The theory of Minimal Massive gravity

Antonio De Felice
Yukawa Institute for Theoretical Physics, YITP, Kyoto U.

3rd Korea-Japan Workshop on Dark Energy
KASI, Daejon , April 7, 2016
[with prof. Mukohyama, PLB 2016; 1512.04008 (JCAP)]

YUKAWA INSTITUTE FOR THEORETICAL PHYSICS

Introduction/motivation

- Massive gravity as an alternative to dark energy
- How to remove BD ghosts?
- dRGT action able to remove it.
- However, cosmology not viable
- One of the expected “good” modes becomes a ghost
- How to remove this mode?

[ADF, E. Gumrukcuoglu, S. Mukohyama: PRL 2012]

Theory of Minimal Massive gravity (part I)

[ADF, S. Mukohyama: arXiv:1506.01594; PLB752 2016]

- Reconsider dRGT in vielbein formalism $g_{\mu\nu}=e^A{}_\mu e^B{}_\nu \eta_{AB}$
- Introduce ADM vielbein

$$\bar{e}^0{}_0=N,\bar{e}^I{}_0=\bar{e}^I{}_k N^k,\bar{e}^0{}_i=0,\bar{e}^I{}_j, \quad \bar{e}^A{}_\mu=\begin{pmatrix} N & 0 \\ N^k e^I{}_k & e^I{}_j \end{pmatrix}$$

- Do not boost/rotate: break Lorentz inv. [BLI: Comelli et al JHEP '13]
- Consider dRGT Lagrangian but substitute ADM vielbein

Precursor action

- ADM fiducial vielbein $$e^A{}_\mu = \begin{pmatrix} N & 0 \\ N^k e^I{}_k & e^I{}_j \end{pmatrix}, \quad E^A{}_\mu = \begin{pmatrix} M & 0 \\ M^k E^I{}_k & E^I{}_j \end{pmatrix}$$
- Build 3D metric $g_{ij} = e^I{}_i e^J{}_j \delta_{IJ}$
- We define $$\mathscr{L}_{pre} = \mathscr{L}_{GR} + \frac{1}{2} M_P^2 m^2 \sum_{n=0}^{4} c_n \mathscr{L}_n$$
- $$\mathscr{L}_0 = \frac{1}{24} \varepsilon^{\mu\nu\varrho\sigma} \varepsilon_{ABCD} E^A{}_\mu E^B{}_\nu E^C{}_\varrho E^D{}_\sigma \qquad \mathscr{L}_1 = \frac{1}{6} \varepsilon^{\mu\nu\varrho\sigma} \varepsilon_{ABCD} E^A{}_\mu E^B{}_\nu E^C{}_\varrho e^D{}_\sigma$$

$$\mathscr{L}_2 = \frac{1}{4} \varepsilon^{\mu\nu\varrho\sigma} \varepsilon_{ABCD} E^A{}_\mu E^B{}_\nu e^C{}_\varrho e^D{}_\sigma \qquad \mathscr{L}_3 = \frac{1}{6} \varepsilon^{\mu\nu\varrho\sigma} \varepsilon_{ABCD} E^A{}_\mu e^B{}_\nu e^C{}_\varrho e^D{}_\sigma$$

$$\mathscr{L}_4 = \frac{1}{24} \varepsilon^{\mu\nu\varrho\sigma} \varepsilon_{ABCD} e^A{}_\mu e^B{}_\nu e^C{}_\varrho e^D{}_\sigma$$

Degrees of freedom for precursor action

- We consider $e^I{}_j$ as the dynamical variables
- N, N^i Lagrange multipliers
- Unitary gauge. Fiducial veilbein: given functions of time
- In phase space there are 2*9 = 18 variables
- Hamiltonian formalism

Precursor Hamiltonian

- Primary Hamiltonian

$$H^{(p)}_{pre}=\int d^3x\left[-N R_0-N^i R_i+m^2 M H_1+\alpha_{MN} P^{[MN]}\right]$$

where

$$R_0=R_0^{GR}-m^2 H_{0,}\quad R_i=2\gamma_{ik} D_j \pi^{kj},\quad \pi^{jk}=\delta^{IJ}\Pi^j{}_I e_J{}^k$$

$$H_0=\sqrt{\tilde{\gamma}}(c_1+c_2 Y_I{}^I)+\sqrt{\gamma}(c_3 X_I{}^I+c_4),\quad Y_I{}^J=E_I{}^k e^J{}_k$$

$$H_1=\sqrt{\tilde{\gamma}}\left[c_1 Y_I{}^I+\frac{1}{2}c_2(Y_I{}^I Y_J{}^J-Y_I{}^J Y_J{}^I)\right]+c_3\sqrt{\gamma},\quad X_I{}^J=e_I{}^k E^J{}_k,$$

$$P^{[MN]}=e^M{}_j \Pi^j{}_I \delta^{IN}-e^N{}_j \Pi^j{}_I \delta^{IM}$$

Secondary precursor Hamiltonian

- Time derivative of the primary constraints
- $\dot{P}^{[MN]} \approx 0 \;\Rightarrow\; Y^{[MN]} \approx 0$
- Only two other secondary contraints: $\tilde{C}^{\sigma}, \;\; \sigma = 1,2$

$$\dot{R}_0 = \{R_0, H^{(p)}\} + \frac{\partial R_0}{\partial t} \approx 0, \;\; \dot{R}_i = \{R_i, H^{(p)}\} \approx 0,$$

$$\{R_0, R_0\} \approx 0, \;\; \{R_i, R_j\} \approx 0, \;\; \{R_0, R_i\} \neq 0$$

so

$$H_{pre}^{(s)} = \int d^3x [-N R_0 - N^i R_i + m^2 M H_1 + \alpha_{MN} P^{[MN]} + \beta_{MN} Y^{[MN]} + \tilde{\lambda}^{\sigma} \tilde{C}_{\sigma}]$$

Dof for precursor theory

- No more (tertiary) constraints
- All constraints are of second class
- Therefore 12 s.c. constraints: $R_0, \ R_i, \ P^{[MN]}, \ Y^{[MN]}, \ \tilde{C}^{\sigma}$
- Therefore (9 * 2 – 12) / 2 = 3 modes remaining
- In fact, Gws + 1 scalar mode, on normal branch
- Strong coupling expected

Removing the scalar dof

- Consider that $H_{pre}^{(s)} \approx \bar{H}_1 \equiv \int d^3x\, m^2 M H_1$
- Consider now

$$C_0 \equiv \{R_0, \bar{H}_1\} + \frac{\partial R_0}{\partial t},$$
$$C_i \equiv \{R_i, \bar{H}_1\}$$

- $\tilde{C}_\sigma \approx$ two linear combinations of C_i

Theory of minimal massive gravity (part II)

- The theory is defined by imposing 4 constraints (instead of 2)

$$C_0 \approx 0, \quad C_i \approx 0$$

- Hamiltonian

$$H = \int d^3x \left[-N R_0 - N^i R_i + m^2 M H_1 + \alpha_{MN} P^{[MN]} + \beta_{MN} Y^{[MN]} + \lambda C_0 + \lambda^i C_i \right]$$

- 14 second class constraints
- (9 * 2 – 14) / 2 = 2 dof

What is the physical content?

- Let us study the FLRW background for given $M(t), E^I{}_j = \tilde{a}(t)\delta^I_j$
- Since

$$\dot{R}_0 = \{R_0, H\} + \frac{\partial R_0}{\partial t} = C_0 + \int dy \lambda \{R_0, C_0(y)\} + \dots$$

$\lambda(t) = 0$ on the background

- Friedmann equation

$$3H^2 = \frac{m^2}{2}(c_4 + 3c_3 X + 3c_2 X^2 + c_1 X^3), \quad H \equiv \frac{\dot{a}}{Na}, \quad X \equiv \frac{\tilde{a}}{a}$$

- Two branches exist

$$(c_3 + 2c_2 X + c_1 X^2)(\dot{X} + NHX - MH) = 0.$$

Background

- The theory has the the same background of dRGT
- In particular: existence of self-accelerating sol.

$$X = X_{\pm} = \frac{-c_2 \pm \sqrt{c_2^2 - c_1 c_3}}{c_1}$$

- Existence of normal branch but only with 2 GWs

Perturbations

- No scalar perturbations
- No vector perturbations
- Vector perturbations do exist

$$S=\frac{M_P^2}{8}\sum_\sigma\int d^4x\,N(t)\,a(t)^3\left[\frac{\dot{h}_\sigma^2}{N^2}-\frac{(\partial h_\sigma)^2}{a^2}-\mu^2 h_\sigma^2\right],$$

$$\mu^2=\frac{1}{2}m^2(c_3+c_2X)\left(X-\frac{M}{N}\right)>0$$

Matter fields

- How to couple matter fields?
- Lorentz breaking terms present only in gravity sector
- Matter Lagrangian coupled only to physical metric
- Standard matter Lagrangians (including Fermions) are invariant under a general Lorentz transformation of vielbeins

$$g_{\mu\nu}=e^{A}{}_{\mu}e^{B}{}_{\nu}\eta_{AB}, \quad \bar{\psi}\gamma^{A}D_{A}\psi\rightarrow\bar{\psi}S^{-1}\gamma^{A}\Lambda_{A}{}^{B}S\,D_{B}\psi=\bar{\psi}\gamma^{B}D_{B}\psi$$

Lagrangian of MTMG

- Having defined the theory, can we find the Lagrangian?
- As usual, use inverse Legendre transformation
- Find $\dot{e}^I{}_j = \{e^I{}_j, H_T\}$
- Invert to find momentum as function of $\dot{e}^I{}_j$
- Lagrangian: $\mathscr{L} = \int d^3x\, \Pi_I{}^j \dot{e}^I{}_j - H_T$

Metric formalism

- Consider the given external fields

$$\tilde{\gamma}_{ij}=\delta_{IJ}E^{I}{}_{i}E^{J}{}_{j}, \quad \tilde{\xi}^{i}{}_{j}=\frac{1}{M}E_{L}{}^{i}\dot{E}^{L}{}_{j}, \quad \gamma_{ij}=\delta_{IJ}e^{I}{}_{i}e^{J}{}_{j},$$

- Define the tensors

$$\kappa^{m}{}_{l}\kappa^{l}{}_{n}=\tilde{\gamma}^{ms}\gamma_{sn}, \quad k^{m}{}_{j}\kappa^{j}{}_{n}=\delta^{m}{}_{n},$$

$$\Theta^{ij}=\frac{\sqrt{\tilde{\gamma}}}{\sqrt{\gamma}}\{c_{1}(\gamma^{il}\kappa^{j}{}_{l}+\gamma^{jl}\kappa^{i}{}_{l})+c_{2}[\kappa(\gamma^{il}\kappa^{j}{}_{l}+\gamma^{jl}\kappa^{i}{}_{l})-2\tilde{\gamma}^{ij}]\}+2c_{3}\gamma^{ij}$$

Precursor Lagrangian

- GR action $S_{GR}=\frac{M_P^2}{2}\int d^4x\,N\sqrt{\gamma}[{}^{(3)}R+K^{ij}K_{ij}-K^2]$
- Precursor action

$$S_{pre}=S_{GR}+\frac{M_P^2}{2}\sum_{i=1}^{4}\int d^4x\,S_i,$$

$$S_1=-m^2c_1\tilde{a}^3(N+M\kappa),$$

$$S_2=-\frac{1}{2}m^2c_2\tilde{a}^3(2N\kappa+M\kappa^2-M\kappa^i{}_j\kappa^j{}_i),$$

$$S_3=-m^2c_3\sqrt{\gamma}(M+Nk),$$

$$S_4=-m^2c_4\sqrt{\gamma}.$$

Lagrangian of MTMG in metric formalism

- The action is found to be

$$S = S_{pre} + \frac{M_P^2}{2}\int d^4x\, N\sqrt{\gamma}\left(\frac{m^2 N\lambda}{4N}\right)^2 (\Theta_{ij}\Theta^{ij} - \Theta^2/2)$$

$$-\frac{M_P^2}{2}\int d^4x\sqrt{\gamma}[\lambda C_0 - (D_n\lambda^i)C^n{}_i] + S_{mat}$$

- where

$$C_0 = \frac{1}{2}m^2 M K_{ij}\Theta^{ij} - m^2 M\left(\frac{\sqrt{\tilde{\gamma}}}{\sqrt{\gamma}}[c_1\xi + c_2(\kappa\xi - \kappa^m{}_n\xi^n{}_m)] + c_3 k^m{}_n\xi^n{}_m\right)$$

$$C^n{}_i = m^2 M\left(\frac{\sqrt{\tilde{\gamma}}}{\sqrt{\gamma}}[c_1\kappa^n{}_i + c_2(\kappa\kappa^n{}_i - \kappa^n{}_l\kappa^l{}_i)] + c_3\delta^n{}_i\right)$$

Phenomenology of MTMG

- The Lagrangian written in 1+ 3 ADM formalism
- Non-trivial constraints, which are not only potential-like
- The Lagrangian is complicated
- The theory is simple
- Why? Only Gws exists (non-linearly, on any background)

FLRW

- The background is exactly the same as of dRGT
- Two branches: self-accelerating and normal branch

$$\left(c_1 X^2 + 2 c_2 X + c_3\right)\left(X H_f - H\right) = 0, \quad H = \frac{\dot{a}}{N a}, \quad H_f = \frac{\dot{\tilde{a}}}{M \tilde{a}}, \quad X = \frac{\tilde{a}}{a}$$

- On the background $\lambda = 0$
- Discussion of the two branches

Self-accelerating branch

- X is constant: effective cosmological constant
- Introduce a perfect fluid, as test matter field
- In cosmological perturbation theory, at linear order, the theory exactly reduces to GR for scalar and vector, but Gws are massive
- At non-linear order? $\mu^2 = \frac{1}{2} m^2 X [c_2 X + c_3 r X (c_1 X + c_2)]$
- The dRGT interesting solution has been healed

Normal branch

- The background is non-trivial

$$3M_P^2H^2=\frac{1}{2}c_4m^2M_P^2+\rho_X+\rho_m\,,\quad \rho_X=\frac{1}{2}m^2M_P^2\left(3c_3X+3c_2X^2+c_1X^3\right)$$

- Perturbation scalar equation

$$\mathscr{L}=M_P^2Na^3Q\left[\frac{1}{N^2}\dot{\delta}_m^2+4\pi G_{eff}\rho_m\delta_m^2\right]$$

- Geff is model dependent

Conclusions

- Stable dRGT-like cosmology, if $\mu^2 > 0$, but $\mu < 10^{-22}$ eV (~10^{-8} Hz)

[Abbott et al, 16]

- No Vainshtein mechanism to implement
- Simpler theory (only tensor modes propagate)
- IR Lorentz-violations in gravity sector
- Phenomenology: self-acc branch: same of GR, except Gws
- Non-trivial for normal branch