

# Nonlinear velocity statistics and redshift-space distortions in peculiar velocity surveys

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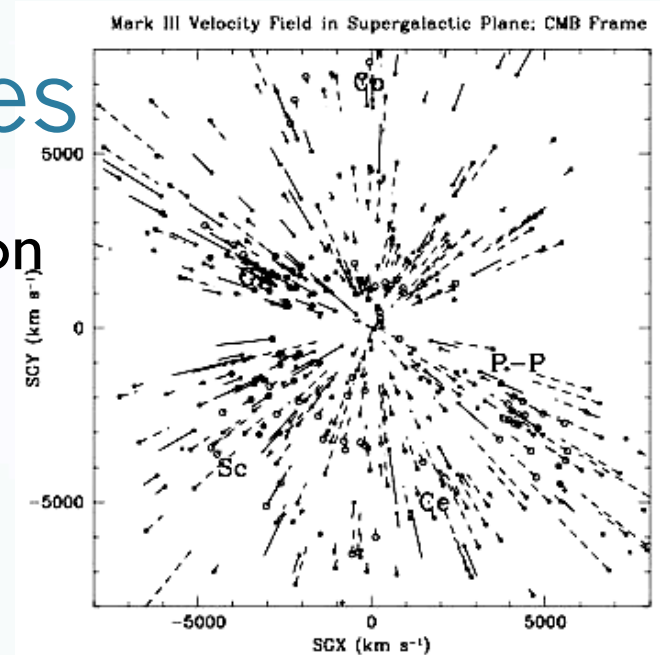
*Okumura, Seljak, Vlah, Desjacques, (2014) JCAP accepted (arXiv:1312.4214)*

# Peculiar velocities of galaxies

- Contains information of 3D mass distribution

Distance to galaxies through spectroscopy  $z = H_0 x + v_{||}$  Radial peculiar velocity

Redshift (redshift space) True distance (real space)



- It's hard to measure peculiar velocities themselves, but by measuring power spectrum in galaxy surveys the effect of peculiar velocities shows up as anisotropy (Redshift-space distortions)

Formula of galaxy power spectrum in linear theory (Kaiser)

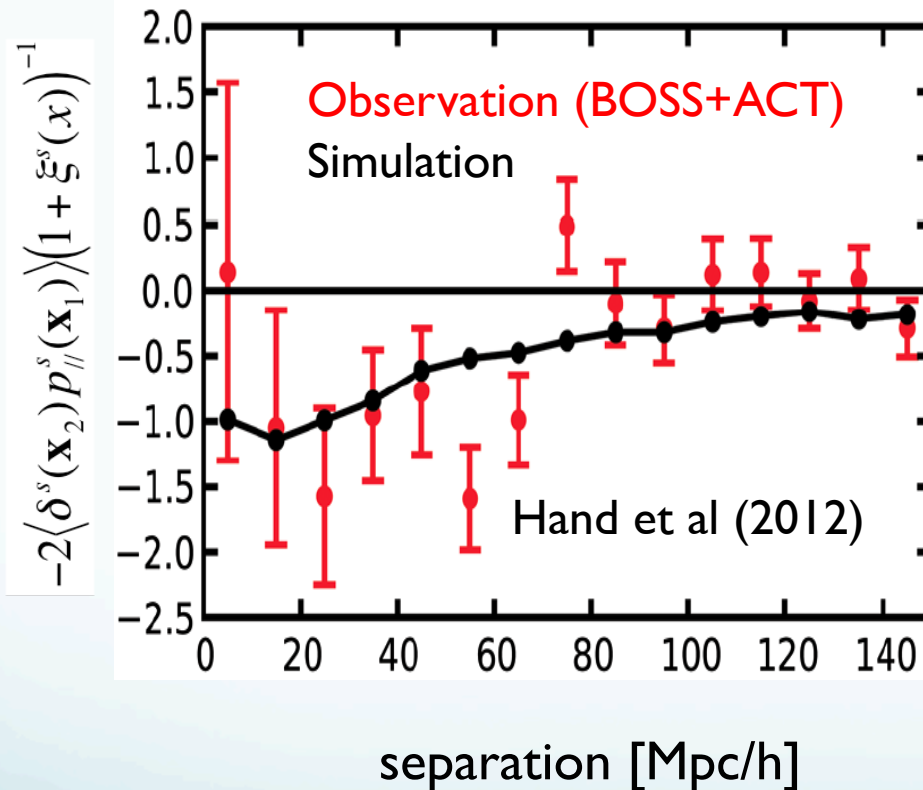
$$P_g^s(k, \mu) = (b + f\mu^2)^2 P_m^r(k)$$

Anisotropy due to RSD ( $\mu$ : direction cosine between wavevector and LoS)

- Theoretical modeling of redshift-space density power spectrum has been extensively studied as an observable.

# Direct measurement of peculiar velocities: kinetic Sunyaev-Zeld'vich (kSZ)

Mean pairwise momentum [Mpc/h]

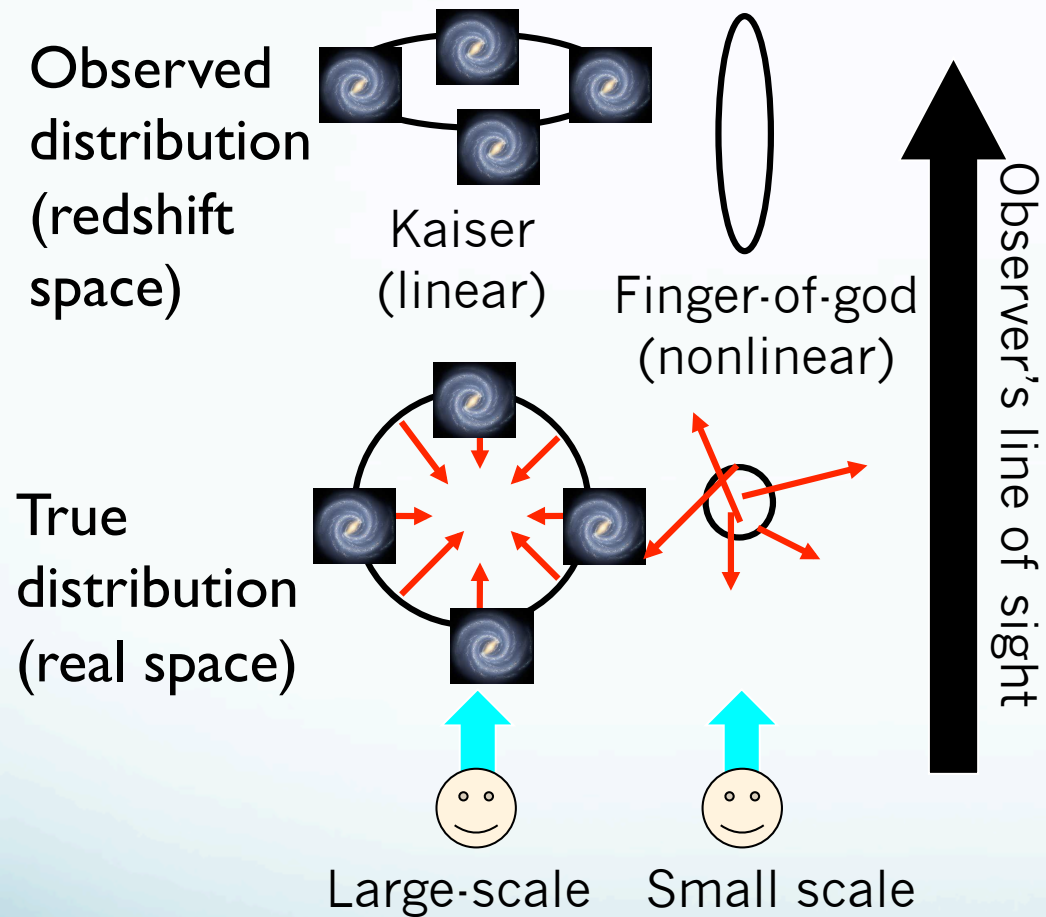


- By combining a galaxy survey (BOSS) and a CMB map (ACT), measured the distortion of the CMB spectrum along the line of observed clusters of galaxies (kSZ effect) and determined peculiar velocities
- Observables are momentum field sampled in redshift-space positions of galaxies, thus affected by (nonlinear) RSD.
- However, such a formulation has been made so far only by linear theory in Fourier space (thus simply Kaiser).

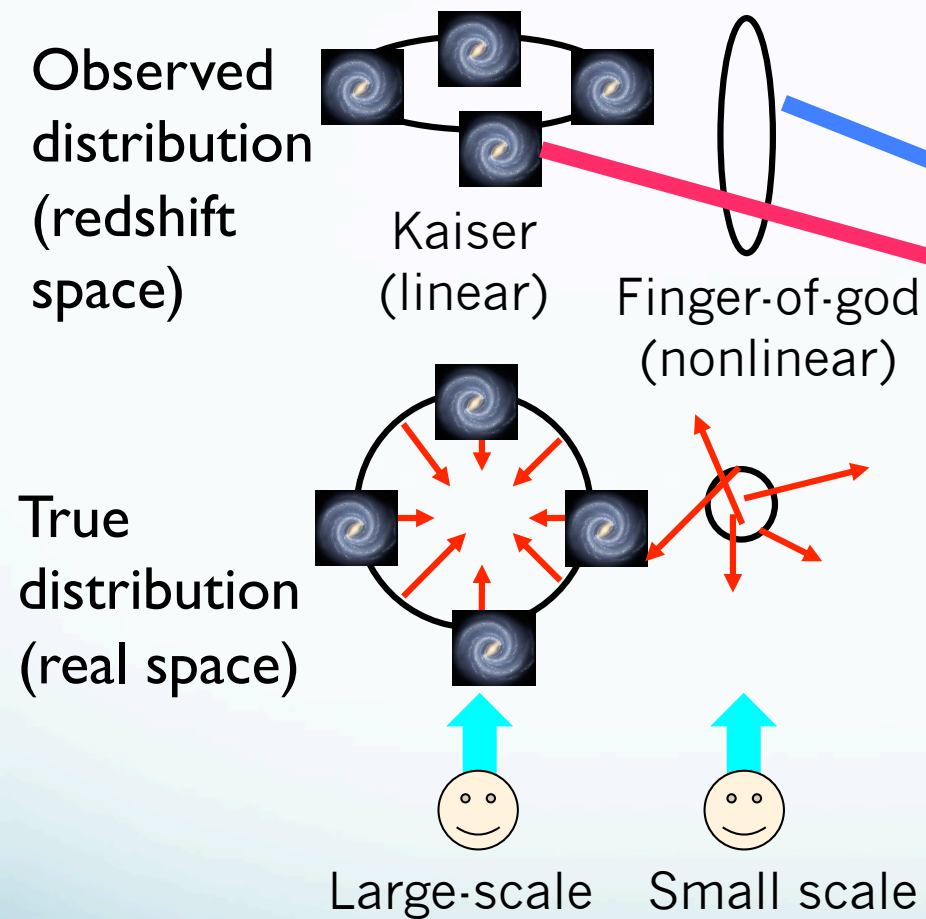
# Purpose and conclusion of this study

- We define the momentum field in redshift space, and derive formulae for peculiar velocity statistics that include nonlinear RSD.
- We compute the statistics using Eulerian 1-loop perturbation theory and linear theory.
- We compare them to the numerical results measured from N-body simulations.
- We find that N-body results cannot be explained by linear theory even in very large scales, while our nonlinear PT improves the accuracy.
  - reference:
  - *Okumura, Seljak, Vlah, Desjacques, (2014) JCAP (arXiv:1312.4214)*

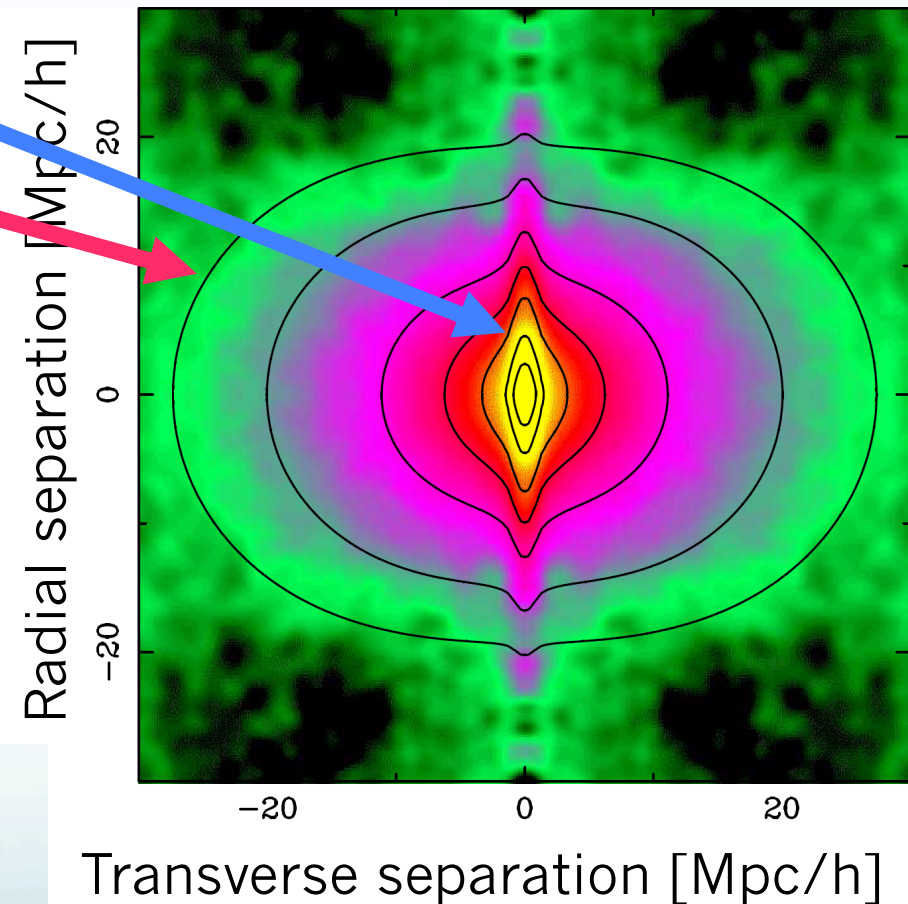
# Redshift-space distortions (RSD)



# Redshift-space distortions (RSD)



2-pt correlation function of galaxies from survey (Peacock et al., 2001 Nature)



Linear power spectrum + FOG damping

$$P_g^s(k, \mu) = (b + f\mu^2)^2 P_m^r(k) \times \exp(-k^2 \mu^2 \sigma_v^2)$$



# Redshift-space power spectrum using phase-space distribution function

- RSD is defined in phase space  $f(\mathbf{x}, \mathbf{p})$  starting from Vlasov equation

$$\vec{s} = \vec{x} + \hat{z} v_{||} / aH = \vec{x} + \hat{z} p_{||} / a^2 m H$$

Distortions due to radial momentum ( $\sim 10 \text{ Mpc/h}$ )

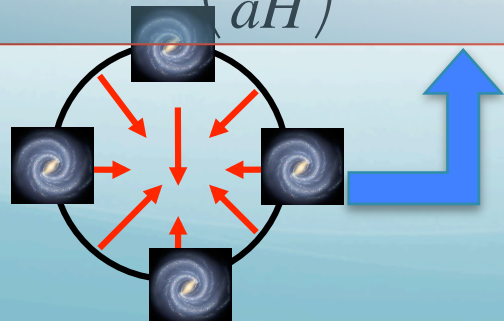
- Redshift-space density field is described by real-space one + infinite sum of moments of mass-weighted velocities

$$\delta^s(\mathbf{k}) = \delta^r(k) + \sum_{L=1} \frac{1}{L!} \left( \frac{ik_{||}}{aH} \right) T_{||}^L(\mathbf{k}) \quad \text{where} \quad T_{||}^L(\mathbf{x}) = \frac{m}{\bar{\rho} a^3} \int d^3 p f(\mathbf{x}, \mathbf{p}) \left( \frac{p_{||}}{am} \right)^L = [1 + \delta(\mathbf{x})] u^L(\mathbf{x})$$

- Power spectrum becomes an infinite sum as well.

$$\langle \delta^s(\mathbf{k}) \delta^s(\mathbf{k}') \rangle = (2\pi)^3 P^s(\mathbf{k}) \delta^D(\mathbf{k} - \mathbf{k}')$$

$$P^s(\mathbf{k}) = P_{00}(\mathbf{k}) - 2i \left( \frac{k_{\mu}}{aH} \right) P_{01}(\mathbf{k}) + \left( \frac{k_{\mu}}{aH} \right)^2 P_{11}(\mathbf{k}) - \left( \frac{k_{\mu}}{aH} \right)^2 P_{02}(\mathbf{k}) + \dots$$



Seljak & McDonald (2011) JCAP (paper I)  
**Okumura**, Seljak, McDonald, Desjacques (2012a) JCAP (II)  
**Okumura**, Seljak, Desjacques (2012,b) JCAP (III)  
Vlah, Seljak, McDonald, **Okumura**, Baldauf (2012) JCAP (IV)  
Vlah, Seljak, **Okumura**, Desjacques (2013) JCAP (V)

# Extension to redshift-space momentum

$$\vec{s} = \vec{x} + \hat{z} v_{||} / aH = \vec{x} + \hat{z} \underline{p_{||} / a^2 m H}$$

Distortions due to radial peculiar velocities ( $\sim 10 \text{ Mpc/h}$ )

- Derivation for density field can be applied to momentum

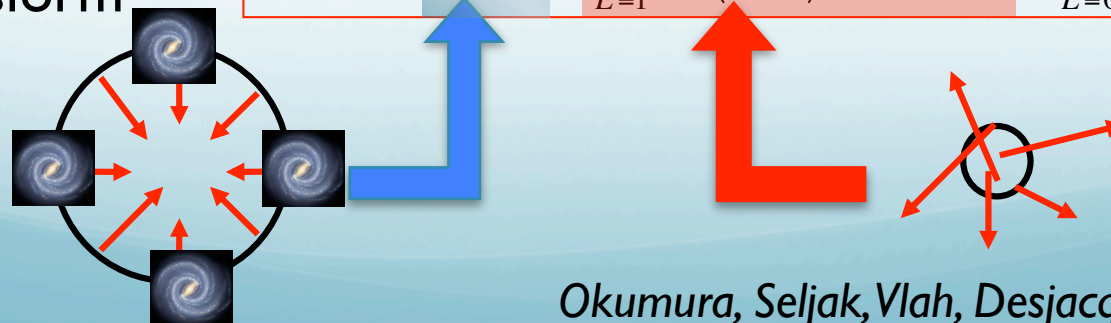
$$\delta^s(\mathbf{k}) = \delta^r(k) + \sum_{L=1} \frac{1}{L!} \left( \frac{ik\mu}{aH} \right) T_{||}^L(\mathbf{k}) \quad \text{where} \quad T_{||}^L(\mathbf{x}) = [1 + \delta(\mathbf{x})] u^L(\mathbf{x})$$

- Redshift-space momentum field can also be described by a sum over real-space velocity moments.

$$p_{||}^s(\mathbf{x}) = [1 + \delta^s(\mathbf{x})] v_{||}^s(\mathbf{x})$$

→ Fourier transform

$$p_{||}^s(\mathbf{k}) = p_{||}(\mathbf{k}) + \sum_{L=1} \frac{1}{L!} \left( \frac{ik\mu}{aH} \right)^L T_{||}^{L+1}(\mathbf{k}) = \sum_{L=0} \frac{1}{L!} \left( \frac{ik\mu}{aH} \right)^L T_{||}^{L+1}(\mathbf{k})$$



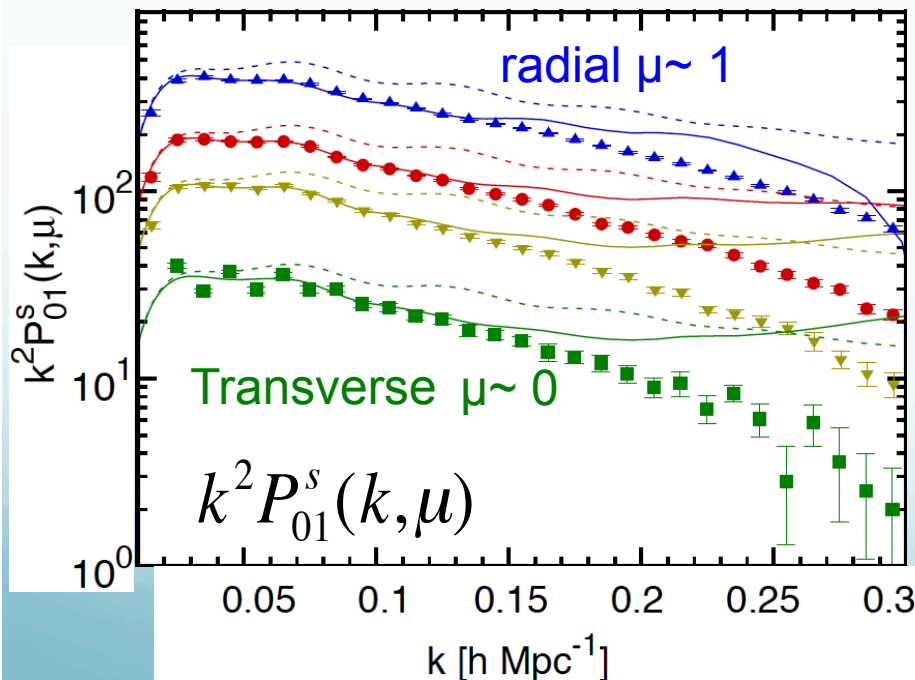


# Expression for density-momentum power

$$\langle \delta^s(\mathbf{k}) p_{||}^{s*}(\mathbf{k}') \rangle = (2\pi)^3 \underline{P_{01}^s(\mathbf{k})} \delta^D(\mathbf{k} - \mathbf{k}')$$

$$-i \frac{k\mu}{\mathcal{H}} P_{01}^s(\mathbf{k}) = -i \frac{k\mu}{\mathcal{H}} P_{01}^r(\mathbf{k}) + \left( \frac{k\mu}{\mathcal{H}} \right)^2 P_{11}^r(\mathbf{k}) - \left( \frac{k\mu}{\mathcal{H}} \right)^2 P_{02}^r(\mathbf{k}) + \frac{i}{2} \left( \frac{k\mu}{\mathcal{H}} \right)^3 P_{03}^r(\mathbf{k}) \\ - 2i \left( \frac{k\mu}{\mathcal{H}} \right)^3 P_{12}^r(\mathbf{k}) + \frac{1}{6} \left( \frac{k\mu}{\mathcal{H}} \right)^4 P_{04}^r(\mathbf{k}) - \left( \frac{k\mu}{\mathcal{H}} \right)^4 P_{13}^r(\mathbf{k}) + \frac{1}{2} \left( \frac{k\mu}{\mathcal{H}} \right)^4 P_{22}^r(\mathbf{k})$$

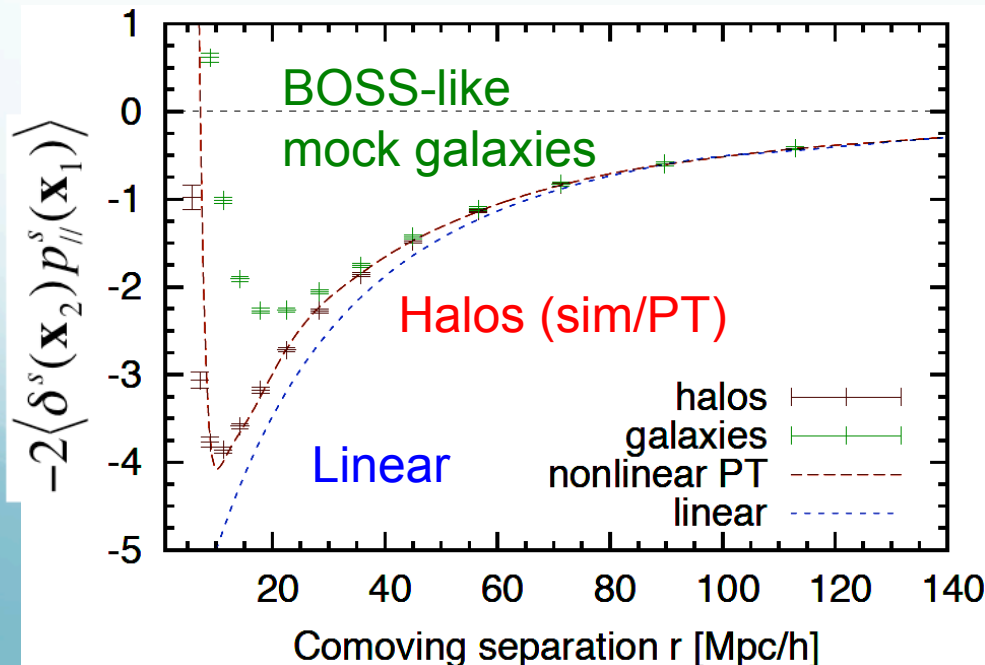
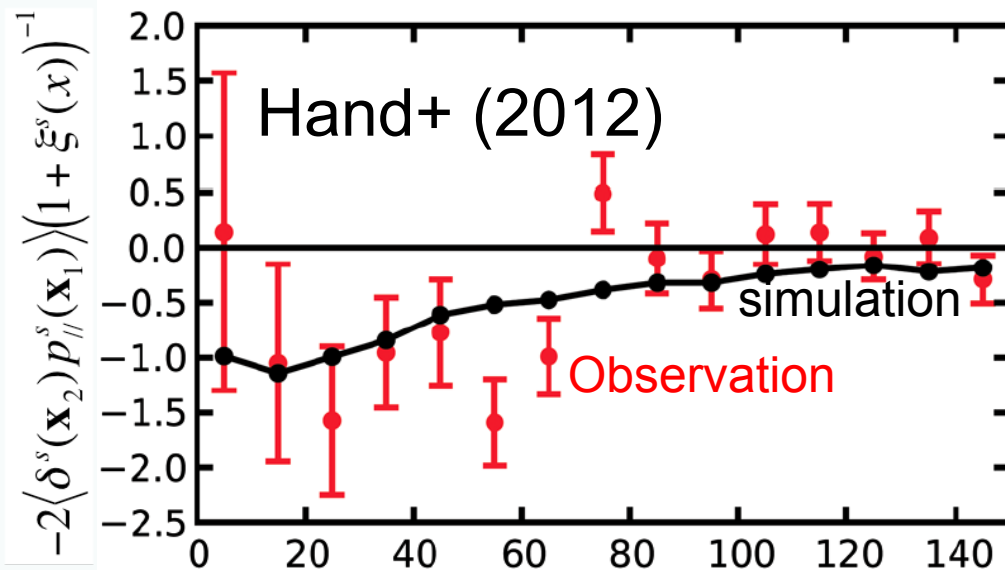
*Halos at  $z=0.5$  ( $b \sim 1.6$ )*



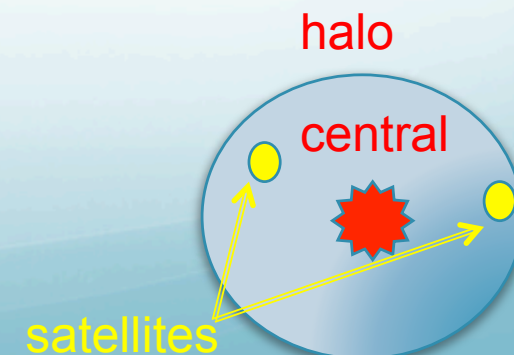
- Data points are simulation results
- Dotted lines are linear theory (Kaiser)  

$$P_{01}^s(k, \mu) = (if\mu/k)(b + f\mu^2)P_m(k)$$
- Solid lines are 1-loop PT + non-local bias + DM spectrum from sim
- Suppression starting at large scales is nonlinear velocity dispersion (RSD)
- No free parameter for RSD

# Redshift-space pairwise infall momentum



- $-2\langle\delta^s(\mathbf{x}_2)p_{||}^s(\mathbf{x}_1)\rangle$ : Fourier tr of  $P_{01}$
- Transition of sign seen in observed data on small scales is caused by nonlinear RSD.
- PT well explains the halo results.
- Finger-of-god for satellites needs to be taken into account.

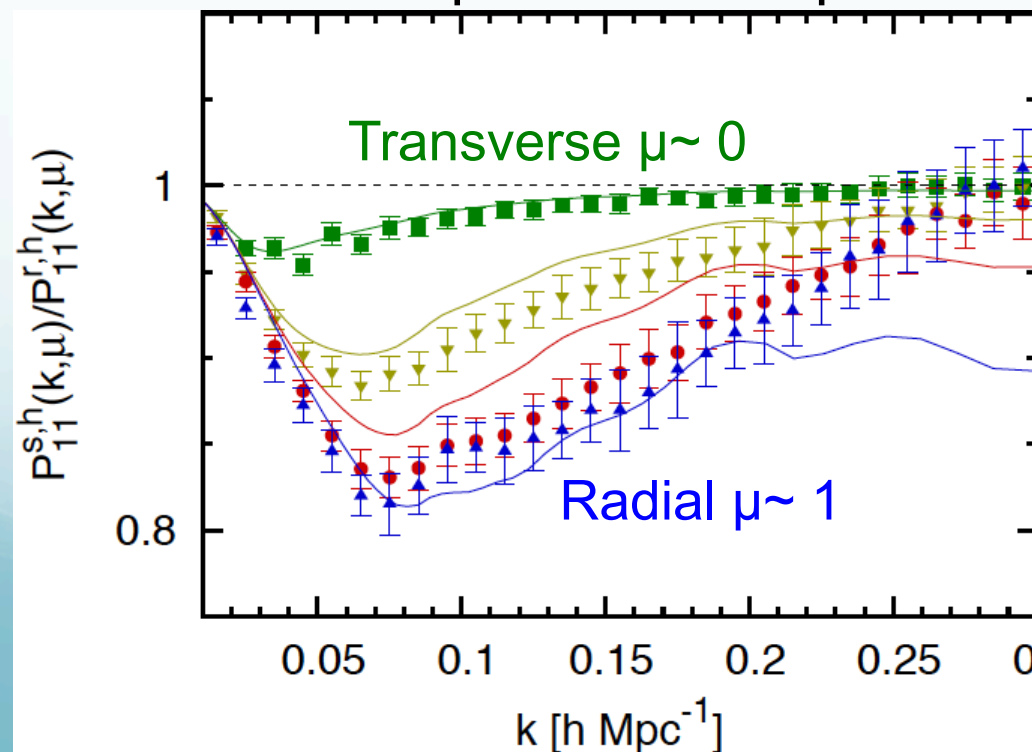


# Momentum-momentum power spectrum

$$\langle p_{||}^s(\mathbf{k}) p_{||}^{s*}(\mathbf{k}') \rangle = (2\pi)^3 \underline{P_{11}^s(\mathbf{k})} \delta^D(\mathbf{k} - \mathbf{k}')$$

$$\left(\frac{k\mu}{\mathcal{H}}\right)^2 P_{11}^s(\mathbf{k}) = \left(\frac{k\mu}{\mathcal{H}}\right)^2 P_{11}^r(\mathbf{k}) - 2i \left(\frac{k\mu}{\mathcal{H}}\right)^3 P_{12}^r(\mathbf{k}) - \left(\frac{k\mu}{\mathcal{H}}\right)^4 P_{13}^r(\mathbf{k}) + \left(\frac{k\mu}{\mathcal{H}}\right)^4 P_{22}^r(\mathbf{k})$$

Ratio of redshift space and real space for halos  $P_{11}^s/P_{11}^r$



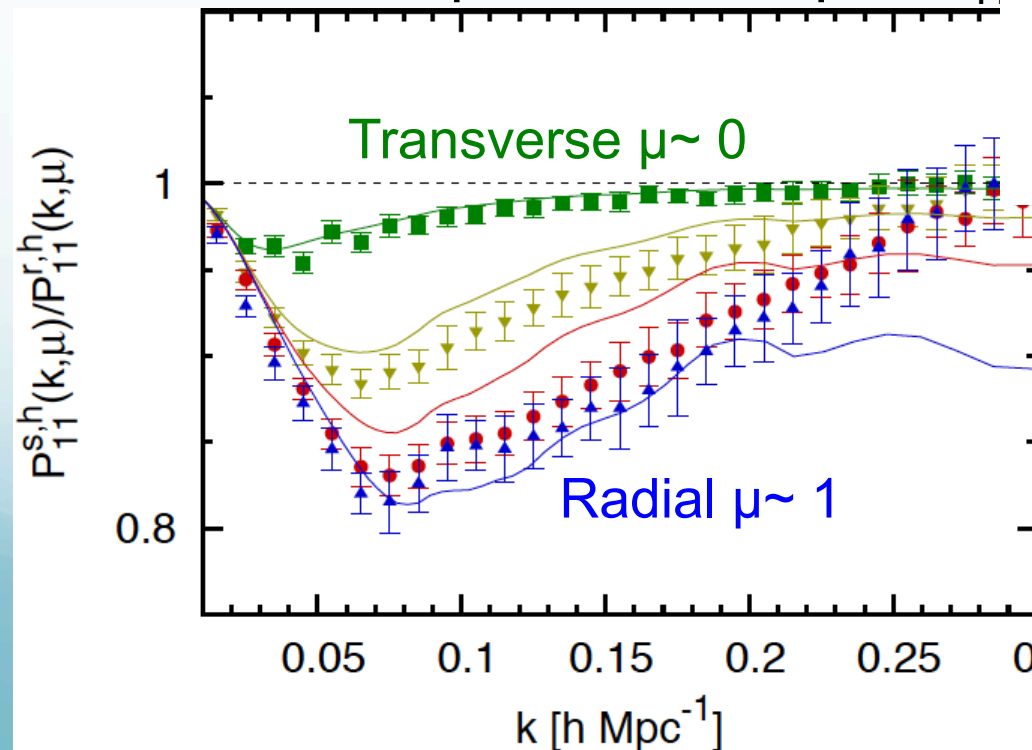
- Unity in linear theory
- Deviation from unity is entirely due to nonlinear RSD ( $\sim 20\%$ )
- Affected by shot noise

# Momentum-momentum power spectrum

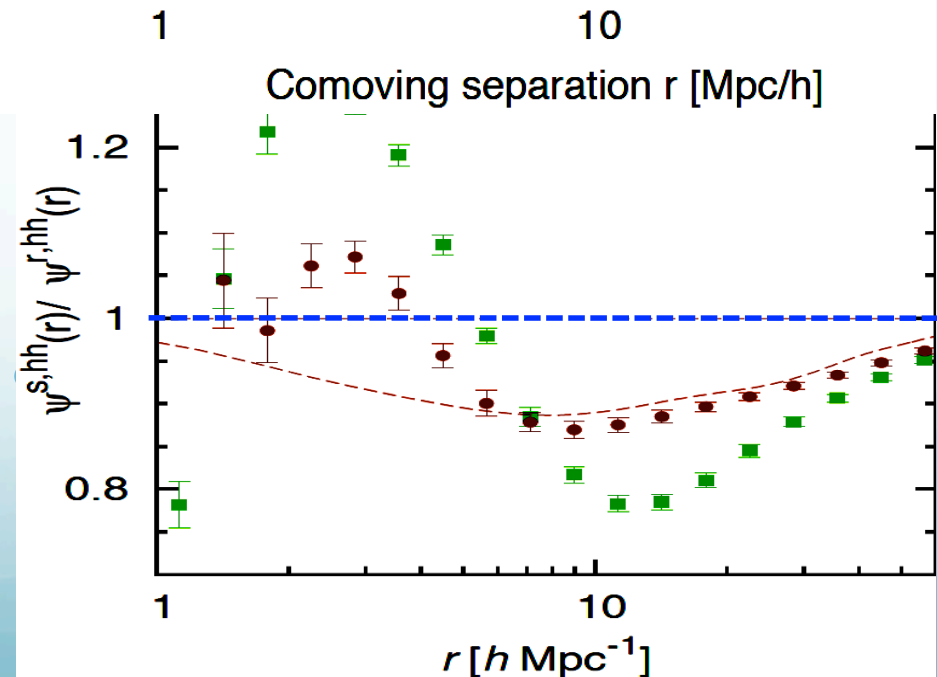
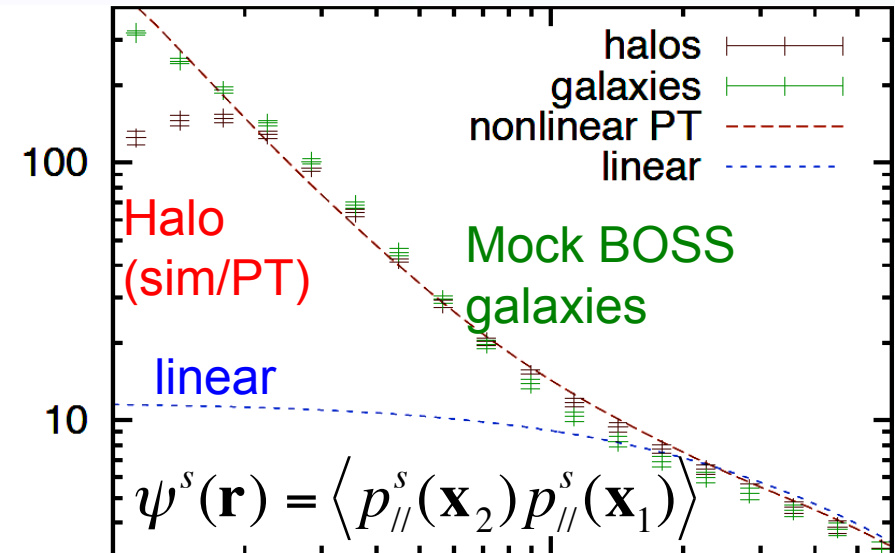
$$\langle p_{||}^s(\mathbf{k}) p_{||}^{s*}(\mathbf{k}') \rangle = (2\pi)^3 \underline{P_{11}^s(\mathbf{k})} \delta^D(\mathbf{k} - \mathbf{k}')$$

$$\left(\frac{k\mu}{\mathcal{H}}\right)^2 P_{11}^s(\mathbf{k}) = \left(\frac{k\mu}{\mathcal{H}}\right)^2 P_{11}^r(\mathbf{k}) - 2i \left(\frac{k\mu}{\mathcal{H}}\right)^3 \psi_{s,h}^r(r) - \left(\frac{k\mu}{\mathcal{H}}\right)^4 P_{13}^r(\mathbf{k}) + \left(\frac{k\mu}{\mathcal{H}}\right)^4 I_{s,h}^r(r)$$

Ratio of redshift space and real space  $P_{11}$



Momentum correlation function



# Conclusions

- Observables from kSZ survey is **momentum field (not velocity field)** in **redshift-space**.
- Expressions for velocity statistics in redshift space were derived.
- Power spectra were computed based on 1-loop nonlinear PT and compared to N-body results for halos/dark matter.
- Linear theory does not work even at very large scales.
- See *Okumura, Seljak, Vlah & Desjacques, (arXiv:1312.4214)* for the detail

## To improve precision

- Better understanding for shot noise is required for momentum auto power spectrum  $P_{11}$
- We have a good model for halos, but not yet for galaxies (1 halo term, Finger-of-God, halo models, etc).