

Halo redshift space distortion

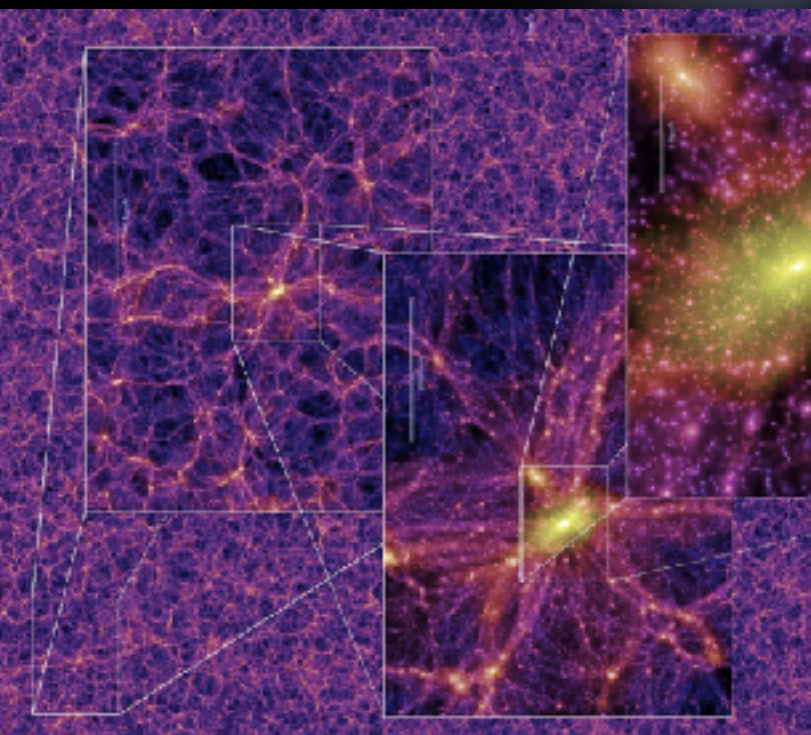
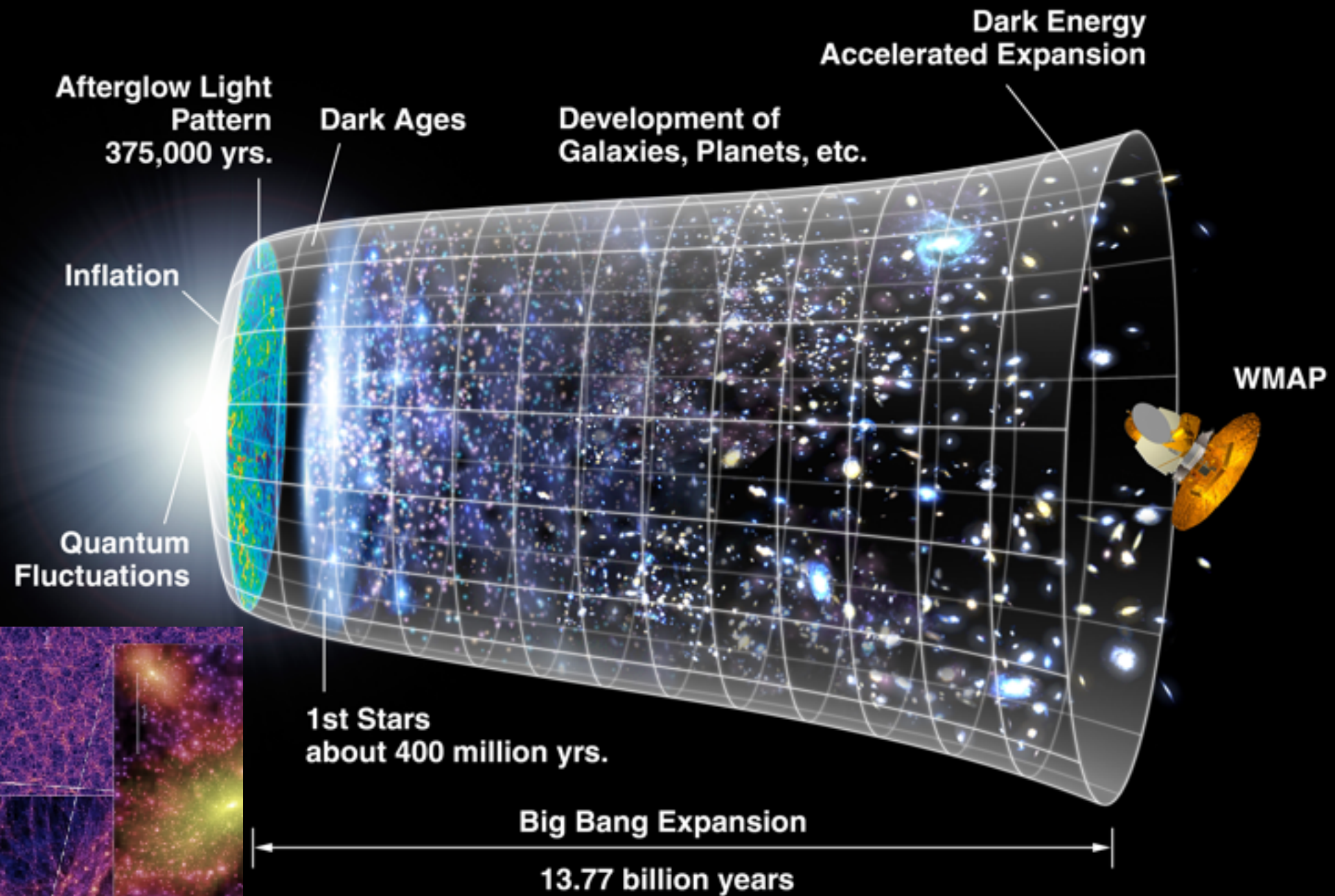
Yi ZHENG (郑逸)

Y. Zheng, Y. Song, arXiv: 1603.00101, JCAP accepted

Y. Zheng, Y. Song and M. Oh, in preparation, 2017

Y. Song, Y. Zheng, M. Oh, and A. Taruya, in preparation, 2017

In general



$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -\kappa T_{\mu\nu},$$

Modified gravity

Dark Energy

1. Background level:

$$ds^2 = c^2 dt^2 - R^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right]$$

$$H^2 = \frac{8\pi G}{3} \left(\sum_i \rho_i \right) - \frac{c^2 k}{R^2}$$

2. Perturbation level:

$$ds^2 = -(1 + 2\psi)dt^2 + (1 - 2\phi)a^2(t)d\vec{x}^2,$$

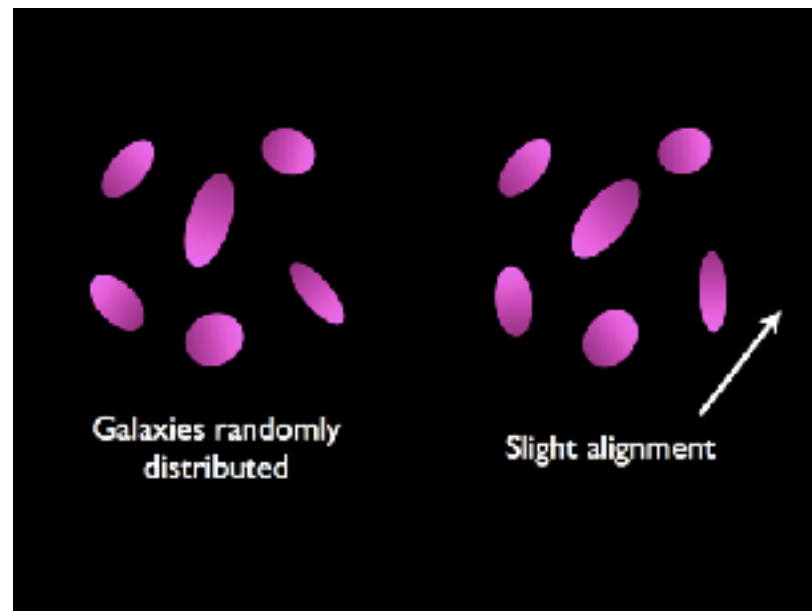
$$k^2 \phi = -4\pi G a^2 \bar{\rho}_{\text{GR}} \left[\delta_{\text{GR}} + 3(1 + w)Ha \frac{\theta_{\text{GR}}}{k^2} \right],$$

$$\simeq -4\pi G a^2 \bar{\rho}_{\text{GR}} \delta_{\text{GR}},$$

$$k^2(\phi - \psi) = 12\pi G a^2 (1 + w) \bar{\rho} \sigma$$

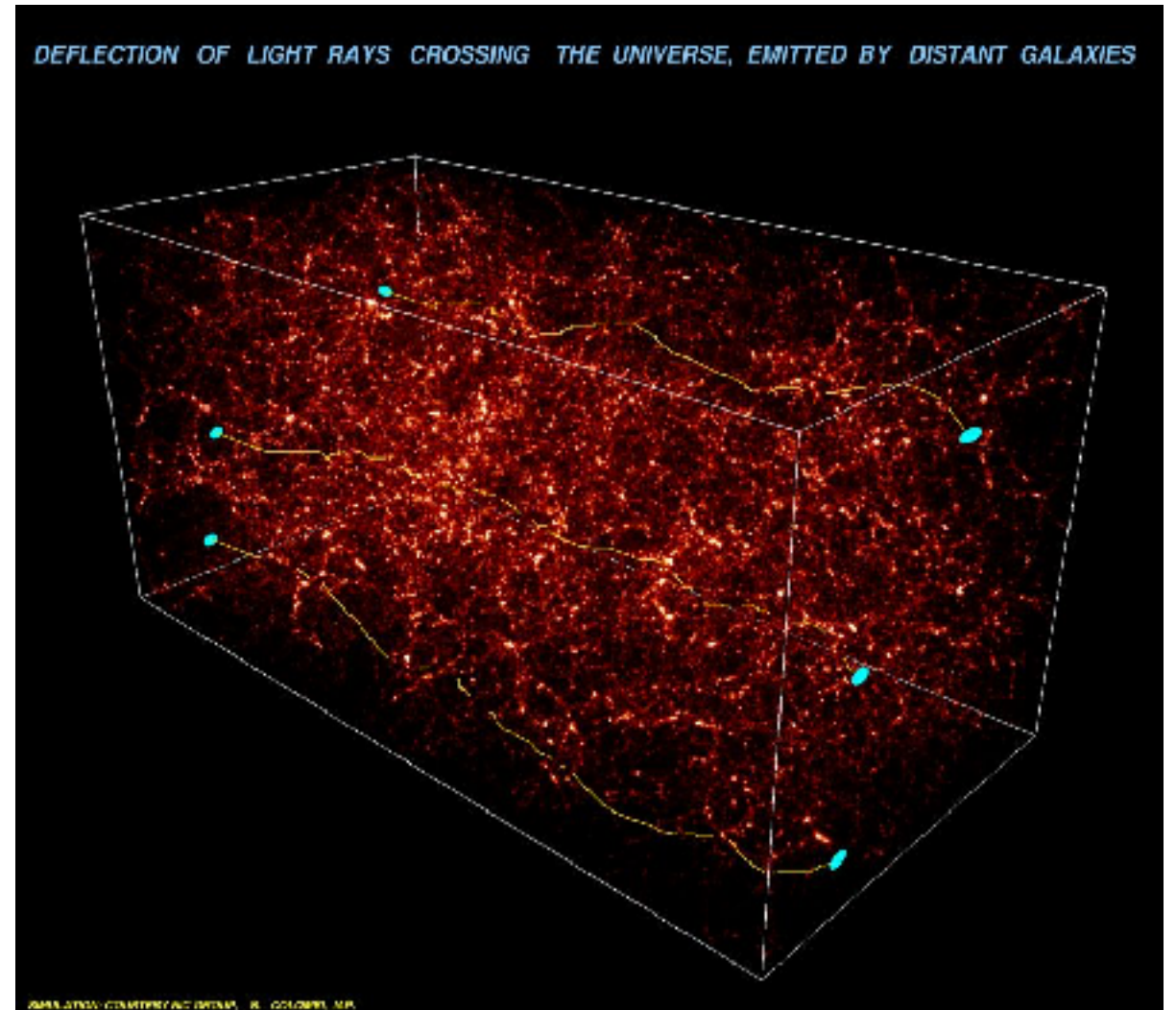
Structure growth

- weak lensing (tomography)



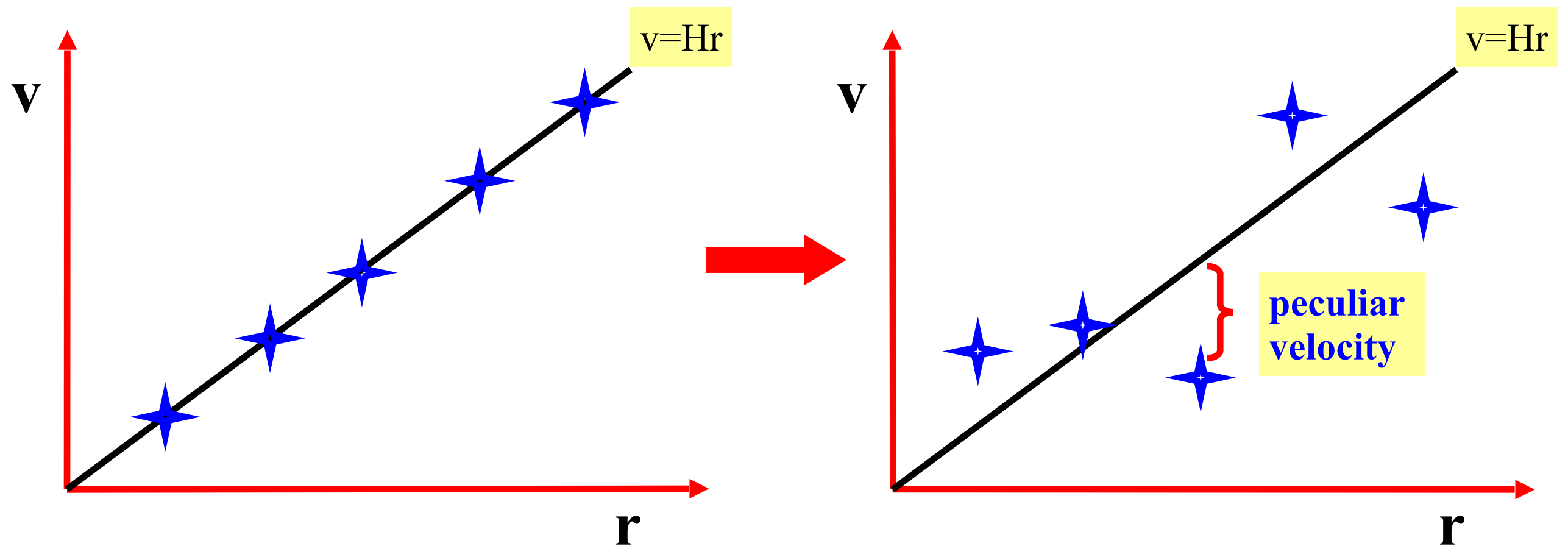
- velocity field

$$\frac{d\delta}{dt} + \frac{1}{a} \nabla \cdot \vec{v} = 0$$



Peculiar velocity: a window to the dark universe

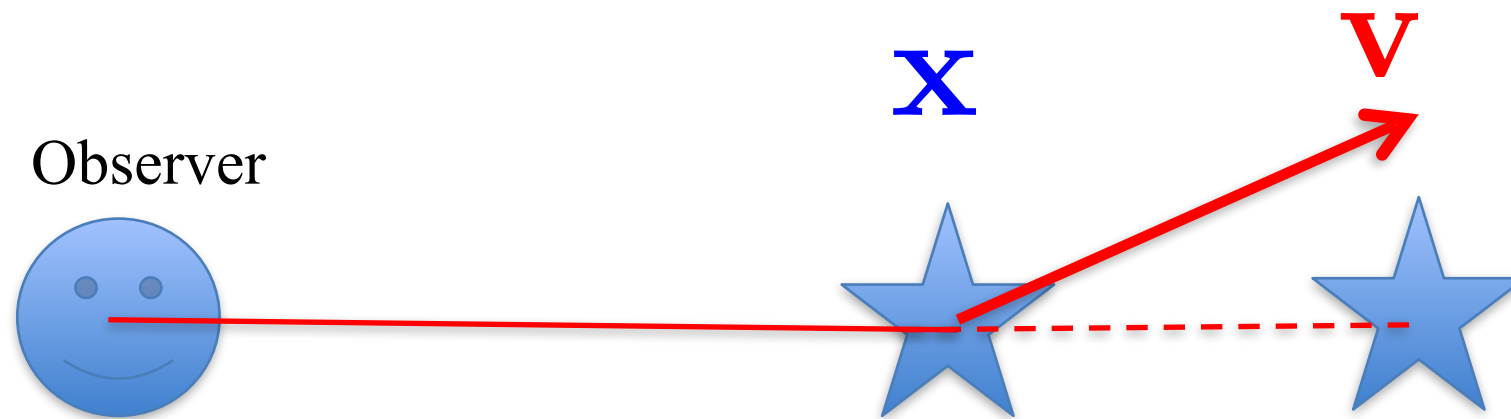
- **Matter distribution in our universe is inhomogeneous**
- **Gravitational attraction arising from inhomogeneity perturbs galaxies and causes deviation from the Hubble flow**



Redshift space distortion

$$z^o = z + v_z$$

$$\mathbf{s} = \mathbf{x} + \frac{v_z}{H} \hat{\mathbf{z}}$$



- Peculiar velocity changes the galaxy redshift and hence distorts the galaxy distribution in an anisotropic way
- Galaxy clustering along the line of sight is different to that perpendicular to the line of sight

RSD Introduction

Redshift space distortion

In observation, galaxy distance is determined by “redshift”

Peculiar velocity of galaxies cause them to appear displaced along the line of sight in redshift space

SDSS DR9, CMASS, 2D correlation function

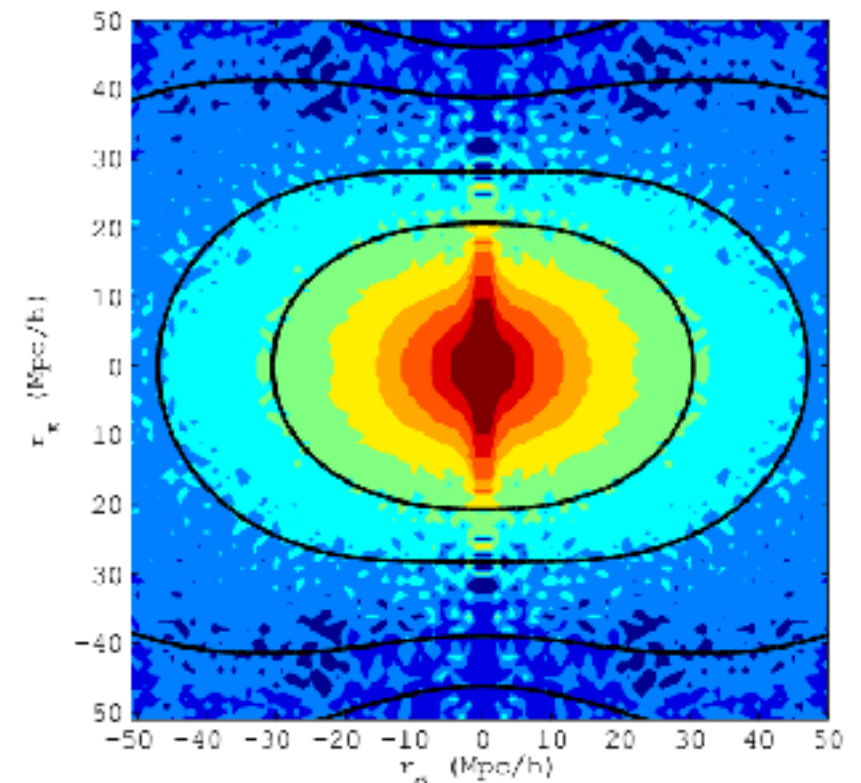
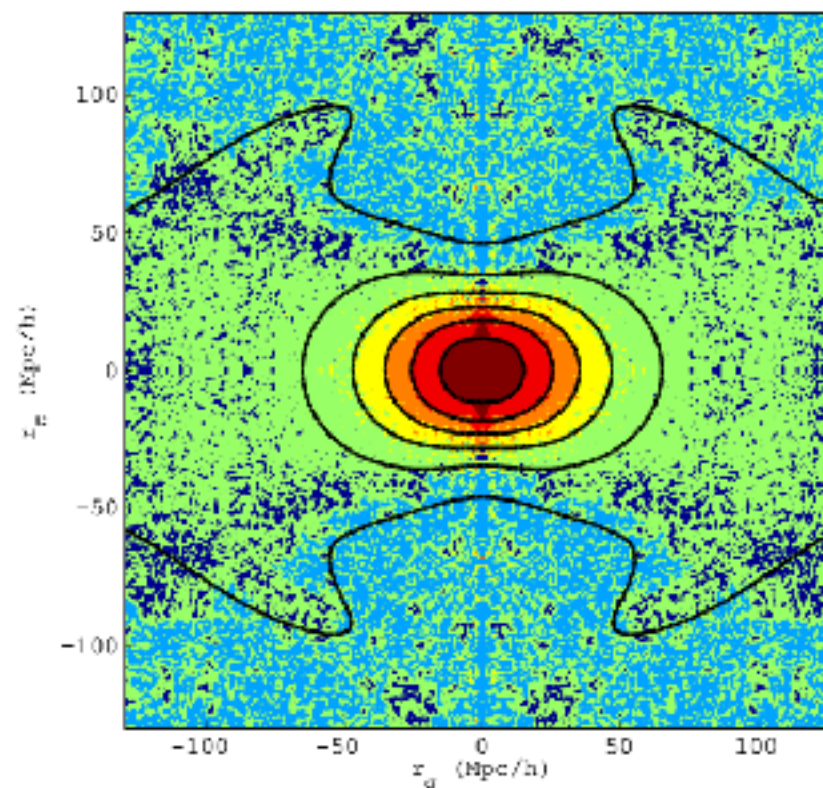
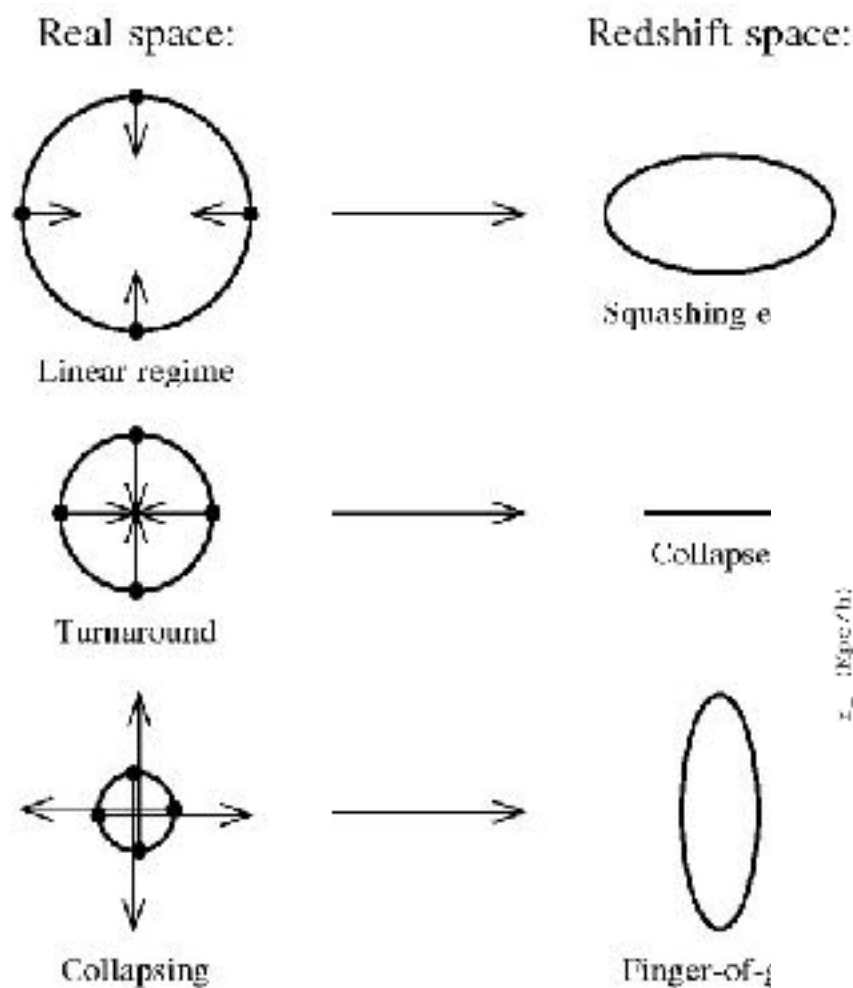
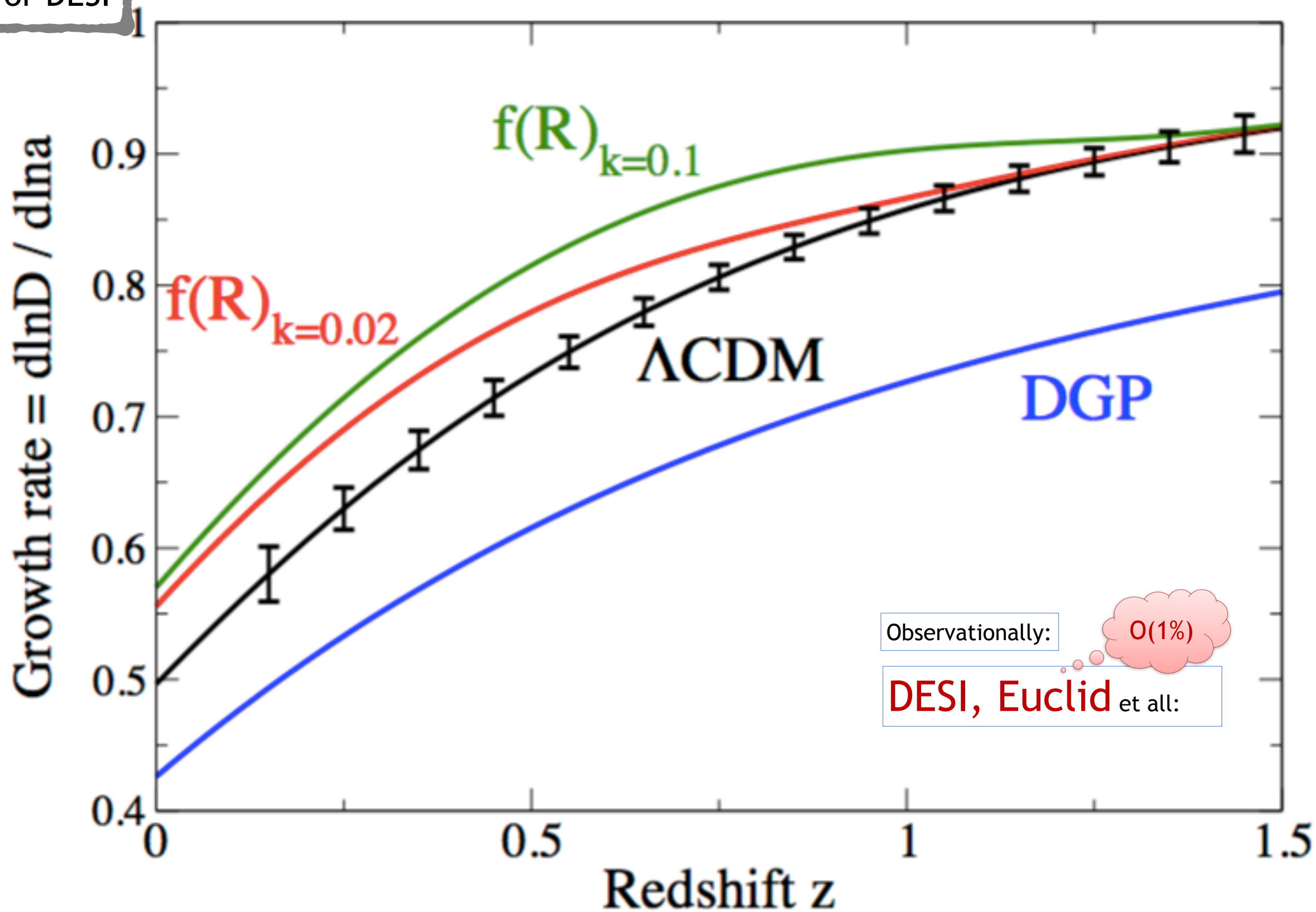


Figure 3. *Left panel:* Two-dimensional correlation function of CMASS galaxies (color) compared with the best fit model described in Section 5.1 (black lines). Contours of equal ξ are shown at [0.6, 0.2, 0.1, 0.05, 0.02, 0]. *Right panel:* Smaller-scale two-dimensional clustering. We show model contours at [0.14, 0.05, 0.01, 0]. The value of ξ_0 at the minimum separation bin in our analysis is shown as the innermost contour. The $\mu \approx 1$ “finger-of-god” effects are small on the scales we use in this analysis.

A. J. S. Hamilton, astro-ph/9708102

For DESI



Redshift space distortion:
small scale modelling
and systematic errors

$$N_k \propto k^3$$

Observationally:

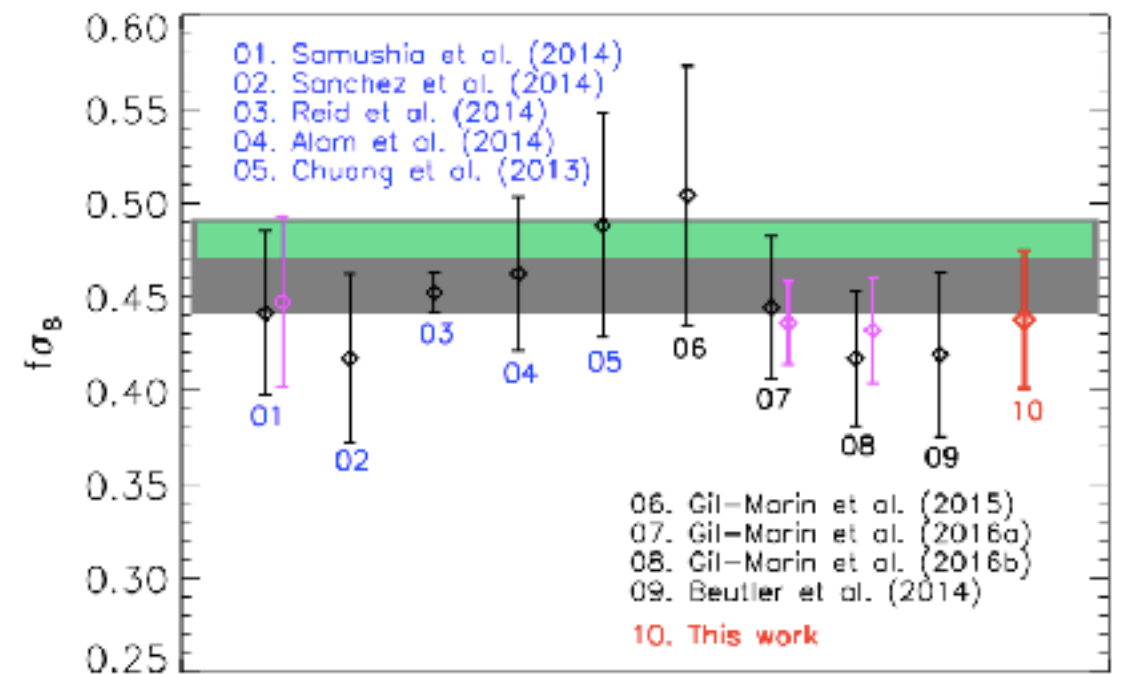
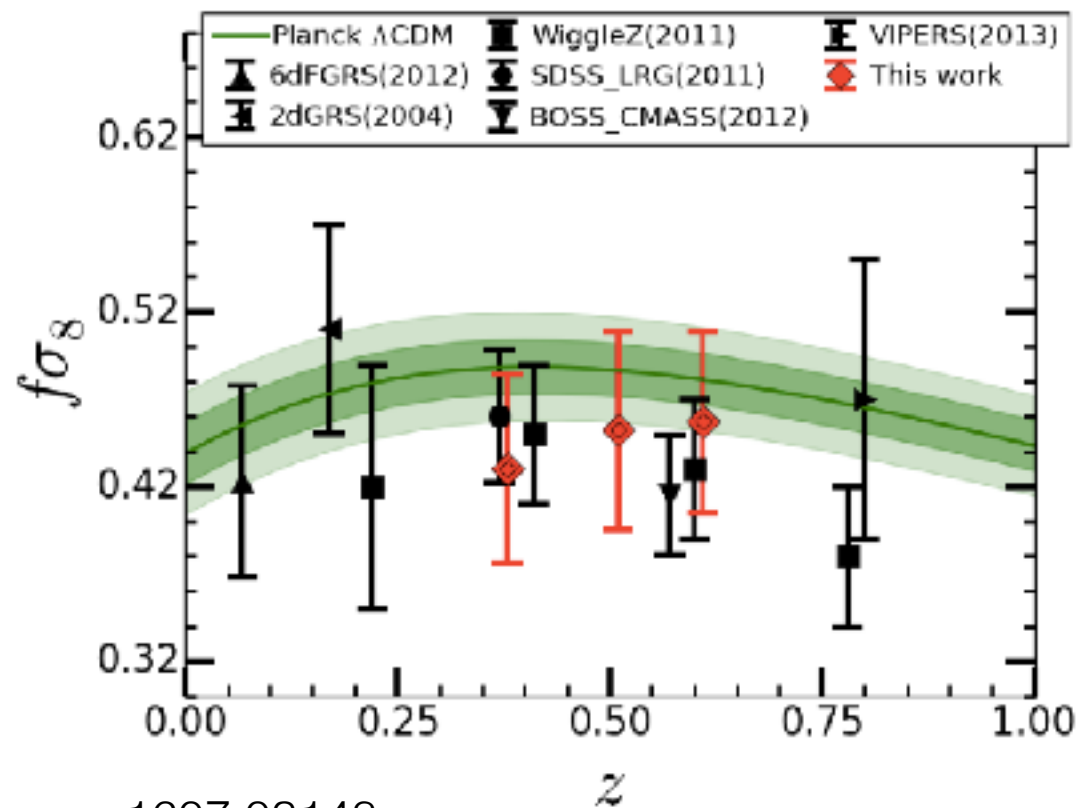
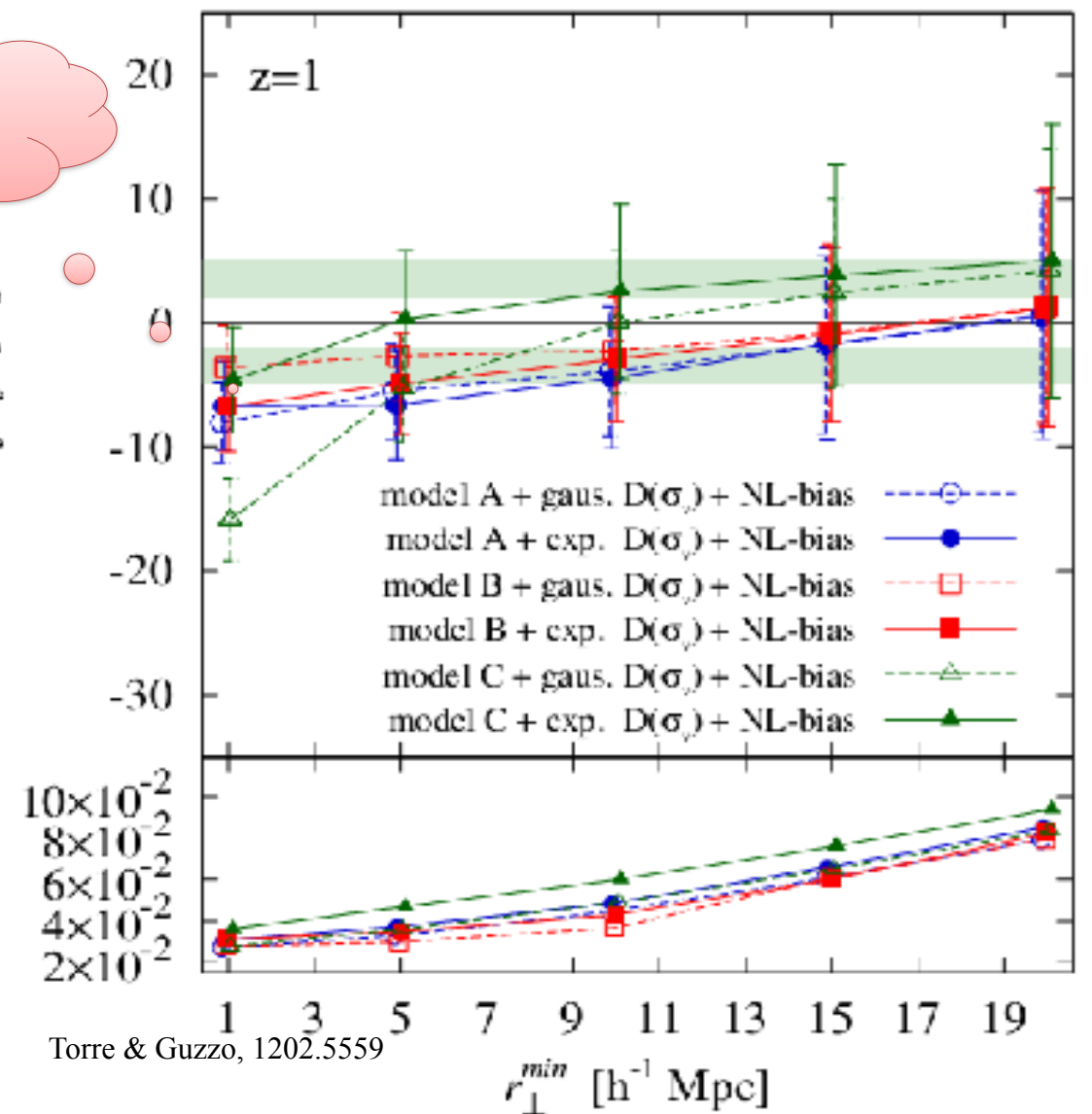
DESI, Euclid et all:

5-10%

O(1%)

$\Delta f/f$ [%]

σ_f



Redshift space distortion modelling: three steps

1. Non-linear mapping of dark matter/halo/galaxy clustering from real space to redshift space

$$P^{(S)}(k, \mu) = \int d^3\mathbf{x} e^{i\mathbf{k}\cdot\mathbf{x}} \langle e^{j_1 A_1} A_2 A_3 \rangle$$

2. Non-linear evolution of density and velocity fields
—— Perturbation theory & High-resolution simulations

$$\delta_n(\mathbf{k}) = \int d^3\mathbf{q}_1 \cdots \int d^3\mathbf{q}_n \delta_D(\mathbf{k} - \mathbf{q}_1 \cdots \mathbf{q}_n) F_n(\mathbf{q}_1, \dots, \mathbf{q}_n) \delta_1(\mathbf{q}_1) \cdots \delta_1(\mathbf{q}_n),$$

$$\theta_n(\mathbf{k}) = \int d^3\mathbf{q}_1 \cdots \int d^3\mathbf{q}_n \delta_D(\mathbf{k} - \mathbf{q}_1 \cdots \mathbf{q}_n) G_n(\mathbf{q}_1, \dots, \mathbf{q}_n) \delta_1(\mathbf{q}_1) \cdots \delta_1(\mathbf{q}_n),$$

F. Bernardeau et al, 2002

3. Galaxy/halo density and velocity bias modelling

$$\delta_g(\mathbf{x}) = b_1 \delta(\mathbf{x}) + \frac{1}{2} b_2 [\delta(\mathbf{x})^2 - \sigma_2] + \frac{1}{2} b_{s^2} [s(\mathbf{x})^2 - \langle s^2 \rangle] + \text{higher order terms},$$

McDonald & Roy, 0902.0991

1. DM Mapping formula

$$P^{(S)}(k, \mu) = \int d^3x e^{i\mathbf{k} \cdot \mathbf{x}} \langle e^{j_1 A_1} A_2 A_3 \rangle$$

$$\delta_D(\mathbf{k}) + P_s(\mathbf{k}) = \int \frac{d^3r}{(2\pi)^3} e^{-i\mathbf{k} \cdot \mathbf{r}} \langle e^{ifk_z \Delta u_z} [1 + \delta(\mathbf{x})] \times [1 + \delta(\mathbf{x}')] \rangle,$$

e.g. Scoccimarro [astro-ph/0407214](#)
Zhang [1207.2722](#)

$$P^{(S)}(k, \mu) = \int \frac{d^3r}{(2\pi)^3} e^{-i\mathbf{k} \cdot \mathbf{r}} \langle e^{ifk_z u_z} [1 + \delta(\mathbf{x})] e^{-ifk_z u'_z} [1 + \delta(\mathbf{x}')] \rangle$$

Taylor expansion!

$$P^{ss}(\mathbf{k}) = \sum_{L=0}^{\infty} \frac{1}{L!^2} \left(\frac{k\mu}{H} \right)^{2L} P_{LL}(\mathbf{k}) + 2 \sum_{L=0}^{\infty} \sum_{L' > L} \frac{(-1)^L}{L! L'!} \left(\frac{ik\mu}{H} \right)^{L+L'} P_{LL'}(\mathbf{k})$$

Seljak&McDonald [1109.1888](#)

TNS [1006.0699](#)
Zheng [1603.00101](#)

$$P^{(S)}(\mathbf{k}) = \int d^3x e^{i\mathbf{k} \cdot \mathbf{x}} \langle e^{-ik\mu f \Delta u_z} \{ \delta(\mathbf{r}) + f \nabla_z u_z(\mathbf{r}) \} \times \{ \delta(\mathbf{r}') + f \nabla_z u_z(\mathbf{r}') \} \rangle,$$

e.g. TNS [1006.0699](#)

$$\langle e^{j_1 A_1} A_2 A_3 \rangle = \exp\{\langle e^{j_1 A_1} \rangle_c\} [\langle e^{j_1 A_1} A_2 A_3 \rangle_c + \langle e^{j_1 A_1} A_2 \rangle_c \langle e^{j_1 A_1} A_3 \rangle_c].$$

Cumulant expansion

$$P^{(S)}(k, \mu) = \int d^3x e^{i\mathbf{k} \cdot \mathbf{x}} \exp\{\langle e^{j_1 A_1} \rangle_c\} [\langle e^{j_1 A_1} A_2 A_3 \rangle_c + \langle e^{j_1 A_1} A_2 \rangle_c \langle e^{j_1 A_1} A_3 \rangle_c].$$

FoG

in terms of j_1

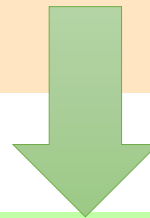
$$\begin{aligned} & \langle e^{j_1 A_1} A_2 A_3 \rangle_c + \langle e^{j_1 A_1} A_2 \rangle_c \langle e^{j_1 A_1} A_3 \rangle_c \\ & \simeq \langle A_2 A_3 \rangle + j_1 \langle A_1 A_2 A_3 \rangle_c \\ & + j_1^2 \left\{ \frac{1}{2} \langle A_1^2 A_2 A_3 \rangle_c + \langle A_1 A_2 \rangle_c \langle A_1 A_3 \rangle_c \right\} + \mathcal{O}(j_1^3). \end{aligned}$$

The 'extended' TNS model

1006.0699 & 1603.00101

Taylor expansion in terms of $j_1 = -ik\mu$

$$\begin{aligned}
 D_{\text{local}}^{\text{FoG}}(k\mu, \mathbf{x}) & \left[\langle e^{j_1 A_1} A_2 A_3 \rangle_c + \langle e^{j_1 A_1} A_2 \rangle_c \langle e^{j_1 A_1} A_3 \rangle_c \right] \\
 & \simeq j_1^0 \langle A_2 A_3 \rangle_c + j_1^1 \langle A_1 A_2 A_3 \rangle_c \\
 & + j_1^2 \left\{ \langle A_1 A_2 \rangle_c \langle A_1 A_3 \rangle_c + \frac{1}{2} \langle A_1^2 A_2 A_3 \rangle_c - \langle u_z u'_z \rangle_c \langle A_2 A_3 \rangle_c \right\} \\
 & + \mathcal{O}(j_1^3), \tag{14}
 \end{aligned}$$

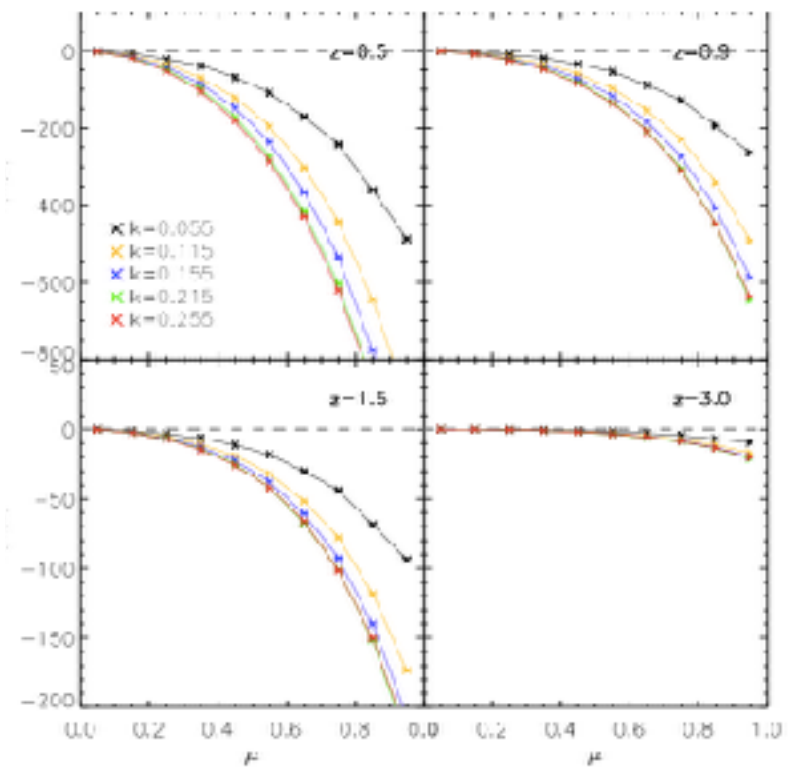
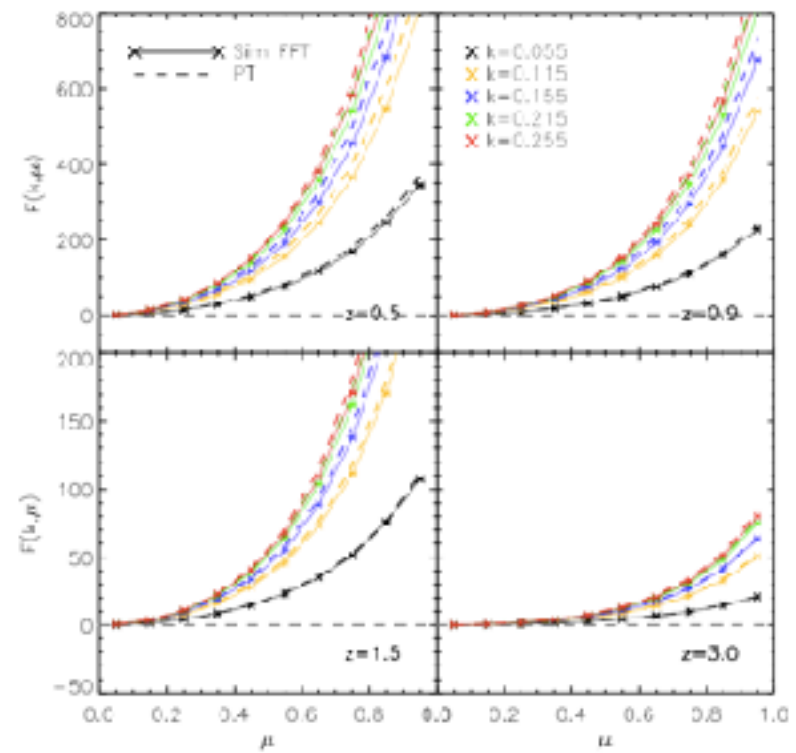
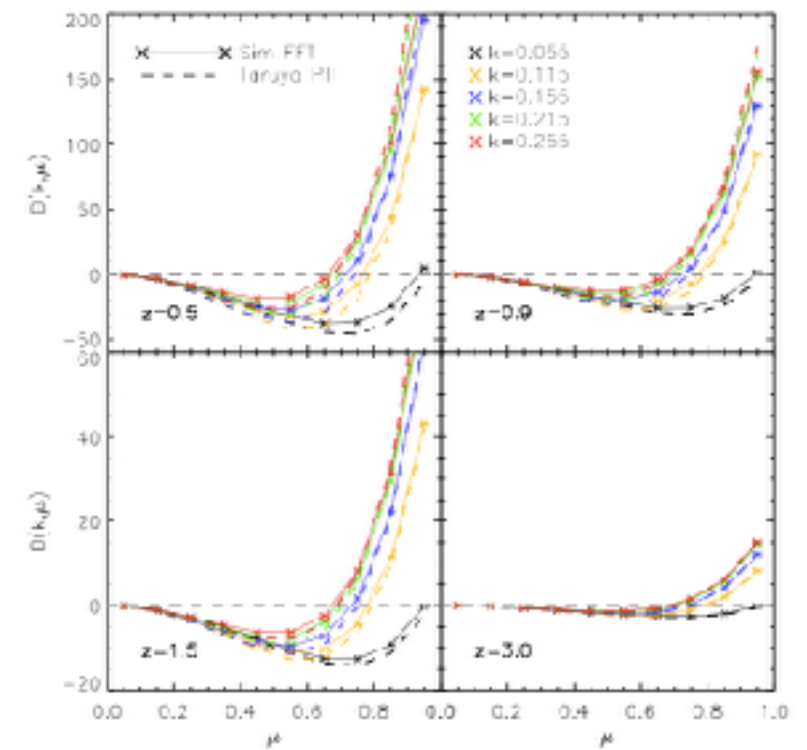
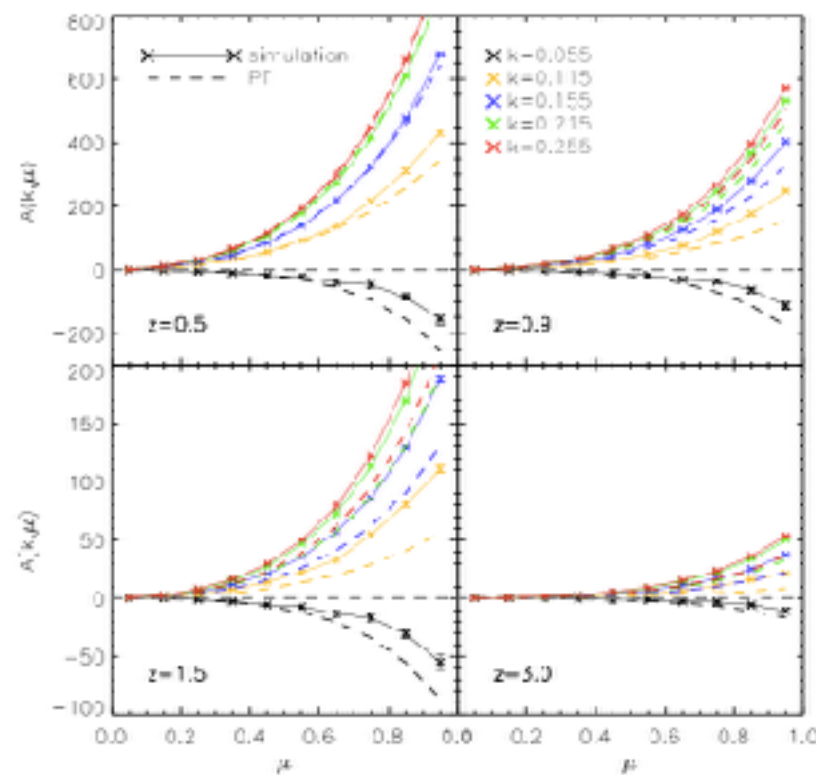
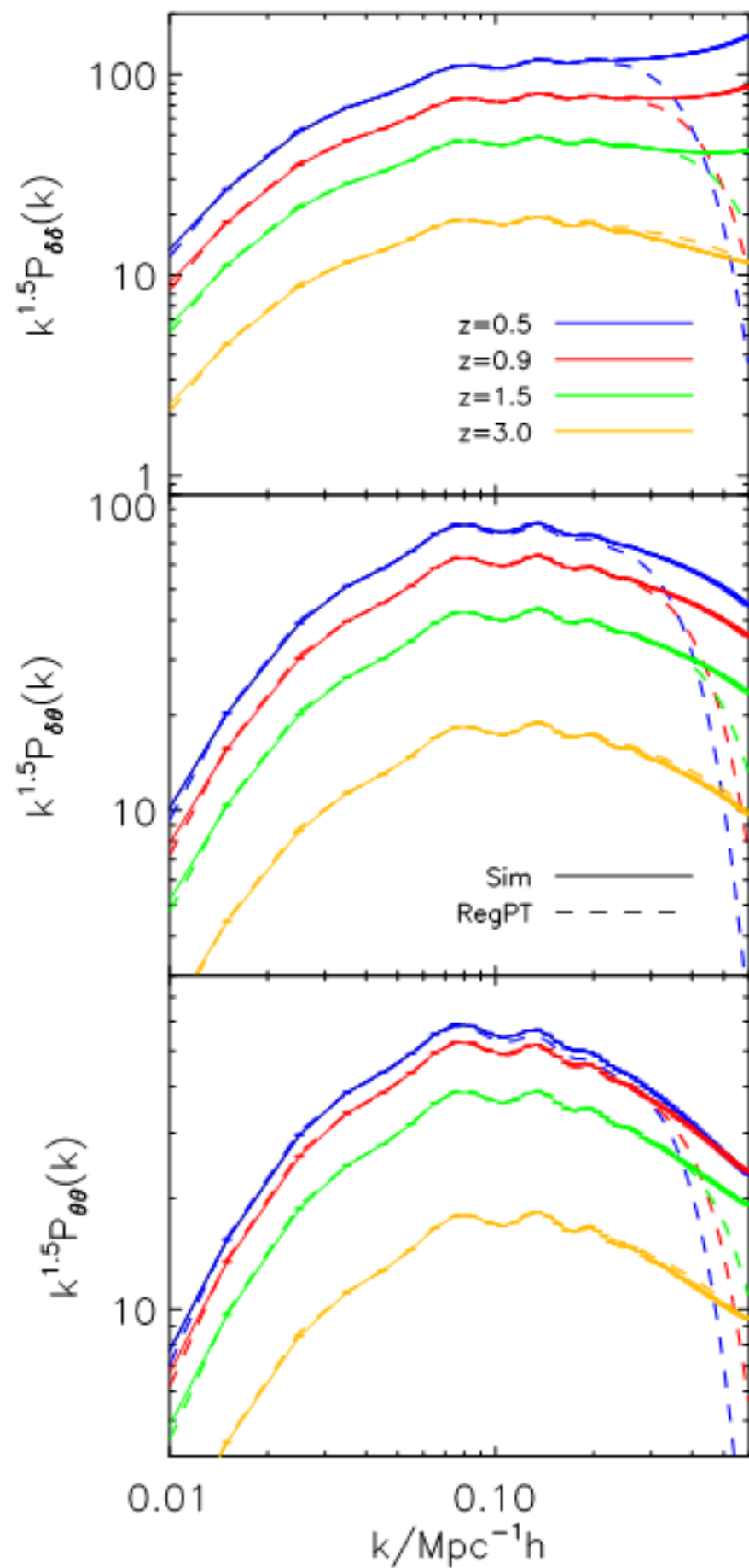


$$\begin{aligned}
 P^{(S)}(k, \mu) & = D^{\text{FoG}}(k\mu\sigma_z) P_{\text{perturbed}}(k, \mu) \\
 & = D^{\text{FoG}}(k\mu\sigma_z) [P_{\delta\delta} + 2\mu^2 P_{\delta\Theta} + \mu^4 P_{\Theta\Theta} \tag{15} \\
 & \quad + A(k, \mu) + B(k, \mu) + T(k, \mu) + F(k, \mu)].
 \end{aligned}$$

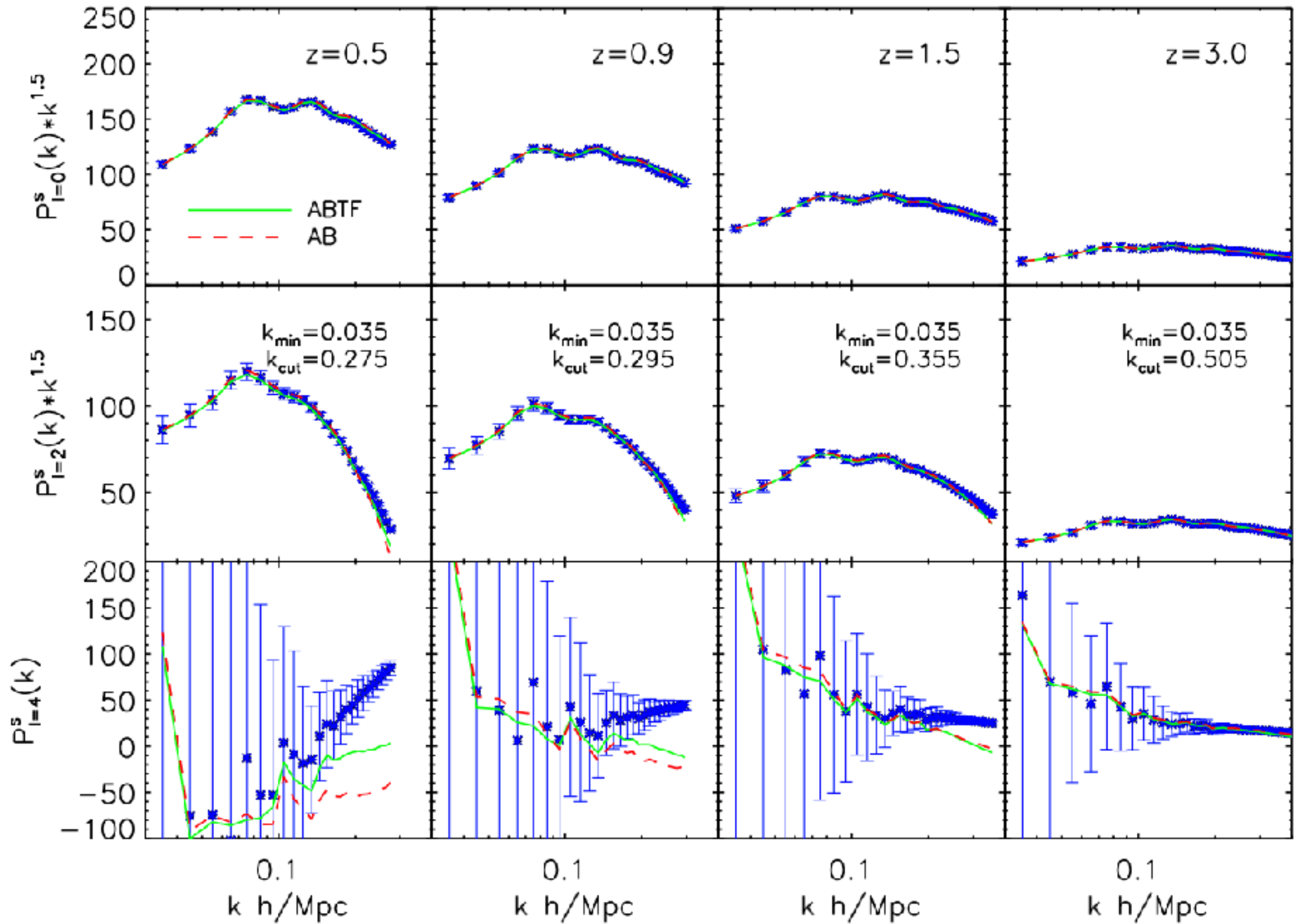
parameter	physical meaning	value
Ω_m	present fractional matter density	0.3132
Ω_Λ	$1 - \Omega_m$	0.6868
Ω_b	present fractional baryon density	0.049
h	$H_0 / (100 \text{ km s}^{-1} \text{ Mpc}^{-1})$	0.6731
n_s	primordial power spectral index	0.9655
σ_8	r.m.s. linear density fluctuation	0.829
L_{box}	simulation box size	$1890 h^{-1} \text{ Mpc}$
N_p	simulation particle number	1024^3
m_p	simulation particle mass	$5.46 \times 10^{10} h^{-1} M_\odot$

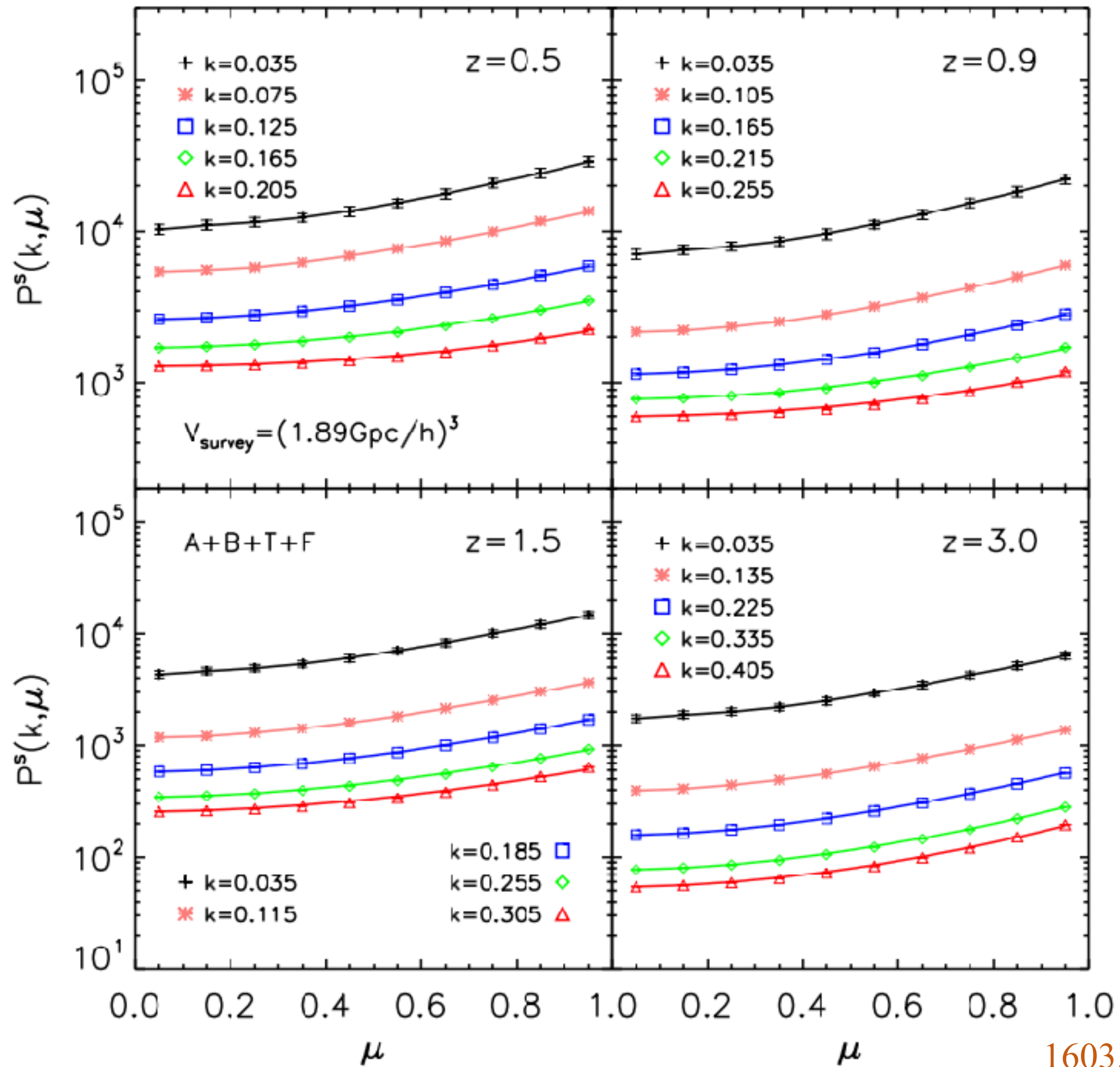
$$\begin{aligned}
 A(k, \mu) & = j_1 \int d^3 \mathbf{x} e^{i\mathbf{k} \cdot \mathbf{x}} \langle A_1 A_2 A_3 \rangle_c \\
 & = j_1 \int d^3 \mathbf{x} e^{i\mathbf{k} \cdot \mathbf{x}} \langle (u_z - u'_z) \\
 & \quad \times (\delta + \nabla_z u_z)(\delta' + \nabla_z u'_z) \rangle_c
 \end{aligned}$$

2-point



• • • • •
 • All higher order terms are important •
 • • • • •





2. Density bias model

$$\delta_g(\mathbf{x}) = b_1\delta(\mathbf{x}) + \frac{1}{2}b_2[\delta(\mathbf{x})^2 - \sigma_2] + \frac{1}{2}b_{s^2}[s(\mathbf{x})^2 - \langle s^2 \rangle] + \text{higher order terms},$$

McDonald&Roy, 0902.0991

$$P_{g,\delta\delta}(k) = b_1^2 P_{\delta\delta}(k) + 2b_2b_1 P_{b2,\delta}(k) + 2b_{s^2}b_1 P_{bs^2,\delta}(k) + b_2^2 P_{b22}(k) + 2b_2b_{s^2} P_{b2s^2}(k) + b_{s^2}^2 P_{bs^22}(k) + 2b_1b_{3nl}\sigma_3^2(k)P^{\text{lin}}(k),$$

$$P_{g\theta}(k) = b_1 P_{\delta\theta}(k) + b_2 P_{b2,\theta}(k) + b_{s^2} P_{bs^2,\theta}(k) + b_{3nl}\sigma_3^2(k)P^{\text{lin}}(k), \quad \text{Hector et.al, 1407.5668}$$

Lagrangian local



$$b_{s^2} = -\frac{4}{7}(b_1 - 1) \quad (\text{Baldauf et al. 2012; Chan et al. 2012}),$$

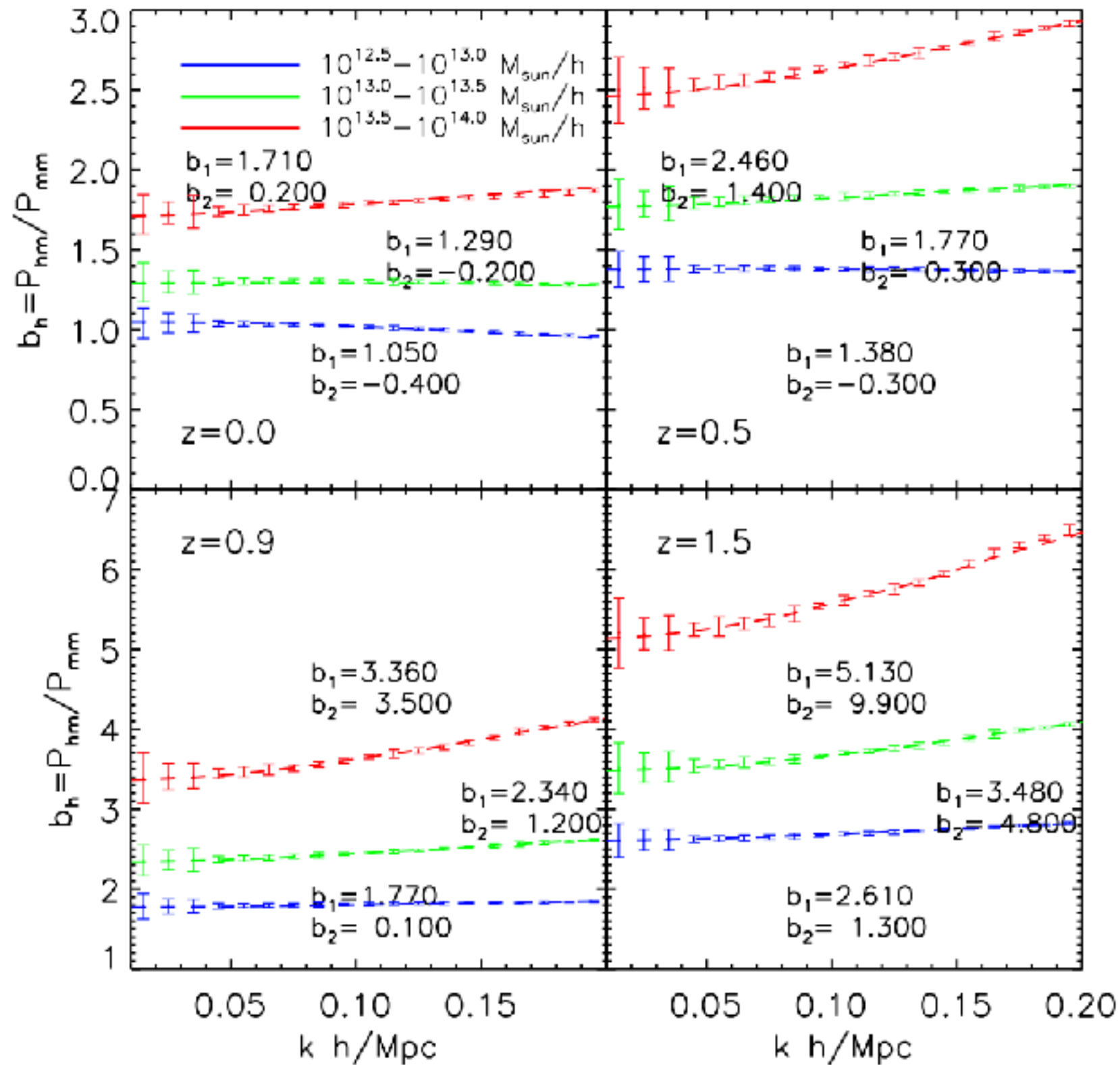
$$b_{3nl} = \frac{32}{315}(b_1 - 1) \quad (\text{Beutler et al. 2014; Saito et al. 2014}).$$

Halo RSD mapping formula test

$$P_h^{(S)}(k, \mu) = D^{\text{FoG}}(k\mu\sigma_z)[(1 + k^2\mu^2\sigma_{z,\text{sim}}^2/2)(P_{\delta_h\delta_h} + 2\mu^2 P_{\delta_h\Theta} + \mu^4 P_{\Theta\Theta}) + b_1^3 A(k, \mu, f/b_1) + b_1^4 \tilde{T}(k, \mu, f/b_1)]$$

No velocity bias, Lagrangian local bias for PS, linear bias for higher order term

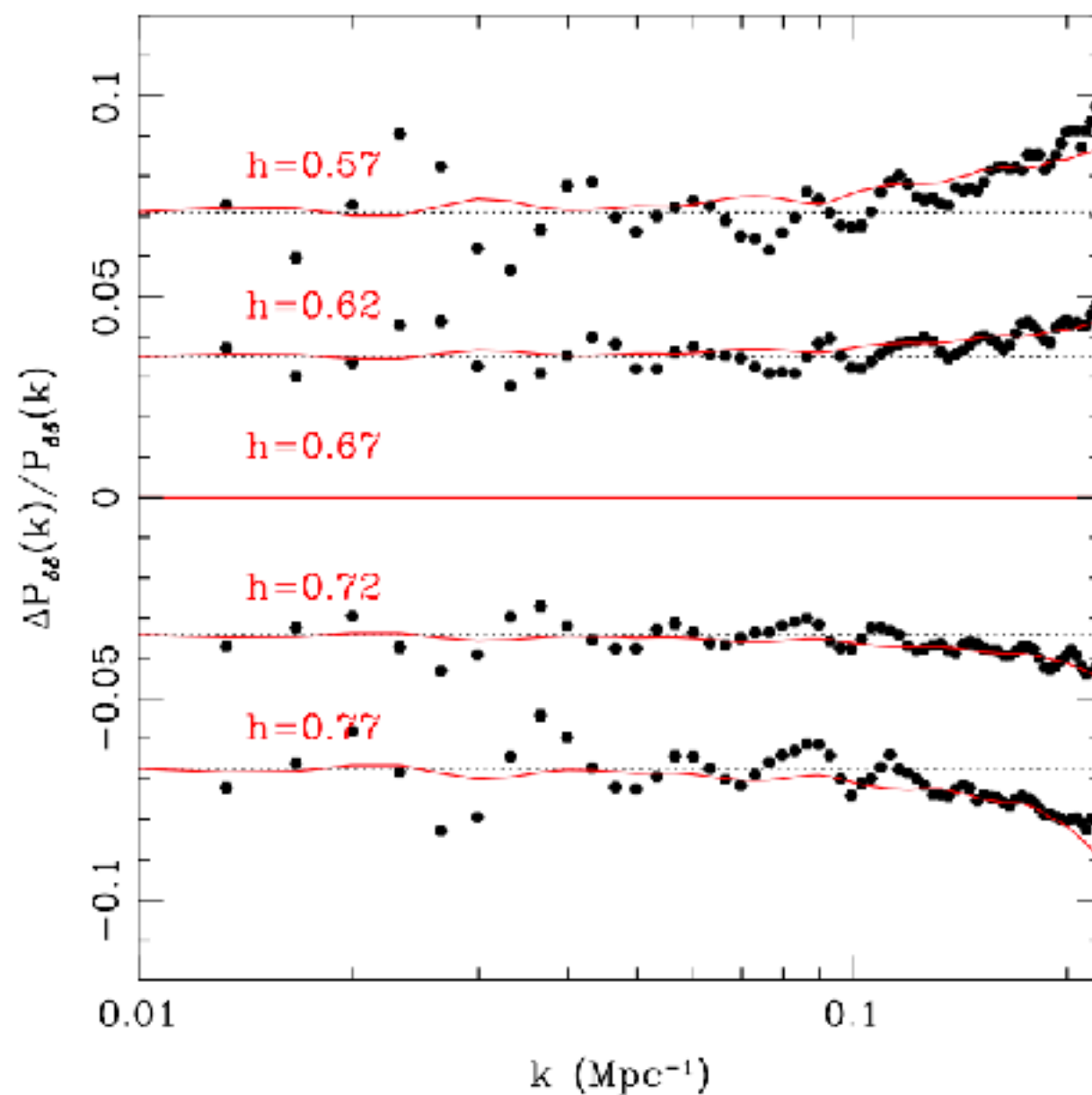
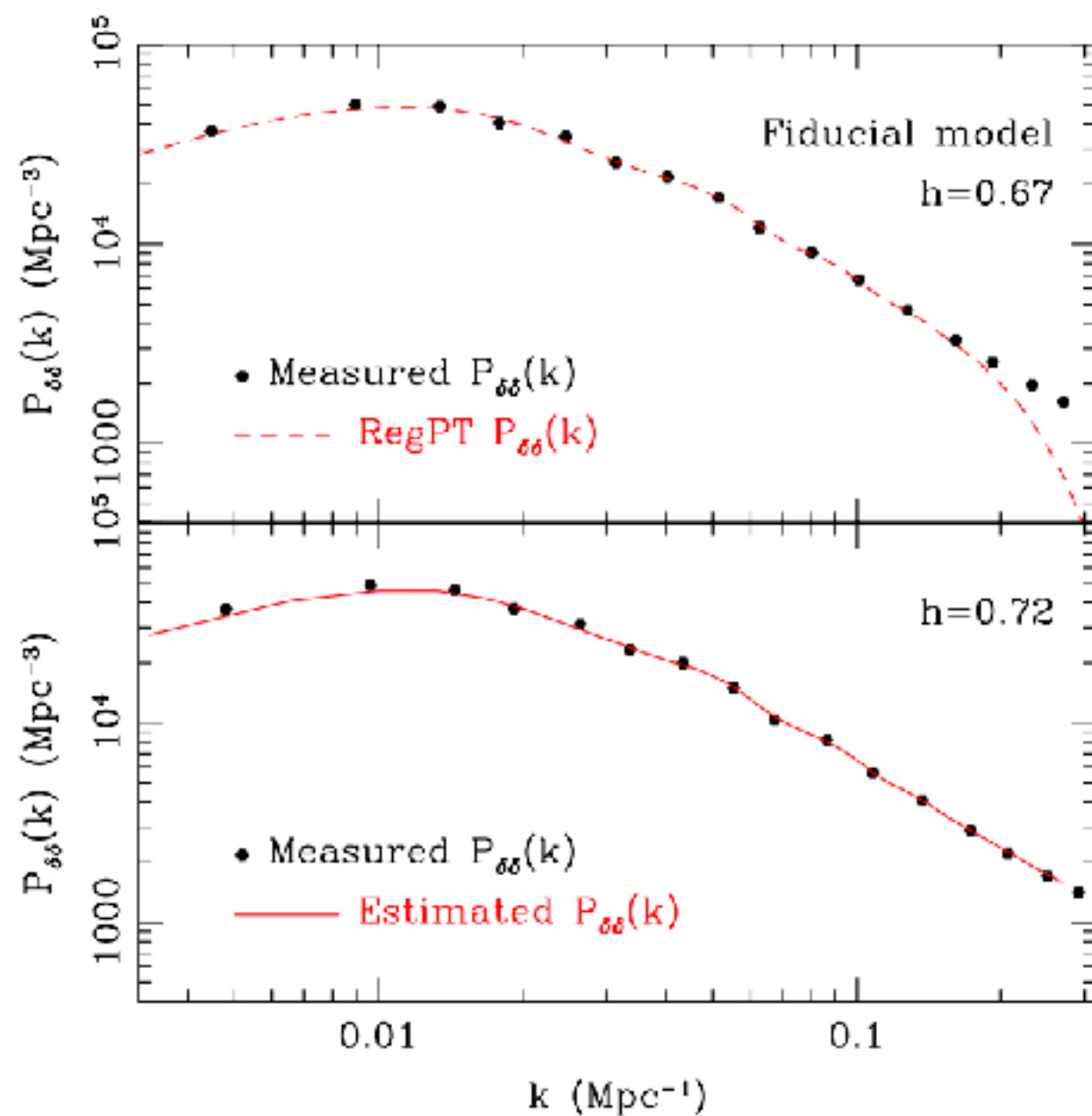
$$P_{\delta_h \delta} = b_1 P_{\delta\delta}(k) + b_2 P_{b2,\delta}(k) + b_{s2} P_{bs2,\delta}(k) + b_{3nl} \sigma_3^2(k) P_m^L(k)$$

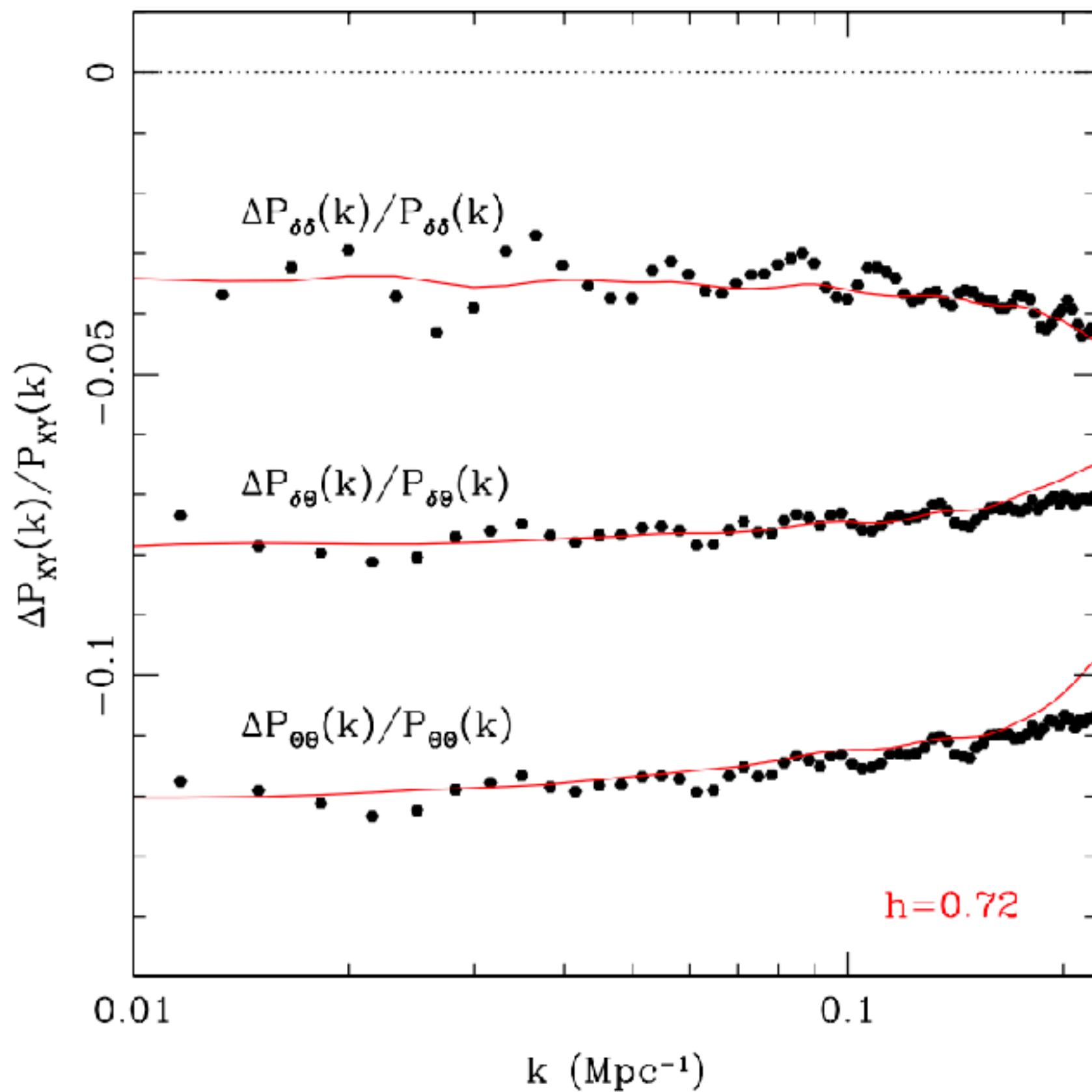


3. Non-linear evolution

Y. Song, Y. Zheng, M. Oh, and A. Taruya, in preparation, 2017

$$P_{XY}(k) = P_{XY}^{\text{th}}(k, z) + P_{XY}^{\text{res}}(k, z)$$

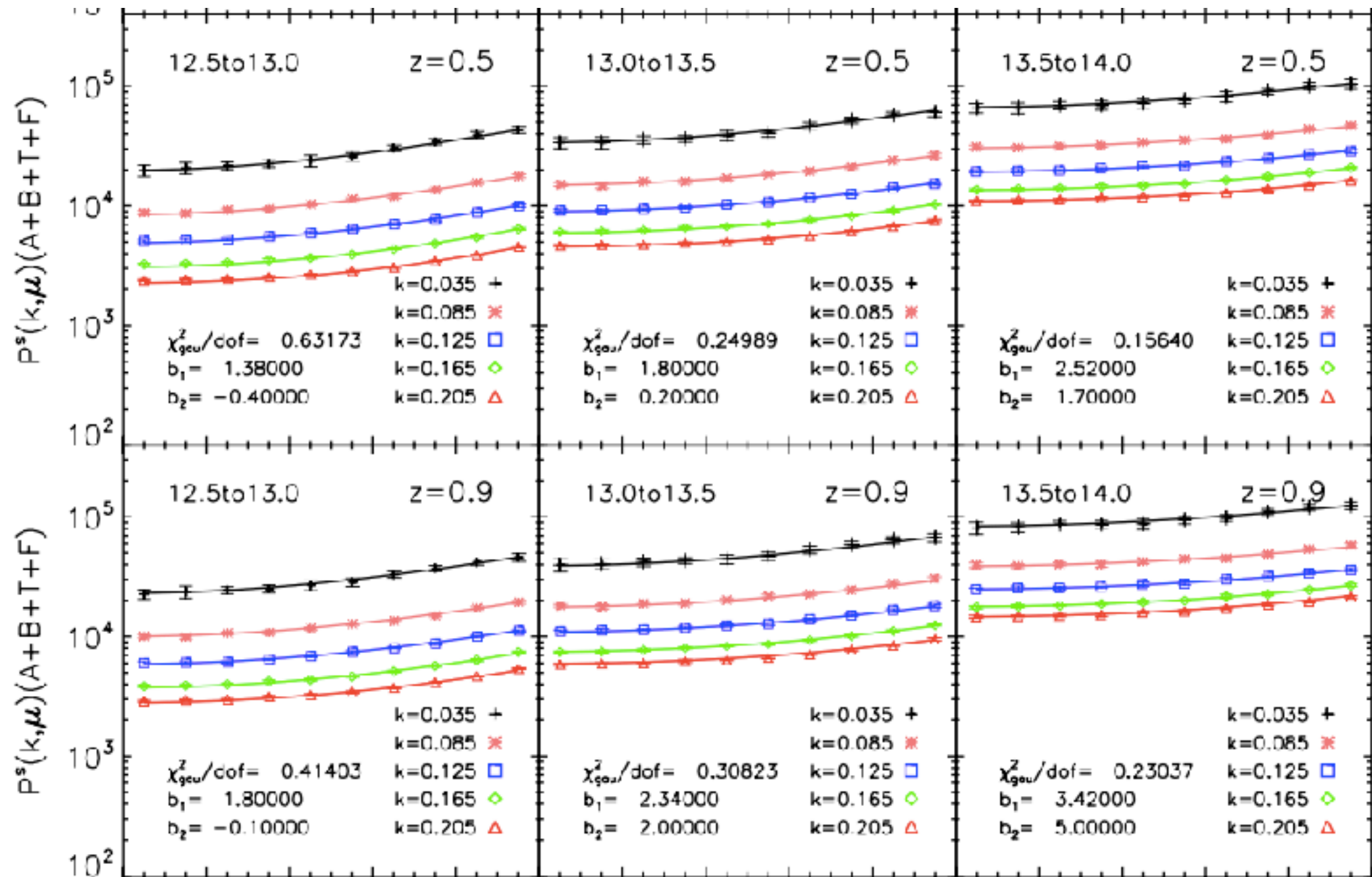




4.Full halo RSD model

b_1, b_2, σ_v

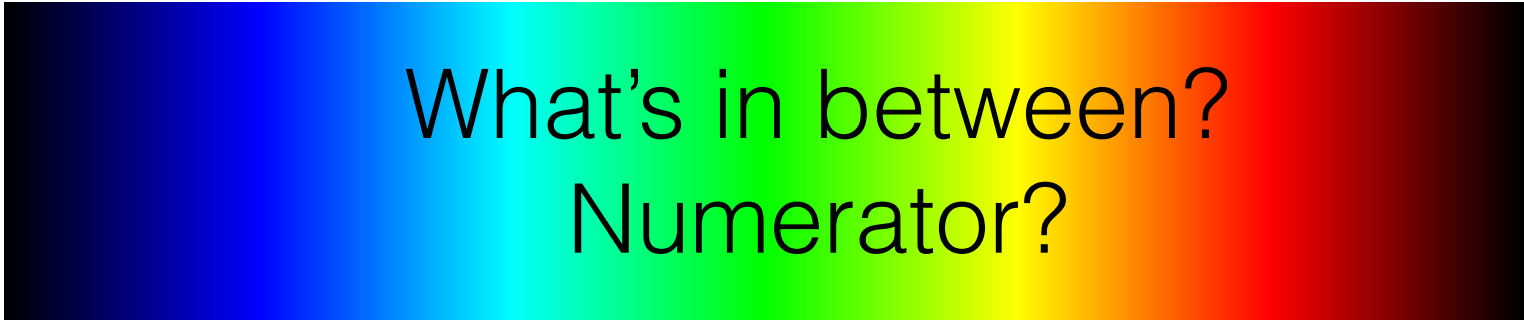
$$P_h^{(S)}(k, \mu) = D^{\text{FoG}}(k\mu\sigma_z) \left[(1 + k^2\mu^2\sigma_{z,\text{sim}}^2/2)(P_{\delta_h\delta_h} + 2\mu^2 P_{\delta_h\Theta} + \mu^4 P_{\Theta\Theta}) + b_1^3 A(k, \mu, f/b_1) + b_1^4 \tilde{T}(k, \mu, f/b_1) \right].$$



Discussion

- Better Taylor expansion of the mapping formula?
Convergence test?
- Density bias model, velocity bias?
- Blind test for RSD model? GreatRSD?
- FoG calculation? 1611.09075

Perturbative
theory: EFTofLSS?



What's in between?
Numerator?

Full numerical,
LSS analysis like CMB?
Quantum computer?