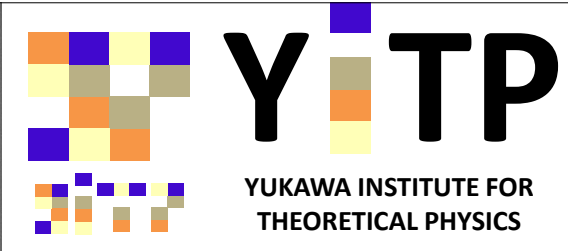


16-18 April 2014

CosKASI conference 2014

KASI, Daejeon



# *Cosmic propagators*

*~powerful tool to characterize large-scale structure~*

A. Taruya  
(YITP, Kyoto Univ.)

# Contents

A fresh look at perturbation theory of large-scale structure  
with a concept of propagators

## Multi-point propagator expansion

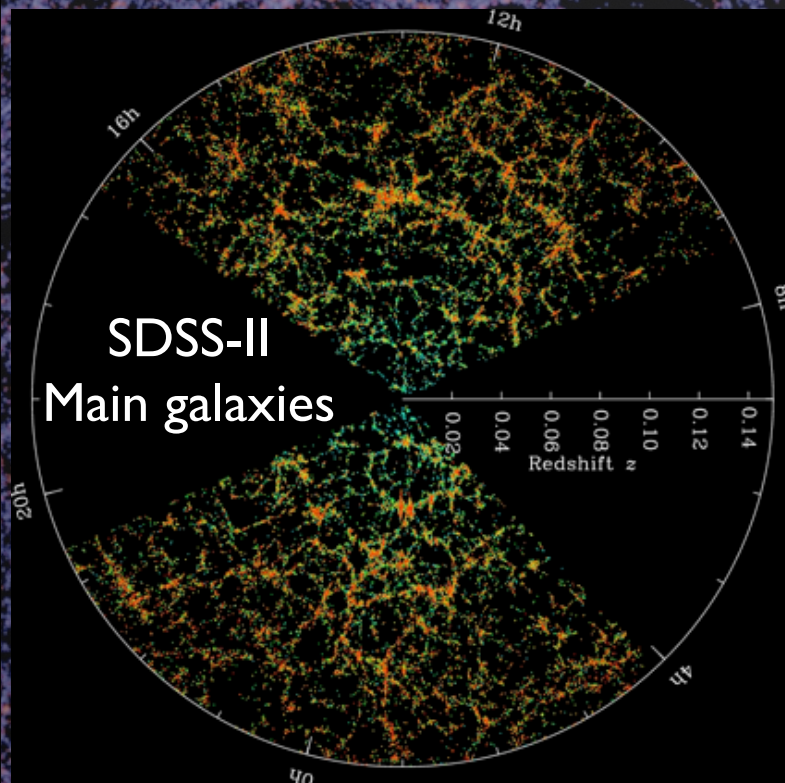
- Dark matter: generic damping tail behaviors  
even in modified gravity models
- Halo/galaxy: scale-dependent & non-local bias

### Collaborators

F. Bernardeau, S. Codis (IAP), T. Hiramatsu (YITP)  
T. Matsubara (KMI), T. Nishimichi (IAP)

# Large-scale structure (LSS)

Spatial inhomogeneity of mass distribution at  $l \sim 10^3$  Mpc

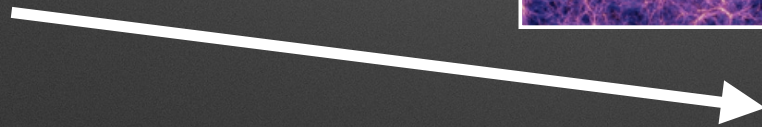
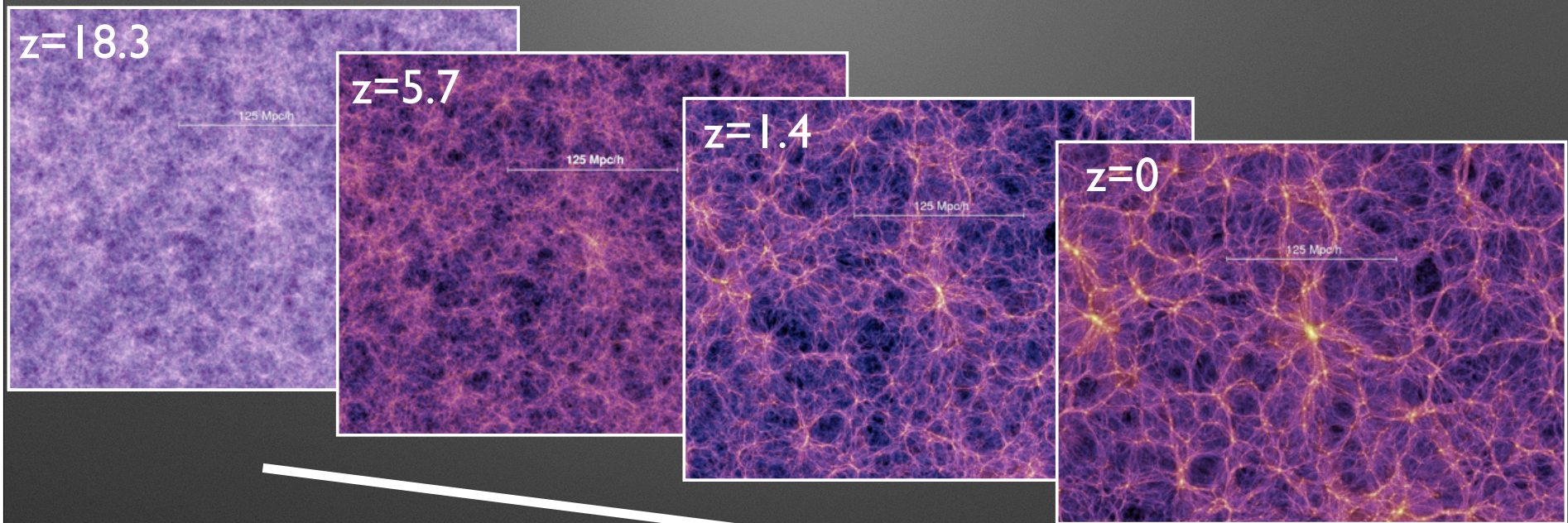


- It is traditionally traced by galaxy clustering via galaxy redshift surveys
- LSS has evolved under the influence of gravity & cosmic expansion

Statistical nature of LSS carries rich cosmological information

# Theoretical issues

How to accurately describe the evolution of LSS



Confronting theory of LSS with precision observations:

Reducing and/or controlling  
non-linear effects

- Non-linear gravity
- Redshift-space distortions
- Galaxy biasing

# Mapping initial cond. to observables

$$\delta_0(\mathbf{k})$$

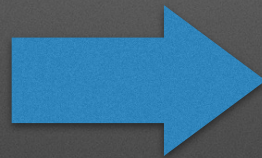
initial density field (Gaussian)

Initial power spectrum

$$P_0(k)$$

from linear theory

(CMB Boltzmann code)



$$\delta(\mathbf{k}; z)$$

Evolved density field (non-Gaussian)

Observables

$$P(k; z)$$

$$B(k_1, k_2, k_3; z)$$

$$T(k_1, k_2, k_3, k_4; z)$$

⋮

of dark matter/galaxies/halos

Concept of '**propagator**' in physics/mathematics may be useful

# Propagator in physics

- ◆ Green's function in linear differential equations
- ◆ Probability amplitude in quantum mechanics

Schrödinger Eq.

$$\left(-i\hbar\frac{\partial}{\partial t} + H_x\right)\psi(x, t) = 0$$

$$G(x, t; x', t') \equiv \frac{\delta\psi(x, t)}{\delta\psi(x', t')}$$

$$\left(-i\hbar\frac{\partial}{\partial t} + H_x\right)G(x, t; x', t') = -i\hbar\delta_D(x - x')\delta_D(t - t')$$

$$\Rightarrow \psi(x, t) = \int_{-\infty}^{+\infty} dx' G(x, t; x', t') \psi(x', t') ; \quad t > t'$$

# Cosmic propagators

Propagator should carry information of  
non-linear evolution & statistical properties

Evolved (non-linear) density field

Crocce & Scoccimarro ('06)

$$\left\langle \frac{\delta \delta_m(\mathbf{k}; t)}{\delta \delta_0(\mathbf{k}')} \right\rangle \equiv \delta_D(\mathbf{k} - \mathbf{k}') \Gamma^{(1)}(k; t) \quad \text{Propagator}$$

Initial density field

Ensemble w.r.t randomness of initial condition

Contain statistical information on *full-nonlinear* evolution

(Non-linear extension of Green's function)

*but*

This is not sufficient to describe nonlinear mode-coupling of LSS

# Multi-point propagators

Bernardeau, Crocce & Scoccimarro ('08)

As a natural generalization,

$$\left\langle \frac{\delta^n \delta_m(\mathbf{k}; t)}{\delta \delta_0(\mathbf{k}_1) \cdots \delta \delta_0(\mathbf{k}_n)} \right\rangle = (2\pi)^{3(1-n)} \delta_D(\mathbf{k} - \mathbf{k}') \Gamma^{(n)}(\mathbf{k}_1, \cdots, \mathbf{k}_n; t)$$

Multi-point propagator

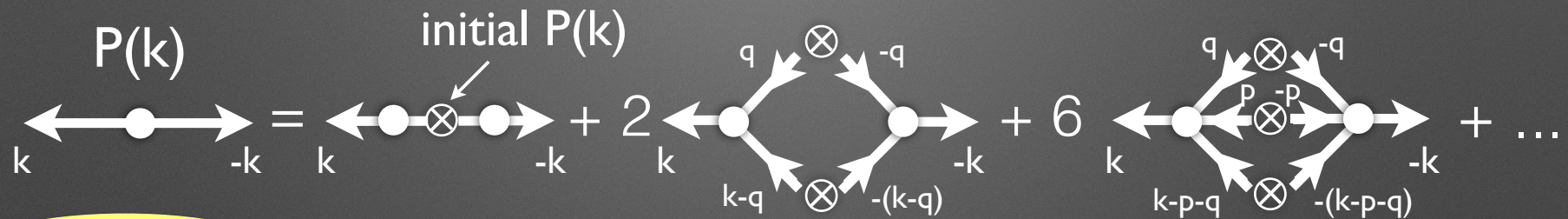
With this multi-point prop.

- Building blocks of a new perturbative theory (PT) expansion  
.....  $\Gamma$ -expansion or Wiener-Hermite expansion
- A good convergence of PT expansion is expected  
(c.f. standard PT)



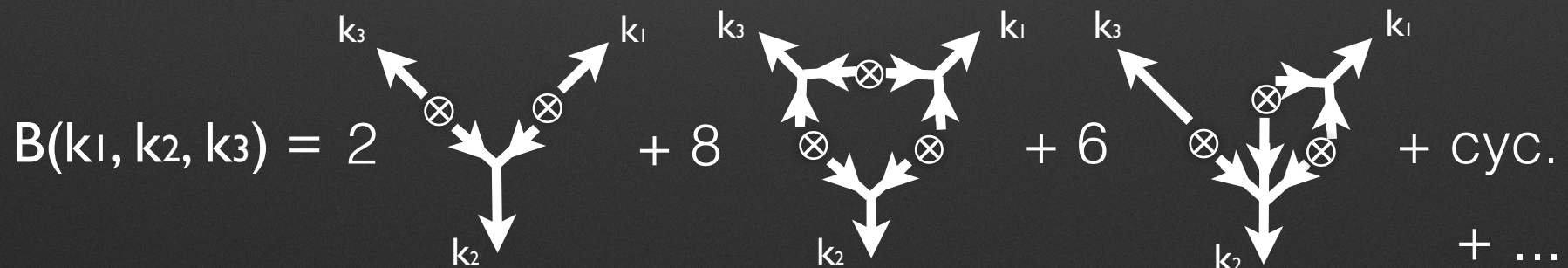
## Power spectrum

$$P(k; t) = \left[ \Gamma^{(1)}(k; t) \right]^2 P_0(k) + 2 \int \frac{d^3 \mathbf{q}}{(2\pi)^3} \left[ \Gamma^{(2)}(\mathbf{q}, \mathbf{k} - \mathbf{q}; t) \right]^2 P_0(q) P_0(|\mathbf{k} - \mathbf{q}|) \\ + 6 \int \frac{d^6 \mathbf{p} d^3 \mathbf{q}}{(2\pi)^6} \left[ \Gamma^{(3)}(\mathbf{p}, \mathbf{q}, \mathbf{k} - \mathbf{p} - \mathbf{q}; t) \right]^2 P_0(p) P_0(q) P_0(|\mathbf{k} - \mathbf{p} - \mathbf{q}|) + \dots$$



## Bispectrum

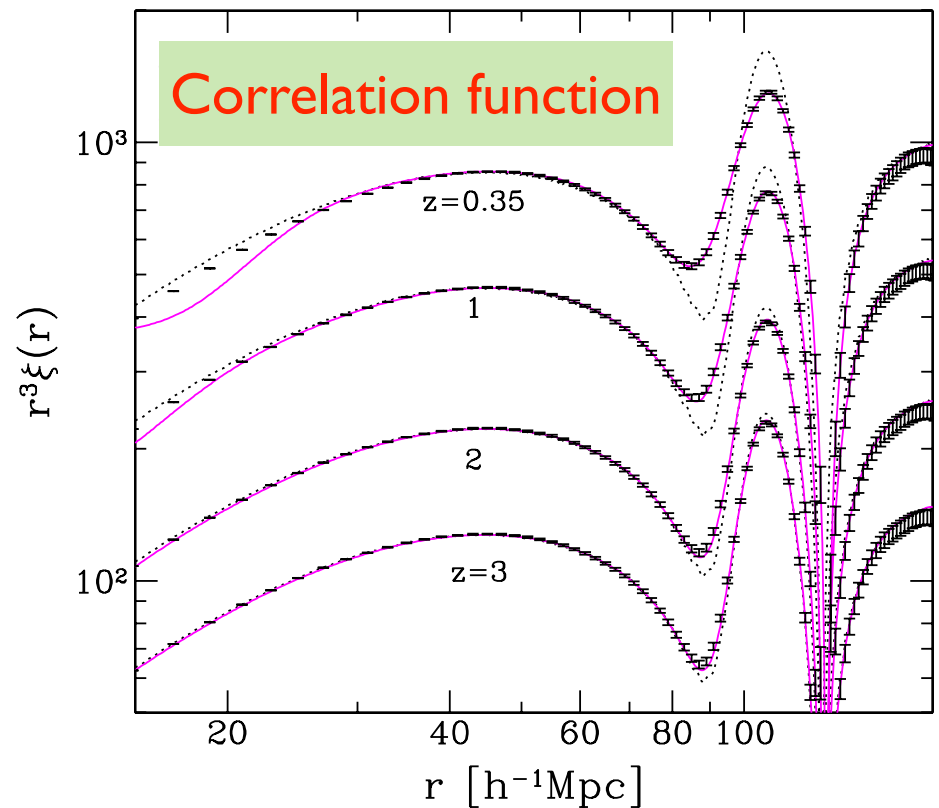
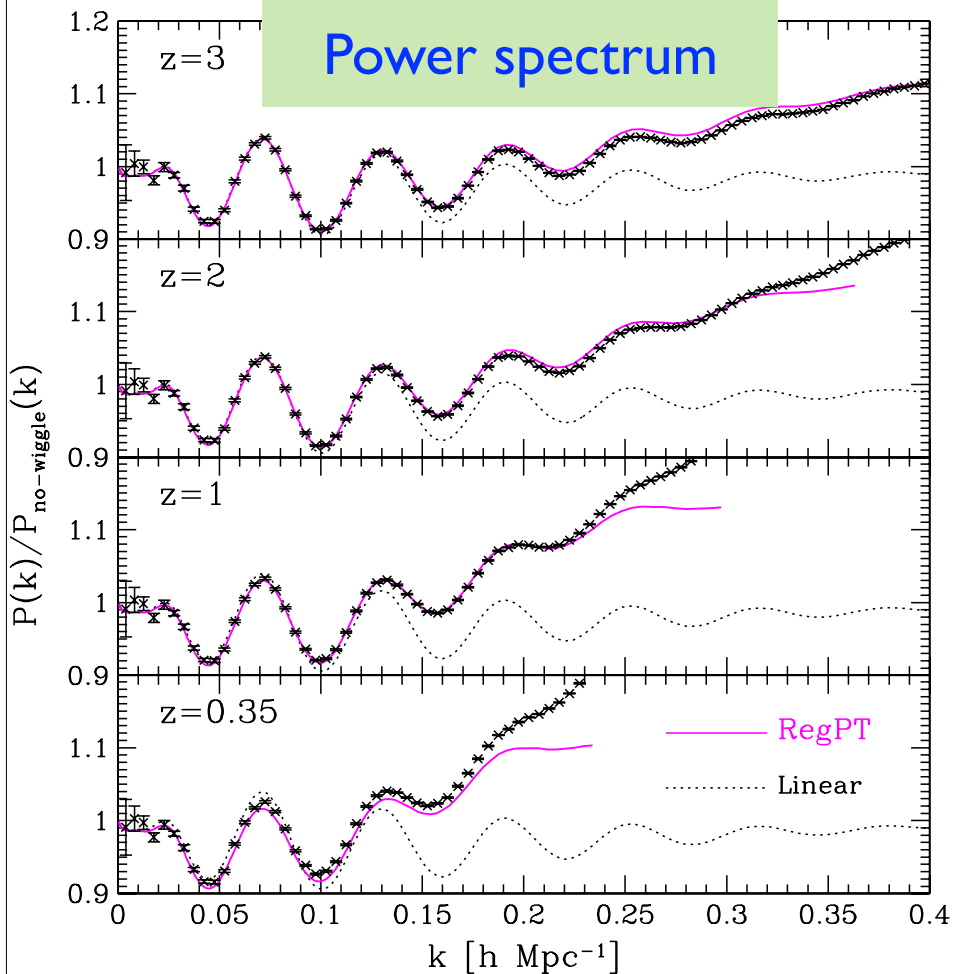
$$B(k_1, k_2, k_3) = 2 \Gamma^{(2)}(\mathbf{k}_1, \mathbf{k}_2) \Gamma^{(1)}(k_1) \Gamma^{(1)}(k_2) P_0(k_1) P_0(k_2) + \text{cyc.} \\ + \left[ 8 \int d^3 q \Gamma^{(2)}(\mathbf{k}_1 - \mathbf{q}, \mathbf{q}) \Gamma^{(2)}(\mathbf{k}_2 + \mathbf{q}, -\mathbf{q}) \Gamma^{(2)}(\mathbf{q} - \mathbf{k}_1, -\mathbf{k}_2 - \mathbf{q}) P_0(|\mathbf{k}_1 - \mathbf{q}|) P_0(|\mathbf{k}_2 + \mathbf{q}|) P_0(q) \right. \\ \left. + 6 \int d^3 q \Gamma^{(3)}(-\mathbf{k}_3, -\mathbf{k}_2 + \mathbf{q}, -\mathbf{q}) \Gamma^{(2)}(\mathbf{k}_2 - \mathbf{q}, \mathbf{q}) \Gamma^{(1)}(\mathbf{k}_3) P_0(|\mathbf{k}_2 - \mathbf{q}|) P_0(q) P_0(k_3) + \text{cyc.} \right].$$



# RegPT: fast PT calculation of $P(k)$ & $\xi(r)$

A public PT code based on multi-point propagators at 2-loop

[http://www2.yukawa.kyoto-u.ac.jp/~atsushi.taruya/regpt\\_code.html](http://www2.yukawa.kyoto-u.ac.jp/~atsushi.taruya/regpt_code.html)



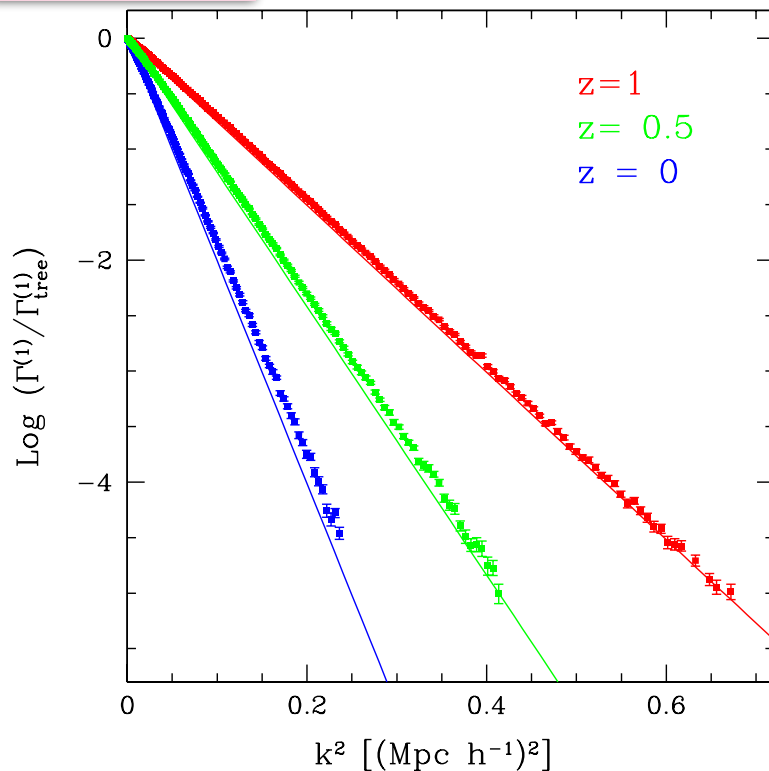
AT, Bernardeau, Nishimichi & Codis ('12)

# Common properties

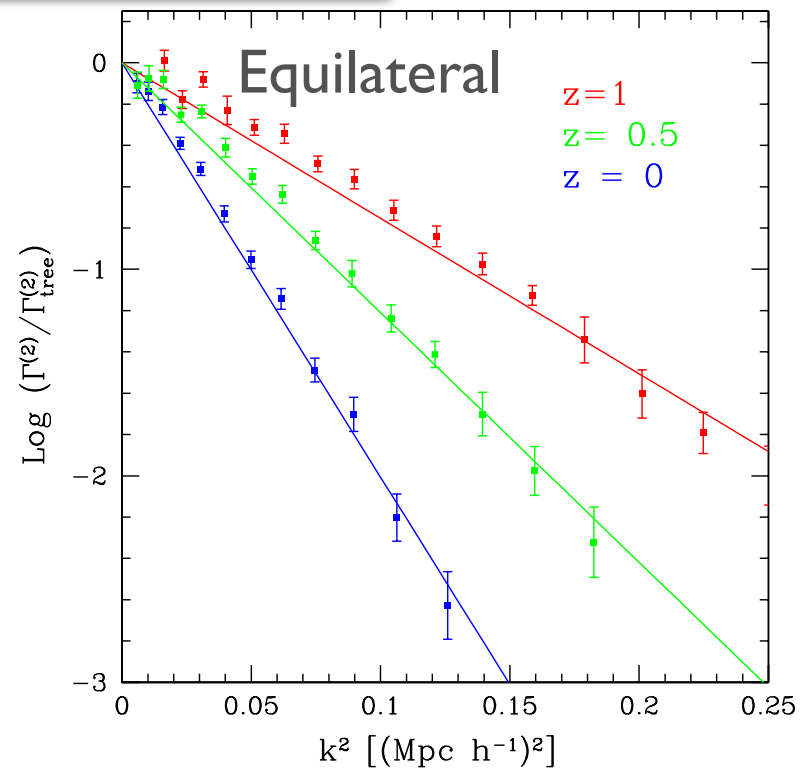
Crocce & Scoccimarro '06, Bernardeau et al. '08

$$\Gamma^{(n)} \xrightarrow{k \rightarrow +\infty} \Gamma_{\text{tree}}^{(n)} e^{-k^2 \sigma_v^2 / 2} \quad ; \quad \sigma_v^2 = \int \frac{dq}{6\pi^2} P_{\theta\theta}(q)$$

$\Gamma^{(1)}(k)$



$\Gamma^{(2)}(k_1, k_2, k_3)$



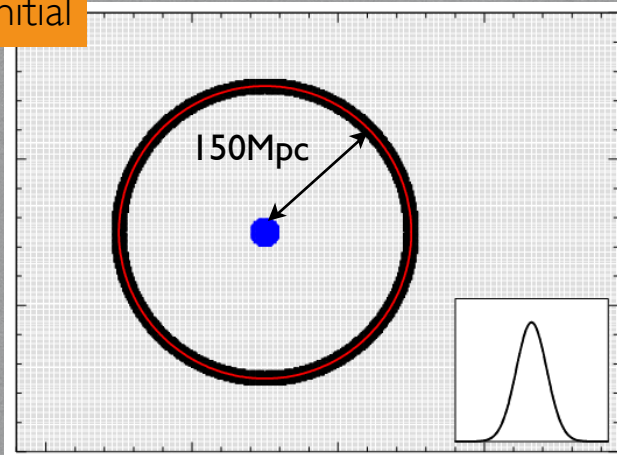
# Origin of Exp. damping

For Gaussian initial condition,

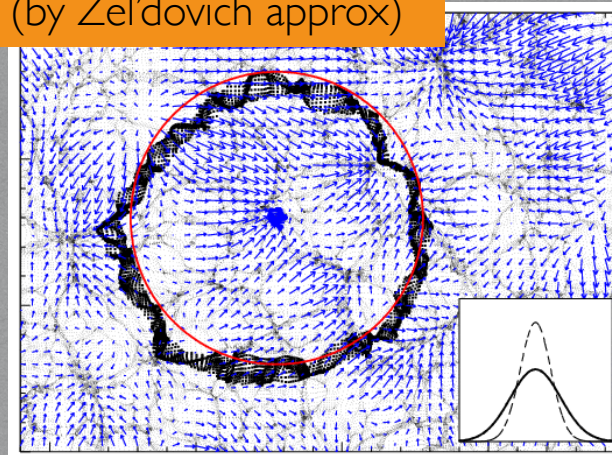
$$\langle \delta_m(\mathbf{k}; t) \delta_0(\mathbf{k}') \rangle = \Gamma^{(1)}(\mathbf{k}; t) \underbrace{\langle \delta_0(\mathbf{k}) \delta_0(\mathbf{k}') \rangle}_{\substack{= P_0(k) \\ \text{initial power spectrum}}}$$

➔ Cross correlation between initial & evolved density fields

initial



evolved (by Zel'dovich approx)



Padmanabhan  
et al. ('12)

Initial structure becomes blurred by the *local* cosmic flow

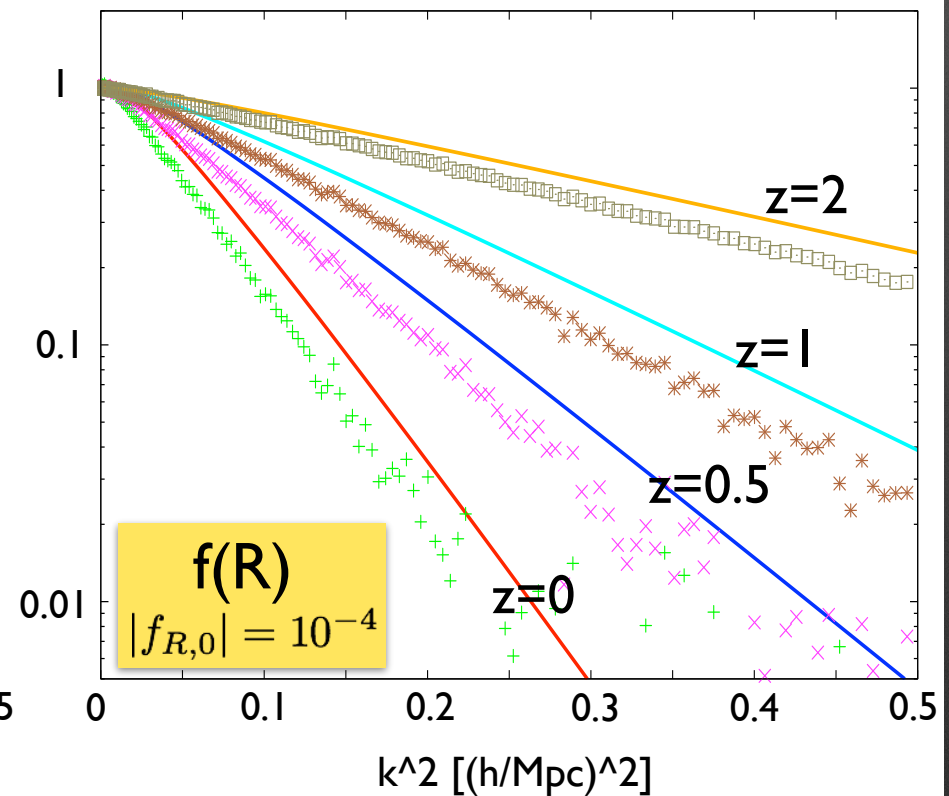
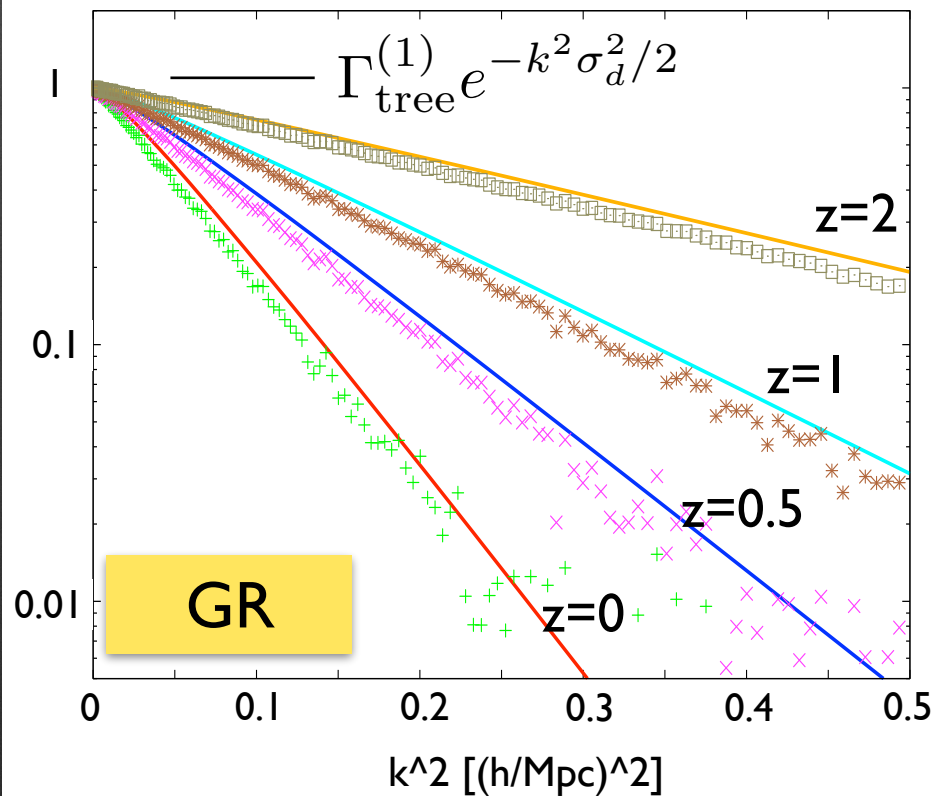
----- origin of Gaussian damping in propagator

# Generic damping behaviors

Exp. damping can appear even in modified gravity models

$$\Gamma^{(1)}(k)$$

Data: Baojiu Li



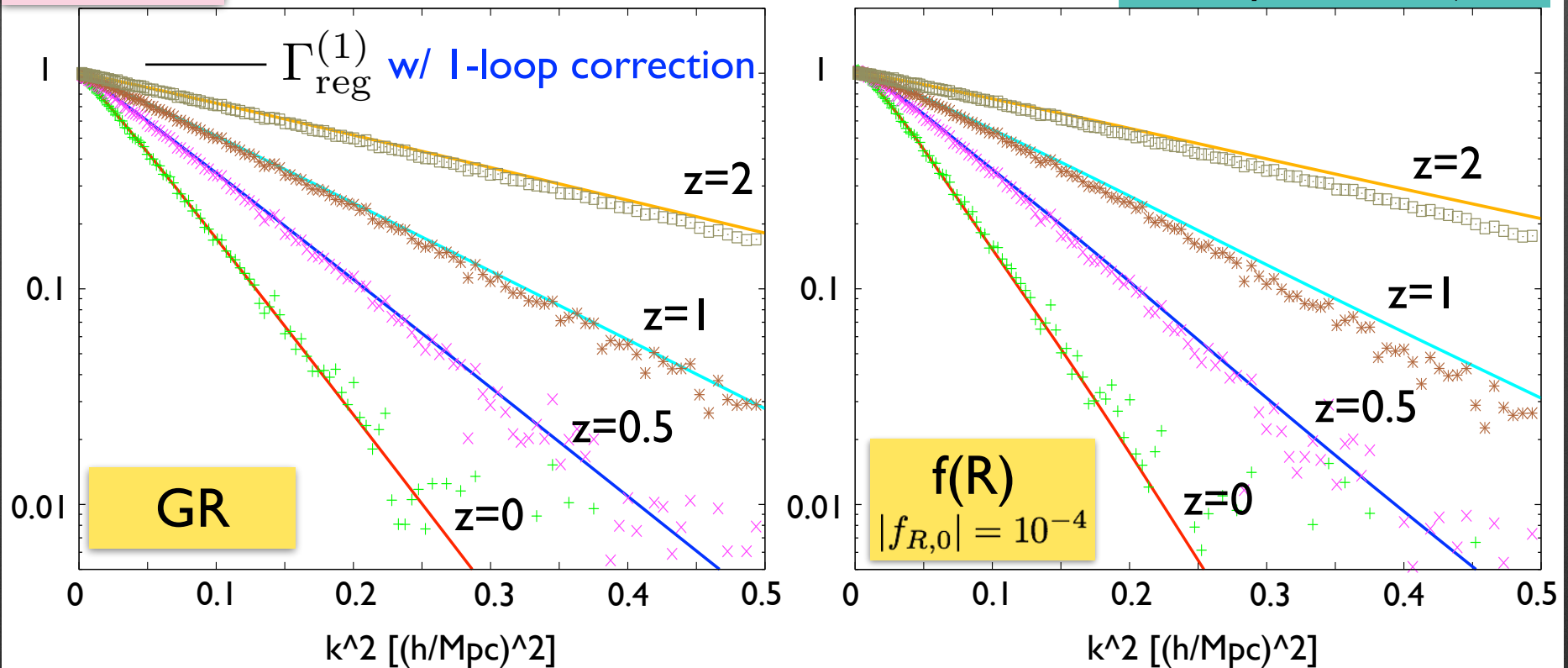
AT, Nishimichi, Hiramatsu, et al. (in prep.)

# Generic damping behavior

Exp. damping generically holds  
not only in GR but also in modified gravity models

$\Gamma^{(1)}(k)$

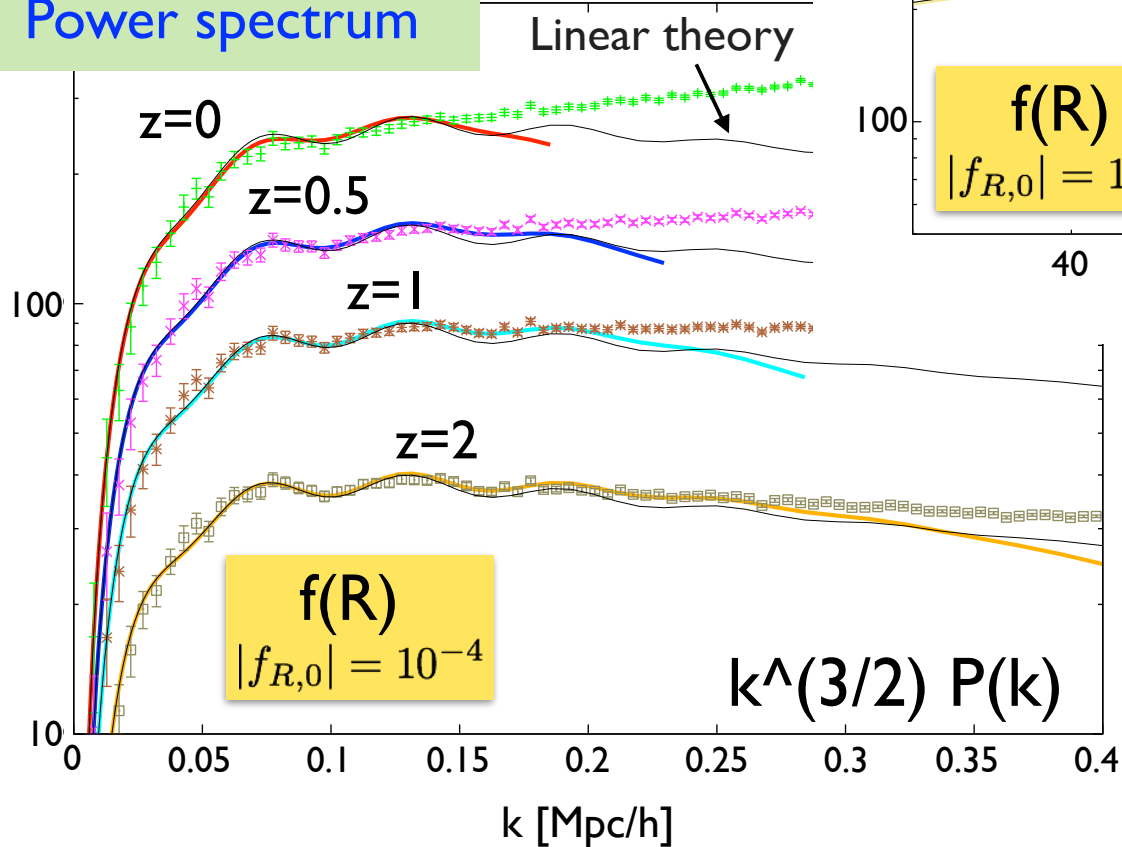
N-body data: Baojiu Li



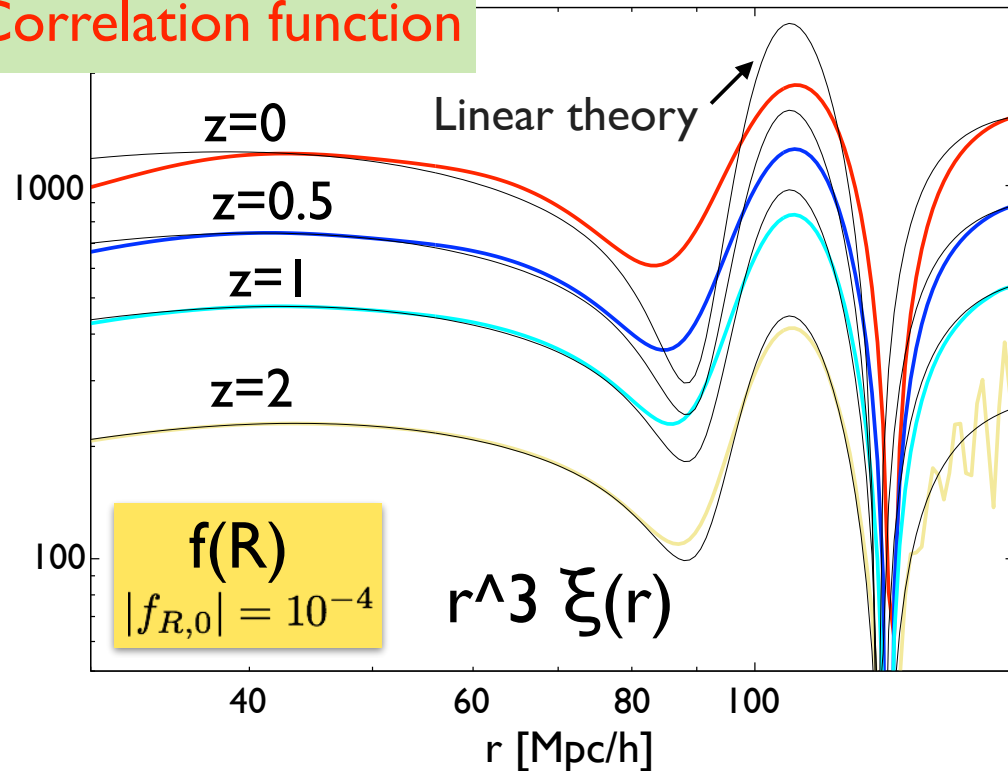
AT, Nishimichi, Hiramatsu, Bernardeau et al. (in prep.)

# PT prediction in $f(R)$ gravity

Power spectrum



Correlation function



based on multi-point  
propagators at  $l$ -loop

# Characterizing halo/galaxy clustering with propagators


Concept of propagator is also useful for halo/galaxy clustering

Matsubara ('11, '12, '13)

## Multi-point propagator of halos

Evolved (non-linear) halo density field

$$\left\langle \frac{\delta^n \delta_{\text{halo}}(\mathbf{k}; t)}{\delta \delta_0(\mathbf{k}_1) \cdots \delta \delta_0(\mathbf{k}_n)} \right\rangle = (2\pi)^{3(1-n)} \delta_{\text{D}}(\mathbf{k} - \mathbf{k}_{12\dots n}) \Gamma_{\text{halo}}^{(n)}(\mathbf{k}_1, \dots, \mathbf{k}_n; t)$$



$$P_{\text{halo}}(k) = \left\{ \Gamma_{\text{halo}}^{(1)}(k) \right\}^2 P_0(k) + 2 \int \frac{d^3 \mathbf{q}}{(2\pi)^3} \left\{ \Gamma_{\text{halo}}^{(2)}(\mathbf{q}, \mathbf{k} - \mathbf{q}) \right\}^2 P_0(q) P_0(|\mathbf{k} - \mathbf{q}|) + \dots$$

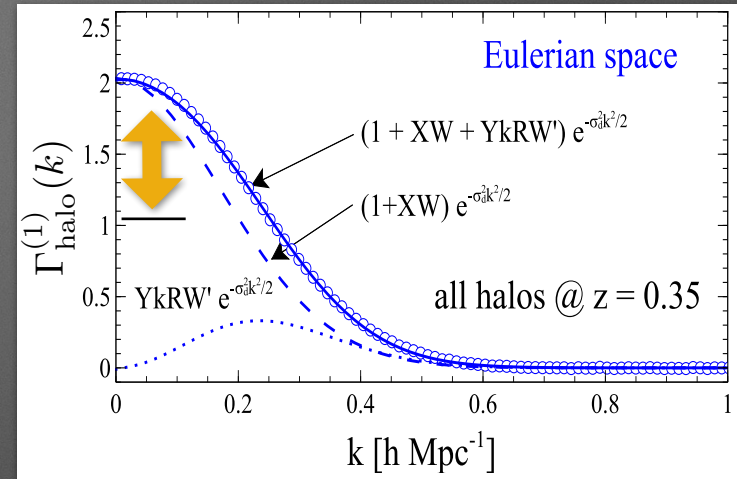
..... integrated PT



# Propagator of halos

$$\Gamma_{\text{halo}}^{(1)} \longrightarrow \{1 + c_1^L(k)\} e^{-k^2 \sigma_d^2 / 2}$$

$c_1^L$  is related to Lagrangian halo bias  $b_1^L$



but more generally, it can be defined as **Lagrangian propagator**:

$$\left\langle \frac{\delta \delta_{\text{halo}}^L(\mathbf{k})}{\delta \delta_0(\mathbf{k}')} \right\rangle = \delta_D(\mathbf{k} - \mathbf{k}') c_1^L(k)$$

halo density field in *Lagrangian space*

It carries information on

*scale-dependent* & *non-local* properties of halo bias

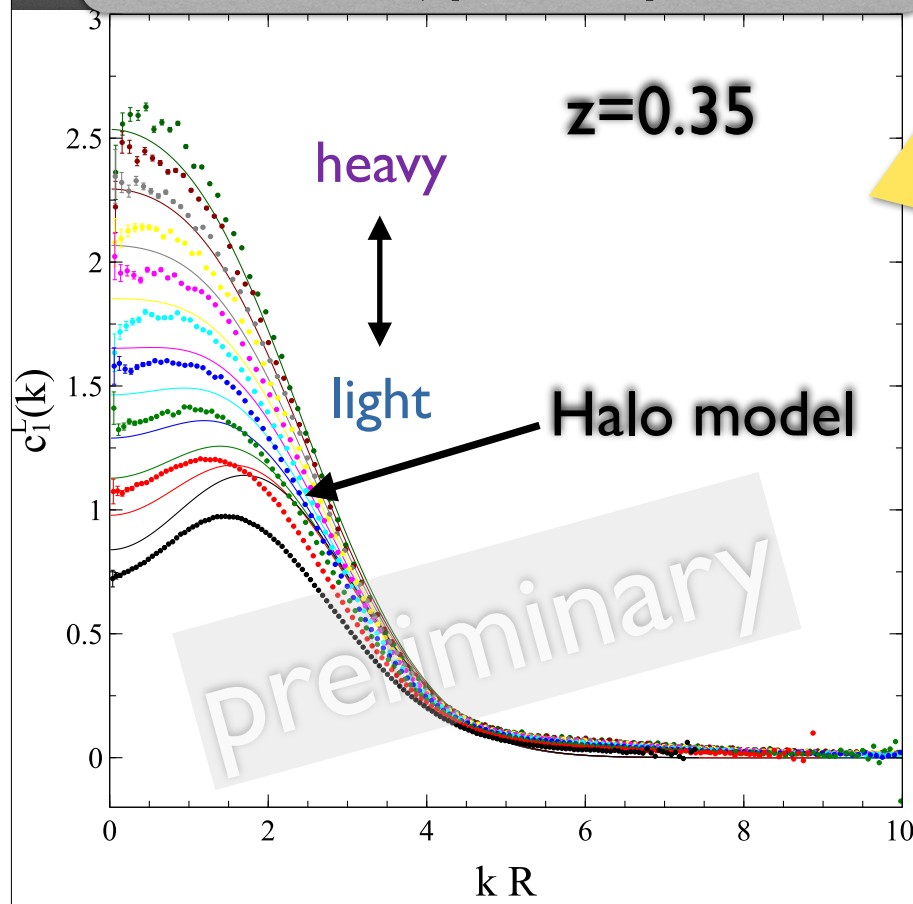
→ directly measurable quantity in N-body simulations

# Lagrangian halo propagator

Nishimichi, Matsubara & AT (in prep.)

10 equal mass bins with mass range

$$2 \times 10^{13} \leq M/[h^{-1} M_{\odot}] \leq 2 \times 10^{14}$$



Lagrangian propagator measured in N-body simulations

Halo model

(Matsubara '12)

$$c_1^L(k) = A(M) W(kR) + B(M) \frac{dW(kR)}{d \ln(kR)}$$

$W(kR)$ : Gaussian filter

halo model qualitatively explains scale-dependent behavior

# Impact on baryon acoustic peak

$$\xi_{\text{halo}}(r) \simeq [(\Gamma_{\text{halo}}^{(1)})^2 \otimes \xi_{\text{lin}}](r) + \dots$$

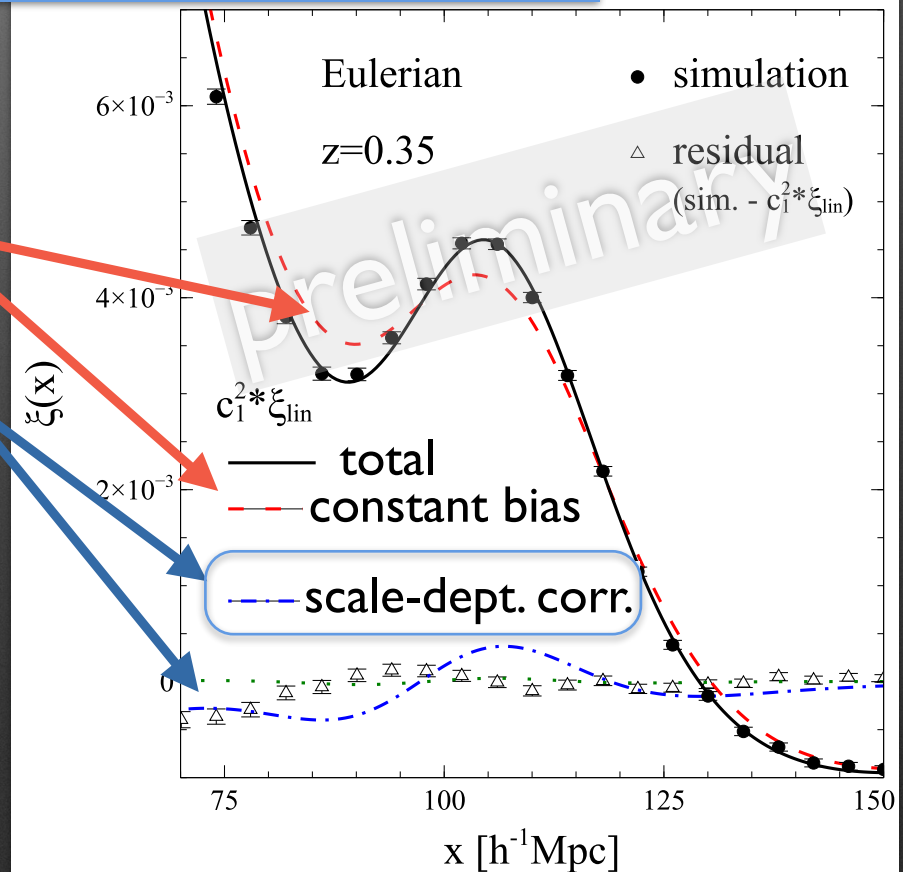
$$\simeq \{1 + c_1^L(k)\} e^{-k^2 \sigma_d^2 / 2}$$

induce *scale-dependent bias* :

- simple constant bias fails
- enhance BAO peak/trough

( c.f. Peak theory by  
Desjacques et al. ('12) )

Nishimichi, Matsubara & AT (in prep.)



# Summary

Cosmic propagators provide a useful way to characterize clustering property of LSS (dark matter/galaxy/halo)

- Dark matter : PT prediction works well not only in GR but also in modified gravity (  $f(R)$  gravity )
- Halo/galaxy : scale-dependent bias around BAO peak

Modeling LSS with PT is still developing, and a further quantitative study on halo/galaxy propagators is worth doing