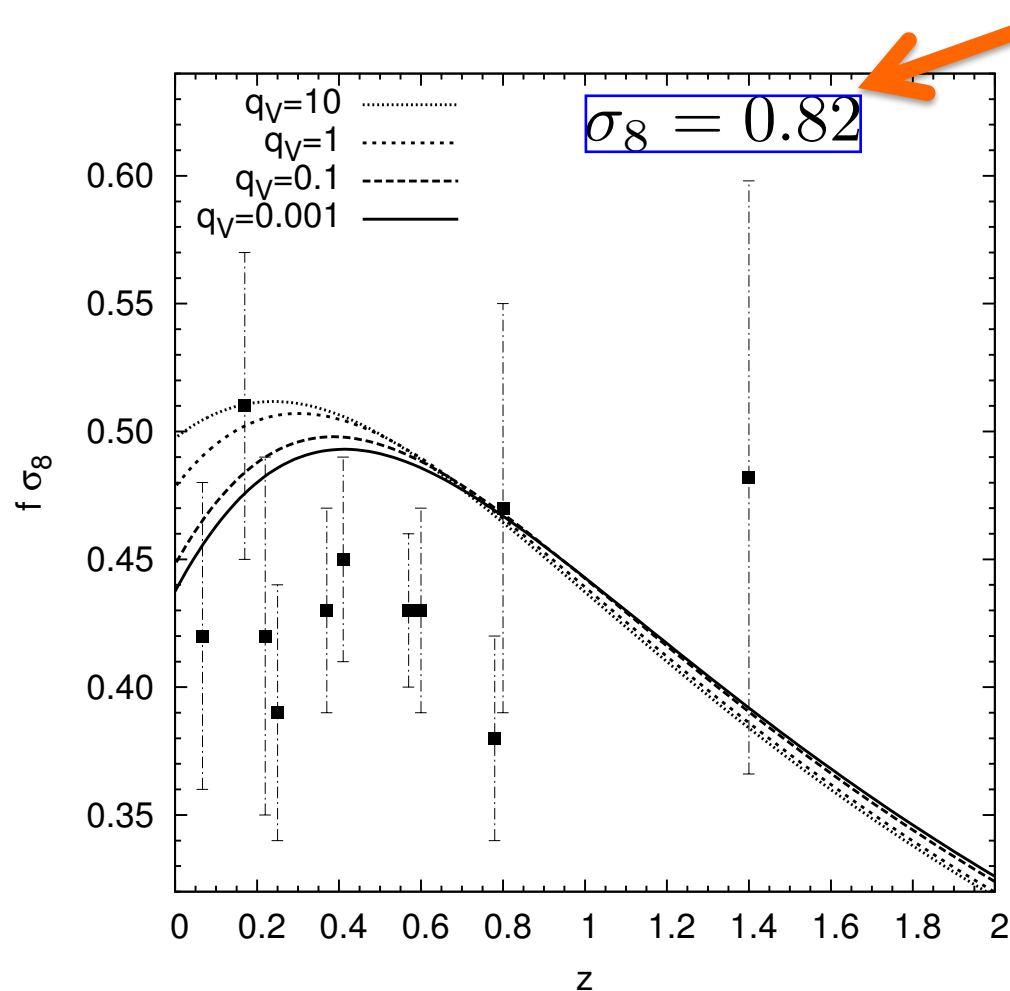
For smaller q_V, the values of $f\sigma_8$ tend to be smaller.

The present $f\sigma_8$ data alone are not sufficient to distinguish between the models with different q_V.

The relization of weak gravity is also limited down to the value around $G_{\text{eff}} \simeq 0.95G$.

Observational constraints including the RSD data

A. De Felice, L. Heisenberg, ST, 1703.09573.

The joint analysis including the RSD data give the bound

$$\Omega_{m0} = 0.299^{+0.006}_{-0.006},$$

$$h = 0.696^{+0.006}_{-0.005},$$

$$s = 0.16^{+0.08}_{-0.08},$$

(95 % CL)

The model with $s > 0$ stills fits the data better than the LCDM. However, the case of weak gravity is not necessarily the best fit.

Best-fit: $\chi^2_{\min} = 625.6$

ΛCDM: $\chi^2_{\min} = 642.7$

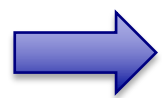# Healthy extension of second-order generalized Proca theories

Heisenberg, Kase, ST, PLB (2016)

The Heisenberg Lagrangian contains the Galileon-like contributions:

$$\mathcal{L}_{i+2}^{\text{Ga}} = g_{i+2}\, \hat{\delta}_{\alpha_1 \cdots \alpha_i \gamma_{i+1} \cdots \gamma_4}^{\beta_1 \cdots \beta_i \gamma_{i+1} \cdots \gamma_4} \nabla_{\beta_1} A^{\alpha_1} \cdots \nabla_{\beta_i} A^{\alpha_i}$$

where

$$\hat{\delta}_{\alpha_1 \cdots \alpha_i \gamma_{i+1} \cdots \gamma_4}^{\beta_1 \cdots \beta_i \gamma_{i+1} \cdots \gamma_4} = \mathcal{E}_{\alpha_1 \cdots \alpha_i \gamma_{i+1} \cdots \gamma_4} \mathcal{E}^{\beta_1 \cdots \beta_i \gamma_{i+1} \cdots \gamma_4}$$

We can consider the generalized Lagrangians like

$$\mathcal{L}_4^{\text{N}} = f_4\, \hat{\delta}_{\alpha_1 \alpha_2 \alpha_3 \gamma_4}^{\beta_1 \beta_2 \beta_3 \gamma_4} A^{\alpha_1} A_{\beta_1} \nabla^{\alpha_2} A_{\beta_2} \nabla^{\alpha_3} A_{\beta_3} ,$$

$$\mathcal{L}_5^{\text{N}} = f_5\, \hat{\delta}_{\alpha_1 \alpha_2 \alpha_3 \alpha_4}^{\beta_1 \beta_2 \beta_3 \beta_4} A^{\alpha_1} A_{\beta_1} \nabla^{\alpha_2} A_{\beta_2} \nabla^{\alpha_3} A_{\beta_3} \nabla^{\alpha_4} A_{\beta_4}$$

⟶ In the scalar limit $A^\mu \to \nabla^\mu \pi$, these recover the Lagrangains of Gleyzes-Langlois-Piazza-Verinizz theories.

(healthy extension of second-order Horndeski theories)

The analysis of linear perturbations on the flat FLRW background and on the anisotropic cosmological background shows that there are no additional ghostly DOF even with these new Lagrangians.

Anisotropic cosmology in beyond-generalized Proca (BGP) theories

Four new Lagrangians :

$$\mathcal{L}_4^N = f_4(X)\hat{\delta}_{\alpha_1\alpha_2\alpha_3\gamma_4}^{\beta_1\beta_2\beta_3\gamma_4} A^{\alpha_1} A_{\beta_1} \nabla^{\alpha_2} A_{\beta_2} \nabla^{\alpha_3} A_{\beta_3} \,,$$

$$\mathcal{L}_5^N = f_5(X)\hat{\delta}_{\alpha_1\alpha_2\alpha_3\alpha_4}^{\beta_1\beta_2\beta_3\beta_4} A^{\alpha_1} A_{\beta_1} \nabla^{\alpha_2} A_{\beta_2} \nabla^{\alpha_3} A_{\beta_3} \nabla^{\alpha_4} A_{\beta_4}$$

$$\tilde{\mathcal{L}}_5^N = \tilde{f}_5(X)\hat{\delta}_{\alpha_1\alpha_2\alpha_3\alpha_4}^{\beta_1\beta_2\beta_3\beta_4} A^{\alpha_1} A_{\beta_1} \nabla^{\alpha_2} A^{\alpha_3} \nabla_{\beta_2} A_{\beta_3} \nabla^{\alpha_4} A_{\beta_4}$$

$$\mathcal{L}_6^N = f_6(X)\hat{\delta}_{\alpha_1\alpha_2\alpha_3\alpha_4}^{\beta_1\beta_2\beta_3\beta_4} \nabla_{\beta_1} A_{\beta_2} \nabla^{\alpha_1} A^{\alpha_2} \nabla_{\beta_3} A^{\alpha_3} \nabla_{\beta_4} A^{\alpha_4}$$

Anisotropic background:

$$ds^2 = -N^2(t)dt^2 + e^{2\alpha(t)}\left[e^{-4\sigma(t)}dx^2 + e^{2\sigma(t)}\left(dy^2 + dz^2\right)\right]$$

with the vector field $\quad A^\mu = \left(\dfrac{\phi(t)}{N(t)}, e^{-\alpha(t)+2\sigma(t)}\, v(t), 0, 0 \right)$
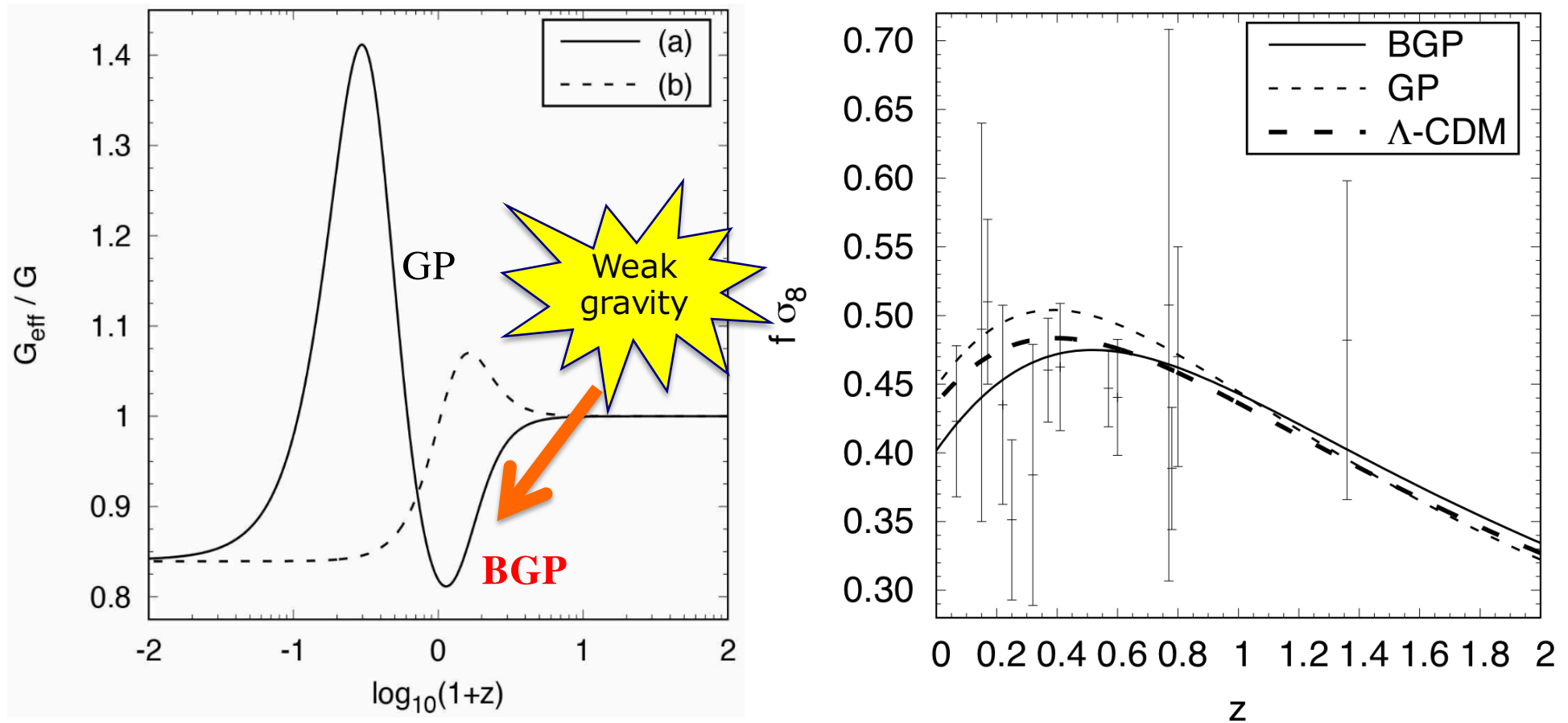
The Hamiltonian constraint is

$$\frac{\partial L}{\partial N} = -\frac{\mathcal{H}}{N} = 0 \quad \Longrightarrow \quad \mathcal{H} = 0 \quad \text{(bounded from below)}$$

No ghost-like Ostrogradski instability

Observational signatures of beyond-generalized Proca theories

The relization of weak gravity like the value $G_{\rm eff} \simeq 0.8G$ is possible.

It remains to be seen whether the BGP theories fit the data better than the LCDM.

Conclusions

1. Generalized Proca theories give rise to interesting cosmological solutions with a late-time de Sitter attractor.

2. We constructed a class of dark energy models in which all the stability conditions are satisfied during the cosmic expanssion history.

3. The joint data analysis of SN Ia, CMB, BAO, H_0, and RSD show that the model in GP theories is favored over the LCDM.

4. The healthy extension of GP theories allows the realization of weak gravity which should be consistent with both RSD and CMB data.

Let's see whether future observations show the signature of vector-tensor theories.