Dark Energy-modelling and testing for it

Ed Copeland -- Nottingham University

1. Issues with pure Lambda
2. Models of Dark Energy
3. Modified Gravity approaches
4. Testing for Dark Energy
5. Hiding Lambda

CosKASI 2014 - Korea - Apr 18th 2014

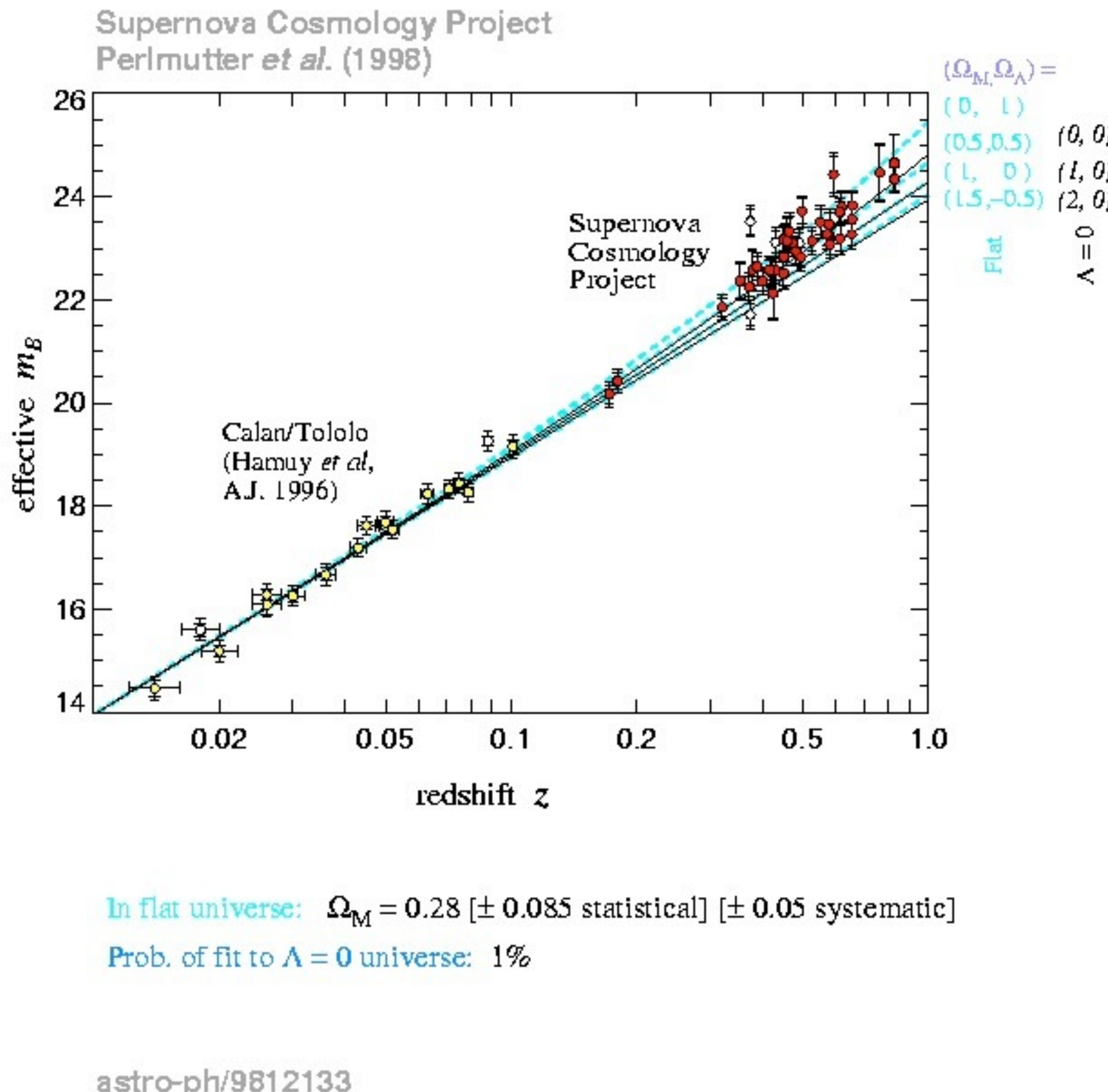
Congratulations to everyone involved with CosKasi

Possible collaborations in Asia:

University of Nottingham has two campuses in Asia

Ningbo campus - China (nr Shanghai)

Malaysian campus



The Universe is accelerating and yet we still really have little idea what is causing this acceleration.

Is it a cosmological constant, an evolving scalar field, evidence of modifications of General Relativity on large scales or something yet to be dreamt up ?

Brief reminder why the cosmological constant is regarded as a problem?

The CC gravitates in General Relativity:

$$\mathcal{L} = \sqrt{-g}\left(\frac{R}{16\pi G} - \rho_{\rm vac}\right)$$

$$G_{\mu\nu} = -8\pi G \rho_{\rm vac} g_{\mu\nu}$$

Now: $\rho_{\rm vac}^{\rm obs} \ll \rho_{\rm vac}^{\rm theory}$

Just as well because anything much bigger than we have and the universe would have looked a lot different to what it does look like. In fact structures would not have formed in it.

Estimate what the vacuum energy should be :

$$\rho_{\text{vac}}^{\text{theory}} \sim \rho_{\text{vac}}^{\text{bare}}$$

$$+$$

zero point energies of each particle

$$+$$

contributions from phase transitions in the early universe

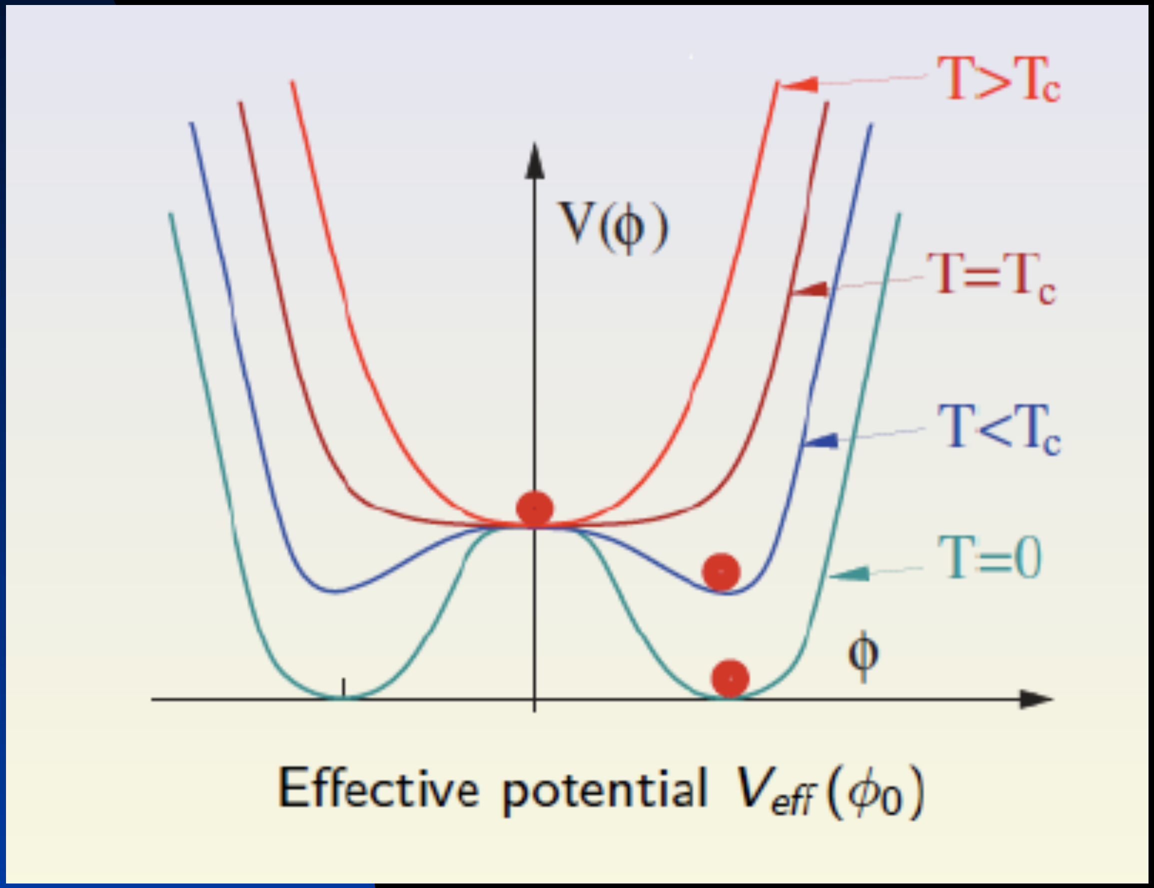
zero point energies of each particle

For many fields (i.e. leptons, quarks, gauge fields etc...):

$$< \rho > = \frac{1}{2} \sum_{\text{fields}} g_i \int_0^{\Lambda_i} \sqrt{k^2 + m^2} \frac{d^3k}{(2\pi)^3} \simeq \sum_{\text{fields}} \frac{g_i \Lambda_i^4}{16\pi^2}$$

where g_i are the dof of the field (+ for bosons, - for fermions).

contributions from phase transitions in the early universe



$$\Delta V_{\rm ewk} \sim (200\ {\rm GeV})^4$$

$$\Delta V_{\rm QCD} \sim (0.3\ {\rm GeV})^4$$

Quantum Gravity cut-off	$-(10^{18}\ \mathrm{GeV})^4$	fine tuning to 120 decimal places
SUSY cut-off	$-(\mathrm{TeV})^4$	fine tuning to 60 decimal places
EWK phase transition	$-(200\mathrm{GeV})^4$	fine tuning to 56 decimal places
QCD phase transition	$-(0.3\mathrm{GeV})^4$	fine tuning to 44 decimal places
Muon	$-(100\mathrm{MeV})^4$	
electron	$-(1\ \mathrm{MeV})^4$	fine tuning to 36 decimal places
	$-(\mathrm{meV})^4$	Observed value of the effective cosmological constant today !

Friedmann: $$H^2 \equiv \frac{\dot{a}^2}{a^2} = \frac{8\pi}{3} G\rho - \frac{k}{a^2} + \frac{\Lambda}{3}$$

a(t) depends on matter.

Energy density $\rho(t)$: Pressure $p(t)$

Related through : $p = w\rho$

w=1/3 – Rad dom: w=0 – Mat dom: w=-1– Vac dom

$$w(a) \equiv \frac{P}{\rho} = w_0 + (1-a)w_a$$ Typical parameterisation

$$\mathbf{H^2(z) = H_0^2\left(\Omega_r(1+z)^4 + \Omega_m(1+z)^3 + \Omega_k(1+z)^2 + \Omega_{\mathrm{de}}\exp\left(3\int_0^z \frac{1+w(z')}{1+z'}dz'\right)\right)}$$

Dark Energy

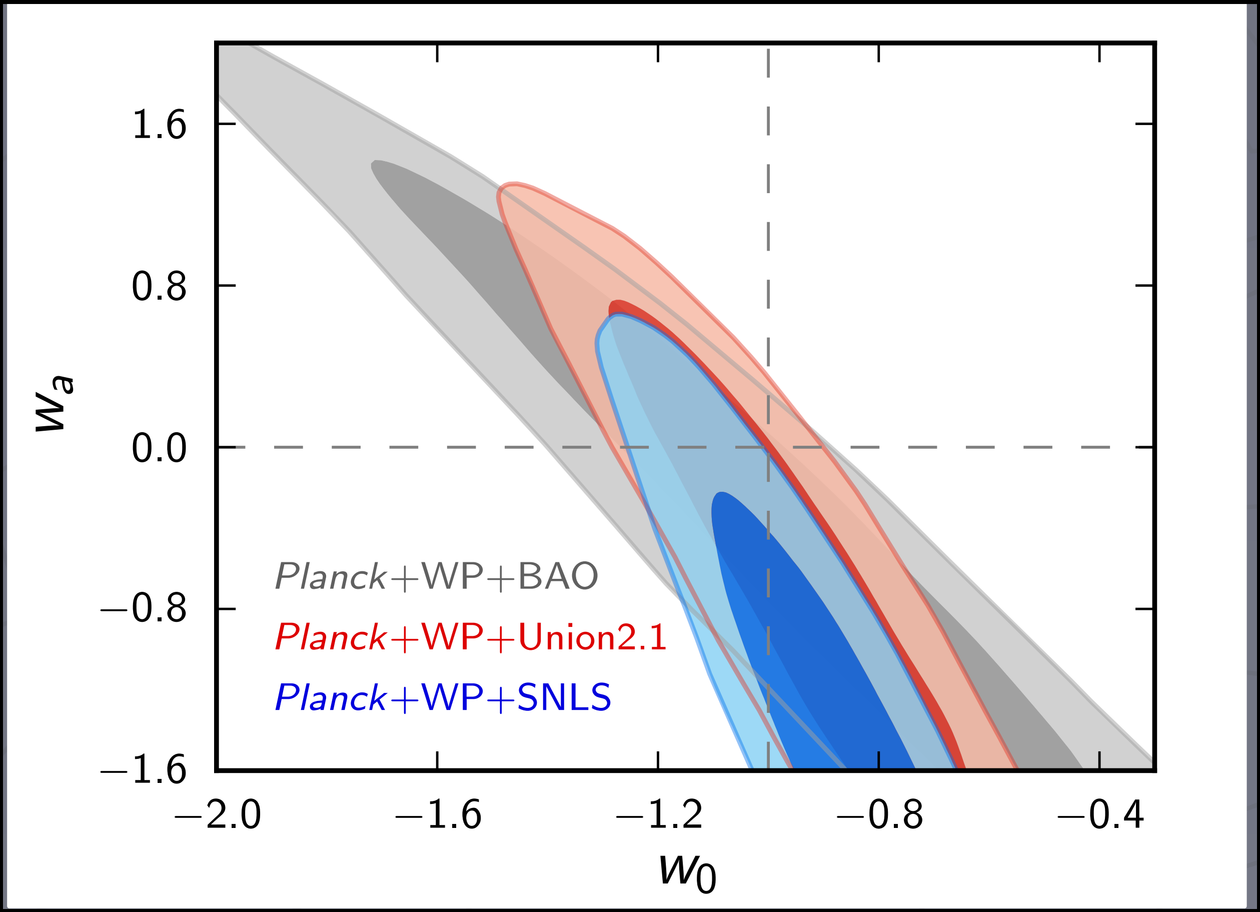
Parameterise eos: $$w(a) \equiv \frac{p}{\rho} = w_0 + (1-a)w_a$$

Planck alone weak constraints on DE because of degeneracy of w with H_0:

Break with other probes including lensing, SN, BAO ...

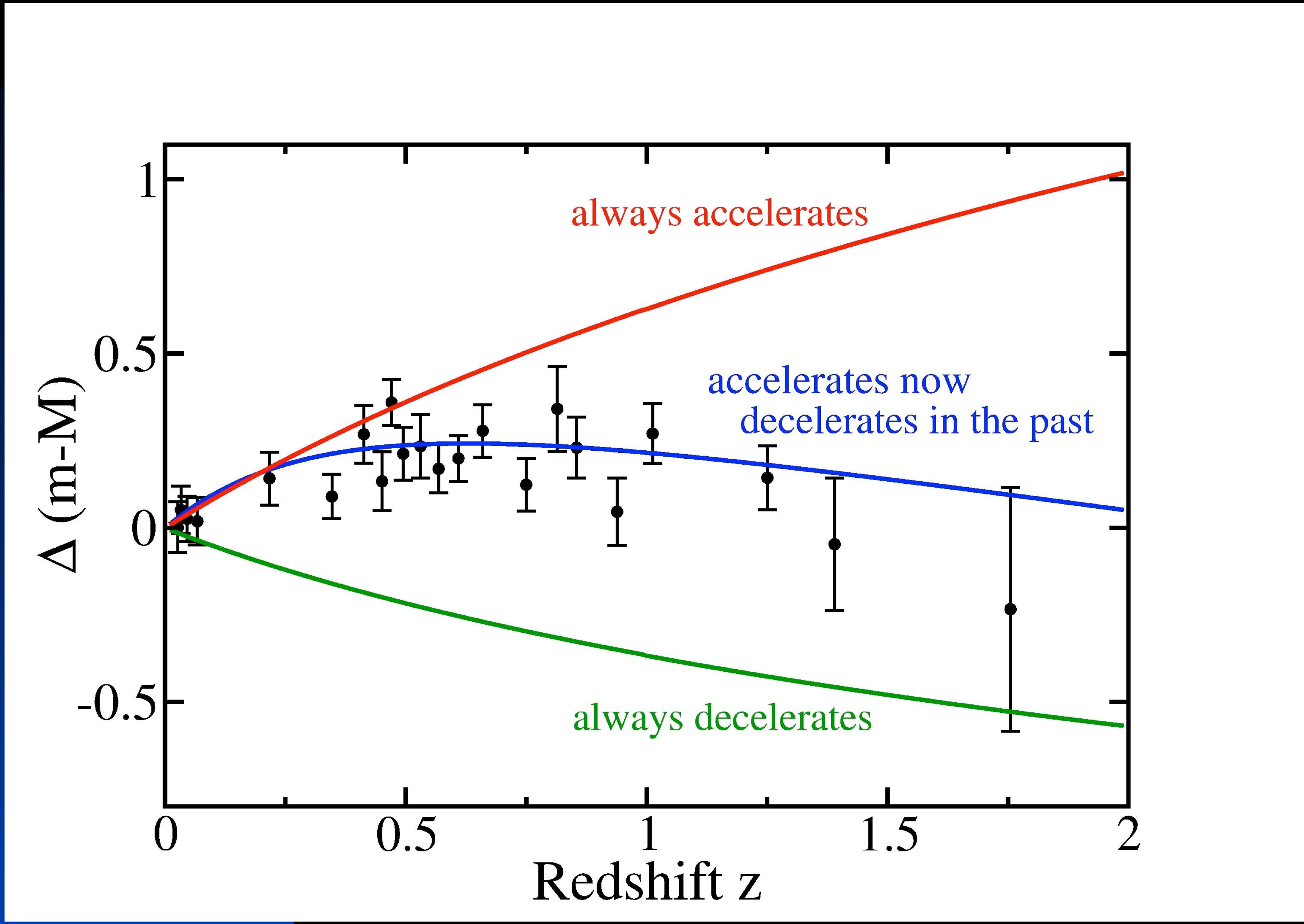
Example - if assume $w_a = 0$

$$w = -1.13 \pm 0.24 \quad (95\%, \textit{Planck} + \text{WP} + \text{BAO})$$
$$w = -1.09 \pm 0.17 \quad (95\%, \textit{Planck} + \text{WP} + \text{Union2.1})$$
$$w = -1.13^{+0.13}_{-0.14} \quad (95\%, \textit{Planck} + \text{WP} + \text{SNLS}),$$
$$w = -1.24^{+0.18}_{-0.19} \quad (95\%, \textit{Planck} + \text{WP} + \text{HST}).$$



How should we parameterise w?

The acceleration has not been forever -- pinning down the turnover will provide a very useful piece of information.



Huterer 2010

Help address cosmic coincidence problem ! A region hopefully DESI will be able to capitalise on as emphasised by George Smoot

Approaches to Dark Energy:

- A true cosmological constant -- but why this value?
- Time dependent solutions arising out of evolving scalar fields -- Quintessence/K-essence.
- Modifications of Einstein gravity leading to acceleration today.
- Anthropic arguments.
- Perhaps GR but Universe is inhomogeneous.
- Hiding the cosmological constant -- its there all the time but just doesn’t gravitate
- Yet to be proposed ...

String - theory -- where are the realistic models?

`No go' theorem: forbids cosmic acceleration in cosmological solutions arising from compactification of pure SUGR models where internal space is time-independent, non-singular compact manifold without boundary --[Gibbons]

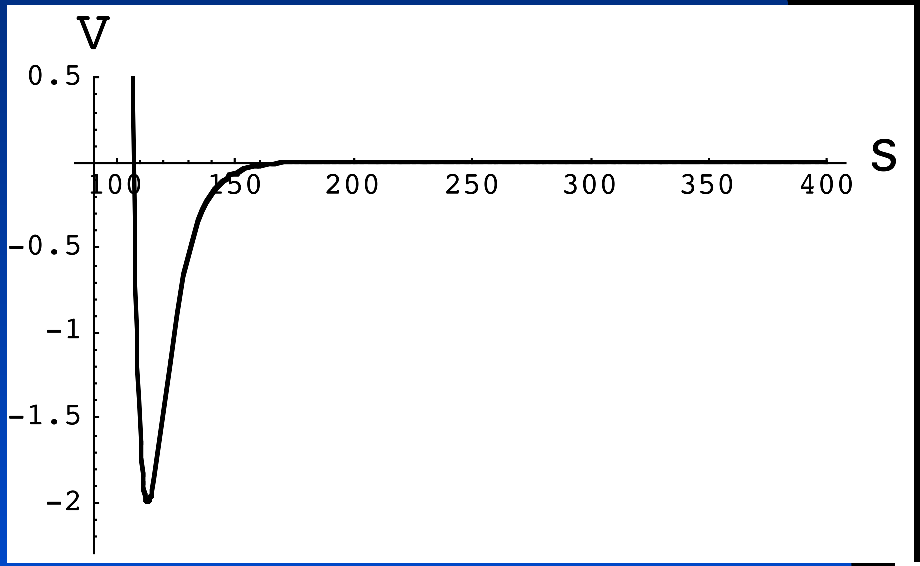
Avoid no-go theorem by relaxing conditions of the theorem.

1. Allow internal space to be time-dependent scalar fields (radion)

2. Brane world set up require uplifting terms to achieve de Sitter vacua hence accn

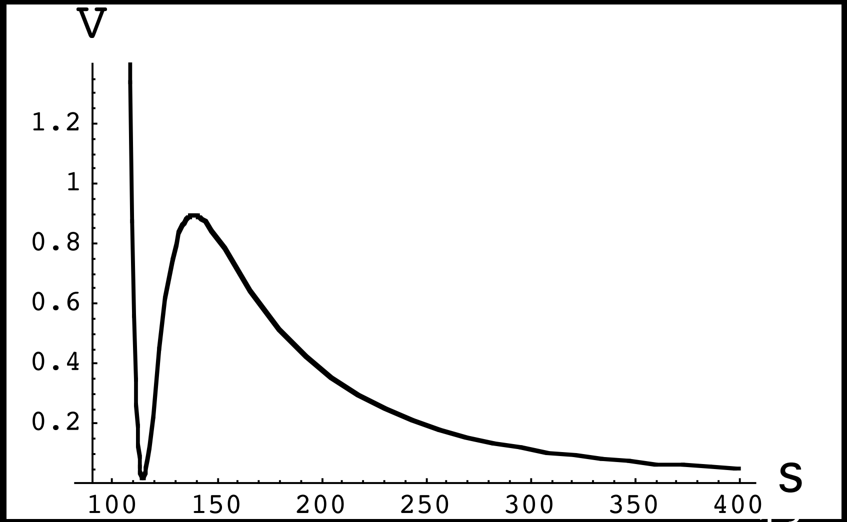
Example of stabilised scenario: Metastable de Sitter string vacua in TypeIIB string theory, based on stable highly warped IIB compactifications with NS and RR three-form fluxes. [Kachru, Kallosh, Linde and Trivedi 2003]

Metastable minima arises from adding positive energy of anti-D3 brane in warped Calabi-Yau space.



AdS minimum

$$V_{\mathrm{KKLT}} = V_{\mathrm{AdS}} + \frac{D}{\sigma^2}$$



Metastable dS minimum

The String Landscape approach

Type IIB String theory compactified from 10 dimensions to 4.

Internal dimensions stabilised by fluxes. Assumes natural AdS vacuum uplifted to de Sitter vacuum through additional fluxes !

Many many vacua ~ 10^{500} !

Typical separation ~ $10^{-500}\ \Lambda_{pl}$

Assume randomly distributed, tunnelling allowed between vacua --> separate universes .

Anthropic : Galaxies require vacua < $10^{-118}\ \Lambda_{pl}$ [Weinberg] Most likely to find values not equal to zero!

[Witten 2008]

Landscape gives a realisation of the multiverse picture.

There isn't one true vacuum but many so that makes it almost impossible to find our vacuum in such a Universe which is really a multiverse.

So how can we hope to understand or predict why we have our particular particle content and couplings when there are so many choices in different parts of the universe, none of them special ?

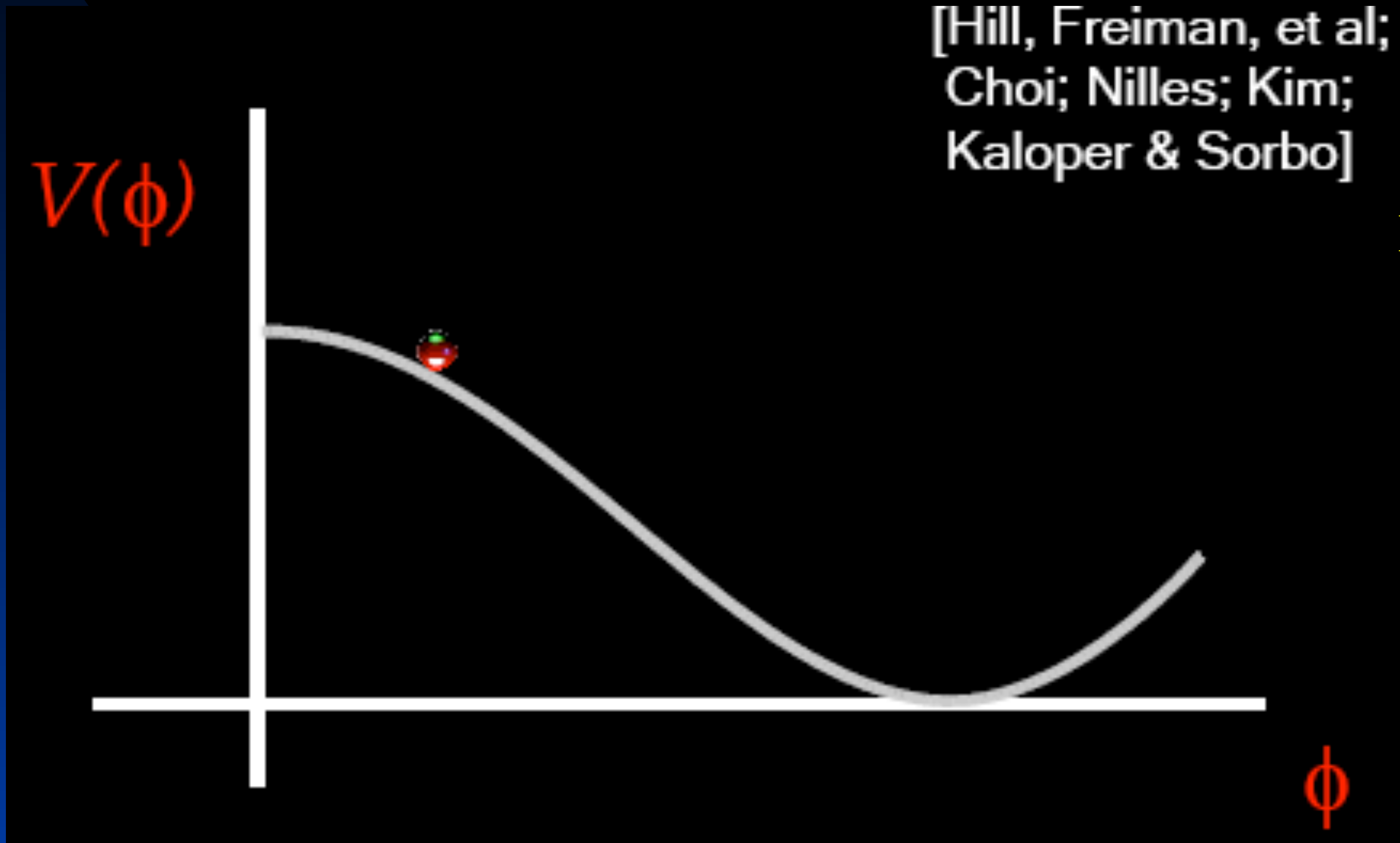
This sounds like bad news, we will rely on anthropic arguments to explain it through introducing the correct measures and establishing peaks in probability distributions.

Or perhaps, it isn't a cosmological constant, but a new field such as Quintessence which will eventually drive us to a unique vacuum with zero vacuum energy -- that too has problems, such as fifth force constraints, as we will see.

Particle physics inspired models?

Pseudo-Goldstone Bosons -- approx sym $\phi \to \phi$ + const.

Leads to naturally small masses, naturally small couplings



Barbieri et al

$$V(\phi) = \lambda^4 (1 + \cos(\phi/F_a))$$

Axions could be useful for strong CP problem, dark matter and dark energy.

Axions could be useful for strong CP problem, dark matter and dark energy.

Strong CP problem intro axion : $m_a = \frac{\Lambda^2_{\rm QCD}}{F_a};\ F_a\ -\ \text{decay constant}$

PQ axion ruled out but invisible axion still allowed: $10^9\ \text{GeV} \leq F_a \leq 10^{12}\ \text{GeV}$

Sun stability CDM constraint

String theory has lots of antisymmetric tensor fields in 10d, hence many light axion candidates.

Can have $F_a \sim 10^{17}$-10^{18} GeV

Quintessential axion -- dark energy candidate [Kim & Nilles].

Requires $F_a \sim 10^{18}$ GeV which can give:

$$E_{\rm vac} = (10^{-3}\ \text{eV})^4 \rightarrow m_{\rm axion} \sim 10^{-33}\ \text{eV}$$

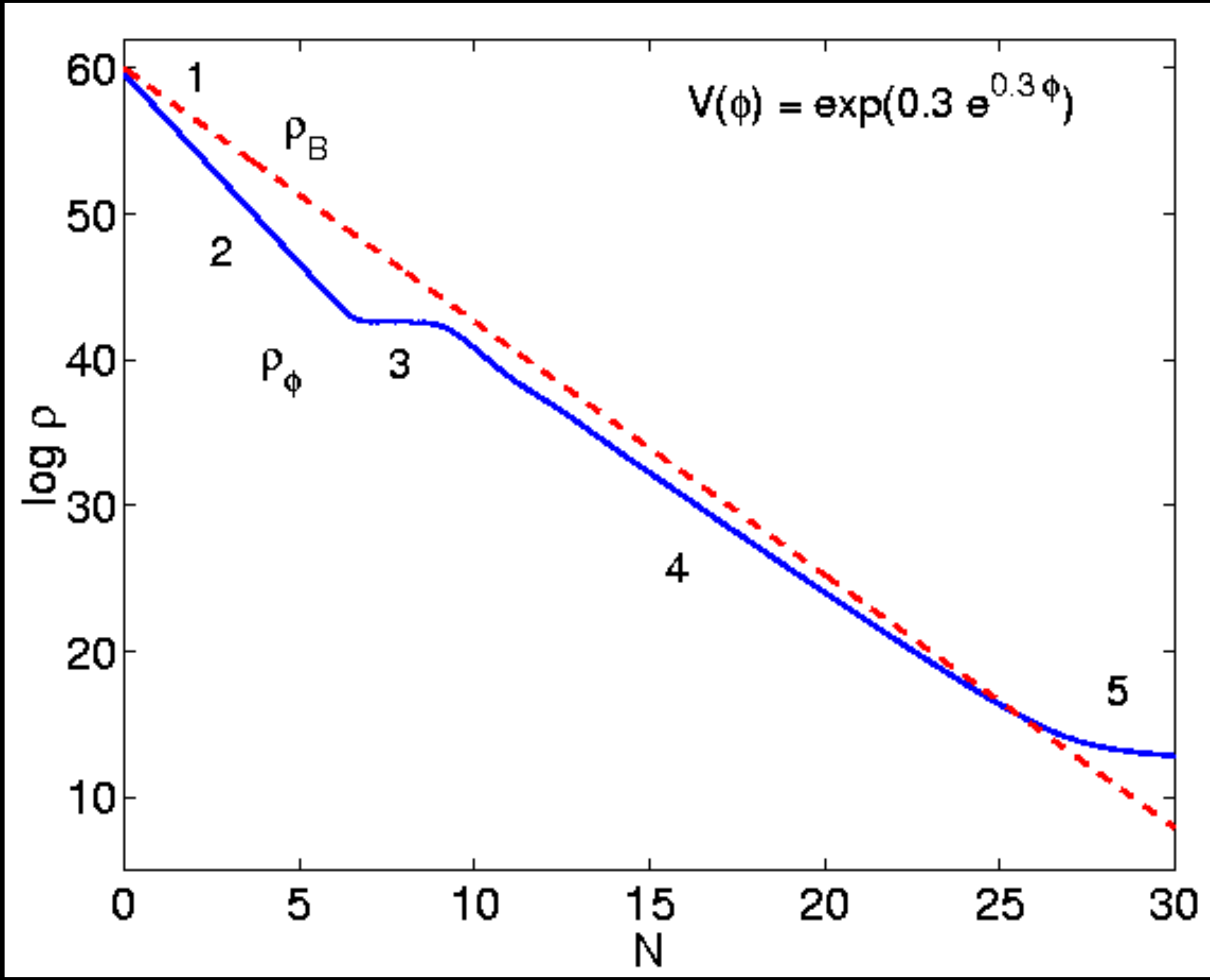
Because axion is pseudoscalar -- mass is protected, hence avoids fifth force constraints

Slowly rolling scalar fields -- Quintessence

As of 14 Mar 2013, can really use this language !

Peebles and Ratra; Wetterich; Ferreira and Joyce

Zlatev, Wang and Steinhardt



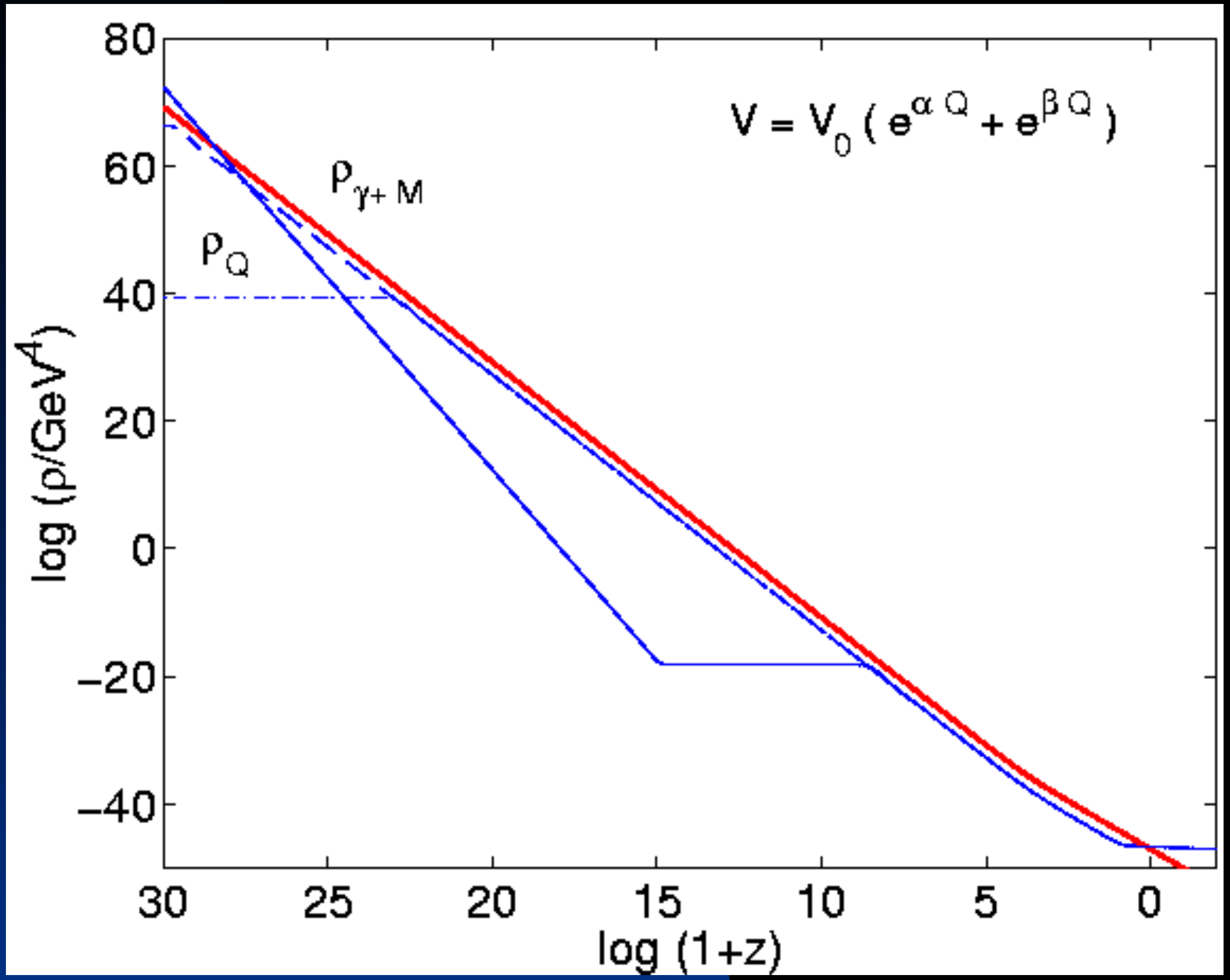
Dashed line - radiation and matter

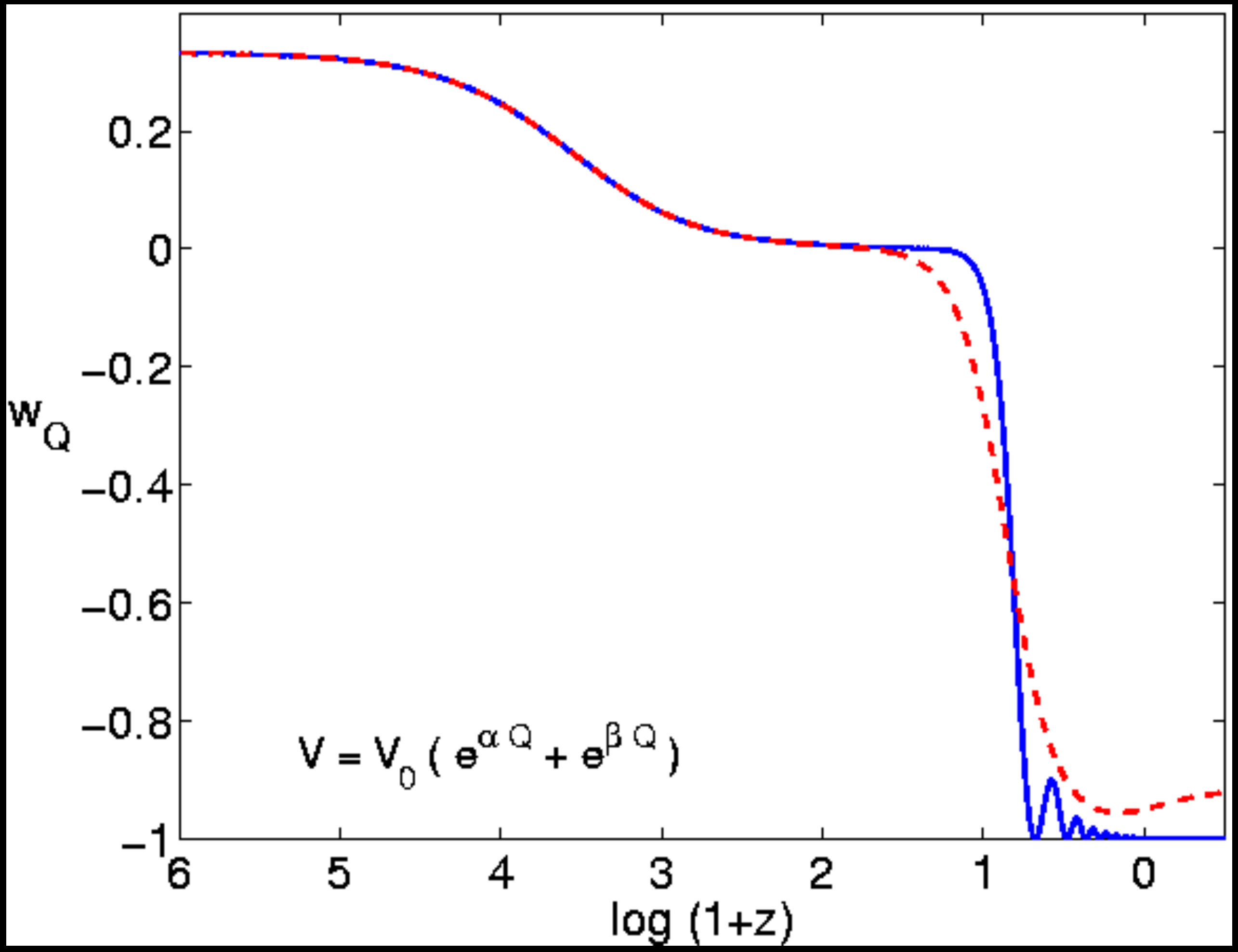
Solid line - Quintessence enters tracking regime (4) and dominates (5)

Nunes

Attractors make initial conditions less important

$$V(\phi) = V_1 + V_2$$
$$= V_{01} e^{-\kappa\lambda_1\phi} + V_{02} e^{-\kappa\lambda_2\phi}$$





$\alpha = 20; \beta = 0.5$

Scaling for wide range of i.c.

Fine tuning: $V_0 \approx \rho_\phi \approx 10^{-47}\ \mathrm{GeV}^4 \approx (10^{-3}\ \mathrm{eV})^4$

Mass: $m \approx \sqrt{\frac{V_0}{M_{pl}^2}} \approx 10^{-33}\ \mathrm{eV}$

Generic issue Fifth force - require screening mechanism!

1. Chameleon fields [Khoury and Weltman (2003) ...]

Non-minimal coupling of scalar to matter in order to avoid fifth force type constraints on Quintessence models: the effective mass of the field depends on the local matter density, so it is massive in high density regions and light (m~H) in low density regions (cosmological scales).

2. K-essence [Armendariz-Picon et al ...]

Scalar fields with non-canonical kinetic terms. Includes models with derivative self-couplings which become important in vicinity of massive sources. The strong coupling boosts the kinetic terms so after canonical normalisation the coupling of fluctuations to matter is weakened -- screening via Vainshtein mechanism

Similar fine tuning to Quintessence -- vital in brane-world modifications of gravity, massive gravity, degravitation models, DBI model, Gallileons, ...

3. Symmetron fields [Hinterbichler and Khoury 2010 ...]

vev of scalar field depends on local mass density: vev large in low density regions and small in high density regions. Also coupling of scalar to matter is prop to vev, so couples with grav strength in low density regions but decoupled and screened in high density regions.

4. Interacting Dark Energy

[Kodama & Sasaki (1985), Wetterich (1995), Amendola (2000) + many others…]

Ex: Including neutrinos -- 2 distinct DM families -- resolve coincidence problem

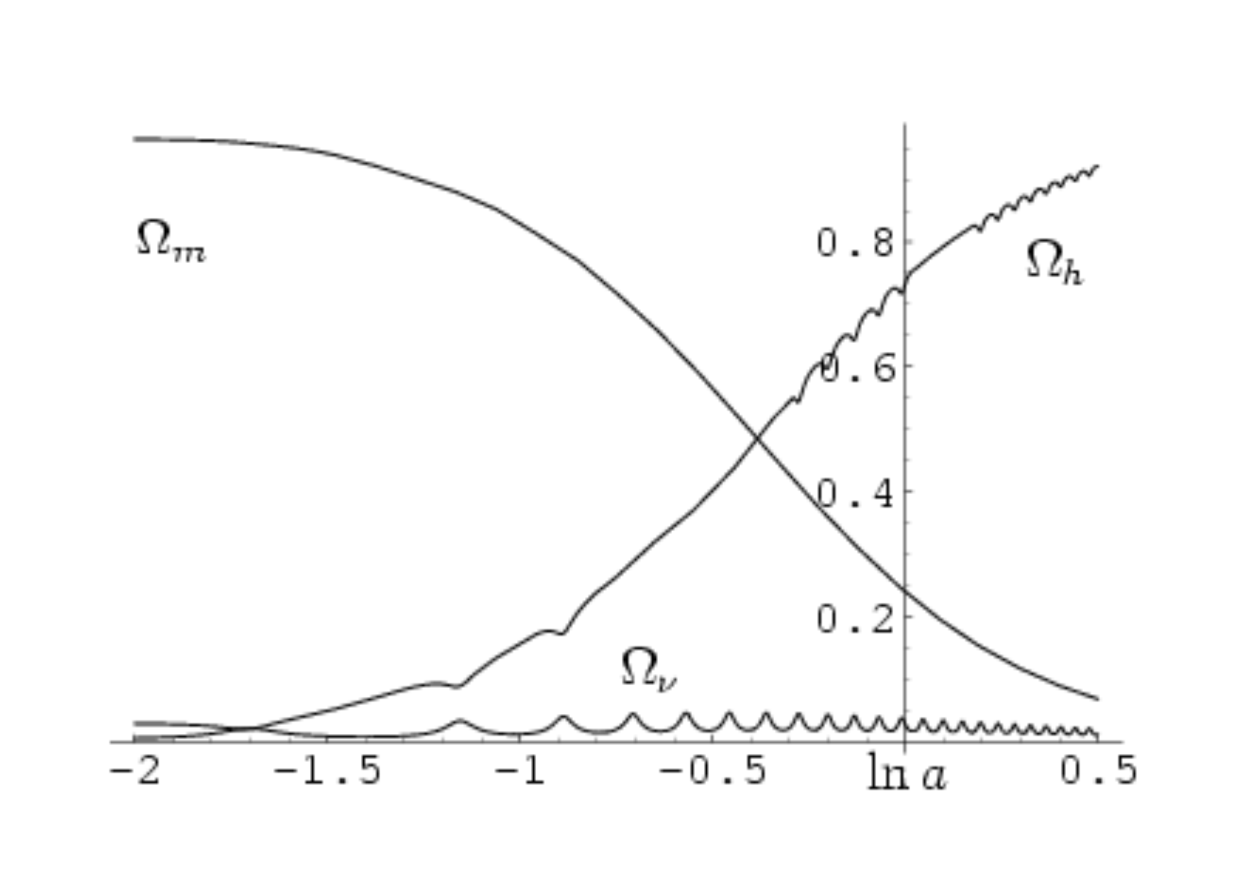
Amendola et al (2007)

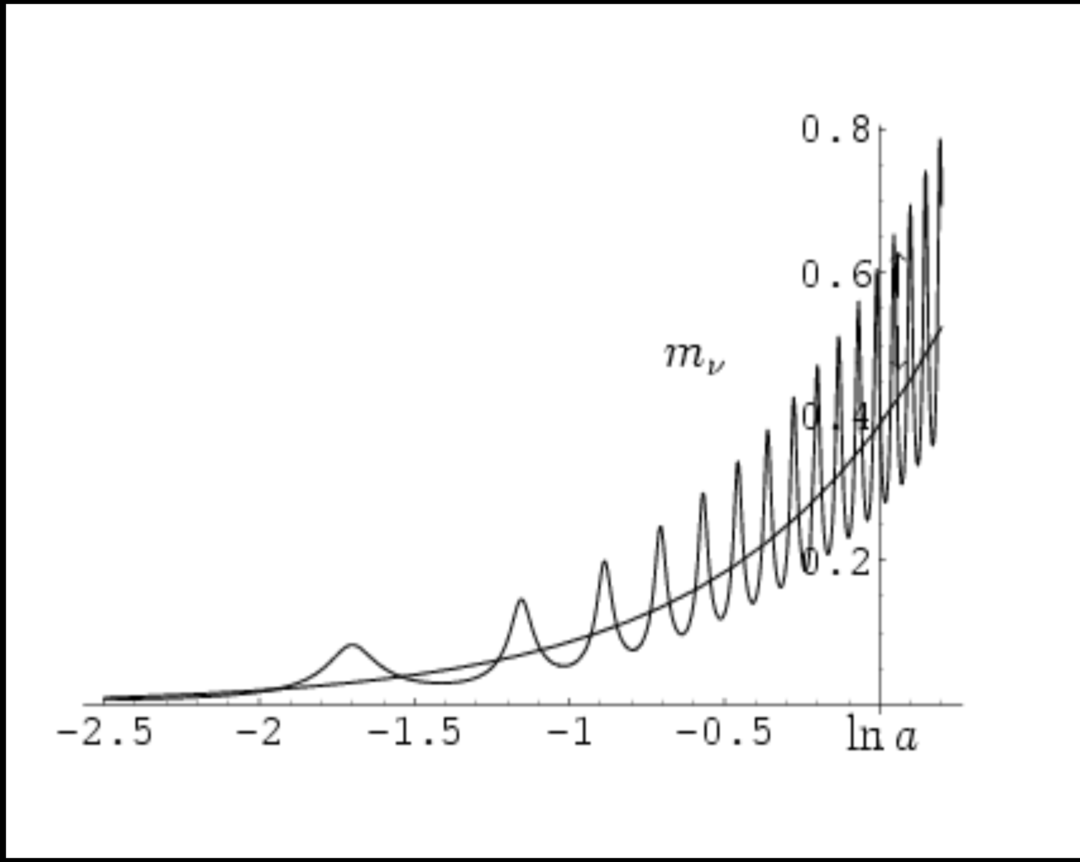
Depending on the coupling, find that the neutrino mass grows at late times and this triggers a transition to almost static dark energy.

Trigger scale set by time when neutrinos become non-rel

$$[\rho_h(t_0)]^{\frac{1}{4}} = 1.07 \left(\frac{\gamma m_\nu(t_0)}{eV}\right)^{\frac{1}{4}} 10^{-3} eV$$

$$w_0 \approx -1 + \frac{m_\nu(t_0)}{12\mathrm{eV}}$$





m_ν

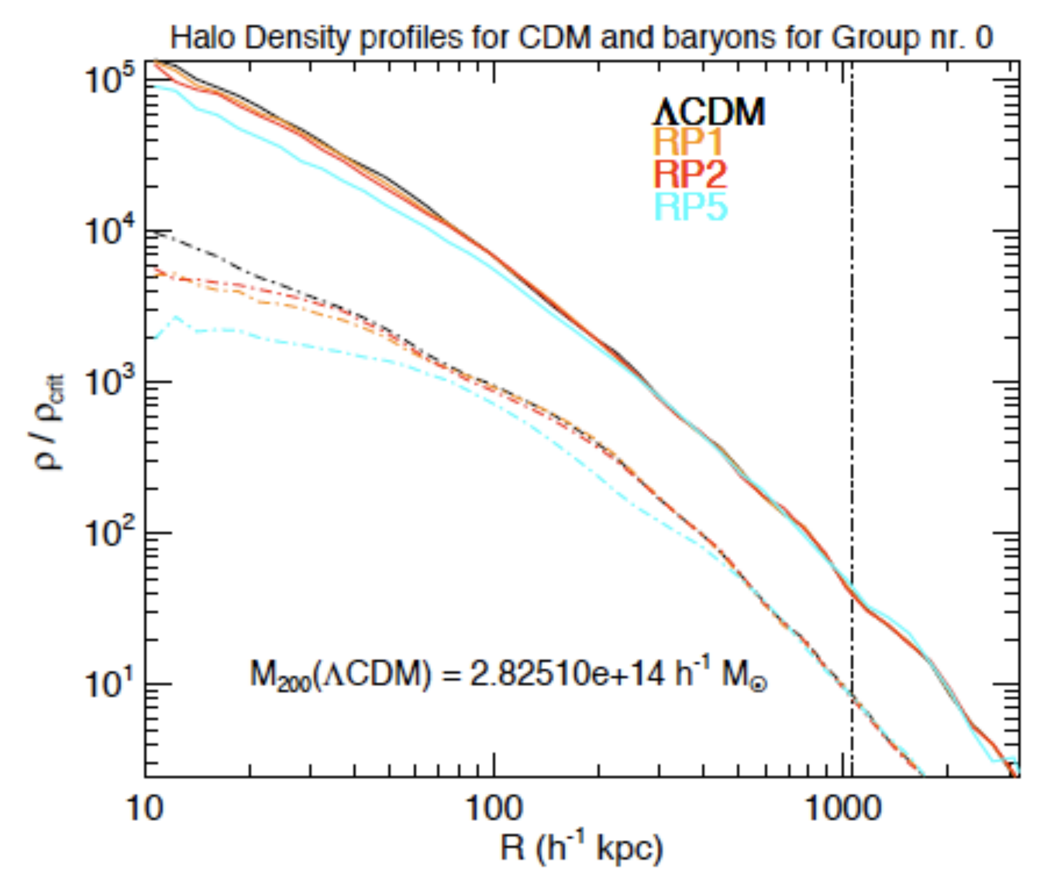
Perturbations in Interacting Dark Energy Models [Baldi et al (2008), Tarrant et al (2010)]

Perturb everything linearly : Matter fluid example

$$\ddot{\delta}_c + \left(2H - 2\beta\frac{\dot{\phi}}{M}\right)\dot{\delta}_c - \frac{3}{2}H^2[(1+2\beta^2)\Omega_c\delta_c + \Omega_b\delta_b] = 0$$

extra friction | **modified grav interaction** | **vary DM particle mass**

Include in simulations of structure formation : GADGET [Springel (2005)]



Density decreases as coupling β increases

Halo mass function modified.

Halos remain well fit by NFW profile.

Density decreases compared to ΛCDM as coupling β increases.

Scale dep bias develops from fifth force acting between CDM particles. enhanced as go from linear to smaller non-linear scales.

Still early days -- but this is where I think there should be a great deal of development (Puchwein et al 2013) -- see Jounghun Lee talk today

Dark Energy Effects

Interactions with standard model particles inevitable even if indirect.

Light scalar fields that interact with std model fields mediate fifth forces

but we dont see any long range fifth forces on earth or in the solar system.

Screening !

Dark energy changes the way photons propagate through B fields. The polarised photon can fluctuate into a DE scalar particle leading to a modification of apparent polarisation and luminosity of the sources.

Two tests [Burgess, Davis, Shaw 2008,2009]

Look for evidence of DE through changes in the scatter of luminosities of high energy sources.

Look for evidence of correlation between poln and freq of starlight .

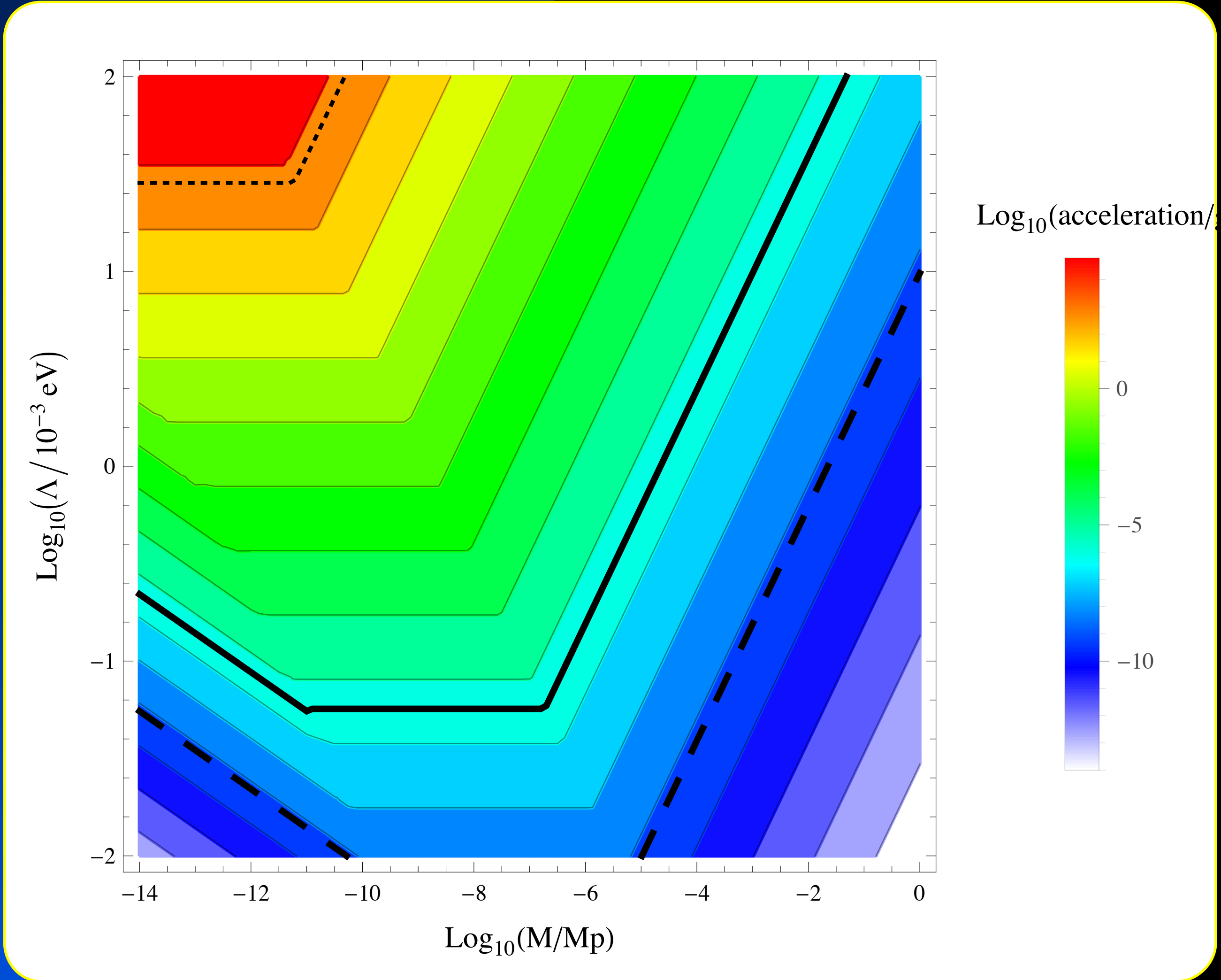
Dark Energy Direct Detection Experiment [Burrage, EC, Hinds]

Atom Interferometry

Idea: Individual atoms in a high vacuum chamber are too small to screen the chameleon field and so are very sensitive to it - can detect it with high sensitivity. Can use atom interferometry to measure the chameleon force - or more likely constrain the parameters !

$$\nabla^2\phi = -\frac{\Lambda^2}{\phi^2} + \frac{\rho}{M}$$

$$F_r = \frac{GM_AM_B}{r^2}\left[1 + 2\lambda_A\lambda_B\left(\frac{M_P}{M}\right)^2\right]$$



$$\lambda_i = 1 \text{ for } \rho_i R_i^2 < 3M\phi_{bg}$$

$$\lambda_i = \frac{3M\phi_{bg}}{\rho_i R_i^2} \text{ for } \rho_i R_i^2 > 3M\phi_{bg}$$

Sph source A and test object B near middle of chamber experience force between them - usually ƛ<<1 in cosmology but for atom ƛ=1 - reduced suppression

Modifying Gravity rather than looking for Dark Energy - non trivial

Any theory deviating from GR must do so at late times yet remain consistent with Solar System tests. Potential examples include:

- f(R), f(G) gravity -- coupled to higher curv terms, changes the dynamical eqns for the spacetime metric. [Starobinski 1980, Carroll et al 2003, ...]

- Modified source gravity -- gravity depends on nonlinear function of the energy.

- Gravity based on the existence of extra dimensions -- DGP gravity

We live on a brane in an infinite extra dimension. Gravity is stronger in the bulk, and therefore wants to stick close to the brane -- looks locally four-dimensional.

Tightly constrained -- both from theory [ghosts] and observations

- Scalar-tensor theories including higher order scalar-tensor lagrangians -- recent examples being Galileon models

- Massive gravity - single massive graviton bounds m>O(1meV) from demand perturbative down to O(1)mm - too large to conform with GR at large distances

[Burrage et al 2013]

Designer f (R) or f(G) models [Hu and Sawicki (2007), ...]

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{R+f(R)}{2\kappa^2} + \mathcal{L}_{\mathrm{m}}\right]$$

Construct a model to satisfy observational requirements:

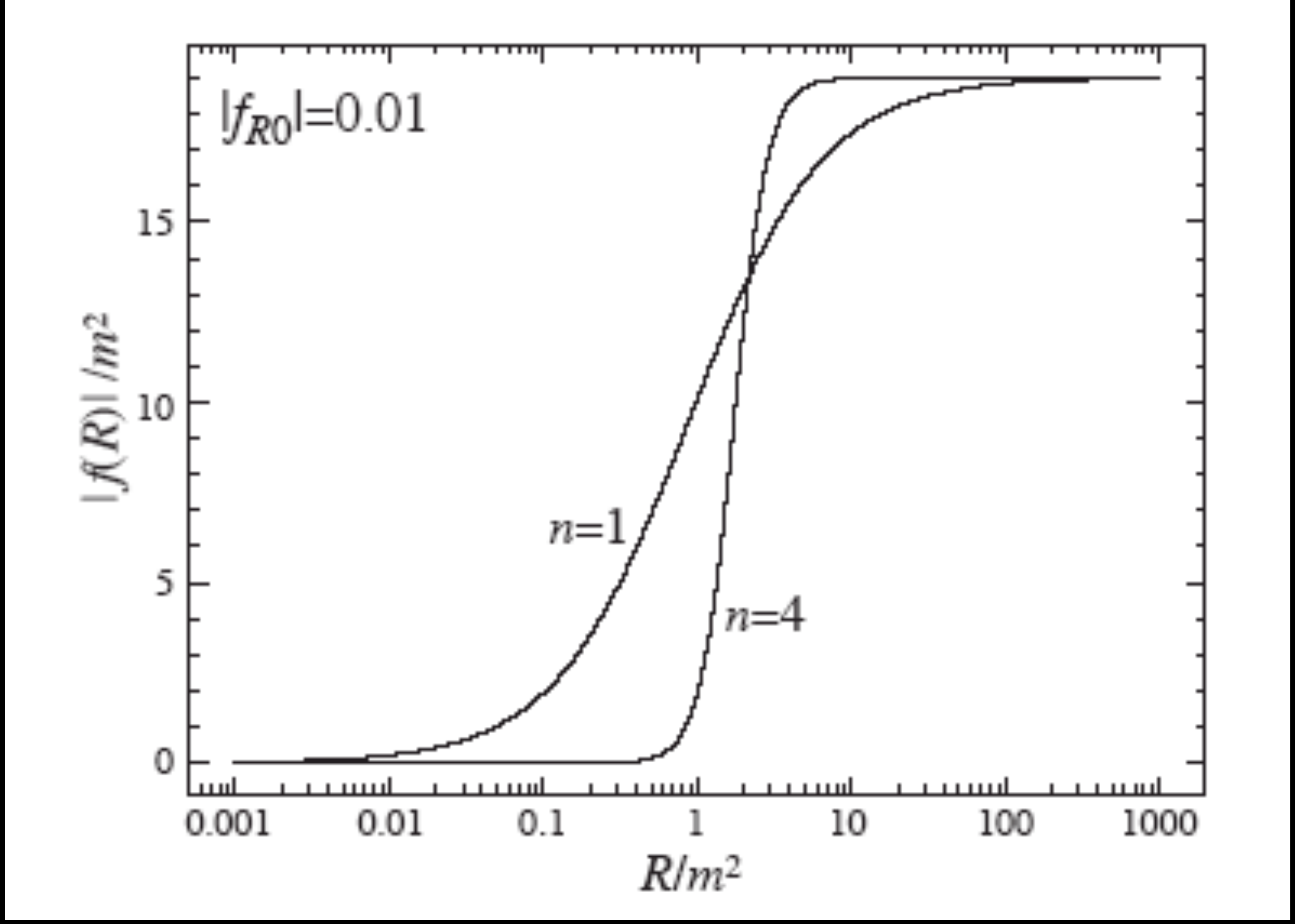
1. Mimic LCDM at high z as suggested by CMB
2. Accelerate univ at low z
3. Include enough dof to allow for variety of low z phenomena
4. Include phenom of LCDM as limiting case.

$$\lim_{R\to\infty} f(R) = \text{const.}\,,$$
$$\lim_{R\to 0} f(R) = 0\,,$$

$$f(R) = -m^2 \frac{c_1(R/m^2)^n}{c_2(R/m^2)^n+1}\,,$$

$$f_{RR} \equiv \frac{\mathrm{d}^2 f(R)}{\mathrm{d}R^2} > 0$$

Effective chameleon mechanism



What should we do to help determine the nature of DE ?

1. We need to define properly theoretically predicted observables, or determine optimum ways to parameterise consistency tests (i.e. how should we parameterise w(z)?)

2. Need to start including dynamical dark energy, interacting dark matter-dark energy and modified gravity models in large scale simulations - [Wyman et al 2013, Li et al 2013 Puchwein et al 2013, Jennings et al 2012, Barreira et al 2012, Brax et al 2013].

3. Include the gastrophysics + star formation especially when considering baryonic effects in the non-linear regimes - `mud wrestling'.

4. On the theoretical side, develop models that go beyond illustrative toy models. Extend Quintessential Axion models. Are there examples of actual Landscape predictions? De Sitter vaccua in string theory is non trivial -[see Burgess et al].

5. Recently massive gravity and galileon models have been developed which have been shown to be free of ghosts. What are their self-acceleration and consistency properties?

6. Will we be able to reconstruct the underlying Quintessence potential from observation?

7. Will we ever be able to determine whether $w \neq -1$?

8. Look for alternatives, perhaps we can shield the CC from affecting the dynamics through self tuning-- The Fab Four

9. Given the complexity (baroque nature ?) of some of the models compared to that of say Λ, should we be using Bayesian model selection criterion to help determine the relevance of any one model.

Things are getting very exciting with DES beginning to take data and future Euclid missions, LSST, as well as proposed giant telescopes, GMT, ELT, SKA - travelling in new directions !

What's the best way to paramterise the DE eqn of state ?

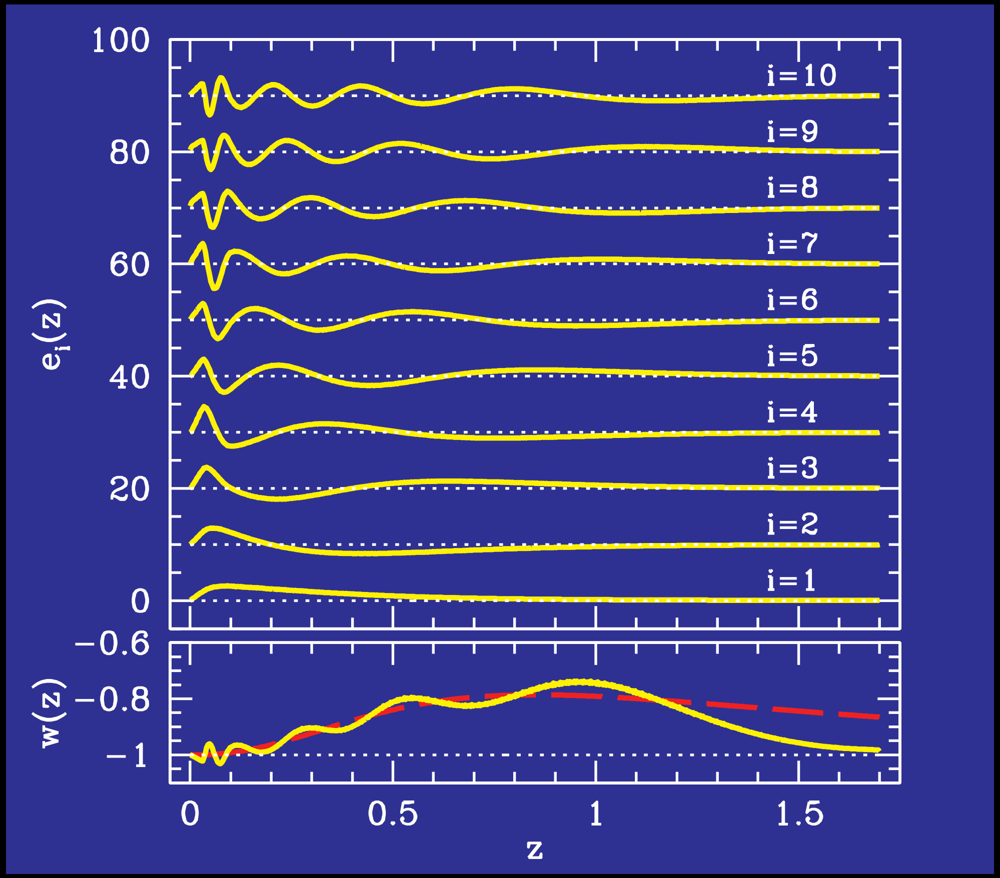
Important for surveys like DES, EUCLID, LSST

1. Principal components -

$$w(z) - w_b(z) = \sum_i \alpha_i e_i(z)$$

w_b – baseline eos

e_i – Fischer matrix eig modes



[Mortonson,Hu and Huterer 2011]

2. w(z)

$$w(z) = w_0 + w_1\left[\frac{z}{1+z}\right] \quad \text{Chevallier – Polarski – Linder}$$

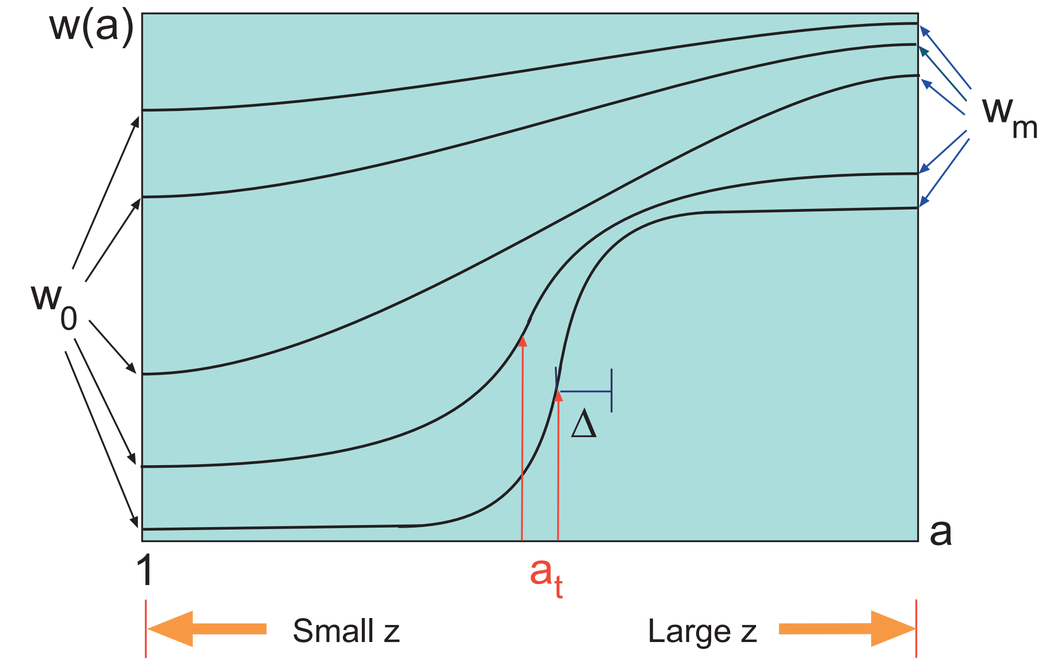
$$w(z) = w_0 + w_1\left[\ln\left(\frac{1}{1+z}\right)\right] \quad \text{Gerke Efstathiou}$$

$$w(z) = w_0 + (w_m - w_0)\Gamma(a, a_t, \Delta)$$

Corasaniti et al

allows for tracker like behaviour although very tight bounds emerging from Planck on allowed density of early dark energy

$$\Omega_e < 0.009$$



3. $w(\Omega_{DE})$

$$w(\Omega_e) = w_0 + w_1\Omega_e + w_2\Omega_e^2$$

Tarrant et al 2013

good for dynamical DE such as Quintessence as long as monotonic evolution.

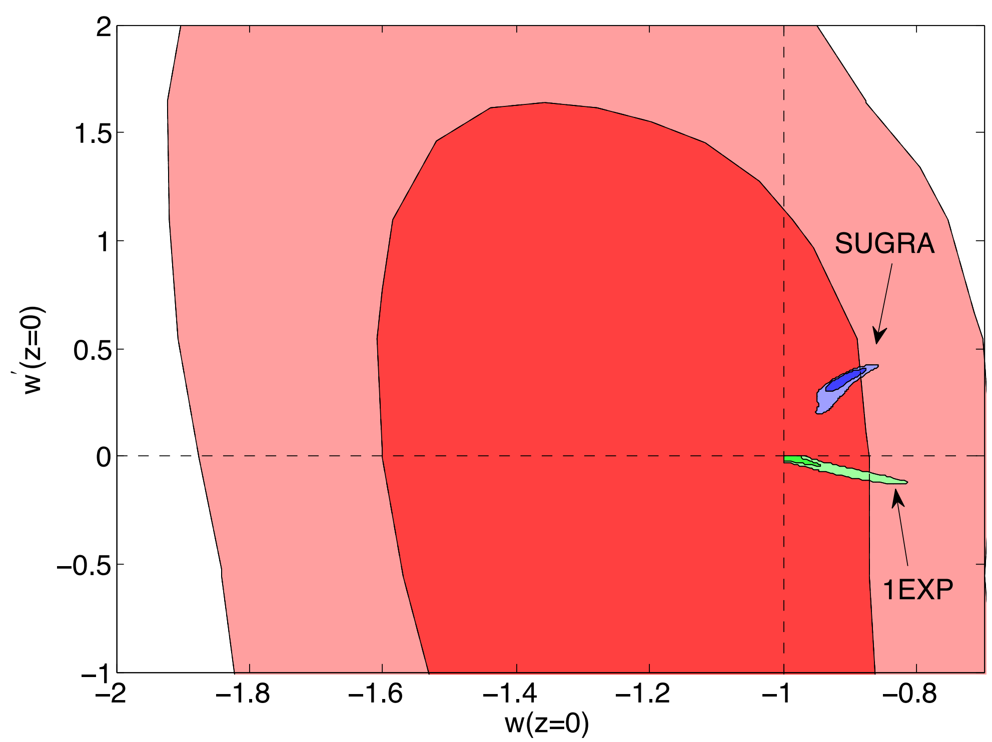


FIG. 5. The 2D 68% (dark shading) and 95% (light shading) marginalised contours in the $w_e|_{z=0}$–$w'_e|_{z=0}$ plane for the 1EXP and SUGRA quintessence models superimposed upon the corresponding contours of the dark energy clock

Self tuning - with the Fab Four

In GR the vacuum energy gravitates, and the theoretical estimate suggests that it gravitates too much.

Basic idea is to use self tuning to prevent the vacuum energy gravitating at all.

The cosmological constant is there all the time but is being dealt with by the evolving scalar field.

with Charmousis, Padilla and Saffin

PRL 108 (2012) 051101; PRD 85 (2012) 104040

with Padilla and Saffin

JCAP 1212 (2012) 026

Horndeski's theory: [G.W. Horndeski, Int. Jour. Theor. Phys. 10 (1974) 363-384

Most general scalar-tensor theory with second order field equations:

$$\begin{aligned}\mathcal{L}_H = \; & \delta^{ijk}_{\mu\nu\sigma}\Big[\kappa_1 \nabla^\mu \nabla_i \phi R_{jk}{}^{\nu\sigma} - \frac{4}{3}\kappa_{1,\rho} \nabla^\mu \nabla_i \phi \nabla^\nu \nabla_j \phi \nabla^\sigma \nabla_k \phi \\ & + \kappa_3 \nabla_i \phi \nabla^\mu \phi R_{jk}{}^{\nu\sigma} - 4\kappa_{3,\rho} \nabla_i \phi \nabla^\mu \phi \nabla^\nu \nabla_j \phi \nabla^\sigma \nabla_k \phi\Big] \\ & + \delta^{ij}_{\mu\nu}\left[(F+2W)R_{ij}{}^{\mu\nu} - 4F_{,\rho} \nabla^\mu \nabla_i \phi \nabla^\nu \nabla_j \phi + 2\kappa_8 \nabla_i \phi \nabla^\mu \phi \nabla^\nu \nabla_j \phi\right] \\ & - 3[2(F+2W)_{,\phi} + \rho\kappa_8]\nabla_\mu \nabla^\mu \phi + \kappa_9(\phi, \rho),\end{aligned}$$

$$\boxed{\rho = \nabla_\mu \phi \nabla^\mu \phi}$$

$\kappa_1,\ \kappa_3,\ \kappa_8,\ \kappa_9 -$ —Four indep func ofϕ andρ

W can be set to zero and F can be derived from κ's.

Equiv to but not as elegant as Deffayet et al, PRD80 (2009) 064015

(see also Kobayashi et al 1105.5723 [hep-th])

The action which leads to self tuning solutions can be rewritten in a more natural way in which we see how the scalar fields couple directly to various curvature invariants:

$$
\begin{aligned}
\mathcal{L}_{john} &= \sqrt{-g} V_{john}(\phi) G^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi \\
\mathcal{L}_{paul} &= \sqrt{-g} V_{paul}(\phi) P^{\mu\nu\alpha\beta} \nabla_\mu \phi \nabla_\alpha \phi \nabla_\nu \nabla_\beta \phi \\
\mathcal{L}_{george} &= \sqrt{-g} V_{george}(\phi) R \\
\mathcal{L}_{ringo} &= \sqrt{-g} V_{ringo}(\phi) \hat{G}
\end{aligned}
$$

where $\hat{G} = R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} - 4 R_{\mu\nu} R^{\mu\nu} + R^2$

and $P^{\mu\nu\alpha\beta} = -\frac{1}{4} \varepsilon^{\mu\nu\lambda\sigma} \, R_{\lambda\sigma\gamma\delta} \, \varepsilon^{\alpha\beta\gamma\delta}$

In other words it can be seen to reside in terms of the four arbitrary potential functions of ϕ coupled to the curvature terms.

Covers most scalar field related modified gravity models studied to date.

Assume no derivative couplings to matter to avoid violation of Equivalence Principle.

Can assume matter only couples to metric.

Begin the Cosmology

$$g_{\mu\nu}dx^\mu dx^\nu = -dt^2 + a^2(t)\left[\frac{dr^2}{1-\kappa r^2} + r^2\, d\Omega_{(2)}\right]$$

Friedmann equation:

$$\mathcal{H}(a,\dot{a},\phi,\dot{\phi}) = \frac{1}{a^3}\left[\dot{a}\frac{\partial L_H^{\text{eff}}}{\partial \dot{a}} + \dot{\phi}\frac{\partial L_H^{\text{eff}}}{\partial \dot{\phi}} - L_H^{\text{eff}}\right] = -\rho_m$$

At most cubic in Hubble parameter H

$$\mu_3 H^3 + \mu_2 H^2 + \mu_1 H + \mu_0 = \rho_m$$

Scalar eom:

$$\mathcal{E}(a,\dot{a},\ddot{a},\phi,\dot{\phi},\ddot{\phi}) \;=\; \frac{\partial L_H^{\text{eff}}}{\partial \phi} - \frac{d}{dt}\left[\frac{\partial L_H^{\text{eff}}}{\partial \dot{\phi}}\right] \;=\; 0$$

Linear in both $\ddot{\phi}$ and $\ddot{a}$.

Self tuning in Horndeski.

1. Vacuum solution is always Minkowski whatever the vacuum energy

2. Solution remains Minkowski even after a phase transition where the vacuum energy changes instantaneously.

In other words the vacuum energy does not gravitate at all because of the influence of the evolving scalar field and curvature.

$$< \rho_m >_{\mathrm{vac}} = \rho_\Lambda, \quad H^2 = -\frac{\kappa}{a^2}, \quad \phi = \phi_\Lambda(t)$$

piecewise constant but discontinuous at transition

``On shell in a" always satisfied

continuous everywhere and not constant

Scalar field eqn of motion should be trivial ``on-shell-in-a"

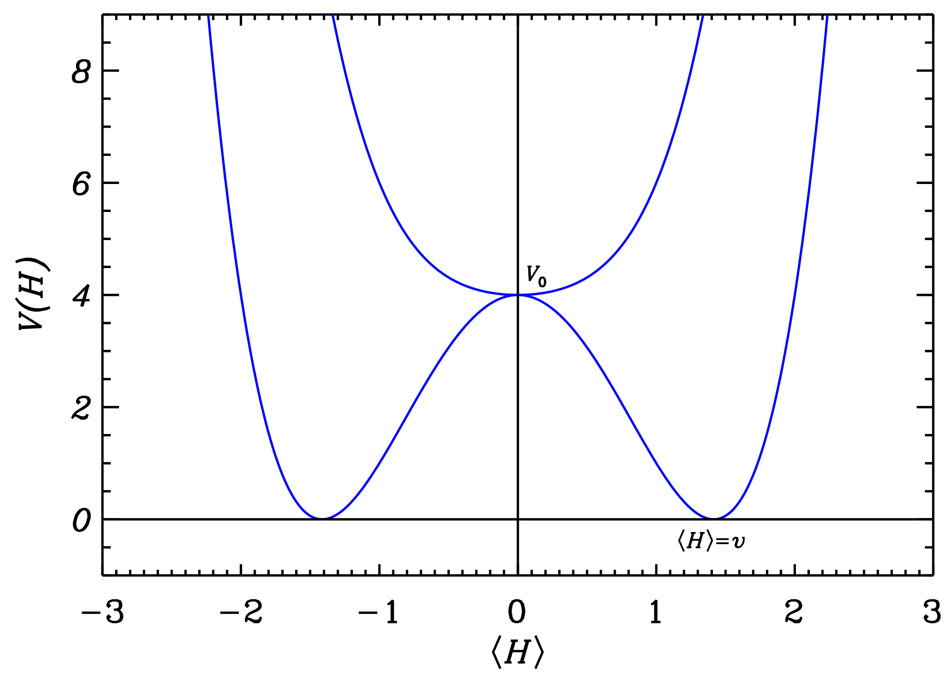
The scalar is completely determined by the vacuum Friedmann equation.

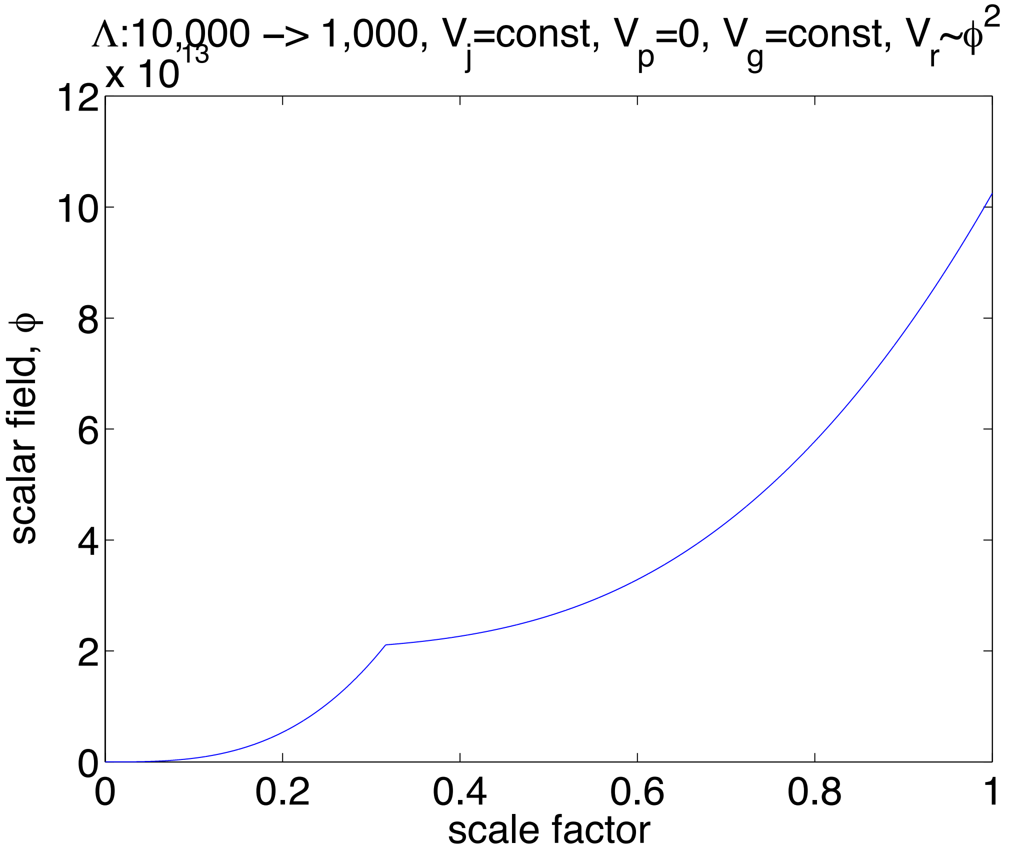
In this self tuning vacuum:

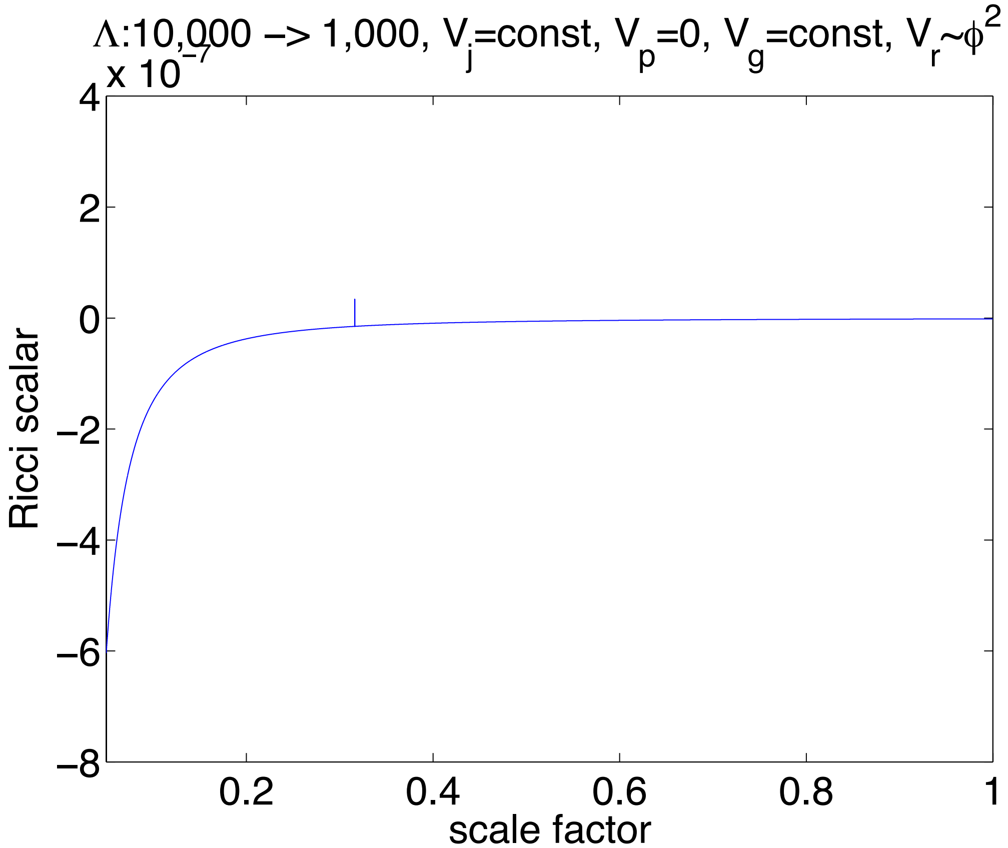
1. the matter tells the scalar how to move - this requires that the ``on-shell-in-a" gravity equation be dependent on $\dot{\phi}$

2. the scalar tells the spacetime not to curve, but crucially only in the vacuum - the scalar equation should not be independent of $\ddot{a}$

A Phase Transition







Some basic cosmological solutions:

$$\cdot V_j \text{ only}, \;\; H^2 \sim \frac{M^2}{a^6}[1 + O(k/a^2)], \;\; \text{stiff fluid}$$

$$\cdot V_p \text{ only}, \;\; H^2 \sim \frac{M^2}{a^6}[1 + O(k/a^2)], \;\; \text{stiff fluid}$$

$$\cdot V_g \text{ only}, \;\; H^2 \sim \frac{M^2}{a^4} - \frac{k}{a^2}, \;\;\; \text{radiation}$$

$$\cdot V_r \text{ only}, \;\; H^2 \sim \frac{M^2}{a^2}[1 + O(k/a^2)], \;\; \text{curvature}$$

In general system is complicated to solve.

Try dynamical systems approach to find scaling solutions.

$$N = \ln(a); \quad x = H^{\alpha}\phi'; \quad y_n = H^{\beta_n} V_n; \quad \sigma = \frac{\sqrt{-k}}{Ha}$$

$$\lambda_n = H^{\gamma_n} \frac{V_n'}{V_n}; \quad h = \ln(H)' \quad \mu_n = \frac{V_n V_n''}{(V_n')^2}$$

For $\mu = \text{const} \rightarrow V \sim \phi^{\frac{1}{1-\mu}},\ e^{A\phi}$

$$x' = \ldots,\ y' = \ldots, \lambda' = \ldots, \mu' = \ldots, \sigma' = \ldots, h' = \ldots$$

fab four cosmology

TABLE I: Examples of interesting cosmological behaviour for various fixed points with $\sigma = 0$.

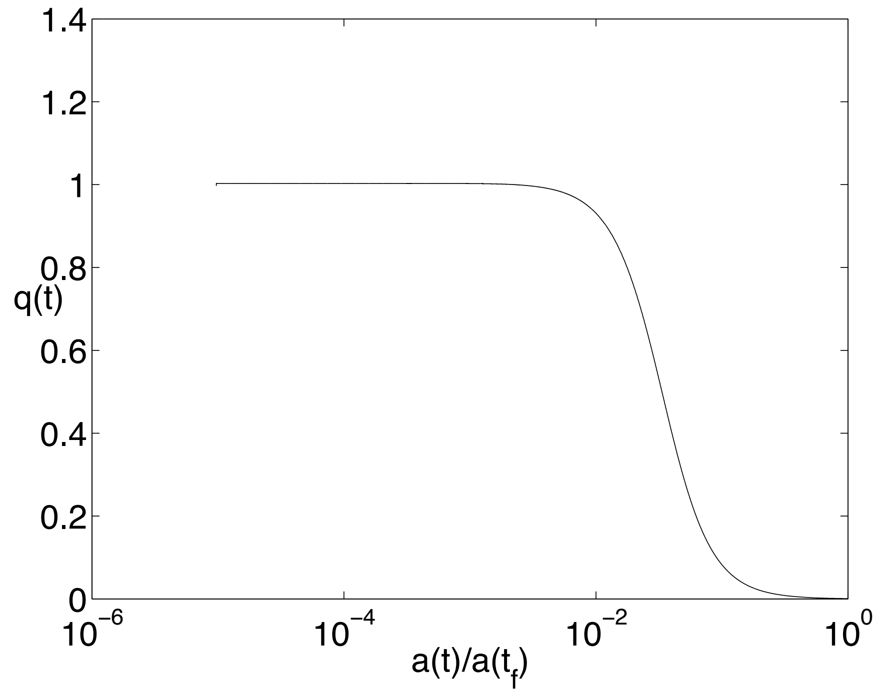
Case	cosmological behaviour	$V_j(\phi)$	$V_p(\phi)$	$V_g(\phi)$	$V_r(\phi)$
Stiff fluid	$H^2 \propto 1/a^6$	$c_1\phi^{\frac{4}{\alpha}-2}$	$c_2\phi^{\frac{6}{\alpha}-3}$	0	0
Radiation	$H^2 \propto 1/a^4$	$c_1\phi^{\frac{4}{\alpha}-2}$	0	$c_2\phi^{\frac{2}{\alpha}}$	$-\frac{\alpha^2}{8}c_1\phi^{\frac{4}{\alpha}}$
Curvature	$H^2 \propto 1/a^2$	0	0	0	$c_1\phi^{\frac{4}{\alpha}}$
Arbitrary	$H^2 \propto a^{2h}, \quad h \neq 0$	$c_1(1+h)\phi^{\frac{4}{\alpha}-2}$	0	0	$-\frac{\alpha^2}{16}h(3+h)c_1\phi^{\frac{4}{\alpha}}$

$$q = -\frac{a\ddot{a}}{\dot{a}^2}$$

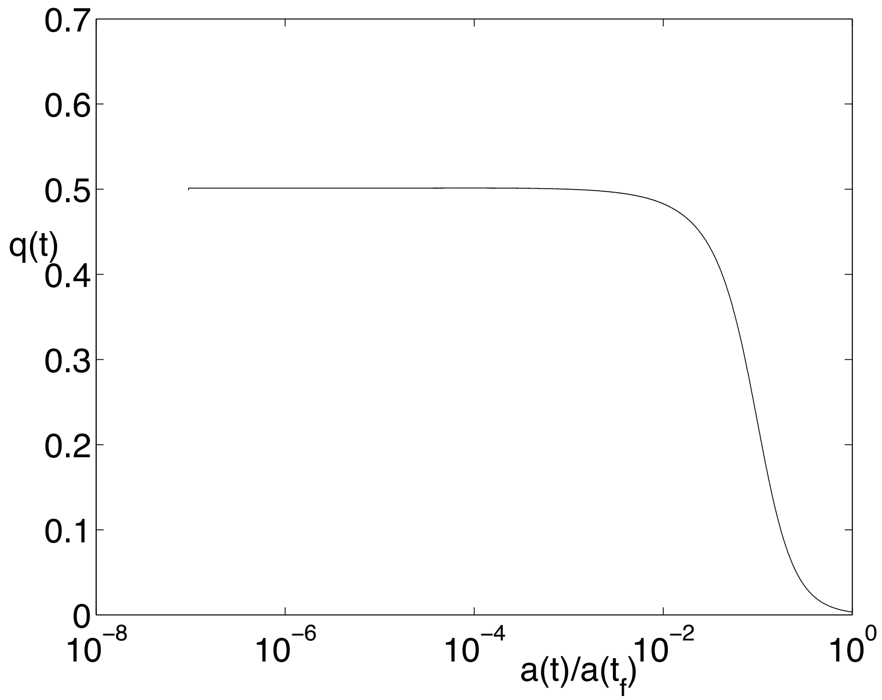
$$a \sim t^p \sim t^{-1/h}$$

$$q = -\frac{p(p-1)}{p^2} = -(1+h)$$

“radiation”



“matter”



Stability?

see Kobayashi et al: 1105.5723; De Felice et al: 1108.4242

$$ds^2 = -N^2dt^2 + \gamma_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$
$$N = 1 + \hat{\alpha},\ N_i = \partial_i \hat{\beta},\ \gamma_{ij} = a^2(t)e^{2\zeta}(\delta_{ij} + h_{ij})$$

Tensor pertns: $\frac{1}{8}\int dt d^3x a^3 \left[\mathcal{G}_T \dot{h}_{ij}^2 - \frac{\mathcal{F}_T}{a^2}(\nabla h_{ij})^2\right]$

Scalar pertns: $\int dt d^3x a^3 \left[\mathcal{G}_S \dot{\xi}^2 - \frac{\mathcal{F}_S}{a^2}(\nabla \xi)^2\right]$

Can find stable $F_T>0$, $G_T>0$, $F_S>0$, $G_S>0$:

Also true for radiation and inflation ...

But can we put them together somehow ?

Non trivial - work in progress with Ruth Gregory

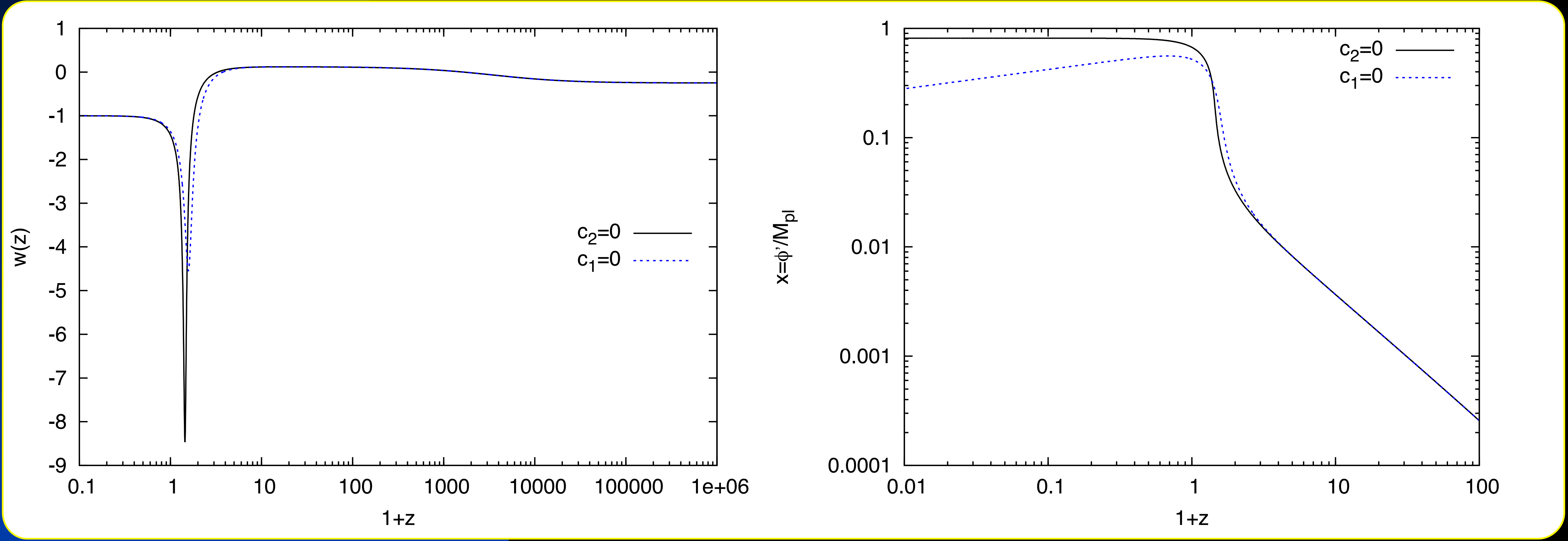
Fab 5 version -

Appleby, De Felice and Linder

JCAP 1210 (2012) 060

Add non-linear combination of standard kinetic term and derivative coupling to Einstein tensor - find lasting accelerating solutions whilst still self tuning away a cosmological constant.

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{pl}^2}{2}R + c_1 X + f\left(c_2 X + \frac{c_G}{M^2}G^{\mu\nu}\phi_\mu\phi_\nu\right)\right] + S_m \qquad X \equiv -\frac{1}{2}g^{\mu\nu}\phi_\mu\phi_\nu$$



See evolution from matter regime to late time de Sitter attractor solution, and how scalar field reacts accordingly.

Possible to have a self tuning `classical' solution in which the system adjusts itself to the Minkowski vacuum irrespective of the magnitude of the cosmological constant and whether it changes. It relies on breaking the assumption of Poincare invariance demanded by Weinberg in his original no-go theorem. In particular we have to have the scalar field evolving in time.

Remains to be seen whether we can satisfy solar system tests and obtain realistic cosmological solutions. See Kaloper and Sandora [arXiv:1310.5058]

The role of quantum corrections remains to be evaluated (although initial calculations suggest they can be controlled). They could spoil the party, although we note the crucial role played in the geometrical structure of the model.

There is always the question of stability of the solutions

Gregory Hormdeski left physics in 1981 having obtained a faculty position at Waterloo, Canada. He was on leave in Amsterdam and went to a Van Gogh exhibition.

His love of art was too strong and the inspiration he took from Van Gogh overpowering. He now works from his studio in Santa Fe.

Summary

1. Depending on your faith in string landscape approach we have a solution to CC problem.

2. Quintessence type approaches require light scalars which bring with them fifth force constraints that need satisfying.

3. Need to screen this which leads to models such as axions, chameleons,non-canonical kinetic terms etc.. -- these have their own issues.

4. Alternatively could consider modified gravity such as massive gravity but this brings with it constraints.

5. Not had time to mention approaches like PPF formalism or new couplings such as scalar field to velocity components.

9. Fab Four - we have a way of living with a large changing cosmological constant - that has to be interesting !