Unitarity and Electroweak oblique corrections

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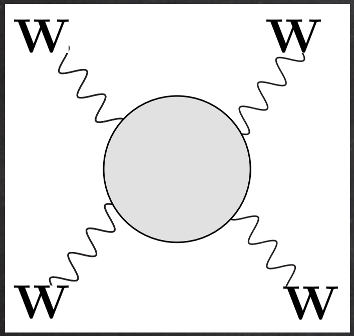
Does unitarity imply finiteness of electroweak oblique corrections?
R. Nagai, M. Tanabashi, KT, Phys. Rev. D91, 034030 (2015)

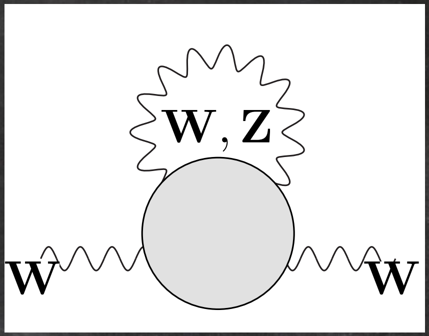
What this talk is about

I am not going to discuss DM (-_-;)...



Unitarity

?



EW oblique corrections

Higgs 2014

Citation: K.A. Olive *et al.* (Particle Data Group), Chin. Phys. **C38**, 090001 (2014) (URL: http://pdg.lbl.gov)

H^0 $J = 0$

Mass $m = 125.7 \pm 0.4$ GeV

H^0 Signal Strengths in Different Channels

Combined Final States $= 1.17 \pm 0.17$ (S = 1.2)

$WW^* = 0.87^{+0.24}_{-0.22}$

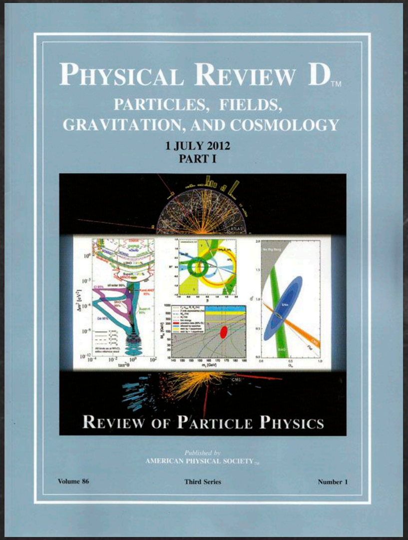
$ZZ^* = 1.11^{+0.34}_{-0.28}$ (S = 1.3)

$\gamma\gamma = 1.58^{+0.27}_{-0.23}$

$b\bar{b} = 1.1 \pm 0.5$

$\tau^+\tau^- = 0.4 \pm 0.6$

$Z\gamma < 9.5$, CL = 95%



(Not yet included in PDG2014)

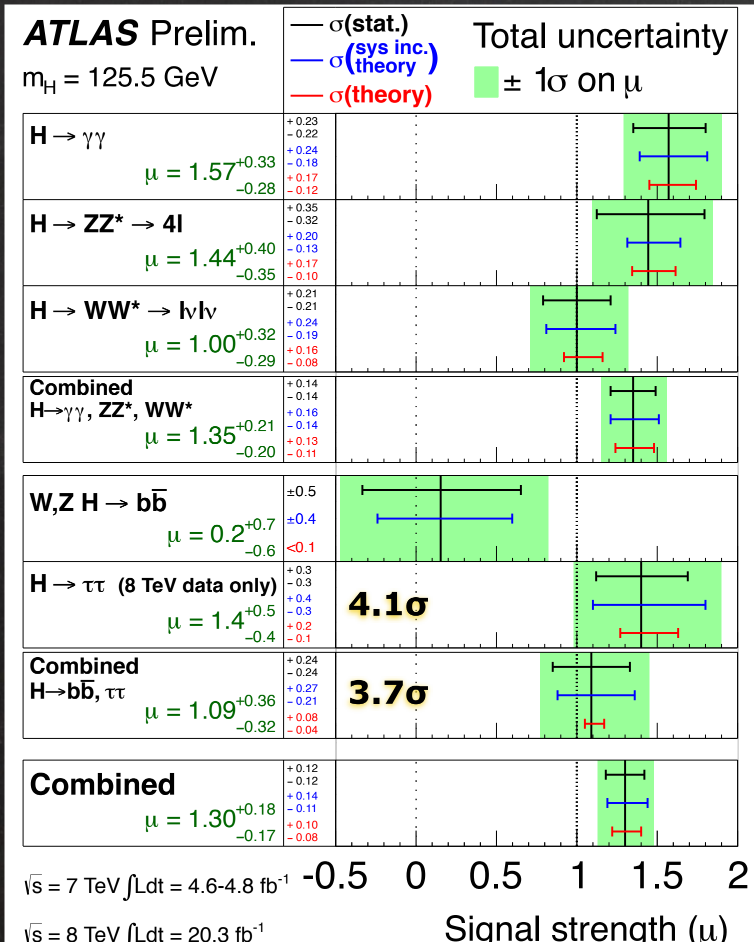
Diphoton-Excess : > 2σ → 1σ

Discrepancy of Mh in ZZ & γγ : 2.5 σ → within 2σ

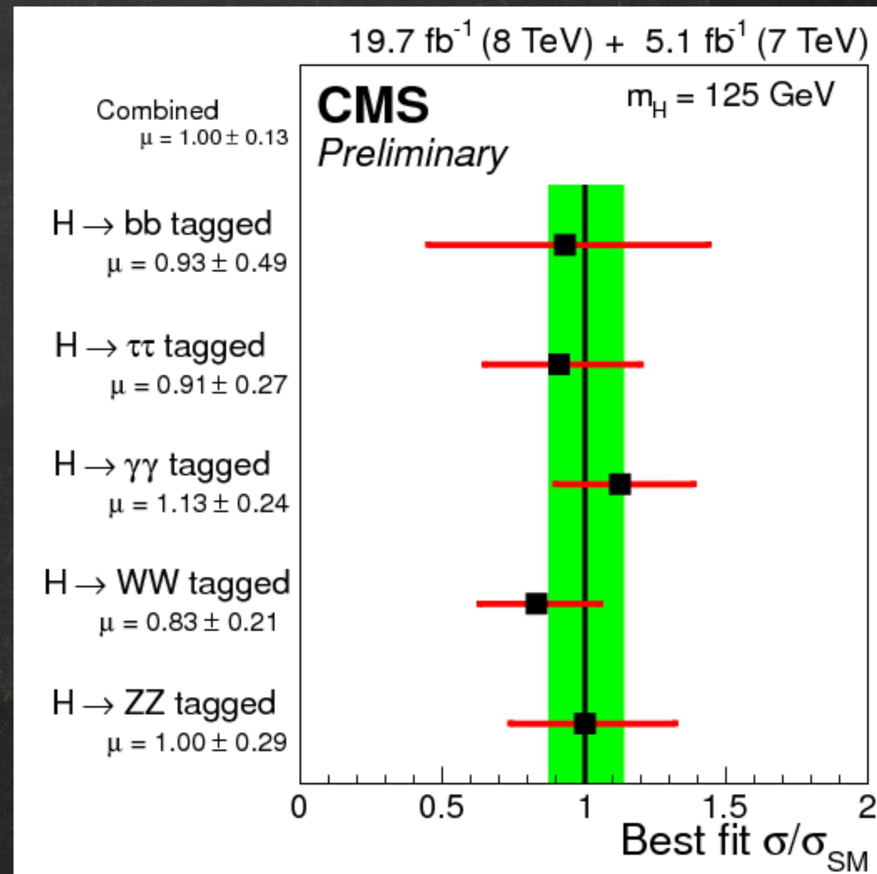
Fermionic decay channels (ττ & bb) : 2σ → > 4σ

Signal strength

$$\mu_{XX} = \frac{\sigma \times \mathcal{B}(h \to XX)}{\sigma^{\rm SM} \times \mathcal{B}^{\rm SM}(h \to XX)}$$



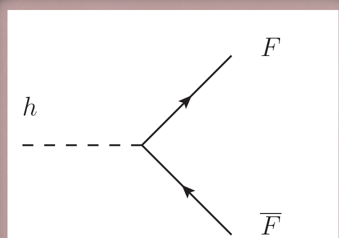
Combined $\mu = 1.30^{+0.18}_{-0.17}$



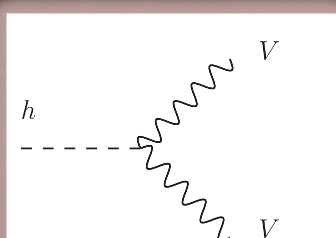
Relation between μ and Higgs Couplings

Scaling Factors

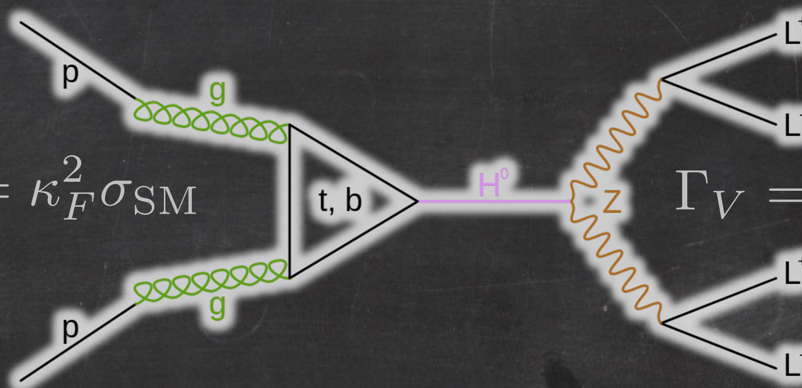
$$\kappa_F = \lambda_{hFF}/\lambda_{hFF}^{\rm SM}$$



$$\kappa_V = \lambda_{hVV}/\lambda_{hVV}^{\rm SM}$$



$$\sigma = \kappa_F^2 \sigma_{\rm SM}$$



$$\Gamma_V = \kappa_V^2 \Gamma_V^{\rm SM}$$

$$\mu \equiv \frac{\sigma \times \mathcal{B}}{\sigma_{\rm SM} \times \mathcal{B}_{\rm SM}} \simeq \kappa_V^2$$

$$\mathcal{B}_V \equiv \frac{\Gamma_V}{\Gamma_{\rm tot}} = \frac{\kappa_V^2 \Gamma_V^{\rm SM}}{\kappa_F^2 \Gamma_F^{\rm SM} + \kappa_V^2 \Gamma_V^{\rm SM}} \simeq \frac{\kappa_V^2}{\kappa_F^2} \mathcal{B}_V{}^{\rm SM}$$

Note : fermionic (& gluonic via Yukawa) decay dominate (~75%)

Relation between μ and Higgs Couplings

Scaling Factors

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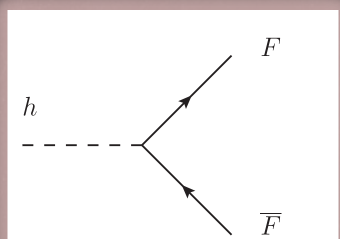

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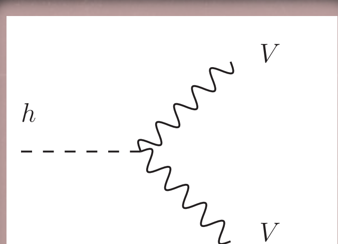
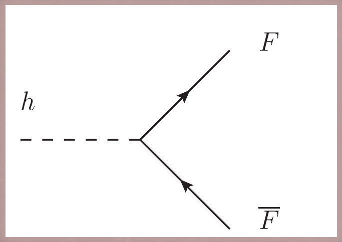
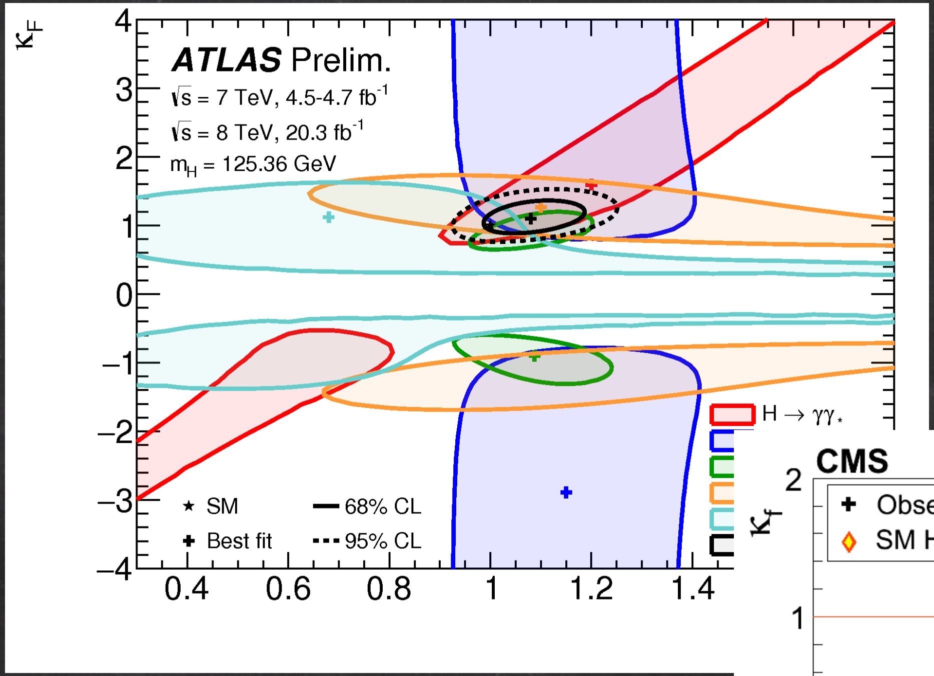


Production	Decay	LO SM	
VH	H→bb	$\sim(K_V^2 x K_F^2)/K_F^2$	$\sim K_V^2$
ttH	H→bb	$\sim(K_F^2 x K_F^2)/K_F^2$	$\sim K_F^2$
VBF/VH	H→ττ	$\sim(K_V^2 x K_F^2)/K_F^2$	$\sim K_V^2$
ggH	H→ττ	$\sim(K_F^2 x K_F^2)/K_F^2$	$\sim K_F^2$
ggH	H→zz	$\sim(K_F^2 x K_V^2)/K_F^2$	$\sim K_V^2$
ggH	H→ww	$\sim(K_F^2 x K_V^2)/K_F^2$	$\sim K_V^2$
VBF/VH	H→ww	$\sim(K_V^2 x K_V^2)/K_F^2$	$\sim K_V^4/K_F^2$
ggH	H→γγ	$\sim K_F^2(8.6K_V-1.8K_F)^2/K_F^2$	$\sim K_V^2$
VBF	H→γγ	$\sim K_V^2(8.6K_V-1.8K_F)^2/K_F^2$	$\sim K_V^4/K_F^2$

Higgs Coupling @ LHC

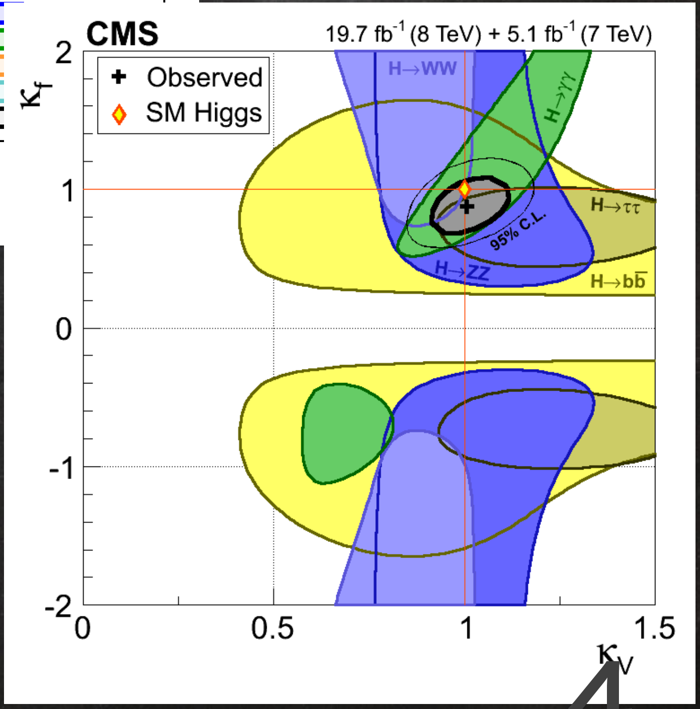
$$\kappa_F = \lambda_{hFF}/\lambda_{hFF}^{\rm SM}$$





$$\kappa_V = 1.15 \pm 0.08$$

$$\kappa_F = 0.99^{+0.17}_{-0.15}$$



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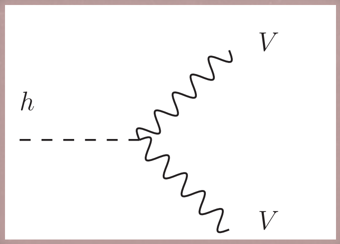


?

Do we arbitrary choose Higgs coupling?

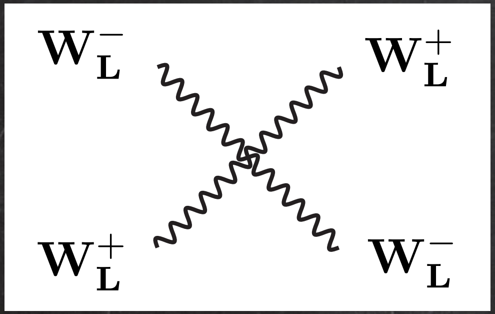
Ad hoc deviations may violate theory consistency ?

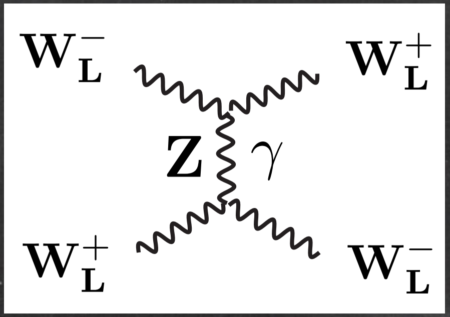
Role of Higgs boson(s)

● Unitarization of Amplitudes

$$\epsilon^{\mu}_{(L)} = \frac{E}{m_W}\begin{pmatrix} \frac{|\vec{p}|}{E} \\ \frac{\vec{p}}{|\vec{p}|} \end{pmatrix}$$

If the Higgs boson is absent





+ crossed.

$$\mathcal{M}_{\times} = g_{WWWW}\left\{+\frac{E^4}{m_W^2}X_1 + \frac{E^2}{m_W^2}X_2\right\}$$

$$\mathcal{M}_{t+u} = g^2_{WWW}\left\{-\frac{E^4}{m_W^2}\left(X_1 + \frac{m_W^2}{E^2}X_2\right) + \frac{E^2}{m_W^2}Y + (\cdots)\right\}$$

Gauge Sym.

$$g_{WWWW} = g^2_{WWW} = g^2$$

cancel E^4 behavior

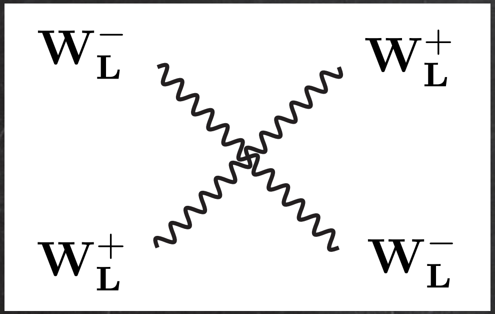
$$\mathcal{M}_{\text{gauge}} = \frac{4E^2}{v^2}Y + (\cdots)$$

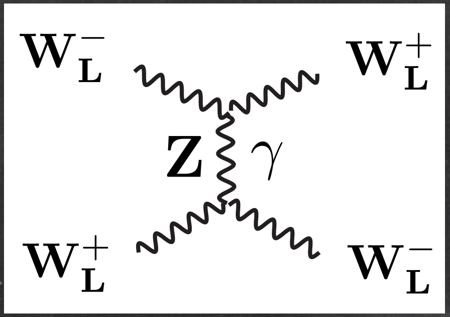
Amplitude violates perturbative unitarity at high energy (>1TeV)

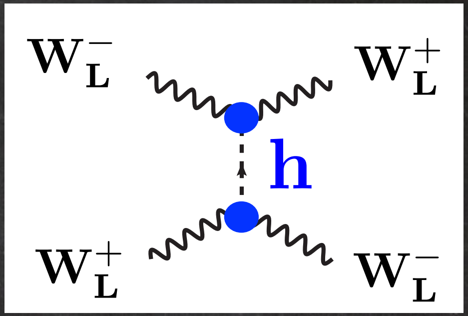
Role of Higgs boson(s)

● Unitarization of Amplitudes

If the Higgs boson exists







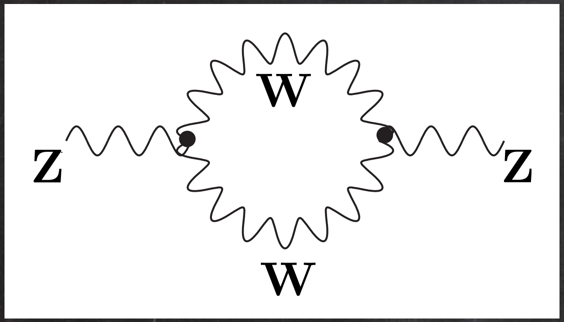
$K_W^2=1$ is required to cancel E^2 behavior

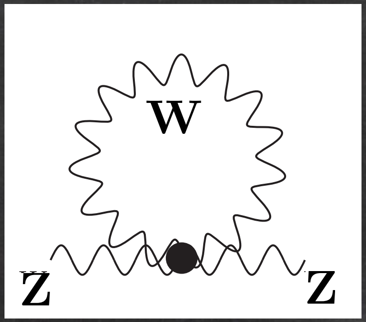
$$\mathcal{M}_{\rm gauge} = \frac{4E^2}{v^2}Y + (\cdots) \qquad \mathcal{M}_{\rm Higgs} = -\kappa_W^2\frac{4E^2}{v^2}Y + (\cdots)$$

Role of Higgs boson(s)

- Stabilization of Quantum Corrections

If the Higgs boson is absent





Electroweak Oblique Parameters (Peskin-Takeuchi parameters) diverge

$$S = \frac{1}{12\pi} \ln \Lambda^2$$

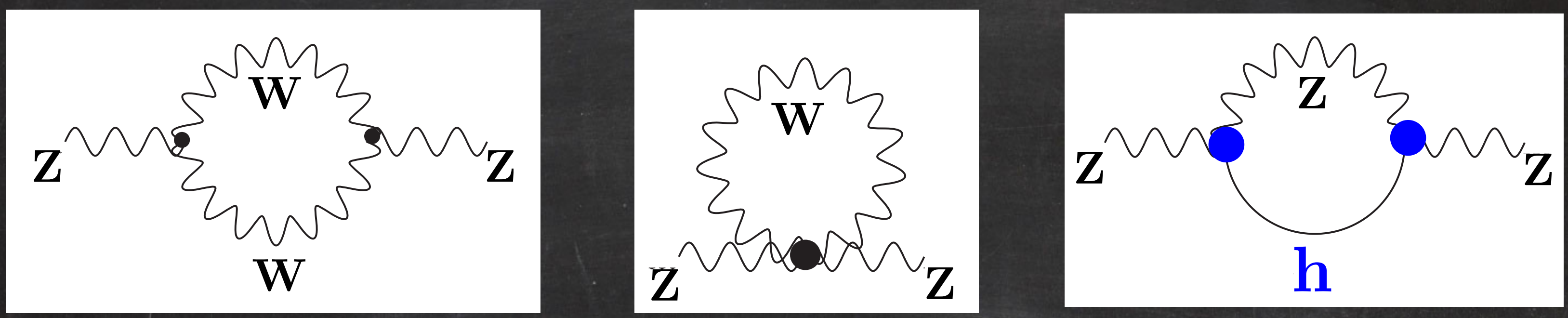
$$\alpha T = -\frac{3}{4} \frac{g_Y^2}{(4\pi)^2} \ln \Lambda^2$$

without Higgs boson(s), we cannot perform EW precision study

Role of Higgs boson(s)

- Stabilization of Quantum Corrections

If the Higgs boson exists



Electroweak Oblique Parameters (Peskin-Takeuchi parameters) diverge

$$S = \frac{1}{12\pi} \ln \Lambda^2 - \kappa_Z^2 \frac{1}{12\pi} \ln \frac{\Lambda^2}{M_h^2}$$

$$\alpha T = -\frac{3}{4} \frac{g_Y^2}{(4\pi)^2} \ln \Lambda^2 + \kappa_Z^2 \frac{3}{4} \frac{g_Y^2}{(4\pi)^2} \ln \frac{\Lambda^2}{M_h^2}$$

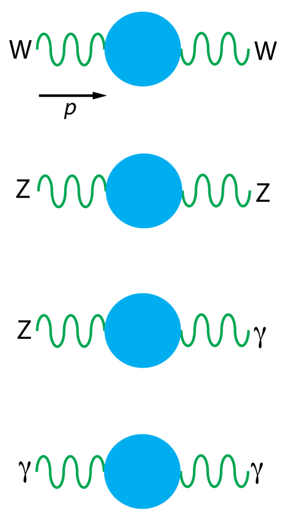
$K_Z^2=1$ is required to cancel $\ln\Lambda^2$

Oblique Parameters

Peskin-Takeuchi 90

Tatsu Takeuchi

- The scale of new physics is large compared to the electroweak scale



$$\Pi_{WW}(p^2) = \Pi_{WW}(0) + p^2\Pi'_{WW}(0) + \cdots$$

$$\Pi_{ZZ}(p^2) = \Pi_{ZZ}(0) + p^2\Pi'_{ZZ}(0) + \cdots$$

$$\Pi_{Z\gamma}(p^2) = p^2\Pi'_{Z\gamma}(0) + \cdots$$

$$\Pi_{\gamma\gamma}(p^2) = p^2\Pi'_{\gamma\gamma}(0) + \cdots$$

Finite combinations (S, T, U) after the renormalization of EW parameters, e.g., α_{EM}, M_Z, G_F

Tatsu Takeuchi

finite) parameters, three linear
are absorbed into the three input
, G_F, and M_Z and are unobservable.

ing (finite) parameters can be taken to be:

$$\alpha S = 4s^2c^2\left[\Pi'_{ZZ}(0) - \frac{c^2-s^2}{sc}\Pi'_{Z\gamma}(0) - \Pi'_{\gamma\gamma}(0)\right]$$

$$\alpha T = \frac{\Pi_{WW}(0)}{M_W^2} - \frac{\Pi_{ZZ}(0)}{M_Z^2}$$

$$\alpha U = 4s^2\left[\Pi'_{WW}(0) - c^2\Pi'_{ZZ}(0) - 2sc\Pi'_{Z\gamma}(0) - s^2\Pi'_{\gamma\gamma}(0)\right]$$

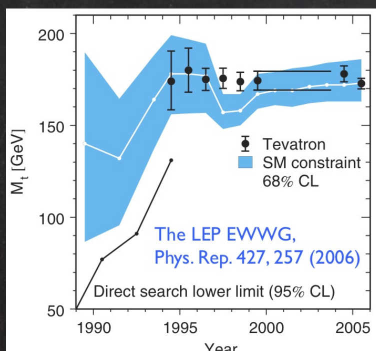
Mass Renormalizations
→ ρ parameter

Wave Function Renormalizations

Success of Oblique Parameters

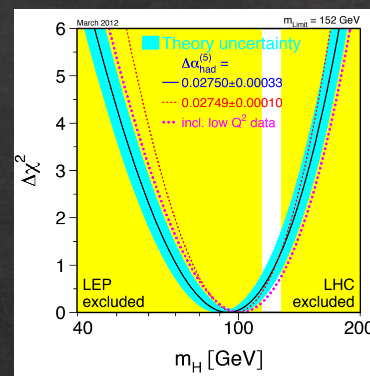
- T (~ρ) [0.1% precision]

Predict M_t



$$\delta\rho_t \simeq \frac{3G_F m_t^2}{8\sqrt{2}\pi^2}$$

Predict M_h



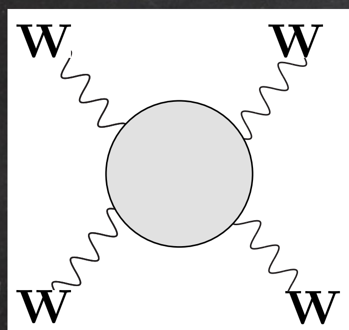
$$\delta\rho_h \simeq -\frac{3G_F m_Z^2 s_W^2}{8\sqrt{2}\pi^2}\left(\log\frac{m_h^2}{m_W^2} - \frac{5}{6}\right)$$

- S Kill the techni-color models, 5^{th} generations, …

SM is tested at loop level !!

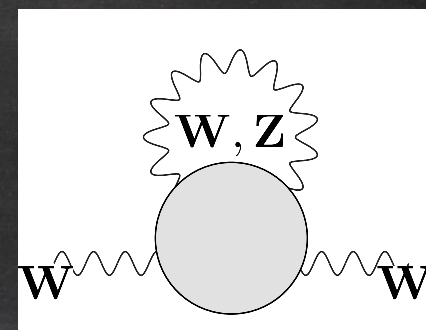
Unitarity vs Oblique Parameters

- h_{SM} (κ_V=1) unitarizes $V_L V_L$ scattering, and stabilize oblique corrections, simultaneously.



Unitarity

?



Finiteness of oblique parameters

What relation in more general framework of the Higgs couplings

Mass-Coupling Relations

- EWSB and Mass generation

→ Mass-Coupling relation is required

$$M_V^2 = \frac{g_V^2}{4} v^2, \quad M_F = \frac{Y_F}{\sqrt{2}} v, \quad M_h^2 = 2\lambda v^2$$

Sources of (possible) coupling deviations :

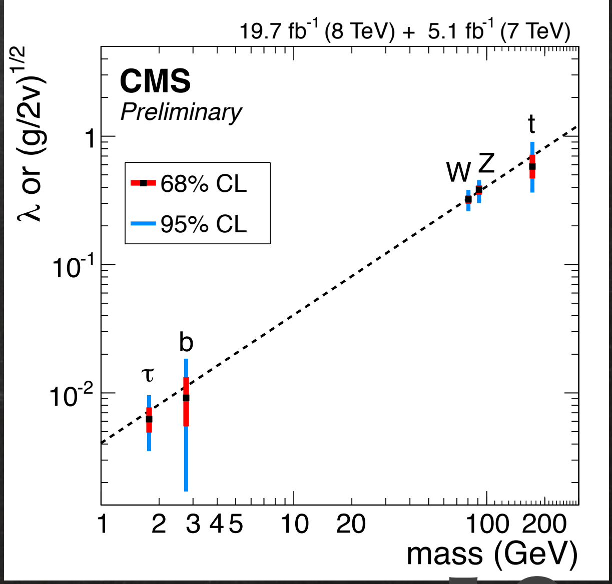
✓ Higgs mixing (universal shift)

✓ VEV mixing (many variations)

⇕

Coupling deviation = 2nd Higgs

New d.o.f. is required for the theory consistency



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Custodial Symmetry

before going to the unitarity and oblique corrections

What is Custodial Symmetry?

$$M_W \simeq M_Z$$

- M_W and M_Z are nearly degenerate
  → Underlying symmetry?

- g'→ 0 limit : $M_W = M_Z (M_{W3})$

- Define the measure of the (custodial) symmetry

$$\rho = \frac{M_W^2}{M_Z^2 c_W^2} = 1$$

ρ=1 is guaranteed by this symmetry

Chiral Sym. Breaking

- Consider 2 component fermions

Φ : 2 x 2 matrix

$$\mathcal{L} = i\,\overline{\Psi}_L \gamma^\mu D_\mu \Psi_L + i\,\overline{\Psi}_R \gamma^\mu D_\mu \Psi_R - y\,(\overline{\Psi}_L \Phi \Psi_R + \overline{\Psi}_R \Phi^\dagger \Psi_L)$$

L is invariant under $SU(2)_L$ x $SU(2)_R$

$SU(2)_L$
$$\begin{cases} \Psi_L \to U_L \Psi_L \\ \Psi_R \to \Psi_R \\ \Phi \to U_L \Phi \end{cases}$$

$SU(2)_R$
$$\begin{cases} \Psi_L \to \Psi_L \\ \Psi_R \to U_R \Psi_R \\ \Phi \to U_R^\dagger \phi \end{cases}$$

	$\overline{\Psi}_L$	Ψ_R	ϕ
L	$\underline{\mathbf{2}}$	$\mathbf{1}$	$\mathbf{2}(\times 2)$
R	$\mathbf{1}$	$\mathbf{2}$	$\underline{\mathbf{2}}(\times 2)$

Different view : $SU(2)_V$ x $SU(2)_A$

Parallel
$$\begin{cases} \Psi_L \to U \Psi_L \\ \Psi_R \to U \Psi_R \\ \Phi \to U \Phi U^\dagger \end{cases}$$

Opposite
$$\begin{cases} \Psi_L \to V \Psi_L \\ \Psi_R \to V^\dagger \Psi_R \\ \Phi \to V \Phi V \end{cases}$$

	$\overline{\Psi}_L$	Ψ_R	ϕ
L+R	$\underline{\mathbf{2}}$	$\mathbf{2}$	$\mathbf{1} + \mathbf{3}$
L-R	$\underline{\mathbf{2}}$	$\underline{\mathbf{2}}$	$\mathbf{1} + \mathbf{3}$

As the global symmetries, they are equivalent.
In the SM, only the $SU(2)_L$ is gauged.

Chiral Sym. Breaking

- Consider 2 component fermions

Φ : 2 x 2 matrix

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$$\begin{cases} \Psi_L \to U_L \Psi_L \\ \Psi_R \to \Psi_R \\ \Phi \to U_L \Phi \end{cases} \qquad \begin{cases} \Psi_L \to \Psi_L \\ \Psi_R \to U_R \Psi_R \\ \Phi \to U_R^\dagger \phi \end{cases}$$

VEV breaks both $SU(2)_L$ and $SU(2)_R$

$$\Phi \longrightarrow \langle \Phi \rangle$$

Chiral Sym. Breaking

- Consider 2 component fermions

Φ : 2 x 2 matrix

$$\mathcal{L} = i\,\overline{\Psi}_L \gamma^\mu D_\mu \Psi_L + i\,\overline{\Psi}_R \gamma^\mu D_\mu \Psi_R - y\left(\overline{\Psi}_L \Phi \Psi_R + \overline{\Psi}_R \Phi^\dagger \Psi_L\right)$$

To keep "Parallel" symmetry : $SU(2)_V$

$$\Phi \to \langle \Phi \rangle = v \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

This is automatically realized in the SM EWSB.

Caution : This is the specialty of the $SU(2)_L$ doublet

Parallel
$$\begin{cases} \Psi_L \to U\,\Psi_L \\ \Psi_R \to U\,\Psi_R \\ \Phi \to U \Phi U^\dagger \end{cases}$$

Opposite
$$\begin{cases} \Psi_L \to V\,\Psi_L \\ \Psi_R \to V^\dagger\,\Psi_R \\ \Phi \to V \Phi V \end{cases}$$

$SU(2)_V$ remains

$SU(2)_A$ is broken

Chiral Sym. Breaking

- Consider 2 component fermions

Φ : 2 x 2 matrix

$$\mathcal{L} = i\,\overline{\Psi}_L \gamma^\mu D_\mu \Psi_L + i\,\overline{\Psi}_R \gamma^\mu D_\mu \Psi_R - y\,(\overline{\Psi}_L \Phi \Psi_R + \overline{\Psi}_R \Phi^\dagger \Psi_L)$$

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Parallel

$$\begin{cases} \Psi_L \to U\,\Psi_L \\ \Psi_R \to U\,\Psi_R \\ \Phi \to U \Phi U^\dagger \end{cases}$$

$SU(2)_V$ remains

Fields are classified by remaining sym.

	$\overline{\Psi}_L$	Ψ_R	ϕ
L+R	$\underline{\mathbf{2}}$	$\mathbf{2}$	$\mathbf{1}+\mathbf{3}$
~~L-R~~	~~$\underline{\mathbf{2}}$~~	~~$\underline{\mathbf{2}}$~~	~~$\mathbf{1}+\mathbf{3}$~~

Why $M_W \simeq M_Z$ in the SM

- Higgs pot. is invariant under SU(2)$_L$ x SU(2)$_R$ $\quad \Phi \to U_L \, \Phi \, U_R^\dagger$

$$\Phi = \begin{pmatrix} \phi^{0*} & \phi^+ \\ \phi^- & \phi^0 \end{pmatrix}$$

Embed 4 components (a complex doublet)

$$V(\Phi) = \frac{1}{2}\mu^2 \, \mathrm{Tr}(\Phi^\dagger \Phi) + \frac{1}{4}\lambda \left[\mathrm{Tr}(\Phi^\dagger \Phi)\right]^2$$

Usual notation :

$$H = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \begin{pmatrix} \varphi_2 + i\,\varphi_3 \\ \varphi_0 + i\,\varphi_1 \end{pmatrix}$$

$$V(H) = \mu^2 \, H^\dagger H + \lambda \, (H^\dagger H)^2$$

$SO(4) \cong SU(2) \times SU(2)$

Correspondence

$$\begin{aligned} H^\dagger H &= |\phi^0|^2 + \phi^+ \phi^- \\ &= \varphi_0^2 + \varphi_1^2 + \varphi_2^2 + \varphi_3^2 \\ &= \tfrac{1}{2}\mathrm{Tr}(\Phi^\dagger \Phi) \end{aligned}$$

EWSB

$\mathbf{1}_{SU(2)_V}$ $\quad$ $\mathbf{3}_{SU(2)_V}$

$$\left(\langle \varphi_0 \rangle + \varphi_0\right)^2 + \varphi_1^2 + \varphi_2^2 + \varphi_3^2$$

$SO(3) \cong SU(2)$

Why $M_W \simeq M_Z$ in the SM

- BEH mechanism

	W	H
L	$\mathbf{3}$	$\mathbf{2} \times 2$
R	$\mathbf{1}$	$\underline{\mathbf{2}} \times 2$
L+R	$\mathbf{3}$	$\mathbf{1} + \mathbf{3}$

$$|D_\mu H|^2 = |(\partial_\mu - i\, g\, \sigma_a W_\mu^a/2)H|^2$$

$$= |\partial_\mu H|^2 + \frac{g^2}{2}\underbrace{(W_1^2 + W_2^2 + W_3^2)}_{\text{SU(2)}_V \text{ triplet after EWSB}} H^\dagger H + \left(+ i\, g\, H^\dagger T_a \partial_\mu H\, W_a^\mu/2 + \text{H.c.}\right)$$

$(\mathbf{1} \times \mathbf{1}) + (\mathbf{3} \times \mathbf{3})$

Kinetic terms of Higgs (& NGB)

$(\mathbf{1} \times \mathbf{1}) + (\mathbf{3} \times \mathbf{3})$

Quartic int of Higgs(NGB)-VV

(1x1) term gets VEV → BEH mech.

$(\mathbf{1} \times \mathbf{3} \times \mathbf{3}_W) + (\mathbf{3} \times \mathbf{1} \times \mathbf{3}_W)$

Higgs(NGB) derivative int

Weak bosons are the triplet under the custodial symmetry

$$M_W = M_Z (M_{W3})$$

Caution : This is the specialty of the SU(2)$_L$ doublet

Why $M_W \simeq M_Z$ in the SM

- Yukawa interations

$$\overline{Q_L} H\, b_R, \overline{Q_L} \tilde{H}\, t_R$$

Forbid

	H	Q_L	t_R	b_R
L	$\mathbf{2} \times 2$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$
R	$\underline{\mathbf{2}} \times 2$	$\mathbf{1}$	~~1~~ $\mathbf{2}$	~~1~~
L+R	$\mathbf{1} + \mathbf{3}$	~~2~~	– $\mathbf{2}$	–

Yukawa int. breaks the custodial symmetry

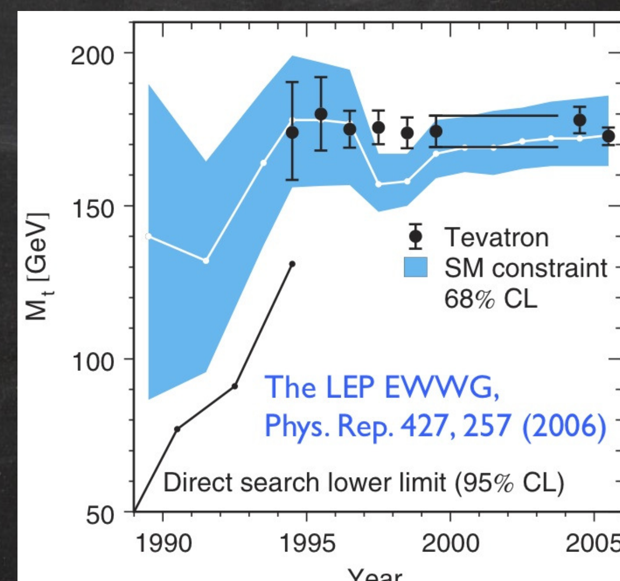
Let us symmetrize it

$$\overline{Q_L}\, \Phi\, Q_R \qquad Q_R \equiv \begin{pmatrix} t_R \\ b_R \end{pmatrix}$$

$M_t = M_b$ is the consequence of the sym.

The breaking effect appears in ρ parameter

$$\delta\rho_t \simeq \frac{3G_F(M_t - M_b)^2}{8\sqrt{2}\pi^2}$$



Why $M_W \simeq M_Z$ in the SM

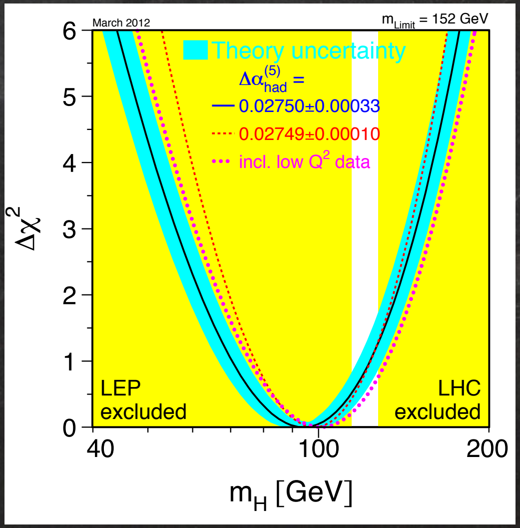
- U(1)$_Y$ also breaks the custodial symmetry

We define the ρ parameter by caring this point

$$\rho = \frac{M_W^2}{M_Z^2 c_W^2} = 1$$

However, the breaking effect appears at 1-loop level

$$\delta\rho_h \simeq -\frac{3g'^2}{64\pi^2}\left(\log\frac{m_h^2}{m_W^2} - \frac{5}{6}\right)$$



?

What happen if we begin with Unitarity ?

The Model (EW chiral Lagrangian w/ Higgs bosons)

- A non-linear realization of $SU(2)_L \times U(1)_Y \rightarrow U(1)_{EM}$

Appelquist-Bernard 1980, 1981

$$\mathcal{L}_\chi = \frac{v^2}{4}\mathrm{tr}[(D_\mu U)^\dagger (D^\mu U)] + \beta\,\frac{v^2}{4}\mathrm{tr}[U^\dagger (D^\mu U)\tau_3]\mathrm{tr}[U^\dagger (D^\mu U)\tau_3]$$

$$\begin{cases} U = \exp(i\,\widetilde{\omega^a}\tau^a) \\ D_\mu U = \partial_\mu U + i\,g\,W_\mu^a \frac{\tau^a}{2} U - i\,g_Y\,U B_\mu \frac{\tau^3}{2} \\ D_\mu U \rightarrow G_L (D_\mu U) G_Y^\dagger \end{cases}$$

Pion (NGBs) absorbed by W, Z

Unitarity Gauge (U=1)

$$\mathcal{L}_\chi \rightarrow +\frac{g^2 v^2}{4} W_\mu^+ W^{-\mu} + \frac{g_Z^2 v_Z^2}{4}\frac{1}{2!} Z_\mu Z^\mu$$

$$\begin{cases} g_Z^2 = g^2 + g_Y^2 \\ v_Z^2 = (1-2\beta)v^2 \end{cases}$$

The Model (EW chiral Lagrangian w/ Higgs bosons)

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ρ parameter (at tree-level)

$$\rho_0 = \frac{G_{NC}}{G_{CC}} = \frac{v^2}{v_Z^2} = \frac{1}{1-2\beta}$$

(ρ_0 is kept arbitrary unlike the SM; $\rho_{0(SM)}=1$)

Unitarity Gauge (U=1)

$$\mathcal{L}_\chi \rightarrow +\frac{g^2 v^2}{4} W_\mu^+ W^{-\mu} + \frac{g_Z^2 v_Z^2}{4}\frac{1}{2!} Z_\mu Z^\mu$$

$$\begin{cases} g_Z^2 = g^2 + g_Y^2 \\ v_Z^2 = (1-2\beta)v^2 \end{cases}$$

The Model (EW chiral Lagrangian w/ Higgs bosons)

- Higgs boson(s) as matter

Appelquist-Bernard 1980, 1981

$$\mathcal{L}_\phi = -\,v \sum_{n=1}^{N} \kappa_{WW}^{\phi_n^0}\, \phi_n^0\, \mathrm{tr}[U^\dagger (D_\mu U)\tau_+]\mathrm{tr}[U^\dagger (D^\mu U)\tau_-] - \frac{v}{4} \sum_{n=1}^{N} \kappa_{ZZ}^{\phi_n^0}\, \phi_n^0\, \mathrm{tr}[U^\dagger (D_\mu U)\tau_3]\mathrm{tr}[U^\dagger (D^\mu U)\tau_3]$$

arbitrary number (N), arbitrary coupling (K_W and K_Z are independent) of scalar bosons

This treatment is more general than the SM with arbitrary multiplets of Higgs fields, such like additional singlet(s), doublet(s) , triplet(s), ⋯, septet(s), ⋯.

K is defined relative to the SM Higgs coupling, i.e., $K^1=1$, $K^n=0$ ($n\geq2$)

Unitarity Gauge (U=1)

$$\mathcal{L}_\phi^{\mathrm{SM}} \to -\,v\, \kappa_{WW}^{\phi_1^0}\, \phi_1^0 \left(i\, \frac{g}{\sqrt{2}}\, W_\mu^+\right)\left(i\, \frac{g}{\sqrt{2}}\, W^{\mu -}\right) - \frac{v}{4}\, \kappa_{ZZ}^{\phi_1^0}\, \phi_1^0 \left(i\, g_Z\, Z_\mu\right)\left(i\, g_Z\, Z^\mu\right)$$

$$\phi_1^0 = h, \kappa_{WW}^{\phi_1^0} = \kappa_{ZZ}^{\phi_1^0} = 1$$

The Model (EW chiral Lagrangian w/ Higgs bosons)

- Higgs boson(s) as matter

Appelquist-Bernard 1980, 1981

$$\mathcal{L}_\phi = -v\sum_{n=1}^{N}\kappa_{WW}^{\phi_n^0}\,\phi_n^0\,\mathrm{tr}[U^\dagger(D_\mu U)\tau_+]\mathrm{tr}[U^\dagger(D^\mu U)\tau_-] - \frac{v}{4}\sum_{n=1}^{N}\kappa_{ZZ}^{\phi_n^0}\,\phi_n^0\,\mathrm{tr}[U^\dagger(D_\mu U)\tau_3]\mathrm{tr}[U^\dagger(D^\mu U)\tau_3]$$

arbitrary number (N), arbitrary coupling (K_W and K_Z are independent) of scalar bosons

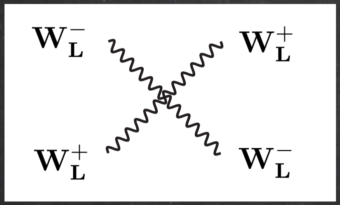
Most general interactions up to mass dimension 4 ;

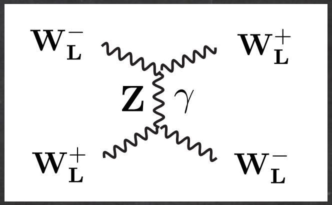
$$\mathcal{L}_{\phi\overleftrightarrow{\partial}\phi} = -\frac{i}{4}\sum_{n,m=1}^{N}\kappa_Z^{\phi_n^0\phi_m^0}\,(\phi_n^0\overleftrightarrow{\partial}_\mu\phi_m^0)\,\mathrm{tr}[U^\dagger(D^\mu U)\tau_3]$$

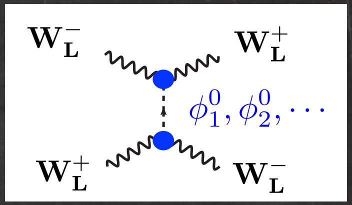
$$\mathcal{L}_{\phi\phi} = -\frac{1}{2}\sum_{n,m=1}^{N}\kappa_{WW}^{\phi_n^0\phi_m^0}\,\phi_n^0\phi_m^0\,\mathrm{tr}[U^\dagger(D_\mu U)\tau_+]\mathrm{tr}[U^\dagger(D^\mu U)\tau_-] - \frac{1}{8}\sum_{n,m=1}^{N}\kappa_{ZZ}^{\phi_n^0\phi_m^0}\,\phi_n^0\phi_m^0\,\mathrm{tr}[U^\dagger(D_\mu U)\tau_3]\mathrm{tr}[U^\dagger(D^\mu U)\tau_3]$$

Implication of Unitarity

$W_L W_L$ scattering :







$$\mathcal{A}(W_L^+ W_L^- \to W_L^+ W_L^-) \sim \frac{\mathbf{u}}{v^2} \left\{ -4 + \frac{3}{\rho_0} + \sum_{n=1}^{N} (\kappa_{WW}^{\phi_n^0})^2 \right\}$$

Absence of perturbative unitarity violation at (very) high energy

$$-4 + \frac{3}{\rho_0} + \sum_{n=1}^{N} (\kappa_{WW}^{\phi_n^0})^2 = 0$$

A “unitarity” sum-rule among Higgs couplings is found!!

Implication of Unitarity

A set of "unitarity sum-rules"

$$\mathbf{W_L^+ W_L^- \to W_L^+ W_L^-} : \quad -4 + \frac{3}{\rho_0} + \sum_{n=1}^{N} (\kappa_{WW}^{\phi_n^0})^2 = 0$$

$$\mathbf{W_L^+ W_L^- \to Z_L Z_L} : \quad \frac{1}{\rho_0} - \rho_0 \sum_{n=1}^{N} \kappa_{WW}^{\phi_n^0} \kappa_{ZZ}^{\phi_n^0} = 0$$

$$\mathbf{W_L^+ W_L^- \to Z_L} \phi_\mathbf{n}^\mathbf{0} : \quad \sum_{m=1}^{N} \kappa_Z^{\phi_n^0 \phi_m^0} \kappa_{WW}^{\phi_m^0} = 0 \quad \text{and} \quad \kappa_{WW}^{\phi_m^0} - \rho_0 \kappa_{ZZ}^{\phi_m^0} = 0$$

$$\mathbf{W_L^+ W_L^-} \to \phi_\mathbf{n}^\mathbf{0} \phi_\mathbf{n'}^\mathbf{0} : \quad \kappa_{WW}^{\phi_n^0} \kappa_{WW}^{\phi_{n'}^0} - \kappa_{WW}^{\phi_n^0 \phi_{n'}^0} = 0 \quad \text{and} \quad \kappa_Z^{\phi_n^0 \phi_{n'}^0} = 0$$

$$\mathbf{Z_L Z_L} \to \phi_\mathbf{n}^\mathbf{0} \phi_\mathbf{n'}^\mathbf{0} : \quad \rho_0 \kappa_{ZZ}^{\phi_n^0} \kappa_{ZZ}^{\phi_{n'}^0} - \kappa_{ZZ}^{\phi_n^0 \phi_{n'}^0} + \sum_{m=1}^{N} \kappa_Z^{\phi_n^0 \phi_m^0} \kappa_Z^{\phi_{n'}^0 \phi_m^0} = 0$$

Minimal conditions for Higgs couplings from Unitarity

Implication of Unitarity

Simplified conditions : (Neutral Higgs bosons only)

$$\kappa_Z^{\phi_n^0 \phi_{n'}^0} = 0$$

Charged Higgs boson(s) are required in order to have CP odd Higgs boson

$$\rho_0 = 1$$

$$\kappa_{WW}^{\phi_m^0} = \kappa_{ZZ}^{\phi_m^0}$$

Custodial Sym. is NOT imposed !!

ρ_0=1 is known as a consequence of the custodial symmetry (at least, many textbooks say so)

$$\sum_{n=1}^{N} (\kappa_{WW}^{\phi_n^0})^2 = 1$$

→ $K_V \leq 1$ ($K_V \geq 1$ is possible if we introduce H^{++})

$$\kappa_{WW}^{\phi_n^0} \kappa_{WW}^{\phi_{n'}^0} = \kappa_{WW}^{\phi_n^0 \phi_{n'}^0}$$

$$\kappa_{ZZ}^{\phi_n^0} \kappa_{ZZ}^{\phi_{n'}^0} = \kappa_{ZZ}^{\phi_n^0 \phi_{n'}^0}$$

Gauge Sym. is NOT imposed for Higgs int.

(Relations between 3- and 4 vertices)

Models without Custodial symmetry

$\rho = 1$ as a guiding principle

- Mass of weak bosons

$$M_W^2 = 2[j(j+1) - Y_\phi^2]\frac{g^2 v_{2j+1}^2}{4} \qquad M_Z^2 = (2Y_\phi)^2 \frac{g_Z^2 v_{2j+1}^2}{4}$$

$$M_W^2 = \frac{g^2 v^2}{4} \qquad M_Z^2 = \frac{g_Z^2 v^2}{4}$$

SM : J=1/2, Y=1/2 $\quad M_W^2 = \frac{g^2 v_2^2}{4}, M_Z^2 = \frac{g_Z^2 v_2^2}{4}$

Pure Triplet Model : J=1, Y=1 $\quad M_W^2 = \frac{g^2 v_3^2}{2}, M_Z^2 = g_Z^2 v_3^2$

Higgs Triplet Model (Sum) : $\quad M_W^2 = \frac{g^2 (v_2^2 + 2v_3^2)}{4}, M_Z^2 = \frac{g_Z^2 (v_2^2 + 4v_3^2)}{4}$

- Let ρ=1 as a guiding principle

$$\rho = \frac{2[j_\phi(j_\phi+1) - Y_\phi^2]}{(2Y_\phi)^2} = 1 \quad \Rightarrow \quad x^2 - 3y^2 = 1$$

Redefine to make them integers

$$x = 2j_\phi + 1, y = 2Y_\phi$$

$x = 2j_\phi + 1, y = 2Y_\phi$

Pell's Equation (in Number Theory)

$$x^2 - ny^2 = 1 \ (n = 3)$$

- ✧ Trivial Solution : (x, y)=(1,0) for arbitrary n — SM singlet : (j,Y)=(0,0)
- ✧ Fundamental Sol. : (x_1, y_1)=(2,1) for n=3 — SM doublet : (j,Y)=(½,½)
- ✧ General Sol. :

$$\begin{cases} x_k = \frac{1}{2}[(x_1 + y_1\sqrt{n})^k + (x_1 - y_1\sqrt{n})^k] \\ y_k = \frac{1}{2\sqrt{n}}[(x_1 + y_1\sqrt{n})^k - (x_1 - y_1\sqrt{n})^k] \end{cases}$$

Bhaskara II (1150)

→ Next Minimal Sol.: (x_2,y_2)=(7,4)

Septet with Y=2

→ Next to Next Minimal Sol. : 26plet w/ Y=15/2

$\rho = 1$ without the Custodial Sym.

$$\rho = \frac{2[j_\phi(j_\phi+1) - Y_\phi^2]}{(2Y_\phi)^2} = 1$$

- Minimal sol. / SM Higgs doublet with Y=½
- Next to ⋯ / Higgs septet with Y=2 (η^-, H^0, H^+, H^{++}, H^{+++}, H^{++++}, H^{+++++})

➡ $\kappa_W = \kappa_Z$ and $v = v_Z$

ρ_0=1 and its quantum stability, K_W=K_Z are no longer the consequence of the Custodial Symmetry.

We want to know what is really needed!!

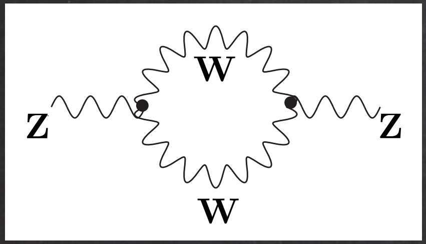
?

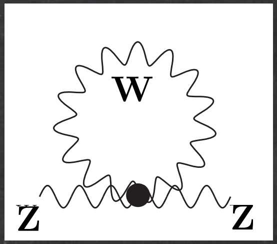
What happen if we begin

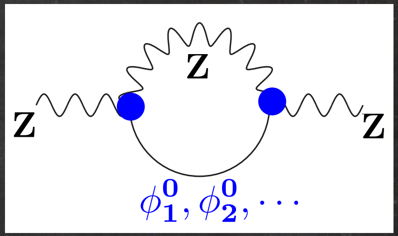
with Oblique Corrections ?

Implication of Finiteness

Oblique Corrections :







$$S = \frac{1}{12\pi} \ln \frac{\Lambda^2}{\mu^2} \left\{ + \frac{2}{\rho_0} - \frac{1}{\rho_0^2} - \rho_0 \sum_{n=1}^{N} (\kappa_{ZZ}^{\phi_n^0})^2 - \frac{1}{2} \sum_{n,m=1}^{N} (\kappa_Z^{\phi_n^0 \phi_m^0})^2 \right\}$$

Absence of divergence in the S parameter
(S [and U] is finite even if $\rho_0 \neq 1$ unlike T)

$$+\frac{2}{\rho_0} - \frac{1}{\rho_0^2} - \rho_0 \sum_{n=1}^{N} (\kappa_{ZZ}^{\phi_n^0})^2 - \frac{1}{2} \sum_{n,m=1}^{N} (\kappa_Z^{\phi_n^0 \phi_m^0})^2 = 0$$

A condition among Higgs couplings is found!!

Implication of Finiteness

A set of conditions (for ρ_0=1)

$$\mathbf{S}\,(\log\boldsymbol{\Lambda}^2):\quad +\frac{2}{\rho_0}-\frac{1}{\rho_0^2}-\rho_0\sum_{n=1}^{N}(\kappa_{ZZ}^{\phi_n^0})^2-\frac{1}{2}\sum_{n,m=1}^{N}(\kappa_{Z}^{\phi_n^0\phi_m^0})^2=0$$

$$\mathbf{T}\,(\boldsymbol{\Lambda}^2):\quad \sum_{n=1}^{N}\Big\{+\kappa_{WW}^{\phi_n^0\phi_n^0}-2(\kappa_{WW}^{\phi_n^0})^2-\kappa_{ZZ}^{\phi_n^0\phi_n^0}+2(\kappa_{ZZ}^{\phi_n^0})^2+\sum_{n,m=1}^{N}(\kappa_{Z}^{\phi_n^0\phi_m^0})^2\Big\}=0$$

$$\mathbf{T}\,(\log\boldsymbol{\Lambda}^2):\quad \sum_{n=1}^{N}\Bigg[\Big\{-\kappa_{WW}^{\phi_n^0\phi_n^0}+(\kappa_{WW}^{\phi_n^0})^2+\kappa_{ZZ}^{\phi_n^0\phi_n^0}-(\kappa_{ZZ}^{\phi_n^0})^2-\sum_{n,m=1}^{N}(\kappa_{Z}^{\phi_n^0\phi_m^0})^2\Big\}\frac{M_{\phi_n^0}^2}{v^2}$$
$$-\frac{3}{4}\Big\{g^2(\kappa_{WW}^{\phi_n^0})^2-g_Z^2(\kappa_{ZZ}^{\phi_n^0})^2\Big\}\Bigg]+\frac{3}{4}(g^2-g_Z^2)=0$$

$$\mathbf{U}\,(\log\boldsymbol{\Lambda}^2):\quad \sum_{n=1}^{N}\Big\{(\kappa_{ZZ}^{\phi_n^0})^2-(\kappa_{WW}^{\phi_n^0})^2\Big\}+\frac{1}{2}\sum_{n,m=1}^{N}(\kappa_{Z}^{\phi_n^0\phi_m^0})^2=0$$

Minimal conditions for Higgs couplings from Finiteness

Implication of Finiteness

Simplified conditions : (for ρ_0=1)

$$\kappa_Z^{\phi_n^0 \phi_{n'}^0} = 0$$

The same condition from unitarity is found !!

$$\kappa_Z^{\phi_n^0 \phi_{n'}^0} = 0$$

Charged Higgs boson(s) are required in order to have CP odd Higgs boson

Implication of Finiteness

Simplified conditions : (for ρ_0=1)

$$(\kappa_{WW}^{\phi_m^0})^2 - \kappa_{WW}^{\phi_m^0\phi_m^0} - (\kappa_{ZZ}^{\phi_m^0})^2 + \kappa_{ZZ}^{\phi_m^0\phi_m^0} = 0$$

$$\sum_{m=1}^{N} (\kappa_{WW}^{\phi_m^0})^2 = \sum_{m=1}^{N} (\kappa_{ZZ}^{\phi_m^0})^2 = 1$$

Weaker conditions as compared to unitarity sum-rules are obtained

$$\rho_0 = 1$$

$$\kappa_{WW}^{\phi_m^0} = \kappa_{ZZ}^{\phi_m^0}$$

} Custodial Sym. is NOT imposed !!

ρ_0=1 is known as a consequence of the custodial symmetry

$$\sum_{n=1}^{N} (\kappa_{WW}^{\phi_n^0})^2 = 1$$

→ $K_V \leq 1$ ($K_V \geq 1$ is possible if we introduce H^{++})

$$\kappa_{WW}^{\phi_n^0}\kappa_{WW}^{\phi_{n'}^0} = \kappa_{WW}^{\phi_n^0\phi_{n'}^0}$$

$$\kappa_{ZZ}^{\phi_n^0}\kappa_{ZZ}^{\phi_{n'}^0} = \kappa_{ZZ}^{\phi_n^0\phi_{n'}^0}$$

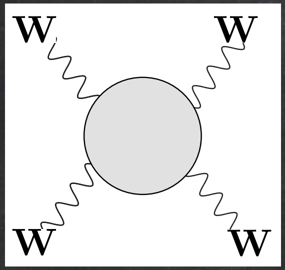
} Gauge Sym. is NOT imposed for Higgs int.

(Relations between 3- and 4 vertices)

Unitarity vs Oblique Parameters

Models with only neutral Higgs boson(s)

Summation inside loops makes conditions weaker



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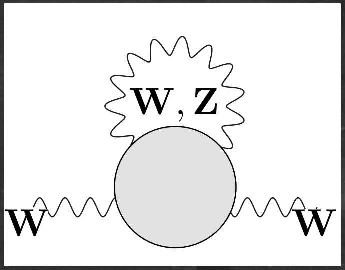


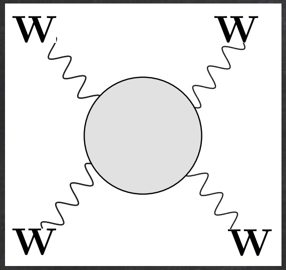
Unitarity

Finiteness of oblique parameters

& $\rho_0 = 1$

Equivalence ?

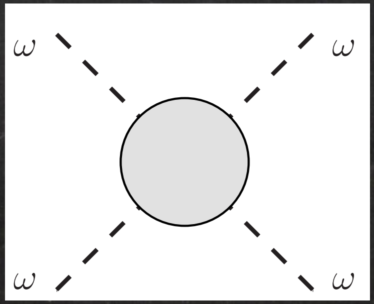
- Renormalizability predicts Unitarity



Renormalizability
+ SSB

Unitarity

$$\mathcal{M}(W_L W_L \to W_L W_L) \simeq \mathcal{M}(ww \to ww) + \mathcal{O}(M_W^2/s)$$



w : NGB absorbed by W

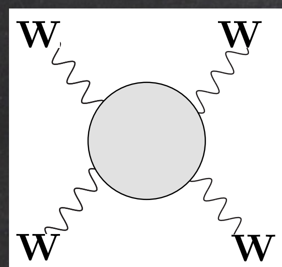
$$\propto \lambda = \frac{m_h^2}{2v^2}$$

$$\lambda(\Phi^\dagger \Phi)^2$$
$$= \lambda(+\tfrac{1}{8}h^4 + w^+ w^- w^+ w^- + \cdots)$$

Unitarity vs Oblique Parameters

Models with only neutral Higgs boson(s)

Summation inside loops makes conditions weaker



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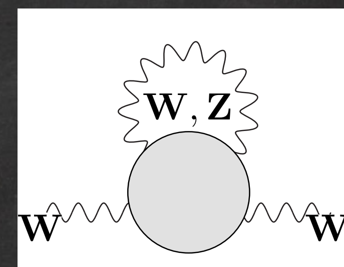


Unitarity

Finiteness of
oblique parameters

However, we don't specify other Higgs Potential
→ Perfect renormalizability is not imposed

?

Is Unitarity too strong requirement?

Perturbative Unitarity may restore at the certain scale !! (Remember 4-Fermi int.)

A Possibility consistent w/ Higgs data

Minimal conditions for finiteness : (for ρ_0=1)

We want to keep the success of EWPT, but, give up the unitarity

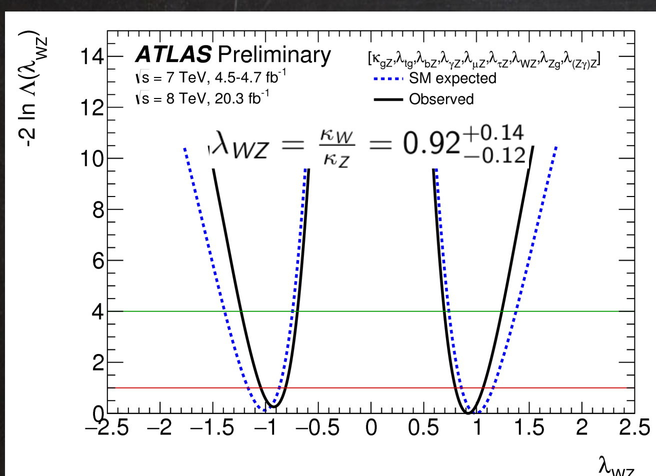
$$\kappa_Z^{\phi_n^0\phi_{n'}^0} = 0$$

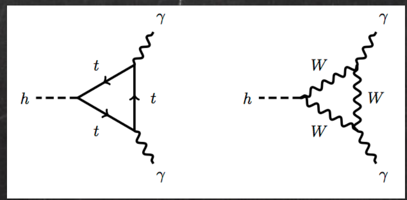
OK : No CP odd Higgs boson is found

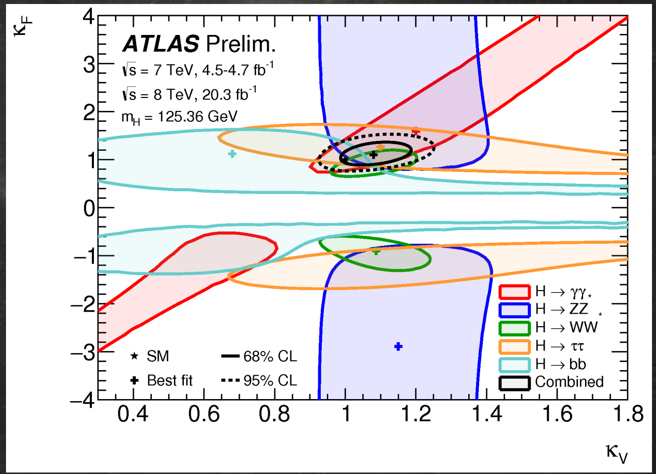
OK : No 4-vertex has been measured

$$(\kappa_{WW}^{\phi_m^0})^2 - \kappa_{WW}^{\phi_m^0\phi_m^0} - (\kappa_{ZZ}^{\phi_m^0})^2 + \kappa_{ZZ}^{\phi_m^0\phi_m^0} = 0$$

$$\sum_{m=1}^{N} (\kappa_{WW}^{\phi_m^0})^2 = \sum_{m=1}^{N} (\kappa_{ZZ}^{\phi_m^0})^2 = 1$$





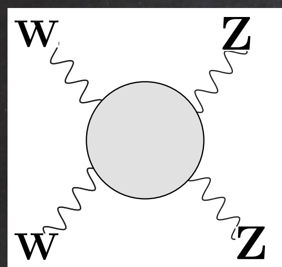


We only know the relative sign between K_W and K_t

$K_W \approx -K_Z$ can explain all Higgs data

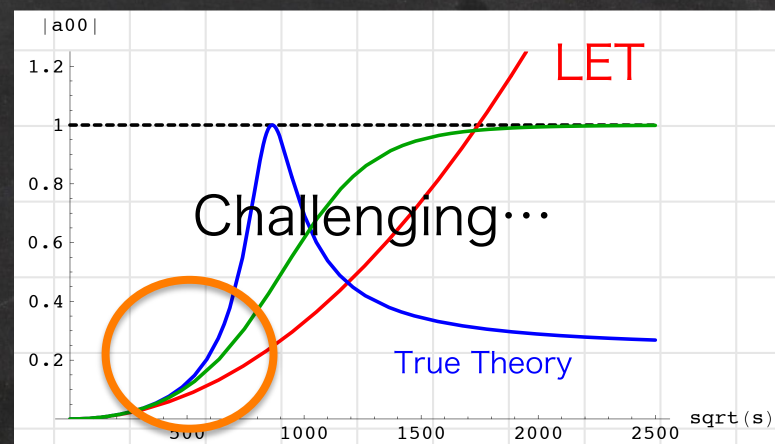
Can we test $K_W \approx -K_Z$?

- Violation of the Unitariry sum-rule(s)



$W_L\, W_L \to Z_L\, Z_L$

$$\propto \frac{E^2}{v^2}$$



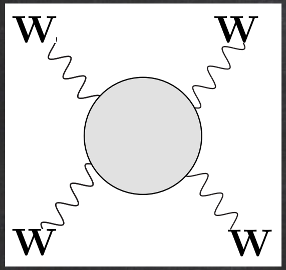
We can see, in principle, an increasing behavior in M_{ZZ} !!

(If I have time) Application to Phenomenology

Application to Phenomenology

Models with only neutral Higgs boson(s)

Summation inside loops makes conditions weaker



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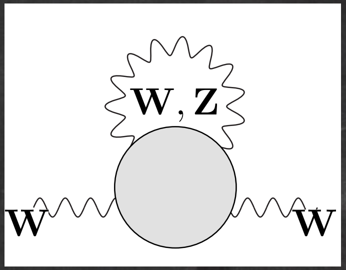


Unitarity

Finiteness of oblique parameters

& $\rho_0 = 1$

Application to Phenomenology

- Lesson from the SM

Lee, Quigg, Thacker (1977)

$$\mathcal{M}_{\rm SM} \propto \frac{M_h^2}{v^2}$$

too large M_h breaks perturbativity

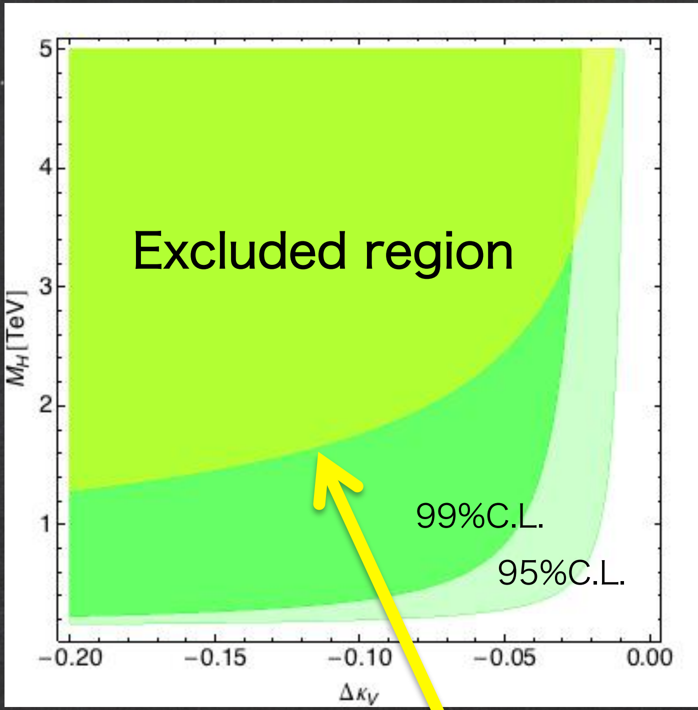
$$\frac{M_h^2}{v^2} \leq 4\pi$$

- A mass bound on the 2nd Higgs boson

when the (SM-like) Higgs coupling is determined

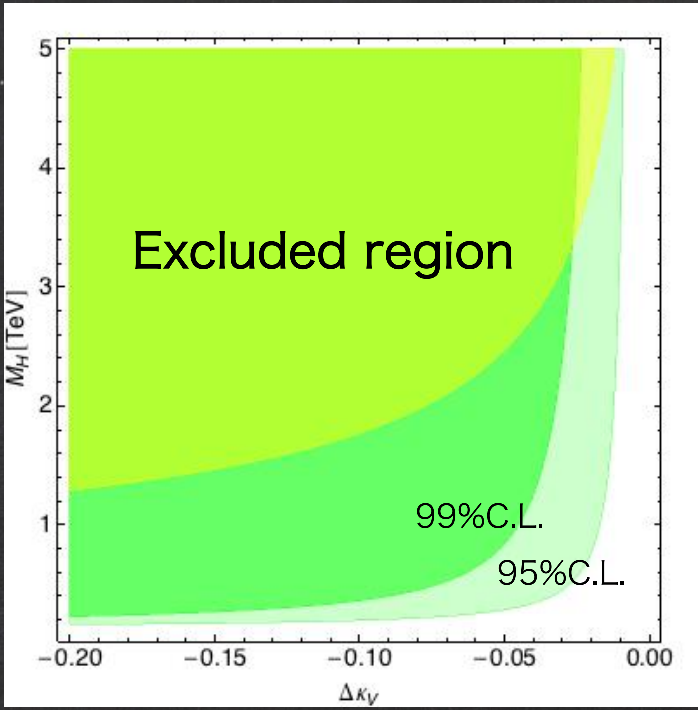
$$\left| \kappa_W^2 \frac{M_h^2}{v^2} + (1 - \kappa_W^2) \frac{M_H^2}{v^2} \right| \lesssim 4\pi$$

Application to Phenomenology



$$\left| \kappa_W^2 \frac{M_h^2}{v^2} + (1 - \kappa_W^2) \frac{M_H^2}{v^2} \right| \lesssim 4\pi$$

Application to Phenomenology



Once unitarity sum-rules are imposed → Finiteness of EW oblique corrections (S, T)

EW precision Tests are applicable (New)

Conclusion

- We discuss the requirement from
  - Perturbative Unitarity
  - Finiteness of quantum corrections

    These conditions may be related each other

- What does the experiment tell us ?
  - Custodial Sym. might be too strong requirement
  - (Tree level) Unitarity might also be too strong requirement
  - [I want to keep the success of the EWPT]
  - An interesting possibility $K_W \approx -K_Z$

- Application of Unitarity sum-rules to Phenomenology

Fin.

Outlook

- Our Framework is very restricted situation!!

Next Step : inclusion of H^+, H^{++}, ⋯

→ Sum-rules from Unitarity and Finiteness are changed drastically!!

Something new conditions would be required

What do the experimental results really tell us ?
(weaker than unitarity, custodial sym.,⋯)

Fin.