

Dark matter in universal extra dimension models

Mitsuru Kakizaki (University of Toyama)

June 10, 2015, CosKASI Dark Matter Workshop 2015 @ KASI

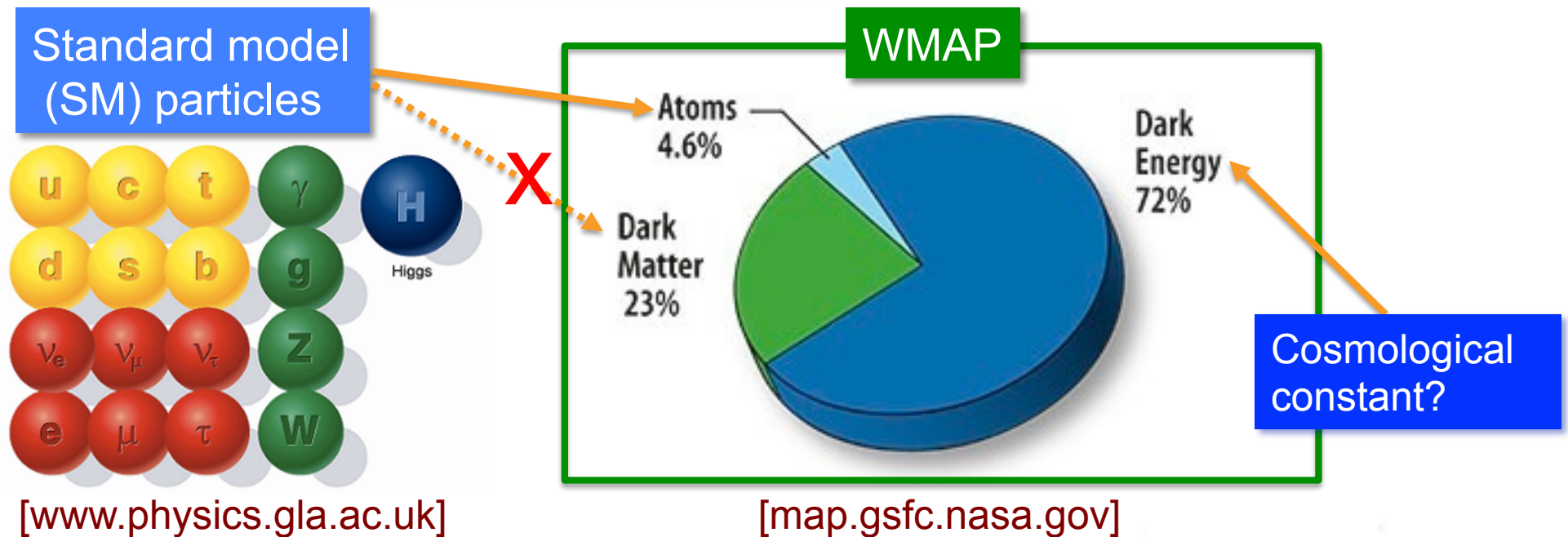
Refs.:

- M.Kakizaki, S.Matsumoto, Y.Sato and M.Senami,
NPB 735, 84 (2006) [hep-ph/0508283]; PRD 71, 123522 (2005) [hep-ph/0502059].
- MK, S.Matsumoto and M.Senami, PRD 74, 023504 (2006) [hep-ph/0605280].
- G.Belanger, MK and A.Pukhov, JCAP 1102, 009 (2011) [arXiv:1012.2577 [hep-ph]].
- G.Belanger, A.Belyaev, M.Brown, MK and A.Pukhov,
PRD87, no. 1, 016008 (2013) [arXiv:1207.0798 [hep-ph]].
- MK, in preparation
- (MK and K.Ishiwata, in preparation)

1. Motivations

- Discovery of a Higgs boson at the CERN LHC
 - The Standard Model (SM) is established as a low-energy effective theory below $O(100)$ GeV

This is not the end of the story



Existence of the dark matter $\longrightarrow$ New physics beyond the SM

Dark matter candidates

- Interesting solution to the DM problem: **Reductionist approach**
 - DM candidates naturally accompany the SM particles
 - DM interactions are related to SM coupling constants

Weak-Scale Supersymmetry

- Vector superfield of the B-boson:

$$V(x^\mu, \theta, \bar{\theta}) = \theta \sigma^\mu \bar{\theta} \underbrace{B_\mu(x^\mu)}_{\text{SM B-boson}} + i\theta^2 \bar{\theta} \bar{\chi}(x^\mu) - i\bar{\theta}^2 \theta \underbrace{\chi(x^\mu)}_{\text{LSP DM}} + \dots$$

- DM interactions are controlled by supersymmetry

Universal Extra Dimensions (UEDs)

← This talk

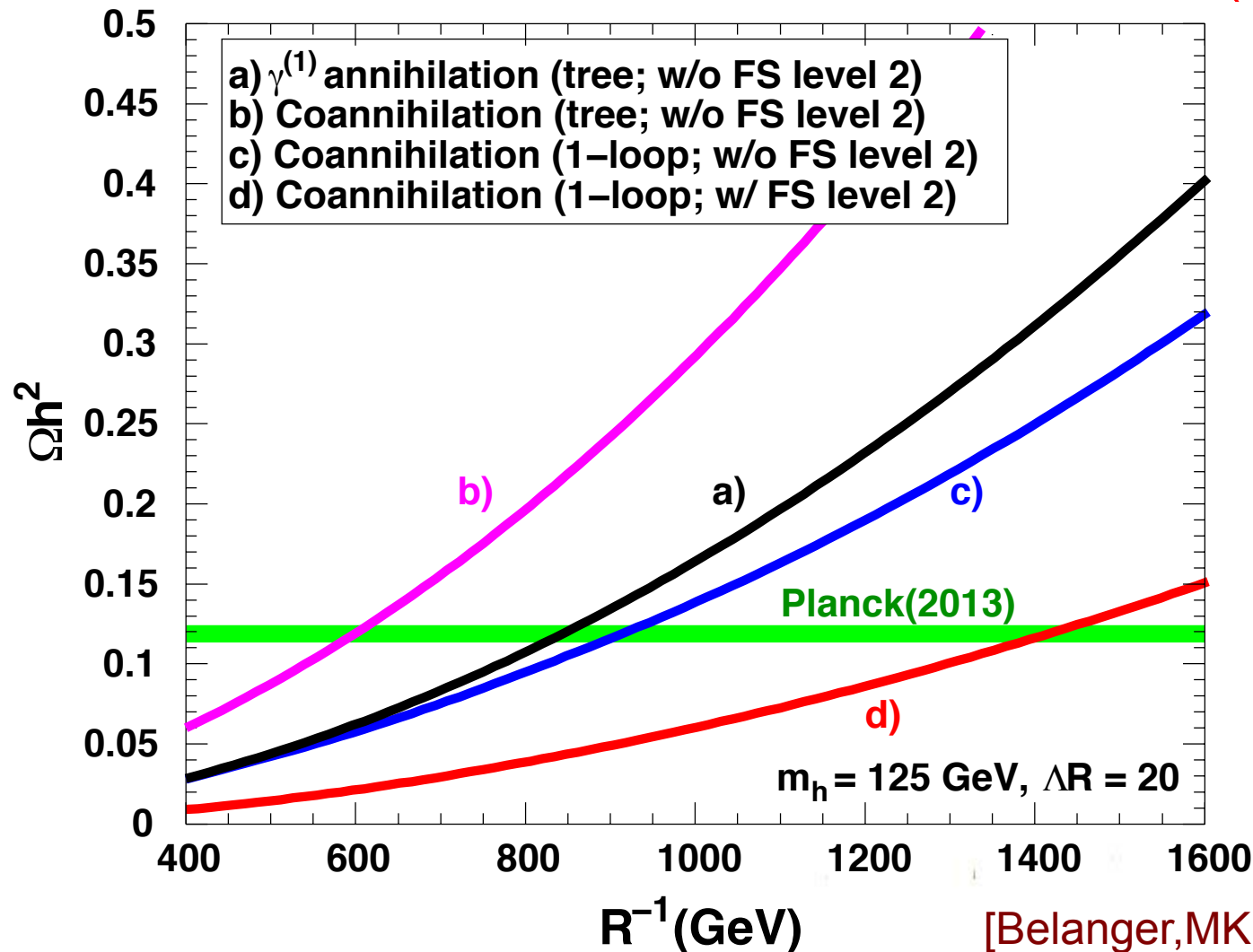
- 5D B-boson field:

$$B_\mu(x^\mu, x^5) = \frac{1}{\sqrt{2\pi R}} \underbrace{B_\mu^{(0)}(x^\mu)}_{\text{SM B-boson}} + \frac{1}{\sqrt{\pi R}} \underbrace{B_\mu^{(1)}(x^\mu)}_{\text{LKP DM}} \cos\left(\frac{x^5}{R}\right) + \dots$$

- DM interactions are controlled by Lorentz invariance

This talk (1/2)

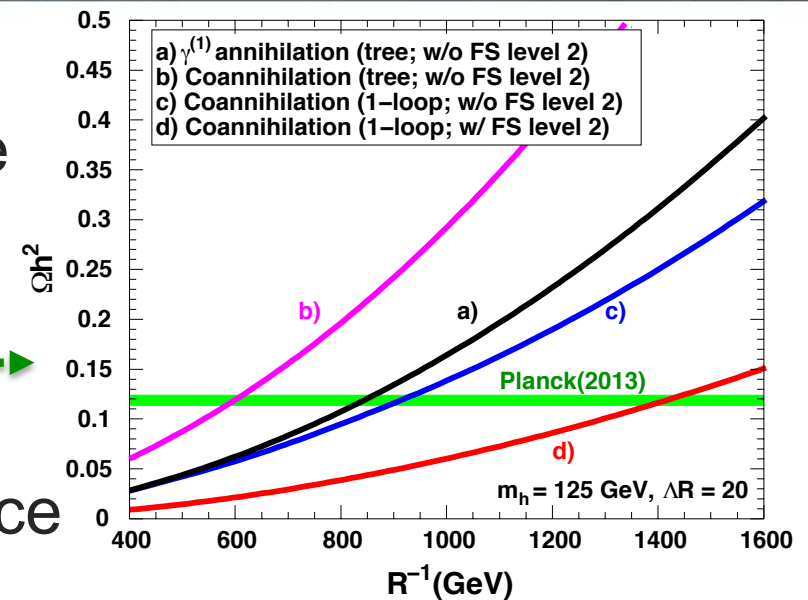
- Goal: Role of 2nd KK modes in the minimal UED (mUED):



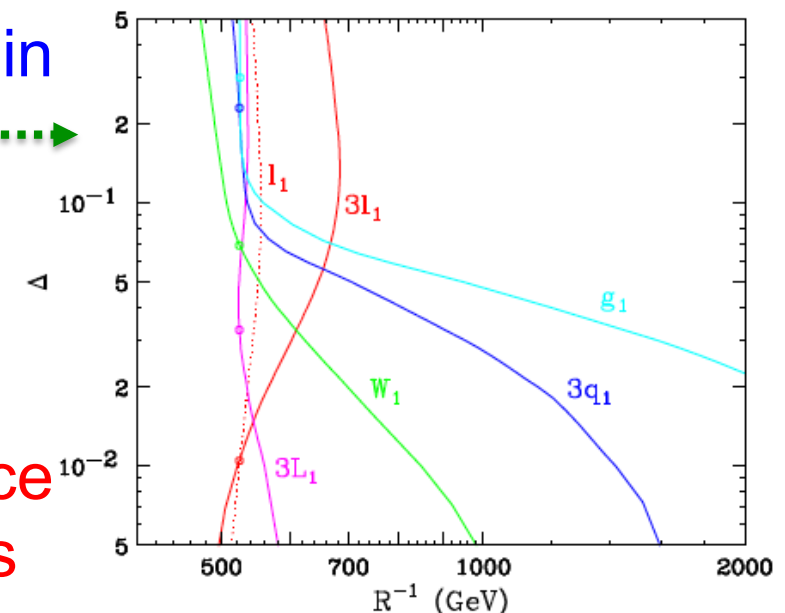
[Belanger,MK,Pukhov(2011);
MK, in preparation]

This talk (2/2)

- Precise computation of the relic abundance is important because the scale of new physics is predicted
- In the minimal UED,
 2^{nd} Kaluza-Klein (KK) modes significantly affect the LKP abundance
 [Belanger,MK,Pukhov(2011)]



- Earlier work on the LKP abundance in phenomenological UEDs:
 - 2^{nd} KK modes are neglected
 [Kong,Matchev(2006)]
- Goal:
 - Computation of the LKP abundance incl. 2^{nd} KK modes in pheno-UEDs



Contents

1. Motivation
2. Review on the minimal universal extra dimension (mUED)
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5. Summary & Discussion

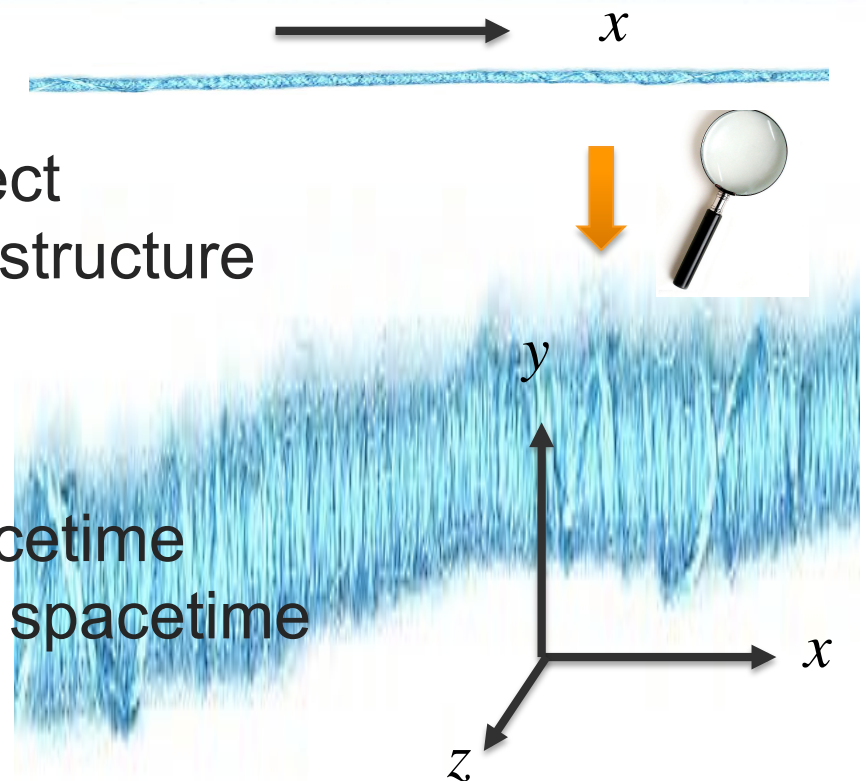
Idea of extra dimensions

- Thread:

- **Macroscopic:** 1-dimensional object
- **Microscopic:** object with internal structure

- Our world :

- **Macroscopic:** 4-dimensional spacetime
- **Microscopic:** higher-dimensional spacetime



- Dispersion relation for a higher dimensional particle:

$$E^2 = \vec{p}^2 + (p_5^2 + p_6^2 + \cdots + m^2)$$

➡ Momentum in the extra dim. = Mass in the 4-d viewpoint

Universal extra dimensions

- **Universal:** All SM particles propagate in flat compact spatial extra dimensions

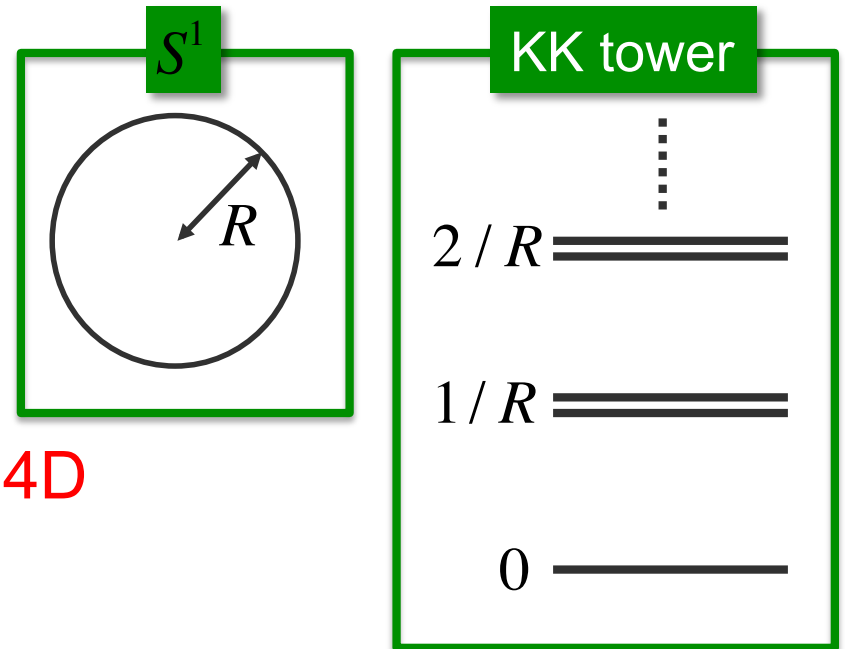
[Appelquist, Cheng, Dobrescu, PRD64, 035002 (2001)]

- S^1 compactification with radius R :

Periodic boundary condition

→ $p_5 = n / R (n = 0, \pm 1, \pm 2, \dots)$

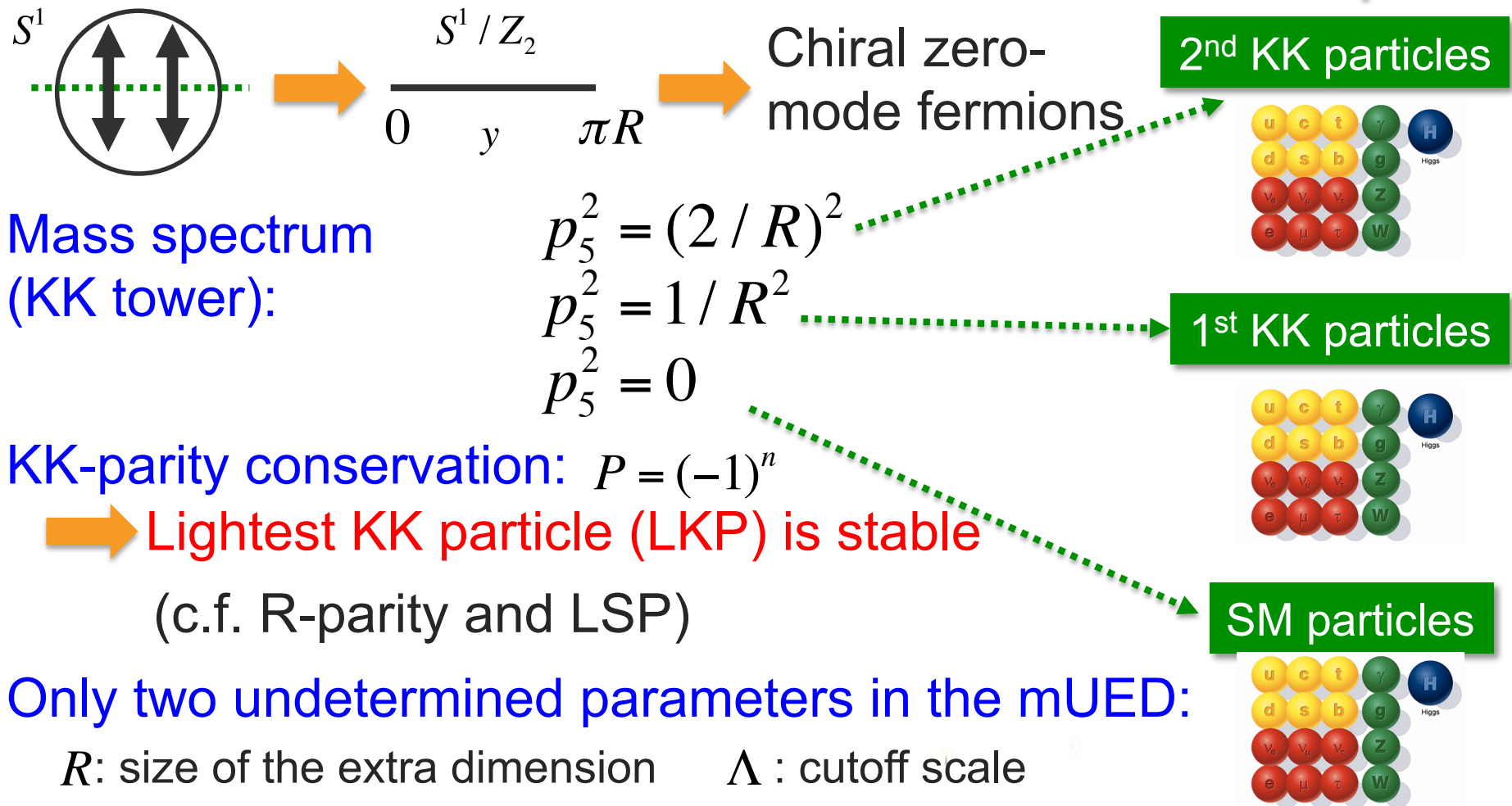
Kaluza-Klein (KK) tower in 4D



- Momentum conservation in the extra dimension
→ Conservation of KK number n at each vertex

Minimal Universal Extra Dimension (mUED)

- All the SM particles propagate in the orbifold: S^1 / Z_2

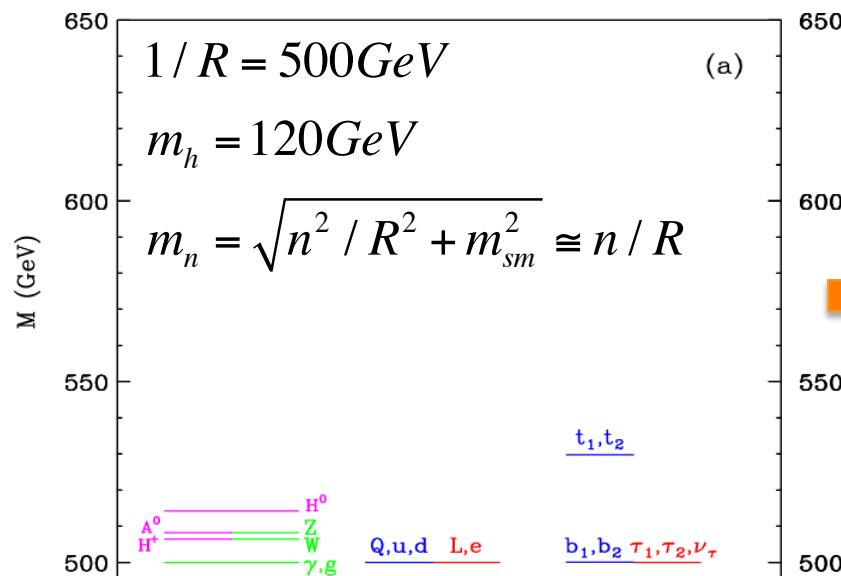


Radiative corrections to KK masses

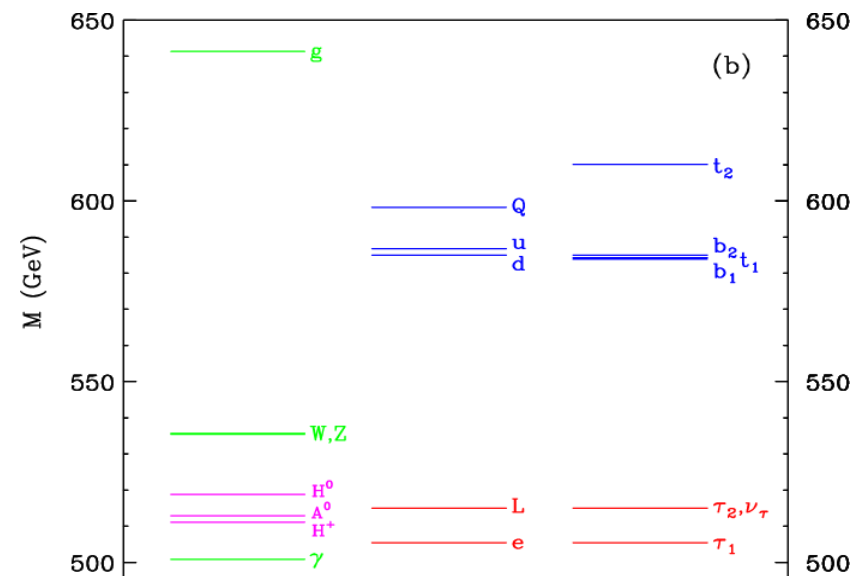
Compactification $\longrightarrow$ 5-dimensional Lorentz invariance
 Orbifolding $\longrightarrow$ Translational invariance in the 5th dimension
 $\longrightarrow$ Radiative corrections generate KK mass splitting

[Cheng, Matchev, Schmaltz, PRD66, 036005 (2002)]

- 1st KK masses at tree level



- 1st KK masses at 1-loop level



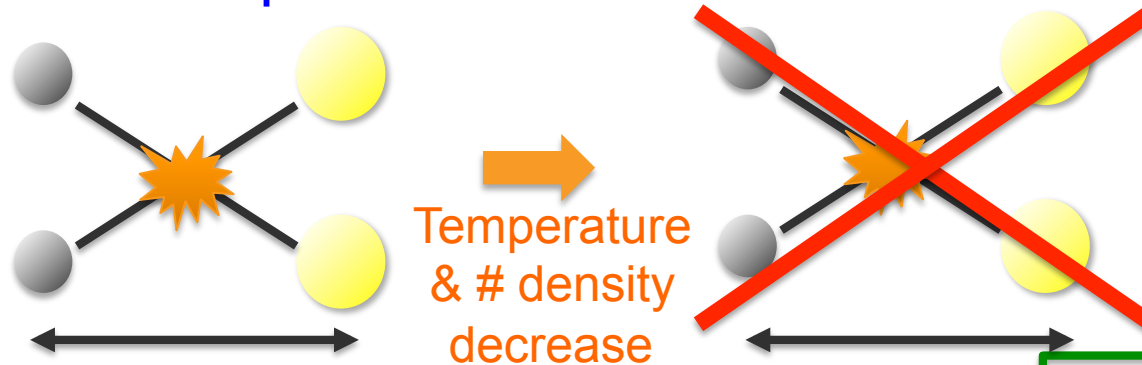
Dark matter candidate in the mUED: 1st KK photon

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WIMP miracle

- Thermal production of WIMPs:



→ Prediction: $\Omega_\chi \approx 0.1 \times \left(\frac{1 \text{ pb}}{\langle \sigma v \rangle} \right)$

- Observed DM abundance:

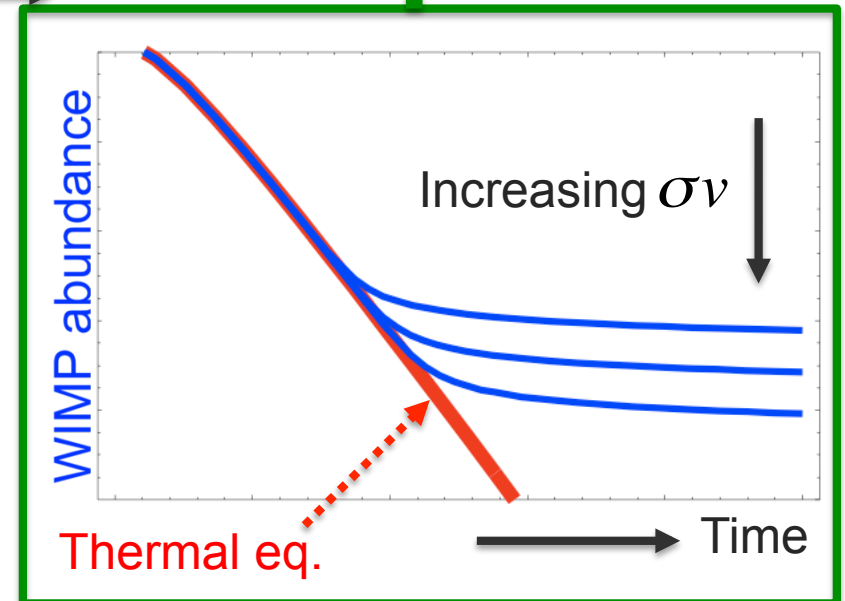
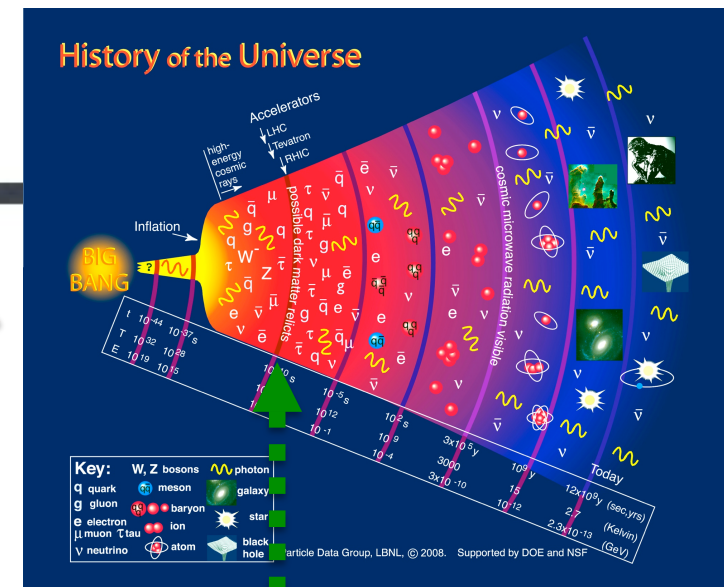
$$\Omega_{DM} \approx 1/4$$

- For WIMP DM:

$$\sigma v \approx \frac{\pi \alpha}{m^2} \approx O(\text{pb})$$

→ There exist new physics at the TeV scale

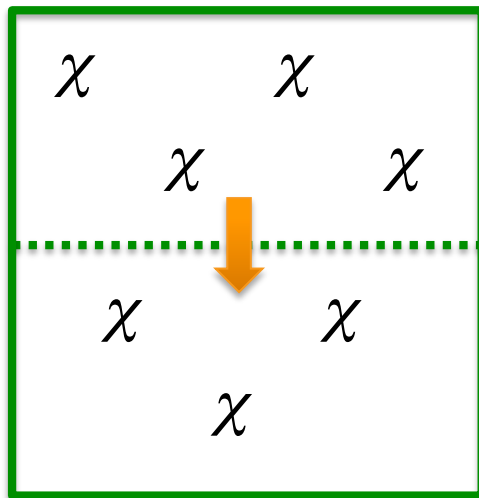
[MSSM, UED, ...]



Including coannihilation

- Only self-annihilation:

$$\chi\chi \rightarrow SM, SM$$

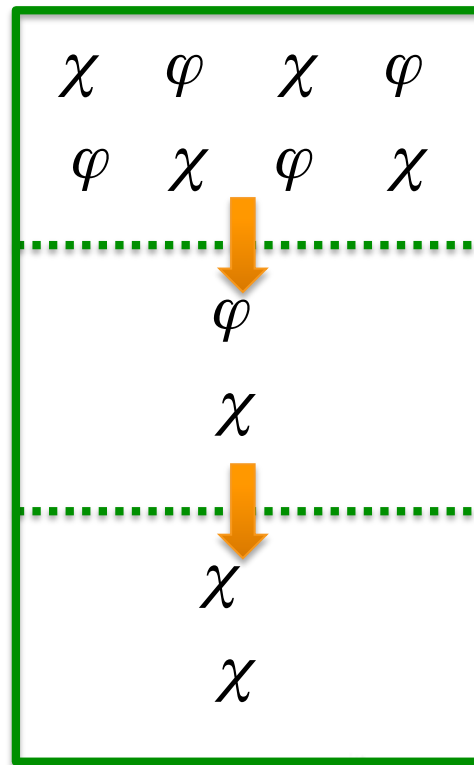


Time

- Incl. coannihilation with next lightest particle φ :

$$\chi\varphi, \varphi\varphi \rightarrow SM, SM$$

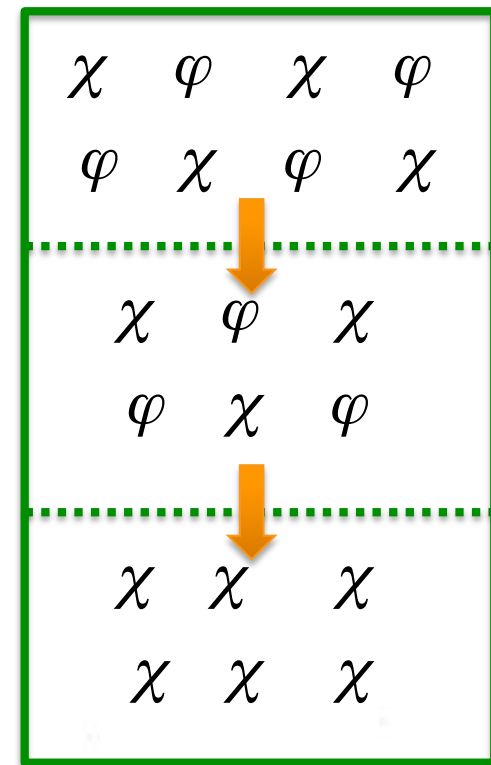
- If $\sigma_{co} \gg \sigma_{self}$
(e.g.: neutralino DM)



$$\Omega_\chi h^2$$

Mitsuru KAKIZAKI

- If $\sigma_{co} \approx \sigma_{self}$
(e.g.: 1st photon DM)

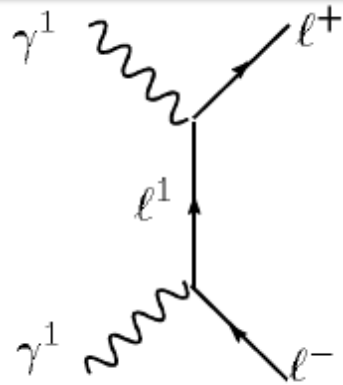


Time

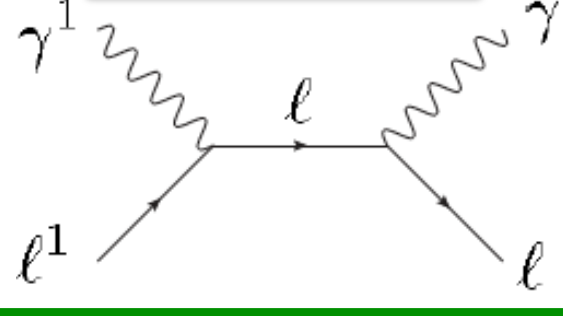
$$\Omega_\chi h^2$$

UED dark matter annihilation processes

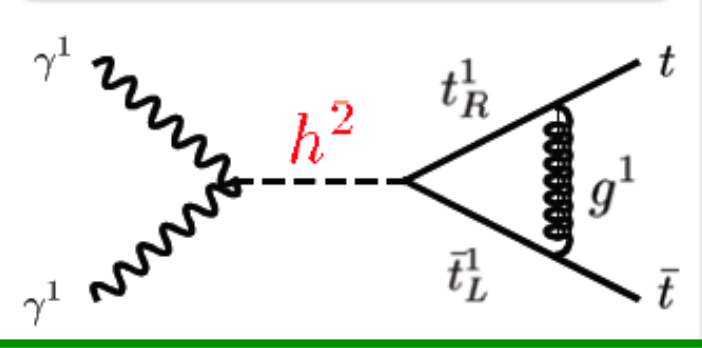
Self-annihilation



Coannihilation

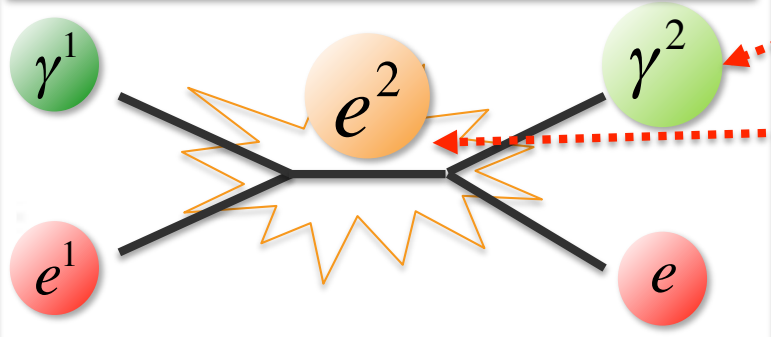


1-loop resonant process



[Servant, Tait, NPB650, 391 (2003); MK, Matsumoto, Sato, Senami, PRD71, 123522 (2005); NPB735, 84 (2006); Burnell, Kribs, PRD73 (2006); Kong, Matchev, JHEP0601 (2006); MK, Matsumoto, Senami, PRD74 (2006) 023504]

Final state level 2 production



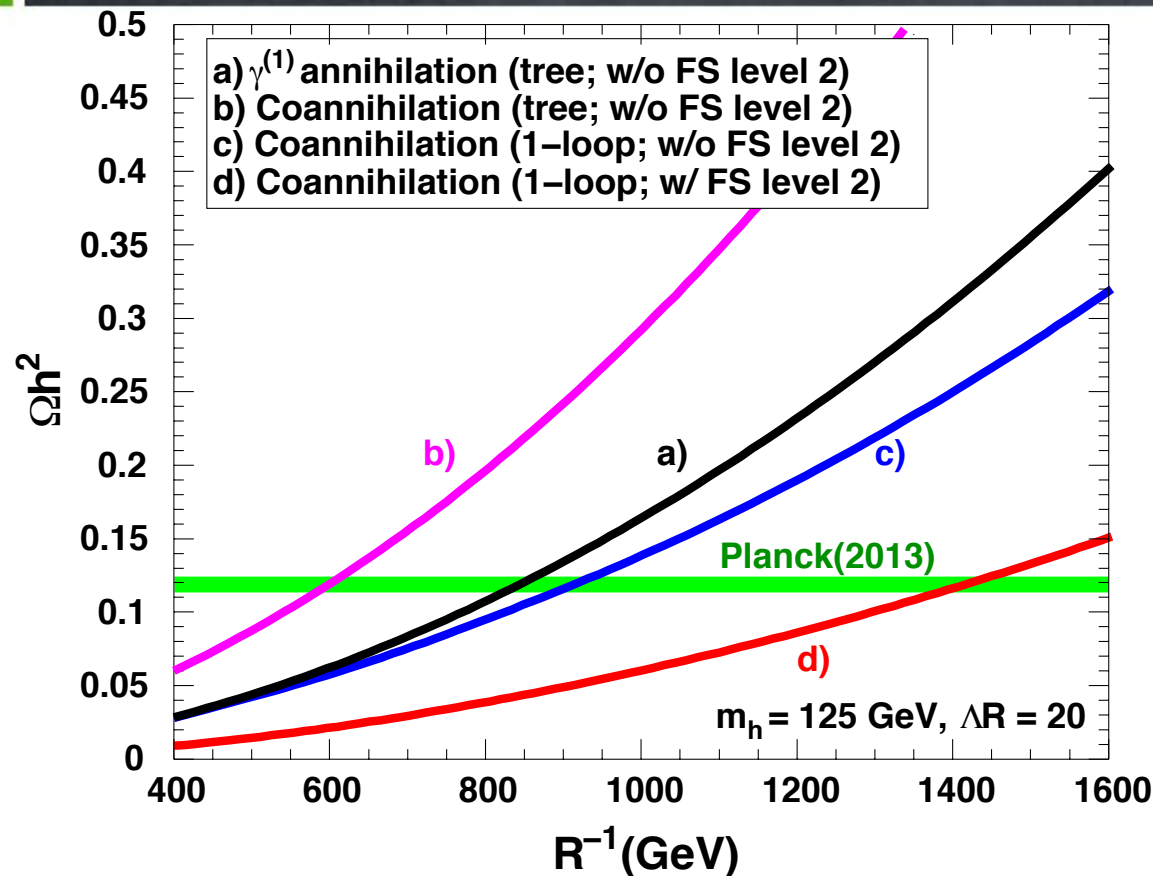
- $\gamma^2, h^2, h^{2\pm}, a^2$ decay mostly into SM particles

- $\sqrt{s} \approx m^{(1)} + m^{(1)} \approx m^{(2)}$

FS level 2 processes enhanced

[Belanger, MK, Pukhov, JCAP1102 (2011) 009]

Comparison of various effects in the mUED



	Self	Co	loop	FS Lv2
a)	X			
b)	X	X		
c)	X	X	X	
d)	X	X	X	X

Compactification scale (GeV)
 $(\cong \gamma^1 \text{ mass})$

[Belanger, MK, Pukhov(2011);
 MK, in preparation]

2nd KK resonance



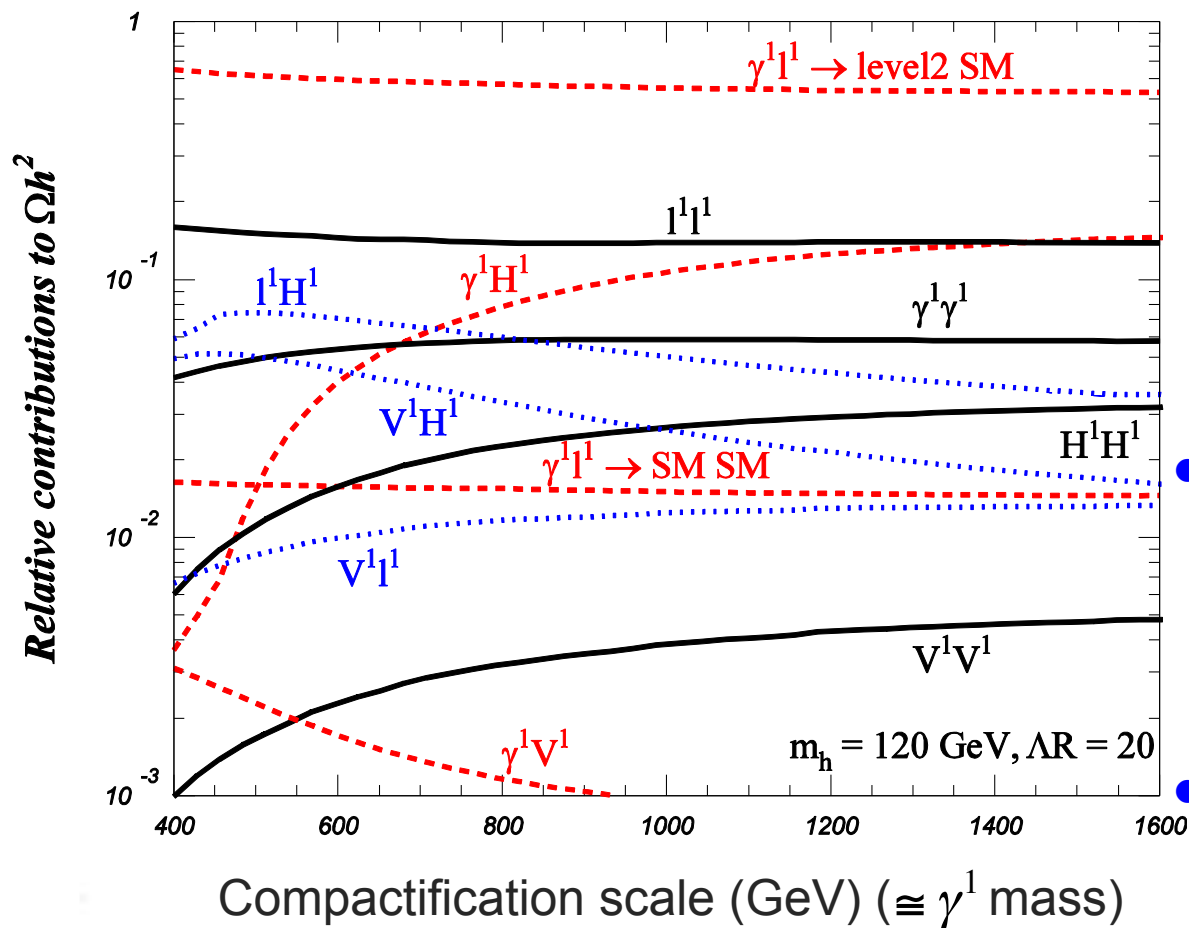
Relic abundance reduced



DM mass pushed above TeV

Relative contributions

- At the freeze-out temperature:



$$\ell^1 = \ell_{L,R}^1, \nu_\ell^1; H^1 = h^1, h^{1\pm}, a^1; V^1 = W^1, Z^1$$

- Important processes:

- $\gamma^1 \ell_{L,R}^1 \rightarrow \ell_{L,R}^2 \rightarrow \gamma^2 \ell$

- $\gamma^1 \nu_\ell^1 \rightarrow \nu_\ell^2 \rightarrow \gamma^2 \nu_\ell$

More than 50%

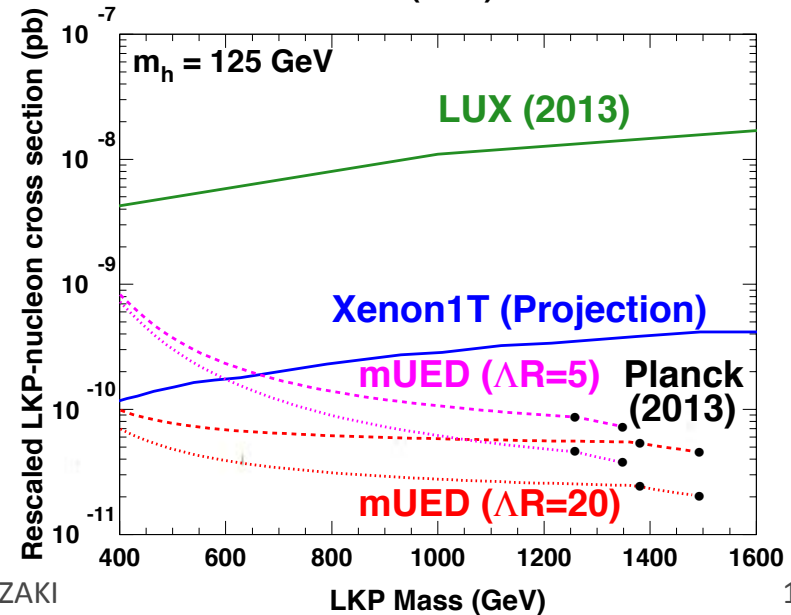
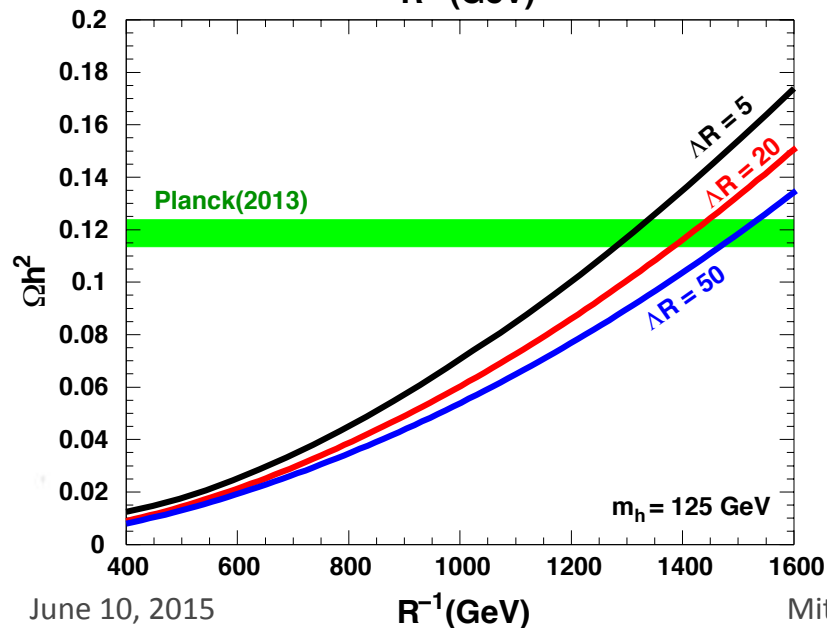
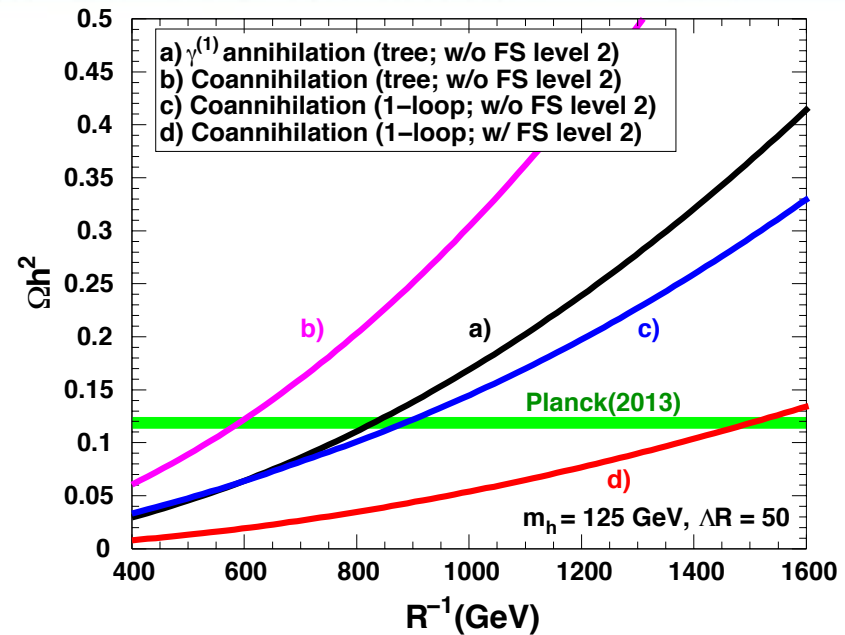
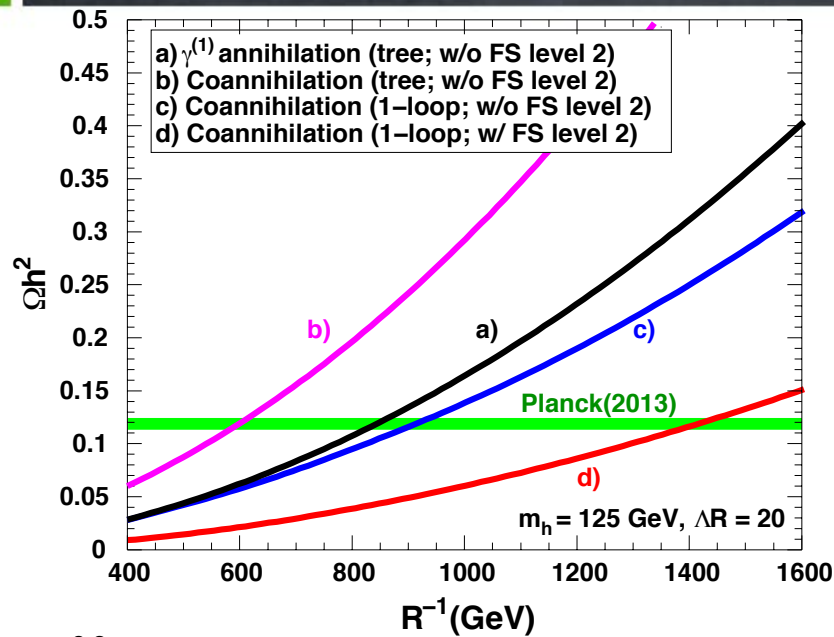
- Loop-induced resonant processes subdominant

- KK quark coannihilation negligible

[Belanger, MK, Pukhov, JCAP1102(2011)009]

Updated Numerical Results in the mUED

[MK, in preparation]



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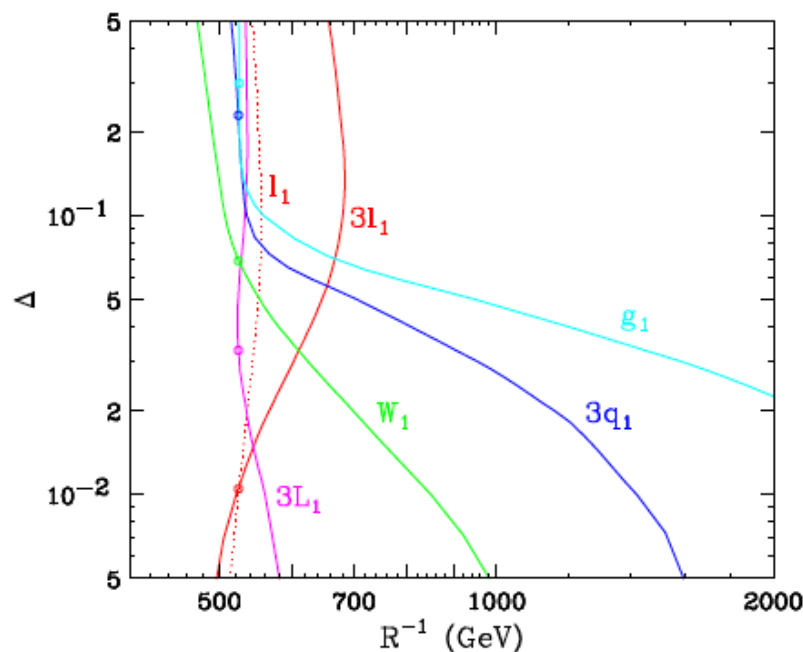
1st KK photon abundance in phenomenological UEDs

- 1st KK photon relic abundance vs mass splitting:

$$\Delta_{e_R} = \frac{m_{e_R^1} - m_{\gamma^1}}{m_{\gamma^1}}$$

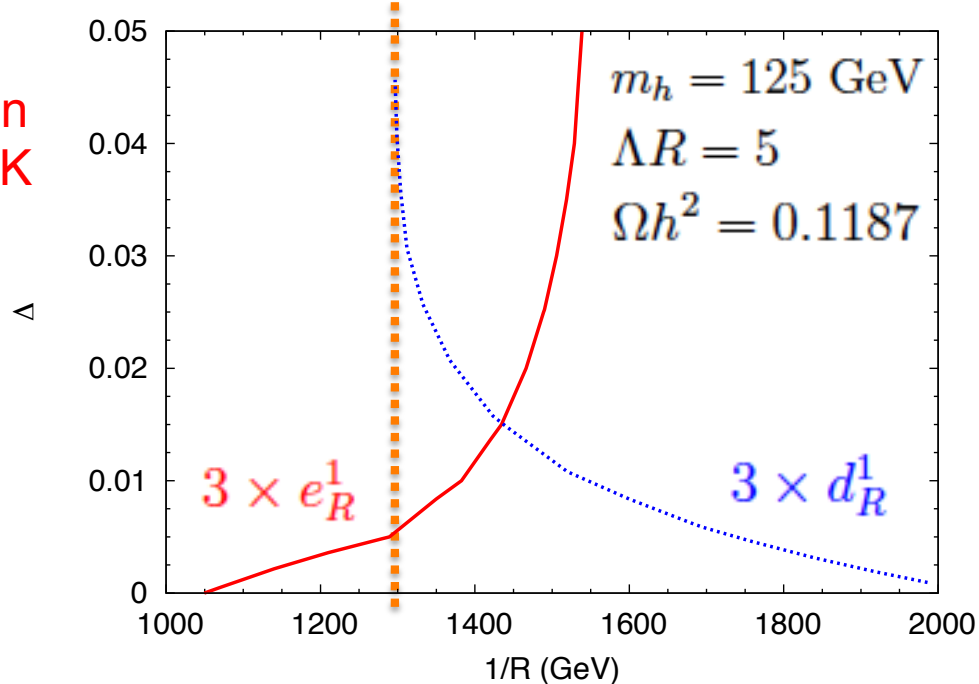
$$\Delta_{d_R} = \frac{m_{d_R^1} - m_{\gamma^1}}{m_{\gamma^1}}$$

mUED



[Kong, Matchev(2006)]

Inclusion
of 2nd KK
modes



The other parameters fixed to the mUED values

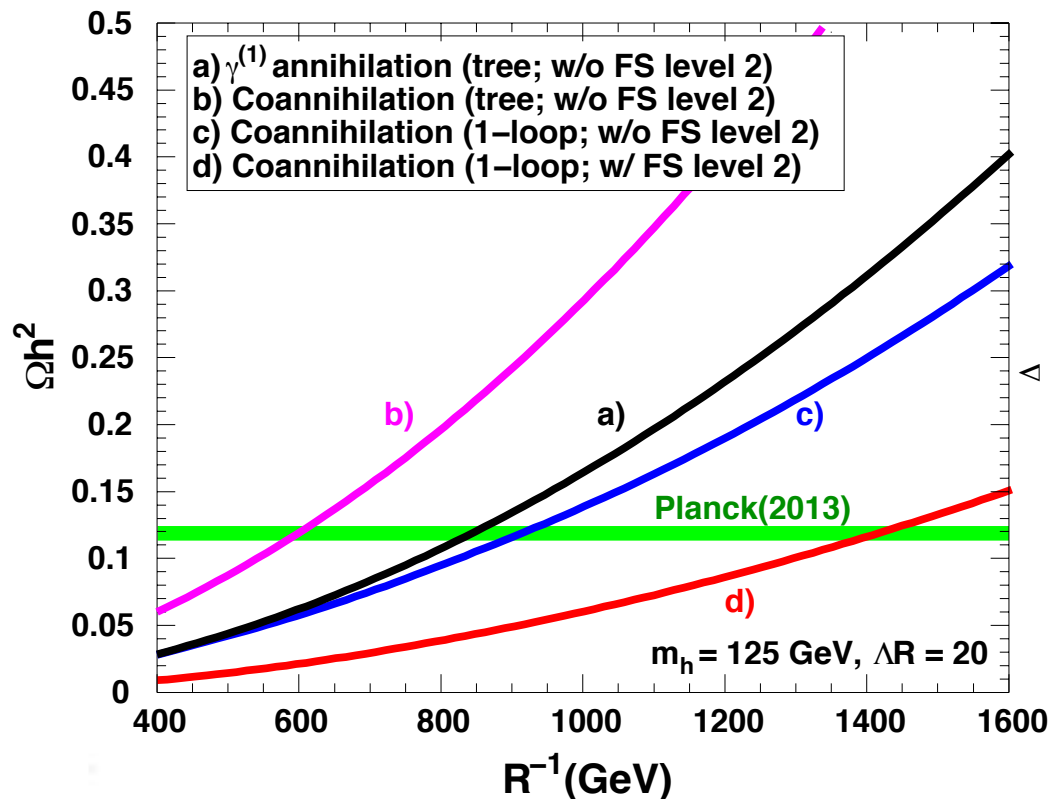
[MK, in preparation]

- The preferred 1st KK photon mass can be shifted upward or downward in pUED

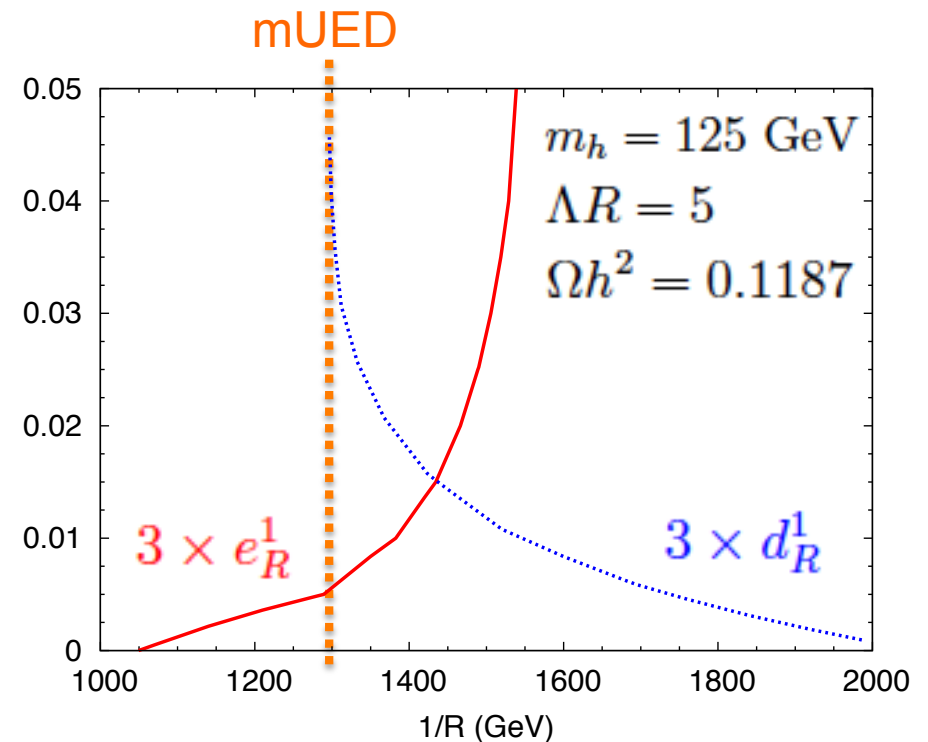
5. Summary

- Relic abundance of the 1st KK photon (mostly 1st KK hypercharge B-boson) in UED models

mUED



Phenomenological UEDs



Preferred 1st KK photon mass in the WIMP scenario: $m > \text{TeV}$

Discussion

1st KK photon DM scenario in 5D UED

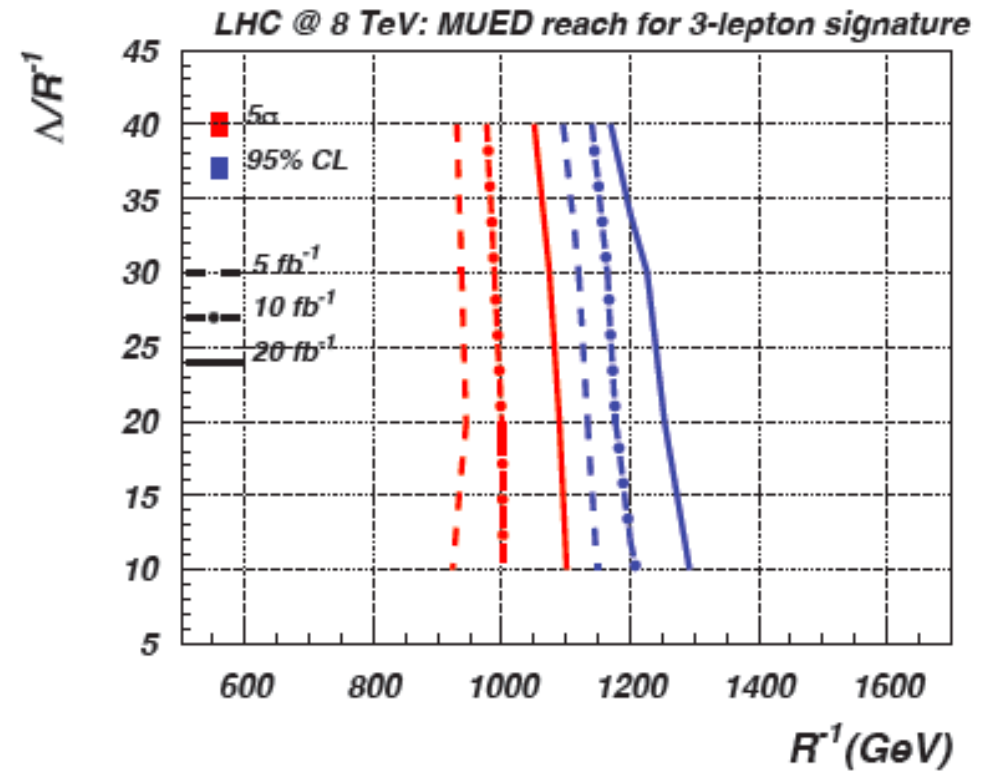
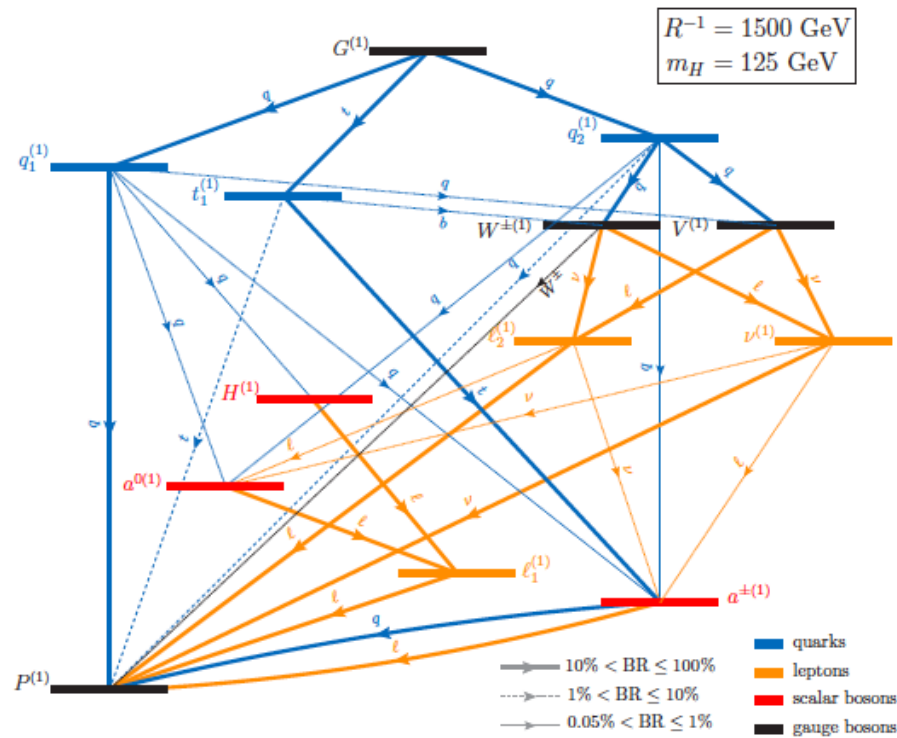
- DM abundance: OK
- EWPO: OK
- FCNC: OK
- LHC Run-I: OK
- Neutrino masses: can be incorporated
[Matsumoto, Sato, Senami, Yamanaka (2007) for Dirac neutrinos]
- Baryon asymmetry: ???
- Inflation: ???
- Accelerated expansion: ???;
- Contribution to muon g-2: too small
- Hierarchy problem: ameliorated
- Little hierarchy problem: ???
- Vacuum stability/triviality: OK; ameliorated by extending the Higgs sector
- Radion stability: ???
- KK-parity: can be anomaly free
- Direct detection: challenging
- Indirect detections: challenging (Non-perturbative effects?)
- LHC Run-II: can cover the WIMP scenario
- Future linear colliders: can test deviations of the Higgs boson couplings

1st KK Z-boson DM scenario in 5D UED

- working with K. Ishiwata

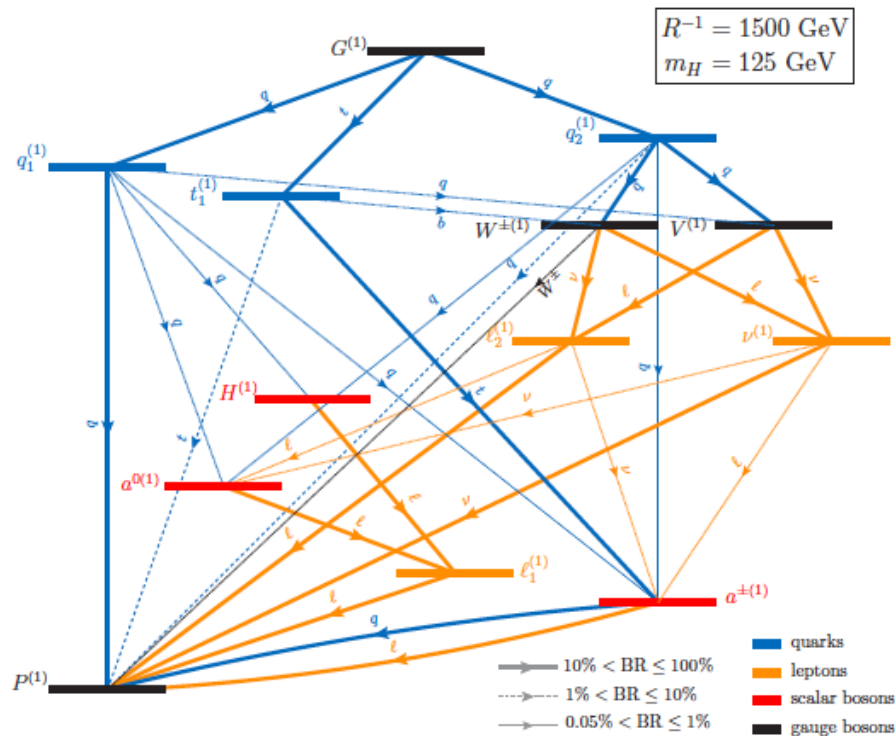
Backup slides

LHC8 constraints

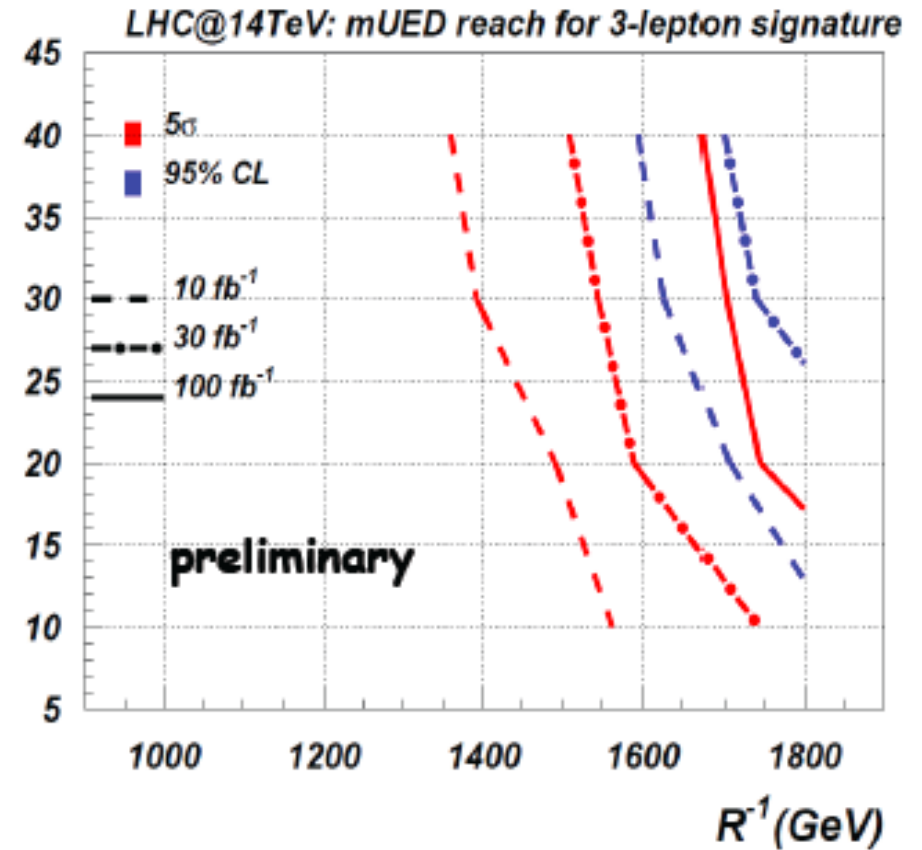


[Belyaev, Brown, Moreno, Papineau (2013)]

LHC14 constraints

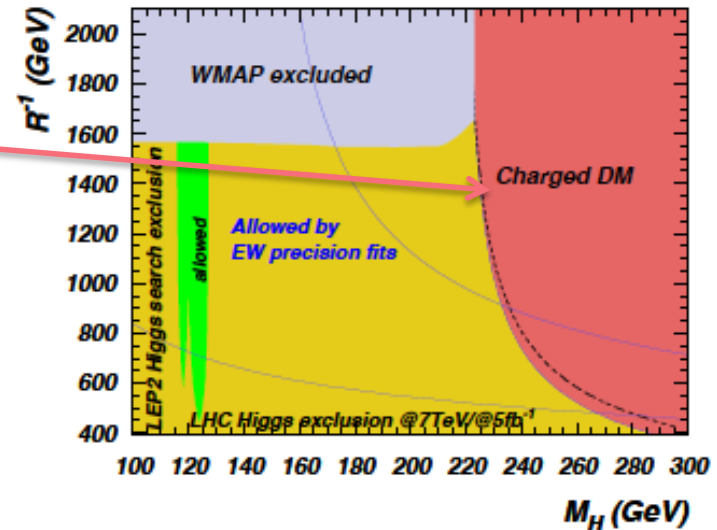
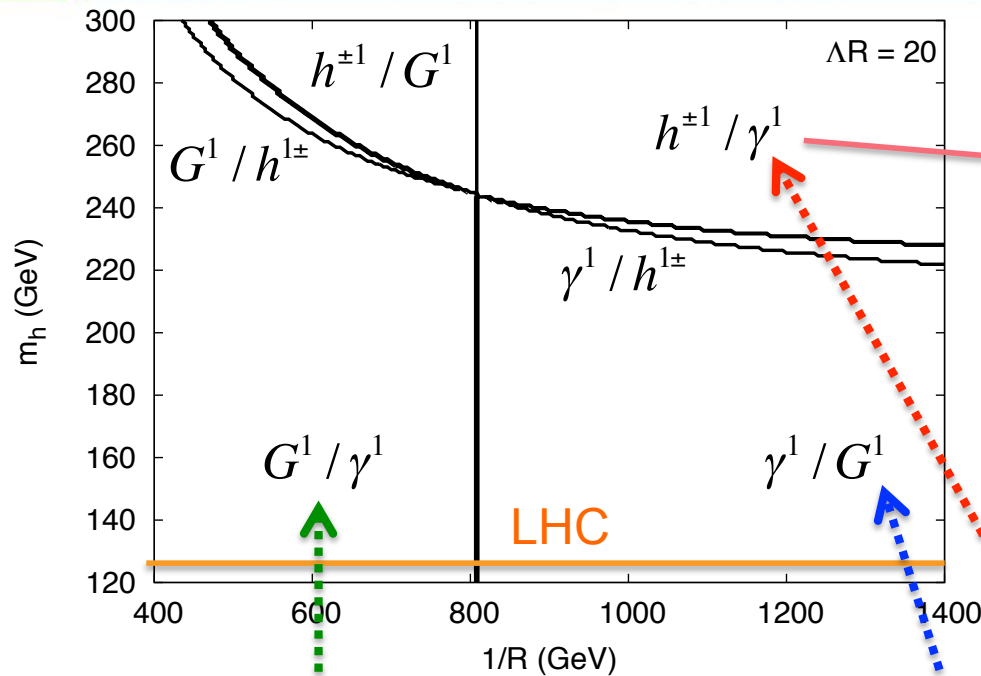


[Belyaev, Brown, Moreno, Papineau (2013)]



[Belyaev' talk at SUSY2013]

LKP/NLKP in the MUED



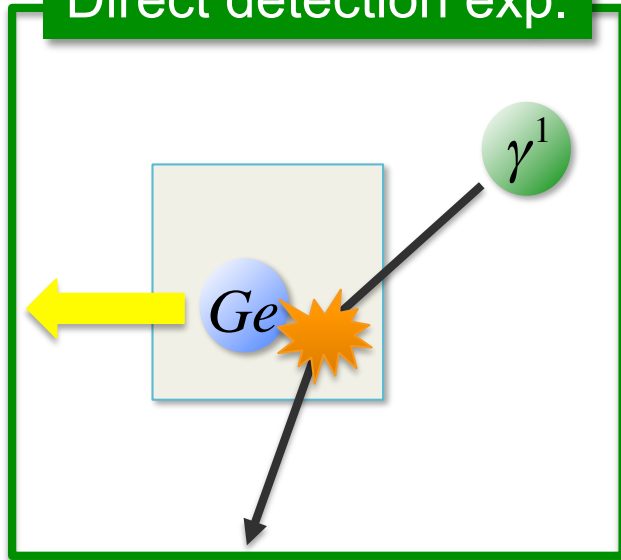
- SuperWIMP DM region (level-1 graviton LKP):
 $1/R < 800 \text{ GeV}, m_h < 240 \text{ GeV}$
 - Disfavored by CMB, diffuse photon spectrum [Feng, Rajaraman, Takayama, PRL91, 011302(2003); PRD68, 085018(2003)]

- CHAMP region ($h^{1\pm}$ LKP):
 $m_h > 240 \text{ GeV}$
 - Excluded by LHC, sea water exp.

- WIMP DM region (γ^1 LKP):
 $1/R > 800 \text{ GeV}, m_h < 220 \text{ GeV}$
 - This talk

Direct detection of the 1st KK photon

Direct detection exp.



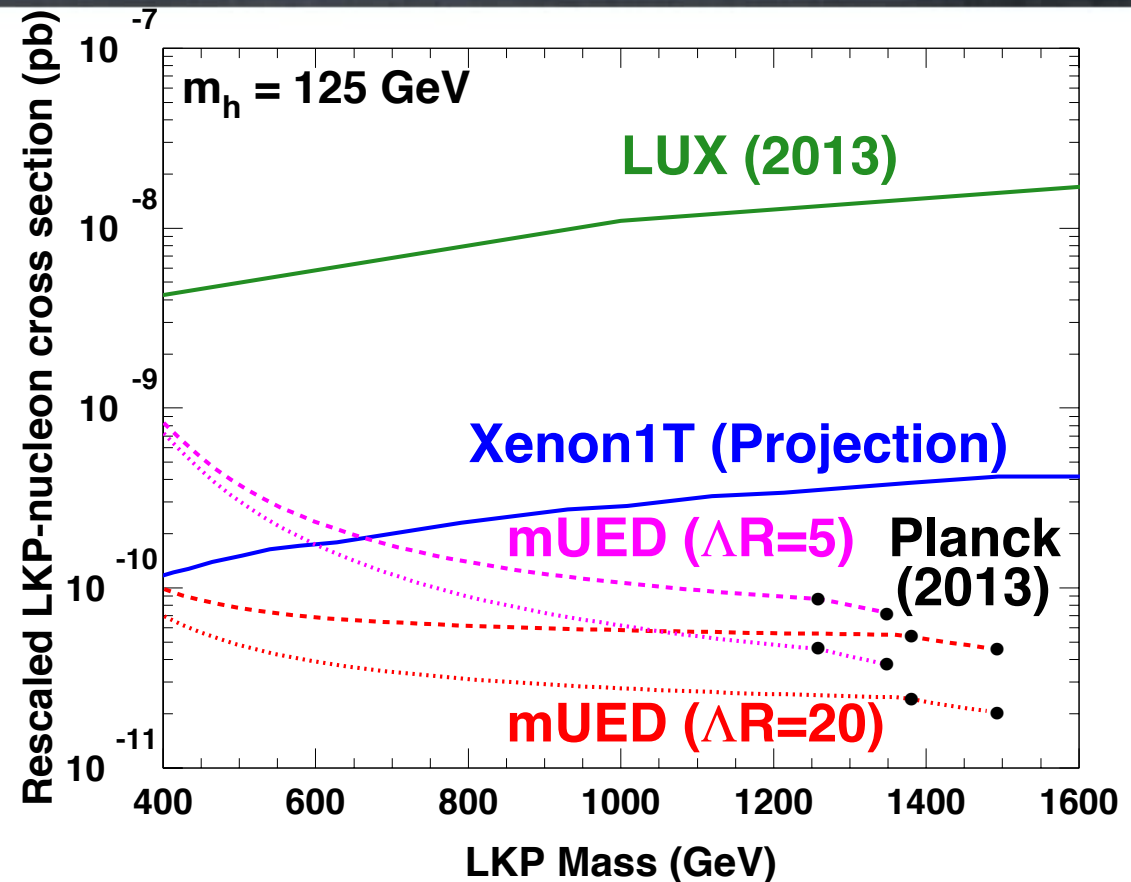
- γ^1 -nucleon cross section in MUED:

$$\sigma_{Ge} = 10^{-11} - 10^{-10} pb$$

[From Belanger, MK, Pukhov, JCAP1102(2011)009; 1st KK photon mass (GeV)
See also Servant, Tait, NPB650, 391(2003)]

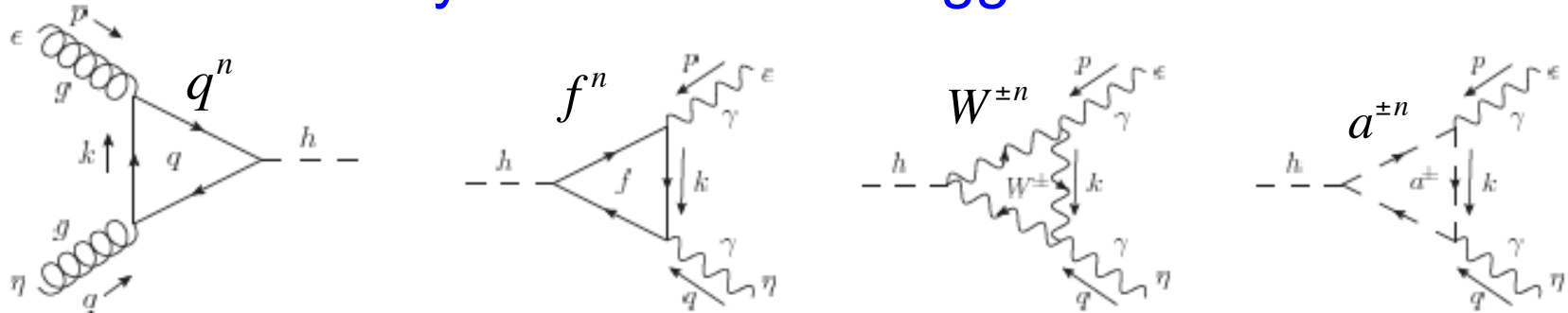
- A complete one-loop computation including KK quarks higher than 1 confirms these results

[Hisano, Ishiwata, Nagata, Yamanaka, PTP, 126, 435(2011)]



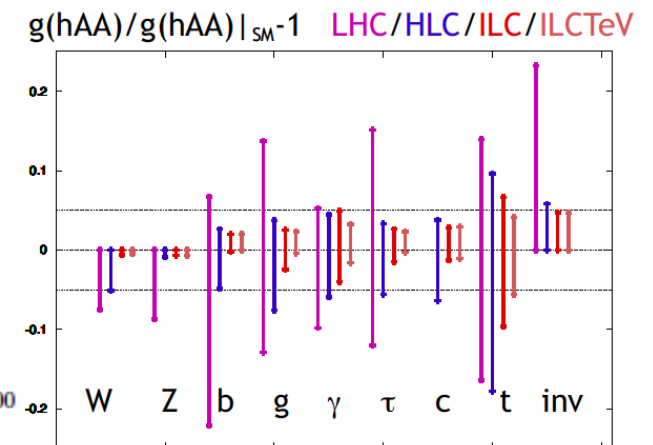
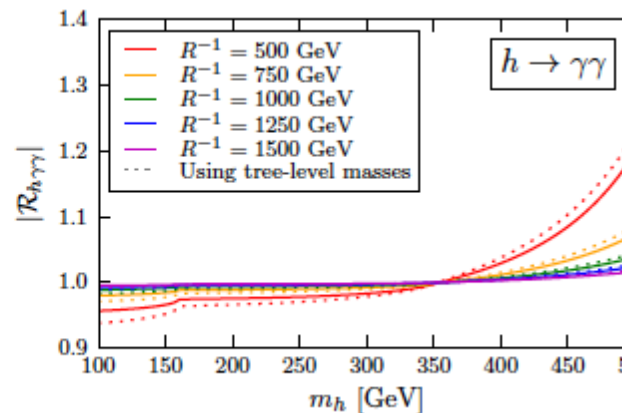
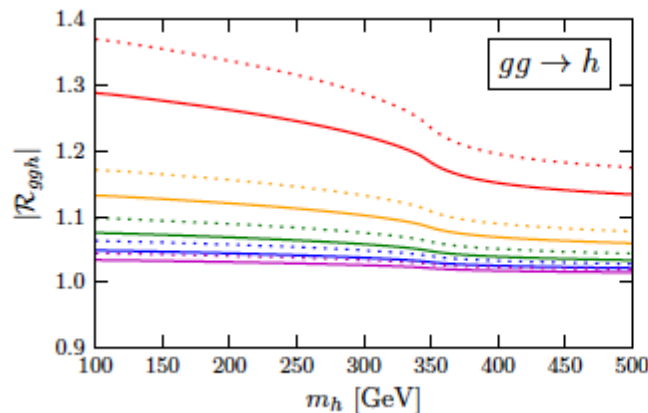
Higgs searches at the LHC

- Production and decay of the SM-like Higgs: $gg \rightarrow h \rightarrow \gamma\gamma$



[Petriello, JHEP05,003(2002)]

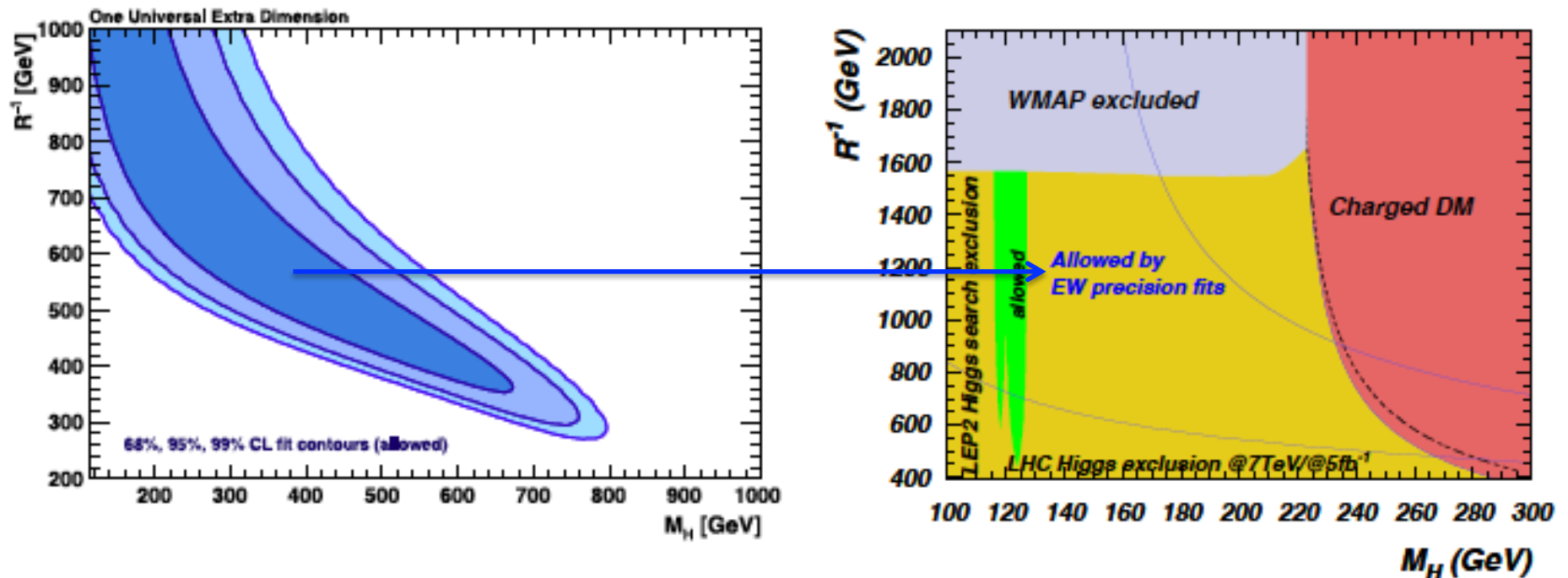
- Deviations of the hgg and $h\gamma\gamma$ couplings from the SM values:



[From Belanger, Belyaev, Brown, MK, Pukhov, arXiv:1207.0798; [Peskin, 1207.2516]
See also Nishiwaki, Oda, Okuda, Watanabe, PLB707, 506(2012)]

Electroweak precision measurements

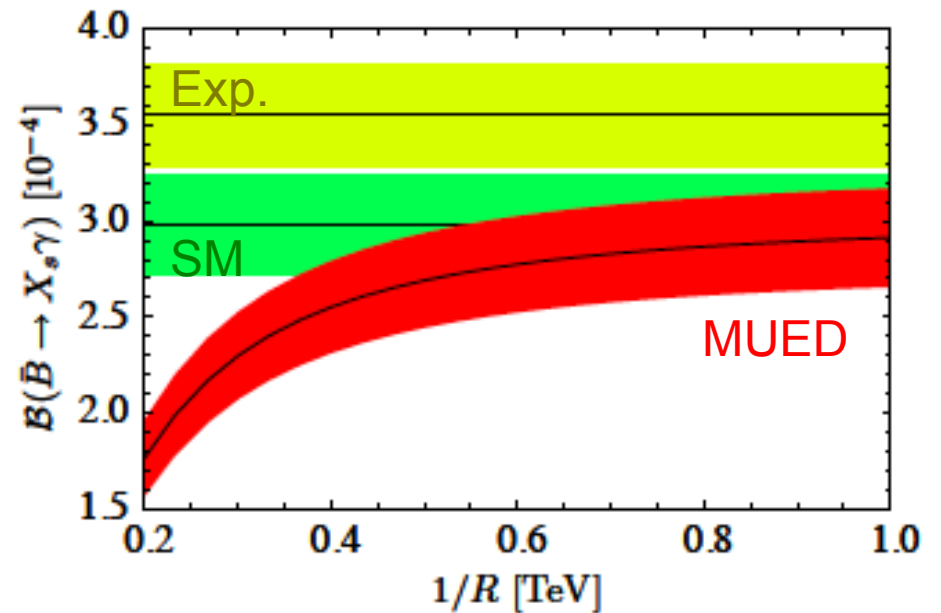
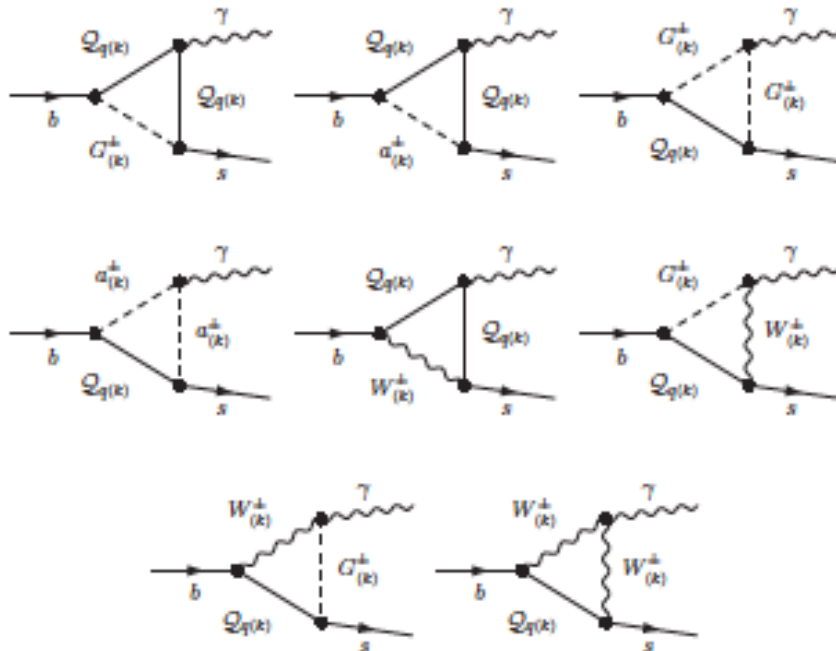
- KK parity $\rightarrow$ Constraints on UEDs are weaker than those on other extra dimension models



[From Baak,Goebel,Haller,Hoecker,Ludwig,Moenig,Schott,Stelzer,
Eur.Phys.J.C72,2003(2012);See also Gogoladze,Macesanu,PRD74 (2006)]

Flavor changing neutral currents

- $B \rightarrow X_s \gamma$

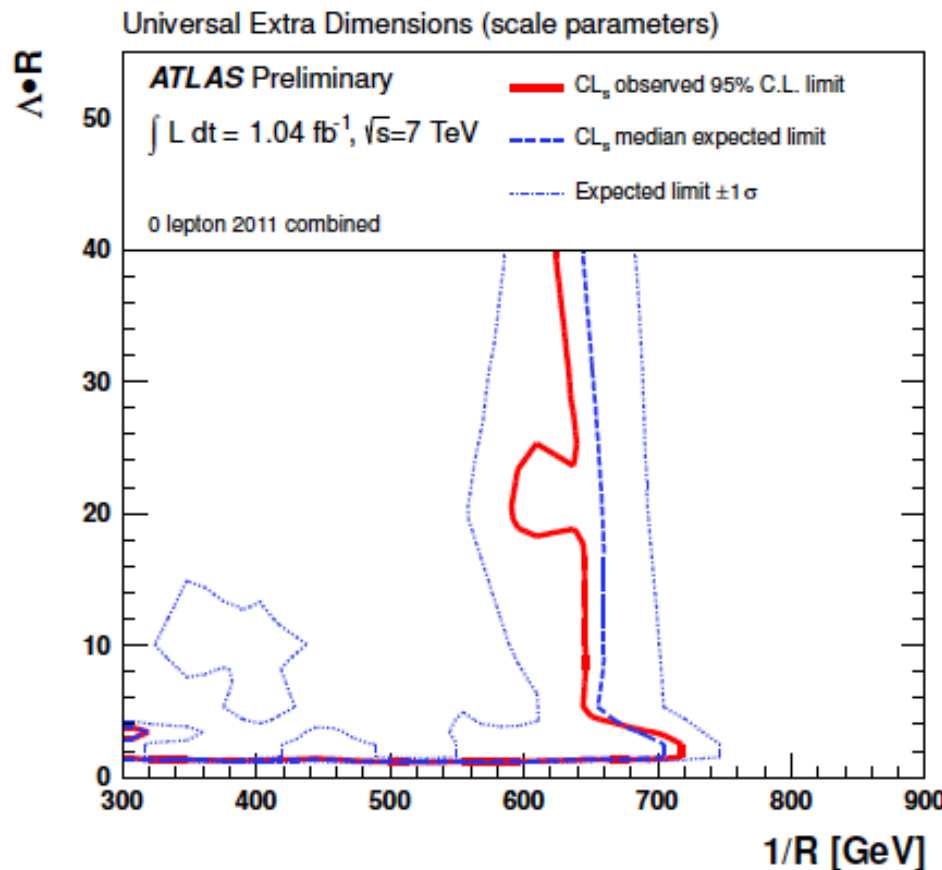


➔ $1/R > 600 \text{ GeV} (95\% \text{ CL})$

[Haisch, Weiler, Phys. Rev. D76, 034014 (2007)]

LHC search for level-1 particles

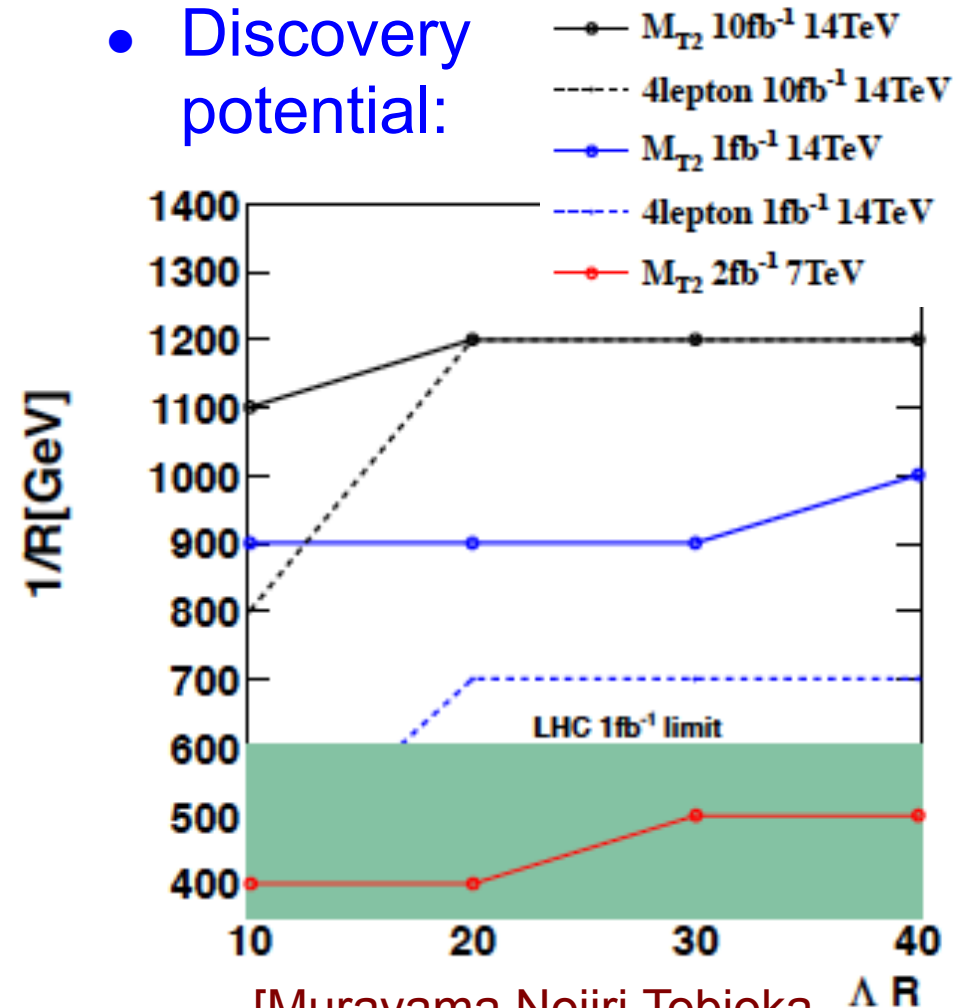
- Exclusion Limits:



[ATLAS-CONF-2011-155]

→ $1/R > 600 \text{ GeV}$

- Discovery potential:

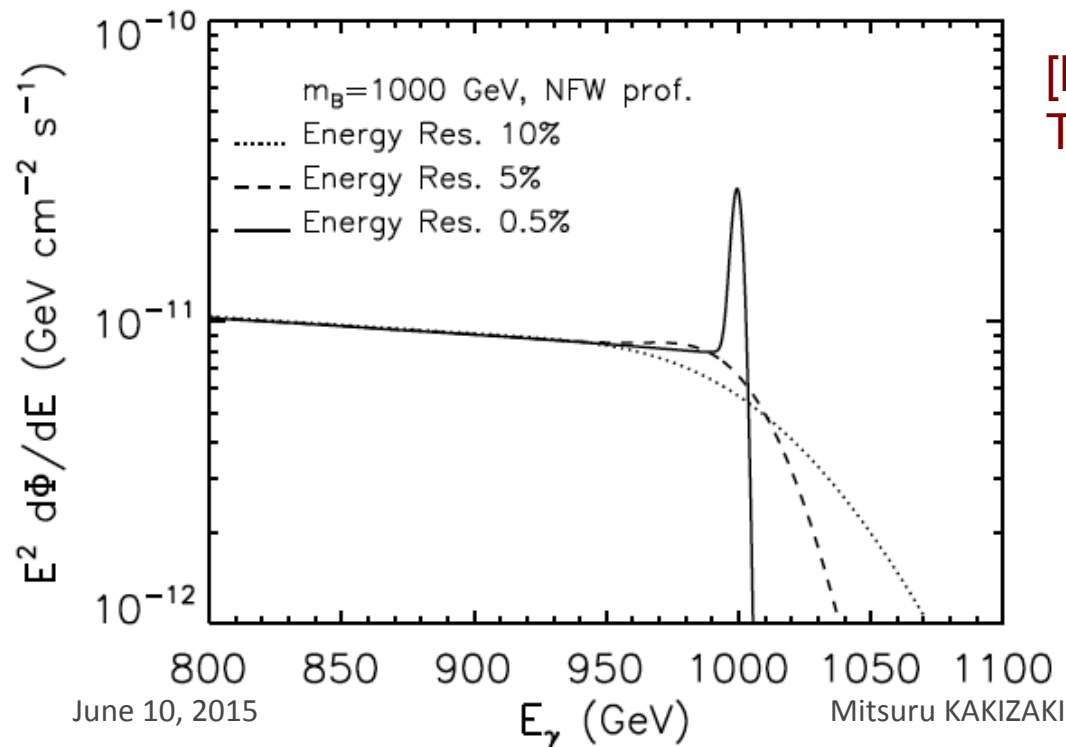
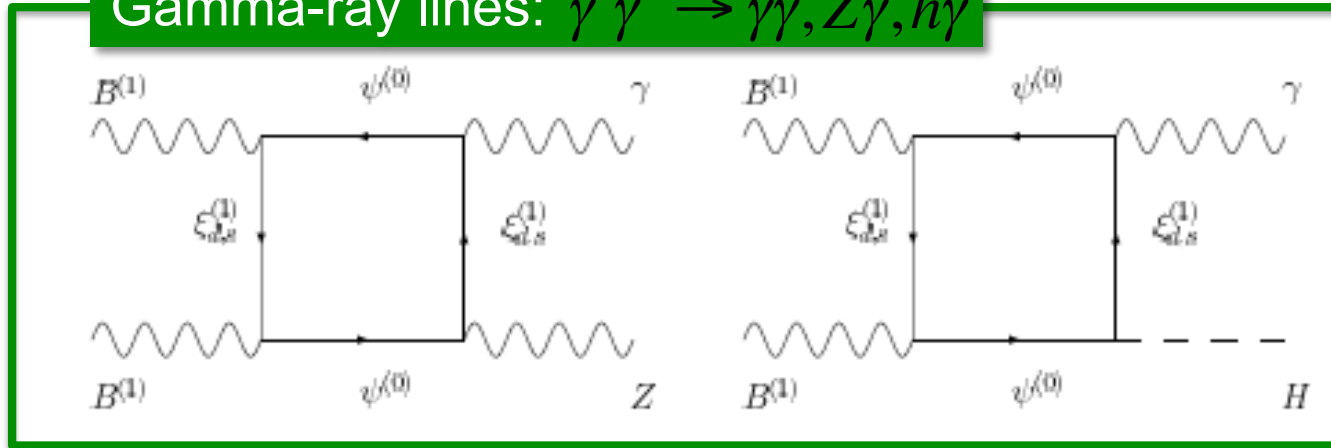


[Murayama, Nojiri, Tobioka, PRD84,094015 (2011)]

→ $1/R \sim 1.2 \text{ TeV}$

Indirect detection (Gamma rays)

Gamma-ray lines: $\gamma^1 \gamma^1 \rightarrow \gamma\gamma, Z\gamma, h\gamma$



[Bertone, Jackson, Shaughnessy, Tait, Vallinotto, JCAP, 1203, 020 (2012)]

Lines are very difficult to distinguish from the continuum

Vacuum stability

- Vacuum stability and triviality bounds:

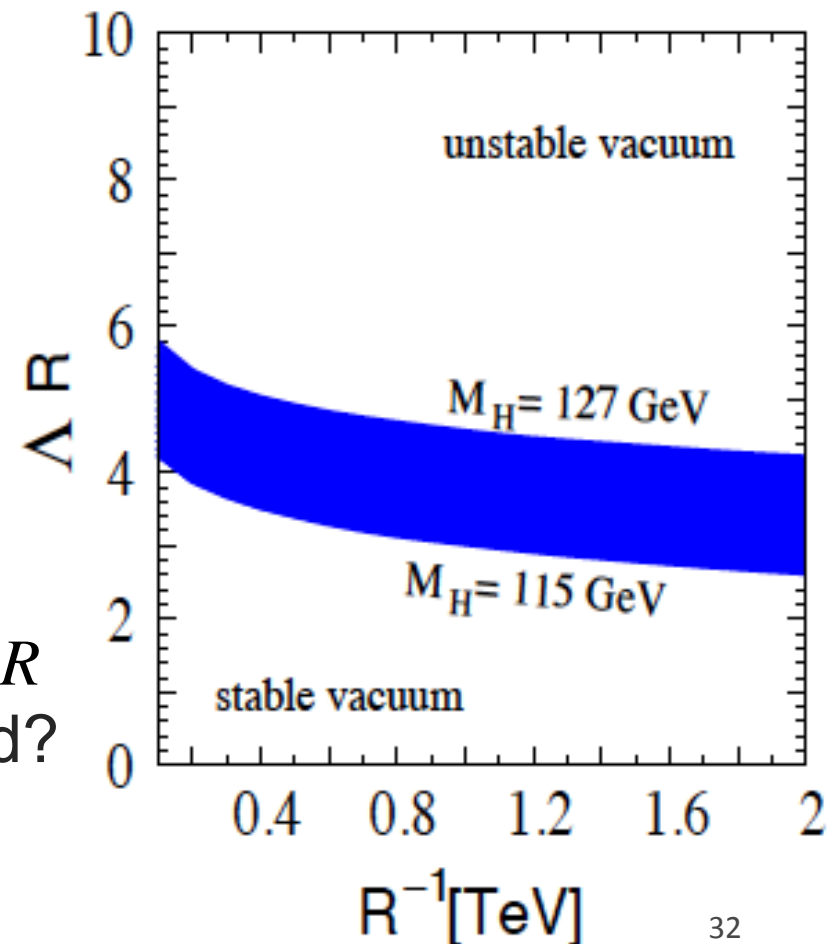
$$148\text{GeV} < m_h < 186\text{GeV} \quad \text{for} \quad 1/R = 1\text{TeV}, \Lambda R = 30$$

[Bhattacharyya,Datta,Majee,Raychaudhuri,NPB760,117(2007)]

- Vacuum stability bounds for a light Higgs:

[From Datta,Raychaudhuri,arXiv:1207.0476;
See also Blennow,Melbeus,Ohlsson,Zhang,
PLB712,419(2012)]

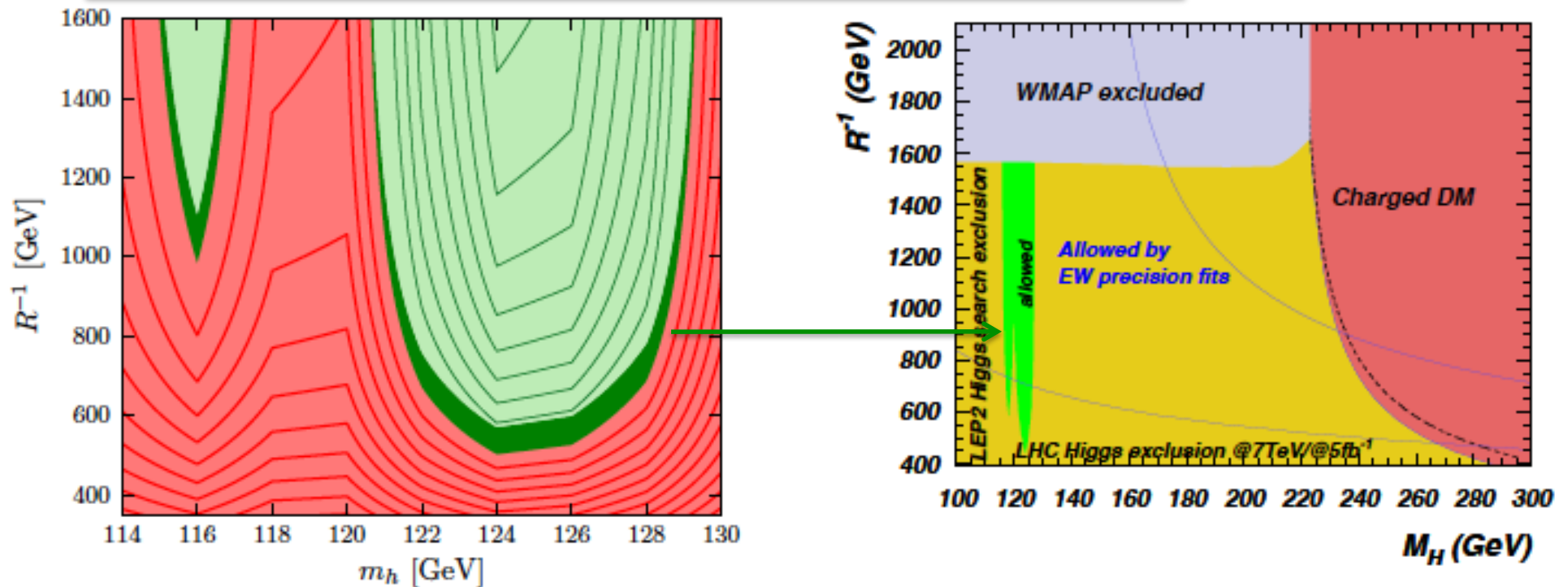
To what extent does the bound on ΛR get relaxed if metastability is allowed?



Limits on the MUED parameters space

- MUED-Higgs properties are similar to those of the SM-Higgs

LHC Higgs exclusion (7 TeV & 8 TeV) as of July 2012



➔ $1/R > 500 \text{ GeV} (95\%CL)$

[From Belanger, Belyaev, Brown, MK, Pukhov, arXiv:1207.0798;
See also Nishiwaki, Oda, Okuda, Watanabe, PLB707, 506(2012)]

Public tools and UEDs

- New physics models are confronted by the LHC and future ILC data
 - ➔ Precise computations of observables are needed to distinguish models
 - Difficulties in universal extra dimensions (UEDs)
 - Many degenerate particles and interactions are involved
 - ➔ Computing UED processes is tedious and error-prone
- Public tools play a crucial role in obtaining reliable results
- Drawbacks in earlier UED implementations in public tools:
 - EW symmetry breaking effects are neglected in many places
 - The Higgs sector is incomplete

Strategy

- **Goal:** Automated computations of UED processes with higher accuracy in anticipation of future exp. data
- **First step:** Automatic generation of 4D Feynman rules from 5D Lagrangian using LanHEP

LanHEP

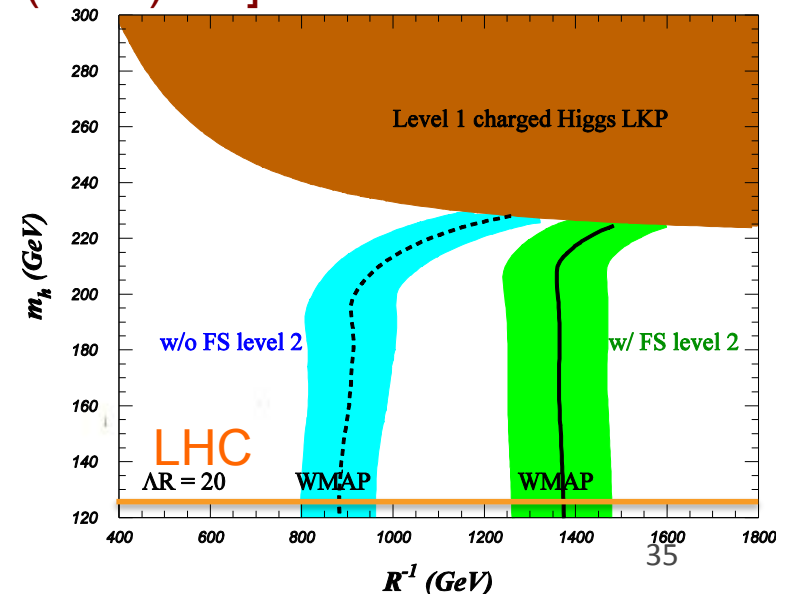
“Package for the automatic generation of Feynman rules in field theory” [\[theory.sinp.msu.ru/~semenov/lanhep.html\]](http://theory.sinp.msu.ru/~semenov/lanhep.html)

- **Second step:** [\[Belanger,MK,Pukhov,JCAP02\(2011\)009\]](#)

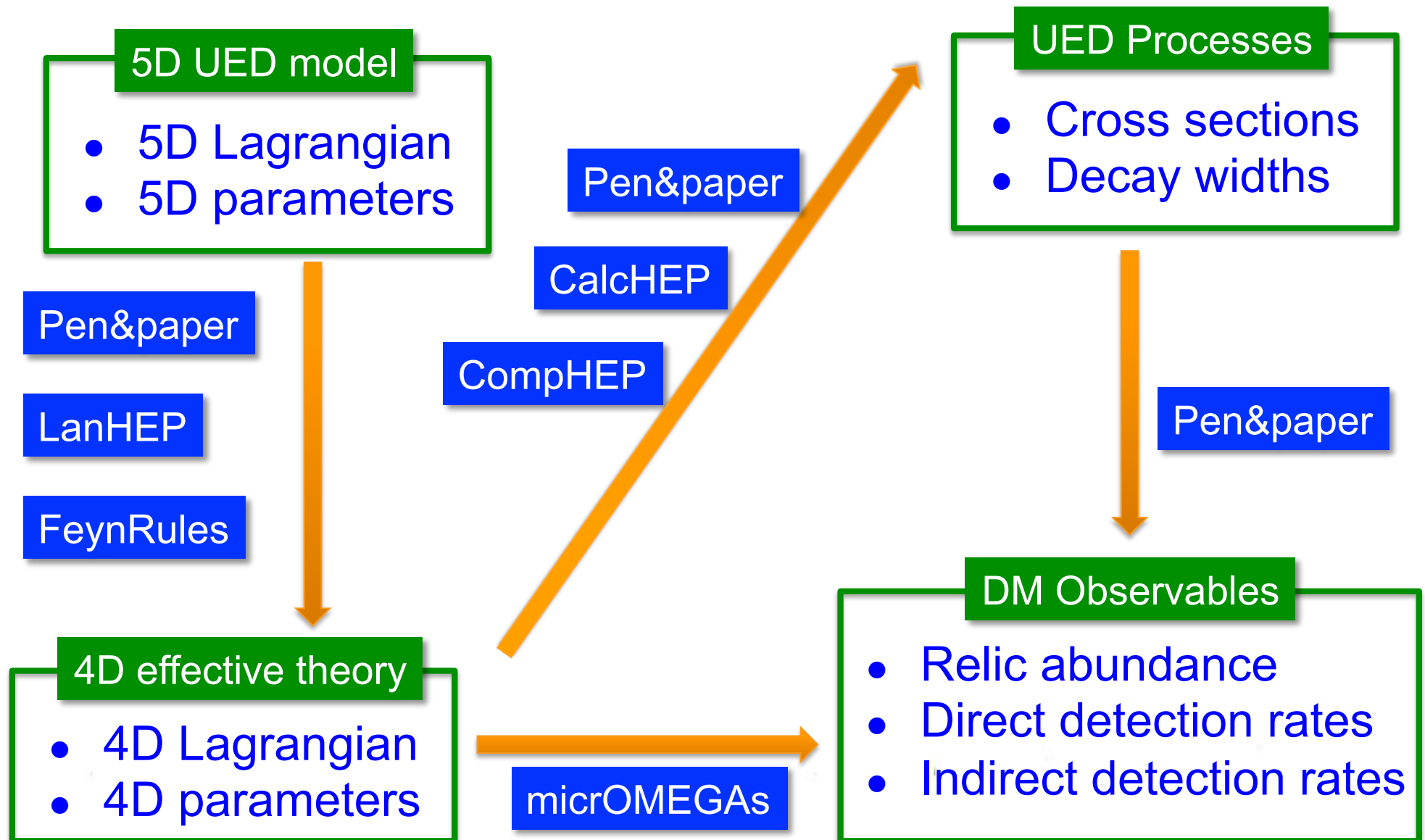
Computing DM observables using micrOMEGAs

micrOMEGAs

“a code for the calculation of Dark Matter Properties” [\[lapth.in2p3.fr/micromegas/\]](http://lapth.in2p3.fr/micromegas/)



Flowchart



Present status of 5D UED implementations

- Public codes:

Datta, Kong, Matchev

[Datta,Kong,Matchev,NJP12(2010)075017;
home.fnal.gov/~kckong/]

- EW symmetry breaking effects are partially included
- Only unitary gauge is available

Feynrules

“Mathematica package to generate Feynman rules for any Lagrangian”

[Christensen et al,arXiv:0906.2474; feynrules.phys.ucl.ac.be]

- The Higgs sector is partially included
- No 2nd KK mode is included
- Only unitary gauge is available

- Our approach:

- Updated LanHEP

[Semenov,arXiv:1005.1909]



Files w/ EWSB
in Feynman gauge as well

Vertices in LanHEP

- Define 5D fields:

Hypercharge B-boson

```
let BB5D = (CW*A - SW*Z)*cos(0)
          + ( (CW1*'~A1' - SW1*'~Z1')*cos(1,invR)
            + (CW2*A2 - SW2*Z2)*cos(2,invR) )*Sqrt2.
```

- Define 5D vertices:

EW interactions of left-handed fermions

```
lterm anti(psi)*(gamma*(i*deriv-EE/SW*taupm*WW5D/2 - Y*(EE/CW)*BB5D))*psi
      where
          psi=l1, Y=-1/2;
          psi=l2, Y=-1/2;
          psi=l3, Y=-1/2;
          psi=q1, Y= 1/6;
          psi=q2, Y= 1/6;
          psi=q3, Y= 1/6.
```

LanHEP

[Belanger,MK,Pukhov,Semenov, in preparation]

- 4D vertices:

```
E    |e    |A    |    |EE
    |G(m3)
```

```
E    |~e11 |~A1 |    | -EE/(4*CW*SW)
    |B01052*G(m3)*(1-G5)+2*CW1*SW*ca1e*G(m3)*(1+G5)
```

Goldstone boson/ghost in LanHEP

- Define 5D Goldstone/ghost fields:
 - KK modes of the 5th component of a gauge boson = Goldstone modes

Goldstone modes of gluons

```
let G5TH = ('~G1.f'*sin(1,invR) + 'G2.f'*sin(2,invR) )*Sqrt2/ZGs.
```

- Define 5D Goldstone/ghost interactions:

```
lterm (- i*SC*f_SU3*(deriv*GhG)*ghG*G5D
      + i*SC*f_SU3*(deriv*GhG)*G5D*ghG)/2.
```



LanHEP

[Belanger,MK,Pukhov,Semenov, in preparation]

- 4D Goldstone/ghost interactions:

```
G2.C |~G1.c |~G1 | | | -SC*Sqrt2/2
      |m3.p1
```

```
G2.C |~G1.c |~G1.f | | | -i*SC*Sqrt2*ZGs*invR
      |1
```

Automatic reconstruction of gluon vertices

- Auxiliary fields are needed to reconstruct 4-point (KK) gluon vertices in CalcHEP, CompHEP and MadGraph:

Auxiliary fields and their interactions in my notes

$$\begin{aligned}t_{\mu\nu}^a &= \frac{1}{\sqrt{2\pi R}} t_{\mu\nu}^{(0)a}(x^\mu) + \sum_{n=1}^{\infty} \frac{1}{\sqrt{\pi R}} t_{\mu\nu}^{(n)a}(x^\mu) \cos \frac{ny}{R} \\v_\mu^a &= \sum_{n=1}^{\infty} \frac{1}{\sqrt{\pi R}} v_\mu^{(n)a}(x^\mu) \sin \frac{ny}{R} \\\mathcal{L}_{\text{aux}}^{(5D)} &= \frac{1}{4} t^{a\mu\nu} t_{\mu\nu}^a + \frac{g_5}{2} f^{abc} t^{a\mu\nu} A_\mu^b A_\nu^c - \frac{1}{2} v^{a\mu} v_\mu^a - g_5 f^{abc} v^{a\mu} A_\mu^b A_\mu^c\end{aligned}$$

[Belanger,MK,Pukhov,
Semenov, in preparation]

Auxiliary fields and their interactions in LanHEP

```
let GT5D = 'G.t'*cos(0) + ('~G1.t'*cos(1,invR) + 'G2.t'*cos(2,invR) )*Sqrt2.
lterm SC*f_SU3*GT5D*G5D*G5D/Sqrt2.

let '~v1' = AuxPrt(1, 8, 0),
    v2 = AuxPrt(1, 8, 0).
let GV5D = ('~v1'*sin(1,invR) + 'v2'*sin(2,invR) )*Sqrt2.
lterm - ZGs*Maux*SC*f_SU3*GV5D*G5D*G5TH.
```

KK mass shifts

- 5D Lorentz violating effects induce mass shifts from n/R :

Vector

$$\mathcal{L}^{(5D)} = -\frac{1}{4}F_{\mu\nu}^a F^{a\mu\nu} - \frac{1}{2}Z F_{\mu 5}^a F^{a\mu 5}$$



$$m_{A(n)}^2 = Z \frac{n^2}{R^2}$$

Fermion

$$\mathcal{L}_{\psi}^{(5D)} = \bar{\psi} i \gamma^{\mu} D_{\mu} \psi - Z_{\psi} \bar{\psi} \gamma^5 D_5 \psi$$



$$m_{\psi(n)} = Z_{\psi} \frac{n}{R}$$

Scalar

$$\mathcal{L}_{\Phi}^{(5D)} = (D^{\mu} \Phi)^{\dagger} (D_{\mu} \Phi) - Z_{\Phi} (D_5 \Phi)^{\dagger} (D_5 \Phi) - \mu_5^2 \Phi^{\dagger} \Phi$$



$$m_{\Phi(n)}^2 = Z_{\Phi} \frac{n^2}{R^2} + \mu^2$$

- The Z-factors are properly introduced in the Goldstone/ghost sector to retain gauge invariance
- The MUED KK spectrum is obtained by adjusting the Z-factors

[Belanger,MK,Pukhov,JCAP02(2011)009]

KK Higgs masses

- Covariant derivatives for the 5D Higgs doublet:

$$D_\mu \Phi = \partial_\mu \Phi + i \frac{g_5}{2} W_\mu^a T^a \Phi + i \frac{g'_5}{2} B_\mu \Phi, \quad [\text{Belanger, MK, Pukhov, JCAP02(2011)009}]$$

$$D_5 \Phi = \partial_5 \Phi + i g_5 \frac{W_5^a}{\sqrt{Z_W}} T^a \Phi + i \frac{g'_5}{2} \frac{B_5}{\sqrt{Z_B}} \Phi$$

- Zero mode Higgs vev: $\langle \Phi^{(0)} \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$

➡ Mixing between the KK Higgs bosons
and KK gauge scalars: $W_5^{\pm(n)}, W_5^{3(n)}, B_5^{(n)}$

Physical modes

Goldstone modes

CP even KK Higgs: $m_{h^{(n)}}^2 = Z_\Phi \frac{n^2}{R^2} + m_h^2$

Charged KK scalar: $m_{a^{\pm(n)}}^2 = \frac{Z_\Phi}{Z_W} m_{W^{(n)}}^2 = Z_\Phi \left(\frac{n^2}{R^2} + \frac{m_W^2}{Z_W} \right), \quad m_{G^{\pm(n)}}^2 = \xi m_{W^{(n)}}^2$

CP odd KK scalar: $m_{a^{0(n)}}^2 = Z_\Phi \left(\frac{n^2}{R^2} + \frac{m_Z^2 c_W^2}{Z_W} + \frac{m_Z^2 s_W^2}{Z_B} \right), \quad m_{G_A^{(n)}}^2 = \xi m_{A^{(n)}}^2, \quad m_{G_Z^{(n)}}^2 = \xi m_{Z^{(n)}}^2$

UED processes

- 4D Lagrangian generated by LanHEP



CalcHEP

- Cross sections & decay widths

$$\overline{\sum} |\mathcal{M}(gg \rightarrow g^* g^*)|^2 = \frac{9g_s^4}{4} \left[3M_1^4 \frac{s^2 + t_3^2 + u_4^2}{t_3^2 u_4^2} - 3M_1^2 \frac{s^2 + t_3^2 + u_4^2}{st_3 u_4} \right. \\ \left. + 1 + \frac{(s^2 + t_3^2 + u_4^2)^3}{4s^2 t_3^2 u_4^2} - \frac{t_3 u_4}{s^2} \right],$$

$$\overline{\sum} |\mathcal{M}(q\bar{q} \rightarrow g^* g^*)|^2 = \frac{2g_s^4}{27} \left[M_1^2 \left(-\frac{4s^3}{t_3^2 u_4^2} + \frac{57s}{t_3 u_4} - \frac{108}{s} \right) \right. \\ \left. + \frac{20s^2}{t_3 u_4} - 93 + \frac{108t_3 u_4}{s^2} \right].$$

[From Smillie, Webber, JHEP 0510, 069 (2005)]

- Analytic 1st KK quark/gluon production cross sections are consistent w/ earlier works both in unitary and Feynman gauges