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Mimetic Horndeski gravity

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FA, N. Bartolo, P. Karmakar and S. Matarrese, JCAP **1509** (2015) 051 [arXiv:1506.08575 [gr-qc]] and arXiv:1512.09374 [gr-qc] (under review).

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Mimetic dark matter

A. H. Chamseddine and V. Mukhanov, JHEP **1311** (2013) 135
A. H. Chamseddine, V. Mukhanov and A. Vikman, JCAP **1406** (2014) 017

- Can we find alternative explanations for the CDM and DE phenomena by considering a different theory of gravity (other than GR)?

  Maybe there are several components of DM. Mimetic dark matter could be one of them.

- The mimetic DM is of gravitational origin.

Consider a conformal transformation of the type: $g_{\mu\nu} = -w\ell_{\mu\nu}$

$$w \equiv \ell^{\rho\sigma}\partial_\rho\Psi\partial_\sigma\Psi$$

Then it follows that: $g^{\mu\nu}\partial_\mu\Psi\partial_\nu\Psi = -1$

Kinematical constraint

and

The theory is invariant under Weyl rescaling as: $\ell_{\mu\nu} \to \Omega^2(x)\ell_{\mu\nu}$

$$g_{\mu\nu} \to g_{\mu\nu}$$

Mimetic dark matter

A. H. Chamseddine and V. Mukhanov, JHEP **1311** (2013) 135

- Let us start with the modified Einstein-Hilbert action as:

$$S = \frac{1}{2}\int d^4x\sqrt{-g(\ell_{\mu\nu},\Psi)}\,[R(g_{\mu\nu}(\ell_{\mu\nu},\Psi)) + \mathcal{L}_m]$$

The equations of motion (eom) are: $G_{\mu\nu} - T_{\mu\nu} = (G-T)\partial_\mu\Psi\,\partial_\nu\Psi, \qquad \nabla_\rho\,[(G-T)\partial^\rho\Psi] = 0$

$\ell_{\mu\nu}$ does not appear explicitly in the equations but ψ does.
The eom for the metric is traceless because of the constraint. It can be written as

$$G_{\mu\nu} = T_{\mu\nu} + \tilde{T}_{\mu\nu} \qquad \tilde{T}_{\mu\nu} = (G-T)\partial_\mu\Psi\partial_\nu\Psi$$

Cf. $\tilde{T}_{\mu\nu} = (\rho + P)u_\mu u_\nu + P g_{\mu\nu}$

$$P = 0$$

- The scalar field imitates dust! With $\rho = G - T \neq 0$ even if $T_{\mu\nu} = 0$

A. H. Chamseddine, V. Mukhanov and A. Vikman, JCAP **1406** (2014) 017

- ✓ By generalizing the previous model by including a potential one can have almost any expansion history. One can produce quintessence, inflation and a nonsingular bouncing universe.
- The speed of sound is zero so one needs to introduce higher-derivative terms to have successful inflation.

Other works

- Why do we have more solutions than in GR? A. Barvinsky, JCAP **1401** (2014) 014

  We are doing a non-invertible transformation. The theory is a conformal extension of GR.
  The theory is free from ghost instabilities if the energy density of the fluid is positive.
  There may exist caustic instabilities cause by the geodesic flow.
  He proposed a modification including a vector field.
  The constraint implies $a_\mu = u^\alpha \nabla_\alpha u_\mu = 0$

A. Golovnev, Phys. Lett. B **728** (2014) 39

- The theory has a dual formulation in terms of a Lagrange multiplier field.

N. Deruelle and J. Rua, JCAP **1409** (2014) 002

- Identified a more general disformal transformation (DT) that leads to mimetic DM.

  Generically, if we are not in that particular case, Einstein's gravity is invariant under DT.

E. A. Lim, I. Sawicki and A. Vikman, *Dust of Dark Energy,* JCAP **1005** (2010) 012

- How can we obtain dust from a scalar field? And how can we obtain "dust with pressure"?
  They discovered the mimetic model, called $\lambda\varphi$ - fluid.

- And several more…

Non-invertibility condition

Disformal transformation

J. D. Bekenstein, Phys. Rev. D **48** (1993) 3641

$$g_{\mu\nu} = A(\Psi, w)\ell_{\mu\nu} + B(\Psi, w)\partial_\mu\Psi\partial_\nu\Psi$$

$$w \equiv \ell^{\rho\sigma}\partial_\rho\Psi\partial_\sigma\Psi$$

$g_{\mu\nu}$ - "physical" metric

$\ell_{\mu\nu}$ - "auxiliary" metric

Ψ - mimetic scalar field

D. Bettoni and S. Liberati, Phys. Rev. D **88** (2013) 084020

$A,\ B$ Free functions obeying some conditions: A>0, should preserve the Lorentzian signature, causal and the inverse metric should exist

- When can we invert the transformation (for a fixed Ψ)? i.e. find $\ell_{\mu\nu}(g_{\alpha\beta})$
  - This is equivalent to ask when can we write $w(g_{\mu\nu})$?

Mimetic disformal transformation

- We found that one cannot invert the transformation if

$$B(\Psi, w) = -\frac{A(\Psi, w)}{w} + b(\Psi)$$

This is the mimetic transformation $b(\Psi)$ is an integration constant

- Then one can find $b(\Psi) g^{\mu\nu} \partial_\mu \Psi \partial_\nu \Psi = 1$

This is called the mimetic constraint

N. Deruelle and J. Rua, JCAP **1409** (2014) 002

✓ This is the same condition as the one found by Deruelle and Rua for the system of eom of mimetic dark matter (i.e. conformal trans. on GR) to be indeterminate as we will see.

- This is a very general result which does not depend on the theory.
  The mimetic constraint is a kinematical constraint valid independently of the dynamics.

- Because the transformation is not invertible it is not surprising that the new theory may contain new solutions.

Disformal transformation method

- Let us perform a DT on a very general scalar-tensor theory and generalize N. Deruelle and J. Rua, JCAP **1409** (2014) 002

(See our paper for the case when the field in the DT is a new field).

$$S = \int d^4x\sqrt{-g}\mathcal{L}[g_{\mu\nu}, \partial_{\lambda_1}g_{\mu\nu}, \dots, \partial_{\lambda_1}\dots\partial_{\lambda_p}g_{\mu\nu}, \Psi, \partial_{\lambda_1}\Psi, \dots, \partial_{\lambda_1}\dots\partial_{\lambda_q}\Psi] + S_m[g_{\mu\nu}, \phi_m]$$

Matter action

One can write the (contracted) eom for the metric as

$$M\begin{pmatrix}\alpha_1\\ \alpha_2\end{pmatrix} = 0, \qquad \text{where} \qquad M = \begin{pmatrix} A - w\frac{\partial A}{\partial w} & -w\frac{\partial B}{\partial w} \\ w^2\frac{\partial A}{\partial w} & -A + w^2\frac{\partial B}{\partial w}\end{pmatrix}$$

where $\alpha_1 \equiv (E^{\rho\sigma}+T^{\rho\sigma})\ell_{\rho\sigma}$ and $\alpha_2 \equiv (E^{\rho\sigma}+T^{\rho\sigma})\partial_\rho\Psi\,\partial_\sigma\Psi$

$$\Omega_\Psi = \frac{\delta\left(\sqrt{-g}\mathcal{L}\right)}{\delta\Psi}, \qquad E^{\mu\nu} = \frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}\mathcal{L})}{\delta g_{\mu\nu}}, \qquad T^{\mu\nu} = \frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}\mathcal{L}_m)}{\delta g_{\mu\nu}}, \qquad \Omega_m = \frac{\delta(\sqrt{-g}\mathcal{L}_m)}{\delta\phi_m}$$

Solving the system: generic case

- If $\det(M) \neq 0$ then the only solution is: $\alpha_1 = \alpha_2 = 0$

  And the eom reduce to:

$$E^{\mu\nu} + T^{\mu\nu} = 0, \qquad \Omega_\Psi = 0, \qquad \Omega_m = 0$$

- We get the same equations of motion by taking the variation with respect to $\ell_{\mu\nu}$ or $g_{\mu\nu}$

- Generically, the theory is invariant under disformal transformations.

- ✓ Not surprising because all we did was a well-behaved invertible change of variables.

Solving the system: mimetic case

- If $\det(M) = w^2 A \frac{\partial}{\partial w}\left(B + \frac{A}{w}\right) = 0$

  then $B(\Psi, w) = -\frac{A(\Psi,w)}{w} + b(\Psi)$

  This leads to the same mimetic transformation

  and the solution is: $\alpha_2 = w\alpha_1$

The eom now read:

$$\begin{cases} E_{\mu\nu}+T_{\mu\nu} = (E+T)\, b\, \partial_\mu\Psi\, \partial_\nu\Psi, & \nabla_\rho\left[(E+T)b\,\partial^\rho\Psi\right] - \frac{\Omega_\Psi}{\sqrt{-g}} = \frac{1}{2}(E+T)\frac{1}{b}\frac{db}{d\Psi} \\ b(\Psi)g^{\mu\nu}\partial_\mu\Psi\partial_\nu\Psi = 1, & \Omega_m = 0 \end{cases}$$

❖ These eom are different from the eom resulting from taking the variation wrt $g_{\mu\nu}$

✓ These are the new eom of mimetic gravity that generalize the mimetic dark matter model.

Formulation with a Lagrange multiplier

$$S_\lambda = \int d^4x\sqrt{-g}\mathcal{L}[g_{\mu\nu}, \partial_{\lambda_1} g_{\mu\nu}, \ldots, \partial_{\lambda_1}\ldots\partial_{\lambda_p} g_{\mu\nu}, \Psi, \partial_{\lambda_1}\Psi, \ldots, \partial_{\lambda_1}\ldots\partial_{\lambda_q}\Psi] + S_m[g_{\mu\nu}, \phi_m]$$
$$+ \int d^4x\sqrt{-g}\lambda\left(b(\Psi)g^{\mu\nu}\partial_\mu\Psi\partial_\nu\Psi - 1\right)$$

For GR+conformal trans.

A. Golovnev, Phys. Lett. B **728** (2014) 39

A. Barvinsky, JCAP **1401** (2014) 014

λ - the Lagrange multiplier field which enforces the kinematical constraint

$b(\Psi)$ is a given potential function

- One obtains the same eom as the DT method before.

  The LM can be found using the eom to be $2\lambda = E + T$

- ✓ Using Horndeski's identity, we showed the field equation is not independent from the other eom. G. W. Horndeski, Int. J. Theor. Phys. **10** (1974) 363
- ✓ The mimetic theory has the same number of derivatives as the original theory if written in terms of $g_{\mu\nu}$.

❖ No higher-derivative ghosts if they did not exist already.

Mimetic Horndeski: the simplest example

$$S_H = \int d^4x\sqrt{-g}\left(\frac{R}{2} + c_2 X\right)$$

$X = -1/2\nabla_\mu\Psi\nabla^\mu\Psi$

No other matter

For a flat FLRW universe the (independent) eom are:

$$b(\Psi)\dot{\Psi}^2 + 1 = 0, \qquad 6H^2 + 4\dot{H} + c_2\dot{\Psi}^2 = 0$$

A solution is: $a(t) = t^{\frac{2}{3(1+\omega)}}, \quad \Psi(t) = \pm\sqrt{-\frac{\alpha}{c_2}}\log\frac{t}{t_0}, \quad b(\Psi) = \frac{c_2}{\alpha}t_0^2 e^{\pm 2\sqrt{-\frac{c_2}{\alpha}}\Psi}$

Integration constant (t_0)

$$\alpha = -\frac{8\omega}{3(1+\omega)^2}$$

→ Constant equation of state (eos)

- The mimetic field can mimic the expansion history of a perfect fluid with eos ω!

  Cf. usual case $\omega = 1$

- ✓ By adjusting, the function $b(\Psi)$ we can mimic the expansion history of a perfect fluid with a fixed sign for the pressure $6H^2 + 4\dot{H} = -2p$

- ○ This is a similar feature as the simple generalization of the original model where one can have almost any expansion history desired.

A. H. Chamseddine, V. Mukhanov and A. Vikman, JCAP **1406** (2014) 017

E. A. Lim, I. Sawicki and A. Vikman, JCAP **1005** (2010) 012

Mimetic cubic Galileon

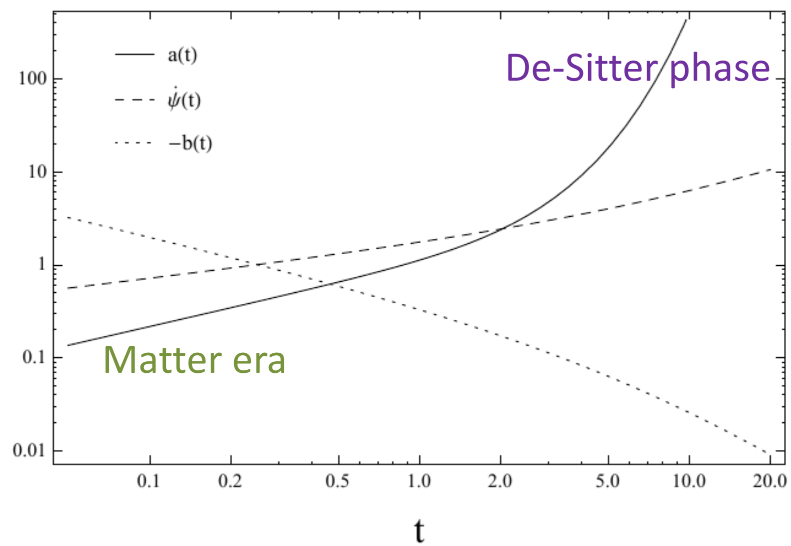
$$S_H = \int d^4x\sqrt{-g}\left(\frac{R}{2} + c_2 X - 2c_3 X\Box\Psi\right)$$

The eom are: $b(\Psi)\dot{\Psi}^2 + 1 = 0, \qquad 6H^2 + 4\dot{H} + \dot{\Psi}^2(c_2 - 4c_3\ddot{\Psi}) = 0$

- Again one can have almost any desired expansion history by choosing an appropriate $b(\Psi)$

The expansion history of a universe with cold dark matter and a positive cosmological constant Λ, $a = a_\star \sinh^{\frac{2}{3}}(\mathcal{C}t)$, is a solution for: $\frac{4c_3}{c_2}\left[-\arctan\left(\pm\sqrt{\frac{3c_2}{8\mathcal{C}^2}}\dot{\Psi}\right) \pm \sqrt{\frac{3c_2}{8\mathcal{C}^2}}\dot{\Psi}\right] = t$

$\mathcal{C} = \sqrt{3\Lambda/4}$



Linear scalar perturbations in Mimetic Horndeski

Arroja et al., 1512.09374

Notation change! $\Psi \to \varphi$

We work in the Poisson gauge $g_{00} = -a^2(\tau)\,(1+2\Phi)\,, \quad g_{0i} = 0, \quad g_{ij} = a^2(\tau)\,(1-2\Psi)\,\delta_{ij}$

$$\varphi(\tau, \mathbf{x}) = \varphi_0(\tau) + \delta\varphi(\tau, \mathbf{x})$$

The first order eom can be simplified to:

$$2b(\varphi_0)\delta\varphi' + \varphi_0' b_{,\varphi}(\varphi_0)\delta\varphi - 2b(\varphi_0)\varphi_0'\Phi = 0$$ Mimetic constraint

$$f_{18}\Psi' + f_{19}\delta\varphi' + \left(f_{20} + \frac{a^2 E^{(0)}}{\varphi_0'}\right)\delta\varphi + f_{21}\Phi = 0$$ 0i eq.

$$f_7\Psi + f_8\delta\varphi + f_9\Phi = 0$$

ij eqs.

$$f_{10}\Psi'' + f_{11}\delta\varphi'' + f_{12}\Psi' + f_{13}\delta\varphi' + f_{14}\Phi' + f_{15}\Psi + f_{16}\delta\varphi + f_{17}\Phi = 0$$

f_i - are given in terms of the Horndeski functions and their derivatives (long expressions)

It can be shown this eq. is redundant.

- ❖ No spatial derivatives appear this implies that the sound speed is also zero even for mimetic Horndeski.

Evolution equations for the perturbations

- By introducing the co-moving curvature perturbation defined as $-\zeta = \Psi + \frac{\mathcal{H}}{\varphi_0'}\delta\varphi$

  The independent set of eom can be written as

$$2b(\varphi_0)\delta\varphi' + \varphi_0' b_{,\varphi}(\varphi_0)\delta\varphi - 2b(\varphi_0)\varphi_0'\Phi = 0$$

$$-f_7\zeta + \left(f_8 - \frac{\mathcal{H}}{\varphi_0'}f_7\right)\delta\varphi + f_9\Phi = 0$$

$$\zeta' = 0$$

❖ Note that the curvature pert. satisfies a simple first order ode. The solution is just a constant!

The equation of motion for the Newtonian potential is

$$\Phi'' + \left(\frac{B_2}{B_3} + \left(\ln\frac{B_3}{B_1}\right)' + \mathcal{H} - \frac{\varphi_0''}{\varphi_0'}\right)\Phi' + \left(\frac{B_1}{B_3}\varphi_0' + \frac{B_1}{B_3}\left(\frac{B_2}{B_1}\right)' + \frac{B_2}{B_3}\left(\mathcal{H} - \frac{\varphi_0''}{\varphi_0'}\right)\right)\Phi = 0$$

B_i are given in terms of the f_i and their derivatives

No spatial Laplacian term → $c_s = 0$

Solutions for the simple models

For the two previous simple models, the previous eq. can be written as:

$$\Phi'' + \Phi' \left(3\mathcal{H} + \tilde{\Gamma}\right) + \Phi \left(\mathcal{H}^2 + 2\mathcal{H}' + \tilde{\Gamma}\mathcal{H}\right) = 0$$

$$\tilde{\Gamma} = \frac{-\mathcal{H}'' + \mathcal{H}\mathcal{H}' + \mathcal{H}^3}{\mathcal{H}' - \mathcal{H}^2}$$

→ Correction to the standard dust eq.
Exactly vanishes for a LCDM expansion history.

- In the limit $\tilde{\Gamma} \to 0$, the perturbations in these models will behave exactly the same as perturbations in LCDM.

This equation and its solution coincide with the result of found for a model where $G_3 = 0$

E. A. Lim, I. Sawicki and A. Vikman, JCAP **1005** (2010) 012

The solution can be found as $\Phi(\tau, \mathrm{x}) = C_1(\mathrm{x}) + \frac{H}{a} C_2(\mathrm{x}) - C_1(\mathrm{x}) \frac{H}{a} \int \frac{da}{H}$

Corresponds to $\zeta = 0$

If $a \propto t^{\frac{2}{3(1+w)}}$ then $\frac{H}{a} \propto a^{\frac{-5-3w}{2}}$ which decays for an expanding universe if $w > -5/3$

Conclusions

- We generalized previous results obtained for GR only and showed that a very general scalar-tensor theory is generically invariant under DT.

- However a particular subset of the DT, when the transformation is not invertible, gives origin to a new theory which is a generalization of the mimetic dark matter scenario.

$$g_{\mu\nu} = A(\Psi, w)\ell_{\mu\nu} + B(\Psi, w)\partial_\mu\Psi\partial_\nu\Psi \qquad B(\Psi, w) = -\frac{A(\Psi,w)}{w} + b(\Psi)$$

- We showed that the mimetic theory can also be derived using a Lagrange multiplier field which imposes the mimetic constraint.

- We proposed some simple toy models in the context of mimetic Horndeski theory.

  - The simplest model can mimic the expansion history of a perfect fluid with a constant eos. (the eos cannot change sign)
  - The mimetic cubic Galileon model can easily mimic the expansion history of a dark matter+Lambda universe.
  - In fact these models can mimic almost any desired expansion.

- ✓ We showed that the mimetic theory does not introduce higher-derivatives when written for $g_{\mu\nu}$. In terms of $\ell_{\mu\nu}$ it may introduce them.

- We obtained and solved the linear eom for scalar perturbations. We found that the sound speed also exactly vanishes for mimetic Horndeski gravity.