Degenerate higher order theories beyond Horndeski

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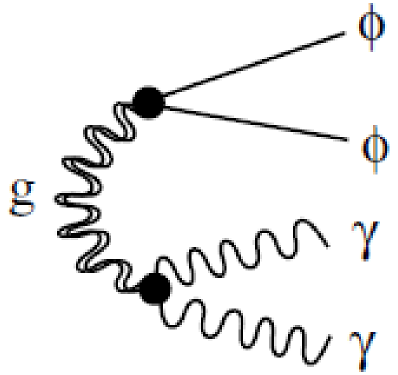
Most general scalar tensor theory in 4D

What is the most general ***viable*** scalar-tensor theory ?

viable = no ghost   ghost carries negative energy



quantum mechanically, particles can be created from vacuum without costing any energy

there is no time scale for
instability in Lorentz invariant theory
the decay is instantaneous

Unless the mass of ghost is above the cut-off scale of the theory,
ghost poses a serious consistency problem

Higher derivative theories

- Ostrogradski ghost

Theories with higher derivatives $L(g,\dot{g},\ddot{g})$

$$\frac{d^2}{d^2t}\left(\frac{\partial L}{\partial \ddot{g}}\right)-\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{g}}\right)+\left(\frac{\partial L}{\partial g}\right)=0$$

We treat $\dot{g}$ as a new variable and $\ddot{g}$ as a new velocity (we need four initial conditions)

If the velocity $\ddot{g}$ can be expressed in terms of momenta $\partial L/\partial \ddot{g}$ (non-degeneracy condition), it can be shown that the Hamiltonian is not bounded from below Woodard astro-ph/0601672

Most general 2nd order Scalar tensor theory

- This was found by Horndeski in 70's

$$^{H}S = \int d^4x \sqrt{-g}\, \left(^{H}\mathcal{L}_2 + {}^{H}\mathcal{L}_3 + {}^{H}\mathcal{L}_4 + {}^{H}\mathcal{L}_5\right)$$

$$
\begin{aligned}
^{H}\mathcal{L}_2 &= K(\phi, X)\,, \\
^{H}\mathcal{L}_3 &= G_3(\phi, X)\Box\phi\,, \\
^{H}\mathcal{L}_4 &= G_4(\phi, X)R - 2G_{4X}(\phi, X)\left[(\Box\phi)^2 - \phi_{\mu\nu}\phi^{\mu\nu}\right]\,, \\
^{H}\mathcal{L}_5 &= G_5(\phi, X)G^{\mu\nu}\phi_{\mu\nu} + \frac{1}{3}G_{5X}(\phi, X)\left[(\Box\phi)^3 - 3\Box\phi\,\phi_{\mu\nu}\phi^{\mu\nu} + 2\,\phi_{\mu\nu}\phi^{\nu\rho}\phi^{\mu}_{\rho}\right]
\end{aligned}
$$

$$X \equiv \partial_\mu\phi\,\partial^\mu\phi \qquad \phi_{\mu\nu} \equiv \nabla_\mu\nabla_\nu\phi$$

"Beyond Horndeski"

- Horndeski

  we can make conformal/disformal transformations

$$\tilde{g}_{\mu\nu} = \Omega^2(X,\phi)g_{\mu\nu} + \Gamma(X,\phi)\partial_\mu\phi\partial_\nu\phi$$

Bekenstein '92

- Beyond Horndeski

Gleyzes, Langlois, Piazza, Vernizzi '14

$$^{BH}S = \int d^4x\,\sqrt{-g}\,\left(^{BH}\mathcal{L}_4 + {}^{BH}\mathcal{L}_5\right) \qquad \phi_\mu = \partial_\mu\phi$$

$$\begin{aligned}
^{BH}\mathcal{L}_4 &= F_4(\phi,\,X)\Big[X\left((\Box\phi)^2 - \phi_{\mu\nu}\phi^{\mu\nu}\right) - 2\left(\Box\phi\,\phi_\mu\phi^{\mu\nu}\phi_\nu - \phi_\mu\phi^{\mu\nu}\phi_{\nu\rho}\phi^\rho\right)\Big], \\
^{BH}\mathcal{L}_5 &= F_5(\phi,\,X)\Big[X\left((\Box\phi)^3 - 3\,\Box\phi\,\phi_{\mu\nu}\phi^{\mu\nu} + 2\phi_{\mu\nu}\phi^{\nu\rho}\phi^\mu_\rho\right) \\
&- 3\Big((\Box\phi)^2\,\phi_\mu\phi^{\mu\nu}\phi_\nu - 2\,\Box\phi\,\phi_\mu\phi^{\mu\nu}\phi_{\nu\rho}\phi^\rho - \phi_{\mu\nu}\phi^{\mu\nu}\phi_\rho\phi^{\rho\sigma}\phi_\sigma + 2\phi_\mu\phi^{\mu\nu}\phi_{\nu\rho}\phi^{\rho\sigma}\phi_\sigma\Big)\Big]
\end{aligned}$$

these terms introduce third order derivatives in the e.o.m but the number of propagation degrees of freedom remains the same

Primary constraint

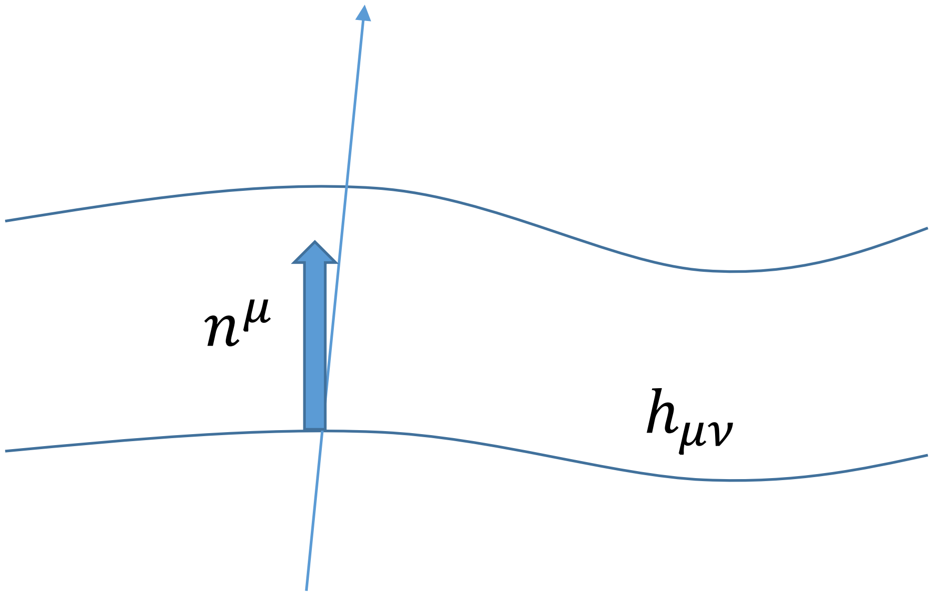
- 1+3 decomposition   Langlois, Noui 1510.06930,1512.06820

$$A_\mu \equiv \nabla_\mu \phi$$

$$A_\mu = -A_* n_\mu + \hat{A}_\nu h^\nu_\mu \qquad A_* \supset \dot{\phi}$$

$$V_* \equiv n^\mu \nabla_\mu A_* = \frac{1}{N}\left(\dot{A}_* - N^\mu D_\mu A_*\right) \qquad V_* \supset \ddot{\phi}$$

$$\pi_* \equiv \frac{1}{\sqrt{-g}} \frac{\delta S}{\delta V_*}$$



if $\pi_* = 0$ (primary constraint), the equations of motion is second order

Beyond Horndeski

- Equations of motions contain higher order derivatives

  $\pi_* \neq 0$

- Gravitational degrees of freedom

$$K_{\mu\nu} = \frac{1}{2N}\left(\dot{h}_{\mu\nu} - D_{(\mu}N_{\nu)}\right) \qquad \pi^{\alpha}_{\mu} \equiv \frac{1}{\sqrt{-g}}\frac{\delta S}{\delta K^{\mu}_{\alpha}}$$

- Primary constraint Langlois, Noui 1510.06930,1512.06820, Hull, Crisostomi, KK, Tasinato 1601.04658

$$A_*\left(2\hat{A}^2 - A_*^2\right)\pi_* - \hat{A}^{\mu}\hat{A}_{\alpha}\,\pi^{\alpha}_{\mu} \approx 0$$

by a suitable re-definition of variable $K^{\mu}_{\alpha} = \bar{K}^{\mu}_{\alpha} - \frac{V_*}{A_*(2\hat{A}^2 - A_*^2)}\hat{A}_{\alpha}\hat{A}^{\mu}$

equations of motion become second order (Lorentz invariance is broken)

Beyond Horndeski + Horndeski

Langlois, Noui 1510.06930,1512.06820, Hull, Crisostomi, KK, Tasinato 1601.04658

- Quartic bH + Quartic H, Quintic bH + Quintic H

  primary constraint still exists

$$\left[A_*\left(2\hat{A}^2 - A_*^2\right) + \frac{2G_{4X}A^2 - G_4}{F_4 A_*}\right]\pi_* - \hat{A}^\mu \hat{A}_\alpha \pi_\mu^\alpha \approx 0$$

$$\left[A_*\left(2\hat{A}^2 - A_*^2\right) - \frac{G_{5X}A^2}{3F_5 A_*}\right]\pi_* - \hat{A}^\mu \hat{A}_\alpha \pi_\mu^\alpha \approx 0$$

- Disformal transformation

$$\bar{g}_{\mu\nu} = g_{\mu\nu} + \Gamma(\phi, X) A_\mu A_\nu$$

$$\begin{aligned} {}^{H}\bar{\mathcal{L}}_4[\bar{G}_4] + {}^{BH}\bar{\mathcal{L}}_4[\bar{F}_4] &= {}^{H}\mathcal{L}_4[G_4] \\ {}^{H}\bar{\mathcal{L}}_5[\bar{G}_5] + {}^{BH}\bar{\mathcal{L}}_5[\bar{F}_5] &= {}^{H}\mathcal{L}_5[G_5] \end{aligned}$$

$$\begin{aligned} \Gamma_{4X} &= \frac{F_4}{X^2 F_4 + 2X G_{4X} - G_4} \\ \Gamma_{5X} &= \frac{3\,F_5}{3\,X^2 F_5 - X G_{5X}}\,, \end{aligned}$$

Beyond Horndeski + Horndeski

Langlois, Noui 1510.06930,1512.06820, Hull, Crisostomi, KK, Tasinato 1601.04658

- Beyond Horndeski alone

  cannot be mapped to Horndeski (beyond H is mapped to beyond H itself)

- Quintic bH+ Quartic H, Quartic bH + Quitic H

  primary constraint is lost

General analysis

Langlois, Noui 1510.06930,1512.06820, Crisostomi, KK, Tasinato 1602.03119

- Quadratic theory

$$\mathcal{L}_{\rm tot} = \sum_{i=1}^{5} \mathcal{L}_i + \mathcal{L}_R$$

$$\begin{aligned}
\mathcal{L}_1[A_1] &= A_1(\phi,\, X)\phi_{\mu\nu}\phi^{\mu\nu}\,,\\
\mathcal{L}_2[A_2] &= A_2(\phi,\, X)(\Box\phi)^2\,,\\
\mathcal{L}_3[A_3] &= A_3(\phi,\, X)(\Box\phi)\phi^\mu\phi_{\mu\nu}\phi^\nu\,,\\
\mathcal{L}_4[A_4] &= A_4(\phi,\, X)\phi^\mu\phi_{\mu\rho}\phi^{\rho\nu}\phi_\nu\,,\\
\mathcal{L}_5[A_5] &= A_5(\phi,\, X)(\phi^\mu\phi_{\mu\nu}\phi^\nu)^2\,,
\end{aligned}$$

$$\mathcal{L}_R[G] = G(\phi,\, X)R$$

- Impose the existence of primary constraint

$$\left(a\, h^\mu_\alpha + b\, \hat{A}_\alpha \hat{A}^\mu\right)\pi^\alpha_\mu + c\,\pi_* + d \approx 0$$

Quadratic theory

Crisostomi, KK, Tasinato 1602.03119, Achour, Langlois, Noui 1602.08398

- Non-minimally coupled theory $\mathcal{L}_R[G] = G(\phi,\, X)R$

Non-minimally coupled theories			
Classification	Free functions	Minkowski limit	Examples
N-I	3	✓ (H, BH)	H, H+Γ (H+BH)$^{(3)}$, H+Ω+Γ$^{(4)}$
N-II	3	✓	
N-III (i)	3	X	
N-III (ii)	3	X	

$$\bar{g}_{\mu\nu} = \Omega(X) g_{\mu\nu} + \Gamma(X)\phi_\mu\phi_\nu$$

$$\bar{L}_H[\bar{G}] = L_H[G] + L_{BH}[F] + L_3[A_3] + L_4[A_4] + L_5[A_5]$$

(N-II) $A_2 = -A_1 = -G/X \qquad A_3 = \dfrac{2\,(G - 2XG_X)}{X^2}$

Cubic theory

Achour, Crisostomi, KK, Langlois, Noui, Tasinato 1608.08135

- Cubic $\sum_{i=1}^{10} b_i L_i^{(3)} + L_G$

$$L_1^{(3)} = (\Box\phi)^3 \,, \quad L_2^{(3)} = (\Box\phi)\,\phi_{\mu\nu}\phi^{\mu\nu} \,, \quad L_3^{(3)} = \phi_{\mu\nu}\phi^{\nu\rho}\phi^{\mu}_{\rho} \,,$$
$$L_4^{(3)} = (\Box\phi)^2\,\phi_\mu\phi^{\mu\nu}\phi_\nu \,, \quad L_5^{(3)} = \Box\phi\,\phi_\mu\phi^{\mu\nu}\phi_{\nu\rho}\phi^\rho \,, \quad L_6^{(3)} = \phi_{\mu\nu}\phi^{\mu\nu}\phi_\rho\phi^{\rho\sigma}\phi_\sigma \,,$$
$$L_7^{(3)} = \phi_\mu\phi^{\mu\nu}\phi_{\nu\rho}\phi^{\rho\sigma}\phi_\sigma \,, \quad L_8^{(3)} = \phi_\mu\phi^{\mu\nu}\phi_{\nu\rho}\phi^\rho\,\phi_\sigma\phi^{\sigma\lambda}\phi_\lambda \,,$$
$$L_9^{(3)} = \Box\phi\,(\phi_\mu\phi^{\mu\nu}\phi_\nu)^2 \,, \quad L_{10}^{(3)} = (\phi_\mu\phi^{\mu\nu}\phi_\nu)^3 \,.$$

$$L_G = f_3\, G_{\mu\nu}\phi^{\mu\nu}$$

Non-minimally coupled theories				
Classification	# dof	Free functions	Minkowski limit	Examples
^{3}N-I	3	f_3, i=1,4	✓ (H, bH)	H, H+bH, $\Gamma\otimes$H$^{(2)}$, $(\Omega,\Gamma)\otimes$H$^{(3)}$
^{3}N-II	3	f_3, i=5,8,10	✓	

Cubic + quadratic theory

Achour, Crisostomi, KK, Langlois, Noui, Tasinato 1608.08135

	^{2}N-I	^{2}N-II	^{2}N-III	^{2}N-IV
^{3}N-I	(1)	X	X	X
^{3}N-II	X	✓	X	X

$$(1).\quad b_4 = \frac{-a_1 f_{3X} X - 6 b_1 f_2 + 6 b_1 f_{2X} X + 2 f_2 f_{3X}}{f_2 X}$$

$$a_3 = \frac{2\big(b_1 \big(9 a_1 f_2 X - 12 a_1 f_{2X} X^2 + 6 f_2 f_{2X} X - 6 f_2^2\big) + 2 f_{3X} (f_2 - a_1 X)^2\big)}{3 b_1 f_2 X^2}$$

$$\bar{g}_{\mu\nu} = \Omega(X) g_{\mu\nu} + \Gamma(X) \phi_\mu \phi_\nu$$

$$L_4^{\rm H} + L_5^{\rm H}. \quad \to (1)$$

Minimally coupled theories			
Classification	Free functions	Minkowski limit	Examples
M-I	3	✓ (BH)	BH, EBH$^{(1)}$, BH+Ω$^{(2)}$
M-II	3	X	
M-III	4	✓	$\mathcal{L}_i (i = 2, 3, 4, 5)$
Non-minimally coupled theories			
Classification	Free functions	Minkowski limit	Examples
N-I	3	✓ (H, BH)	H, H+Γ (H+BH)$^{(3)}$, H+Ω+Γ$^{(4)}$
N-II	3	✓	
N-III (i)	3	X	
N-III (ii)	3	X	

	^{3}M-I	^{3}M-II	^{3}M-III	^{3}M-IV	^{3}M-V	^{3}M-VI	^{3}M-VII
^{2}M-I	(1)	(2)	✓	X	(3)	X	X
^{2}M-II	X	X	✓	✓	X	X	(4)
^{2}M-III	X	X	X	X	✓	✓	(5)

	^{3}M-I	^{3}M-II	^{3}M-III	^{3}M-IV	^{3}M-V	^{3}M-VI	^{3}M-VII
^{2}N-I	(1) & (3)	(1) & (6)	(1)	X	(1) & (4)	X	X
^{2}N-II	X	X	X	X	X	X	(7)
^{2}N-III	(3)	(6)	✓	X	(4)	X	X
^{2}N-IV	(2) & (3)	(2) & (6)	(2)	X	(5)	X	X

Minimally coupled theories				
Classification	# dof	Free functions	Minkowski limit	Examples
^{3}M-I	3	i=1,2,3,4	✓ (bH)	bH, Ω⊗bH$^{(1)}$
^{3}M-II	3	i=1,3, 6	X	
^{3}M-III	3	i=1	X	
^{3}M-IV	3	i=1,4,5,8,10	X	
^{3}M-V	1	i=1,4	X	
^{3}M-VI	1	i=1,4,5,8,9,10	X	
^{3}M-VII	3	i=5,7,8,10	X	
Non-minimally coupled theories				
Classification	# dof	Free functions	Minkowski limit	Examples
^{3}N-I	3	f_3, i=1,4	✓ (H, bH)	H, H+bH, Γ⊗H$^{(2)}$, (Ω, Γ)⊗H$^{(3)}$
^{3}N-II	3	f_3, i=5,8,10	✓	

	^{2}M-I	^{2}M-II	^{2}M-III
^{3}N-I	X	X	X
^{3}N-II	X	X	X

	^{2}N-I	^{2}N-II	^{2}N-III	^{2}N-IV
^{3}N-I	(1)	X	X	X
^{3}N-II	X	✓	X	X

Achour, Crisostomi, KK, Langlois, Noui, Tasinato 1608.08135

Applications

- Cosmological perturbations

  quadratic de Rham, Matas 1604.08638

Non-minimally coupled theories			
Classification	Free functions	Minkowski limit	Examples
N-I	3	✓ (H, BH)	H, H+Γ (H+BH)$^{(3)}$, H+Ω+$\Gamma^{(4)}$
N-II	3	✓	
N-III (i)	3	X	
N-III (ii)	3	X	

  tensor mode is well behaved in N-I and N-III (i)

  scalar perturbations are unstable (gradient instability) in N-III (i)

- Screening

  beyond H breaks the Vainshtein mechanism in the presence of matter

breaking of Vainshtein mechanism

- EH+ beyond Horndeski

$$S = \int d^4x \sqrt{-g} \left[M_{\rm pl}^2 \left(\frac{R}{2} - \Lambda \right) - k_2 \mathcal{L}_2 + f_4 \mathcal{L}_{4,\rm bH} \right]$$

$$\mathcal{L}_2 = \phi_\mu \phi^\mu \equiv X$$

$$\mathcal{L}_{4,\rm bH} = -X \left[(\Box\phi)^2 - (\phi_{\mu\nu})^2 \right] + 2\phi^\mu \phi^\nu \left[\phi_{\mu\nu} \Box\phi - \phi_{\mu\sigma} \phi^\sigma{}_\nu \right]$$

$$\mathrm{d}s^2 = (-1 + 2\Phi)\,\mathrm{d}t^2 + (1 + 2\Psi)\,\delta_{ij}\,\mathrm{d}x^i\,\mathrm{d}x^j$$

$$k_2 = -2\frac{M_{\rm pl}^2 H^2}{v_0^2}\left(1 - \sigma^2\right), \qquad f_4 = \frac{M_{\rm pl}^2}{6 v_0^4}\left(1 - \sigma^2\right)$$

$$\frac{\mathrm{d}\Phi}{\mathrm{d}r} = \frac{G_{\rm N} M}{r^2} + \frac{\Upsilon_1 G_{\rm N} M''}{4}$$

$$\frac{\mathrm{d}\Psi}{\mathrm{d}r} = \frac{G_{\rm N} M}{r^2} - \frac{5\Upsilon_2 G_{\rm N} M'}{4r^2}$$

$$\sigma^2 \equiv \Lambda / (3 M_{\rm pl}^2 H^2)$$

$$\phi(r,t) = v_0 t + \frac{v_0}{2H} \ln\left(1 - H^2 r^2\right)$$

$$G_{\rm N} = \frac{3G}{5\sigma^2 - 2}$$

$$\Upsilon_1 = \Upsilon_2 \equiv \Upsilon = -\frac{1}{3}\left(1 - \sigma^2\right)$$

Kobayashi, Watanabe, Yamauchi, arXiv:1411.4130
KK, Sakstein, arXiv:1502.06872
Saito, Yamauchi, Mizuno, Gleyze, Langlois, arXiv:1503.01448
Babichev, KK, Langlois, Saito, Sakstein, arXiv:1606.06627

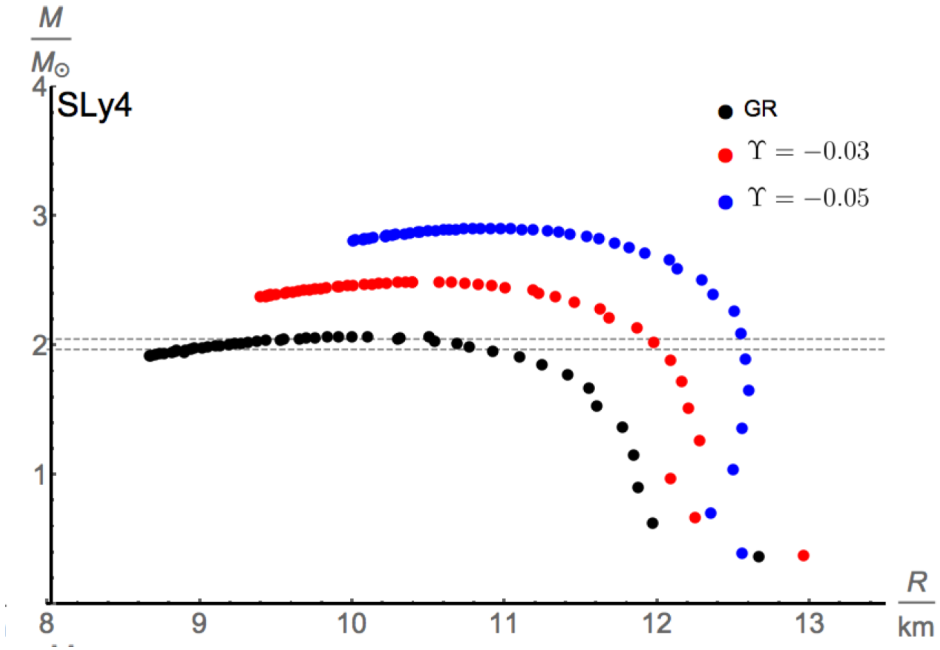
Neutron stars

$$S = \int d^4x \sqrt{-g} \left[M_{\rm pl}^2 \left(\frac{R}{2} - \Lambda \right) - k_2 \mathcal{L}_2 + f_4 \mathcal{L}_{4,\rm bH} \right]$$

$$\mathcal{L}_2 = \phi_\mu \phi^\mu \equiv X$$

$$\mathcal{L}_{4,\rm bH} = -X \left[(\Box\phi)^2 - (\phi_{\mu\nu})^2 \right] + 2\phi^\mu \phi^\nu \left[\phi_{\mu\nu} \Box\phi - \phi_{\mu\sigma} \phi^\sigma{}_\nu \right]$$

$$\phi(r,t) = v_0 t + \frac{v_0}{2H} \ln\left(1 - H^2 r^2\right) + \varphi(r)$$



Babichev, KK, Langlois, Saito, Sakstein, arXiv:1606.06627

Conclusion

- The ``most'' general viable scalar tensor theory has been identified

  this includes the Jordan version of the Honrdeski theory with metric

  $$\tilde{g}_{\mu\nu} = \Omega^2(X,\phi)g_{\mu\nu} + \Gamma(X,\phi)\partial_\mu\phi\partial_\nu\phi$$

  and beyond Horndeski theories

- Some interesting consequences in cosmology/screening have been found