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Conical singularities and the Vainshtein screening in full GLPV theories

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1. Introduction

- **Horndeski theories**
  Horndeski theories are the most general second-order scalar-tensor theories on general backgrounds.

  - **Quintessence and K-essence**

    $$G_2 = G_2(\phi, X)\,, \quad G_3 = 0\,,$$
    $$G_4 = M_{\rm pl}^2/2\,, \quad G_5 = 0\,.$$

  - **f(R) and Brans-Dicke gravity**

    $$G_2 = G_2(\phi, X)\,, \quad G_3 = 0\,,$$
    $$G_4 = F(\phi)\,, \quad G_5 = 0\,.$$

  - **covariant Galileon**

    $$G_2 = c_2 X\,, \quad G_3 = c_3 X\,,$$
    $$G_4 = M_{\rm pl}^2/2 + c_4 X^2\,, \quad G_5 = c_5 X^2\,.$$

$$S = \int d^4x \sqrt{-g} \sum_{i=2}^{5} L_i$$

$$L_2 = G_2(\phi, X)\,, \quad L_3 = G_3(\phi, X)\Box\phi\,,$$
$$L_4 = G_4(\phi, X)R - 2G_{4,X}(\phi, X)\times \left[(\Box\phi)^2 - \phi^{;\mu\nu}\phi_{;\mu\nu}\right]\,,$$
$$L_5 = G_5(\phi, X)G_{\mu\nu}\phi^{;\mu\nu} + \frac{1}{4}G_{5,X}(\phi, X)\times \left[(\Box\phi)^3 - 3(\Box\phi)\phi^{;\mu\nu}\phi_{;\mu\nu} + 2\phi_{;\mu\nu}\phi^{;\mu\sigma}\phi^{;\nu}{}_{;\sigma}\right]\,,$$

$$X \equiv g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi\,,$$
$$;\ :\ \text{covariant derivative}\,,$$
$$G_{i,X} \equiv \partial G_i/\partial X\,.$$

1. Introduction

- **3+1 decomposition**
  Expressing Horndeski Lagrangians in terms of the 3+1 decomposition in unitary gauge $(\phi = \phi(t))$, they are simplified with the following conditions:

$$A_4 = 2XB_{4,X} - B_4\,,$$
$$A_5 = -XB_{5,X}/3\,.$$

Gleyzes, Langlois, Piazza, Vernizzi extended Horndeski theories in the way that the above relations are not necessarily satisfied.

GLPV theories

Gleyzes *et al.* PRL (2015)

$$L = A_2 + A_3 K + A_4(K^2 - \mathcal{S}) + B_4\mathcal{R} + A_5 K_3 + B_5(\mathcal{U} - K\mathcal{R}/2)\,.$$

$K_{\mu\nu}$: extrinsic curvature

$\mathcal{R}_{\mu\nu}$: intrinsic curvature

$n_\mu \propto \nabla_\mu \phi$: unit normal vector

$$K \equiv K^\mu_\mu\,, \quad \mathcal{S} \equiv K^\mu_\nu K^\nu_\mu\,,$$
$$\mathcal{R} \equiv \mathcal{R}^\mu_\mu\,, \quad \mathcal{U} \equiv \mathcal{R}_{\mu\nu}K^{\mu\nu}\,,$$
$$K_3 = 3H(2H^2 - 2HK + K^2 - \mathcal{S})$$

Gleyzes, Langlois, Piazza and Vernizzi, JCAP (2013)

1. Introduction

- **GLPV theories (covariant form)**
  Rewriting GLPV Lagrangians in the covariant form, the deviation from the Horndeski domain exhibits new derivative self-interaction terms:

$$
\begin{aligned}
L_2 &= G_2(\phi, X)\,,\\
L_3 &= G_3(\phi, X)\Box\phi\,,\\
L_4 &= G_4(\phi, X)R - 2G_{4,X}(\phi, X)\left[(\Box\phi)^2 - \nabla^\mu\nabla^\nu\phi\nabla_\mu\nabla_\nu\phi\right]\\
&\quad + \boxed{F_4(\phi, X)\epsilon^{\mu\nu\rho\sigma}\epsilon_{\mu'\nu'\rho'\sigma}\nabla^{\mu'}\phi\nabla_\mu\phi\nabla^{\nu'}\nabla_\nu\phi\nabla^{\rho'}\nabla_\rho\phi\,,}\\
L_5 &= G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi\\
&\quad + \frac{1}{3}G_{5,X}(\phi, X)\left[(\Box\phi)^3 - 3\Box\phi\nabla_\mu\nabla_\nu\phi\nabla^\mu\nabla^\nu\phi + 2\nabla_\mu\nabla_\nu\phi\nabla^\sigma\nabla^\mu\phi\nabla_\sigma\nabla^\nu\phi\right]\\
&\quad + \boxed{F_5(\phi, X)\epsilon^{\mu\nu\rho\sigma}\epsilon_{\mu'\nu'\rho'\sigma'}\nabla^{\mu'}\phi\nabla_\mu\phi\nabla^{\nu'}\nabla_\nu\phi\nabla^{\rho'}\nabla_\rho\phi\nabla^{\sigma'}\nabla_\sigma\phi\,.}
\end{aligned}
$$

<u>They are originated from the deviation from Horndeski theories.</u>

$$F_4 = -(\underline{A_4 + B_4 - 2XB_{4,X}})/X^2\,, \quad F_5 = -(\underline{A_5 + XB_{5,X}/3})/(X|X|^{3/2})\,.$$

They vanish identically in Horndeski theories.

1. Introduction

- **Previous studies of GLPV theories**
  - **3 propagating DOF: 1 scalar + 2 tensor**

    The new derivative interactions never arises higher order time-derivatives and the number of DOF remains the same as Horndeski theories.

    Lin *et al.* JCAP(2014); Deffayet *et al.* PRD(2015).

  - **Partially breaking of the Vainshtein mechanism**

    The new self-interaction terms can break the screening mechanism inside a source such as a star. Kobayashi *et al.* PRD(2015); Saito *et al.* JCAP(2015).

  - **Conical singularities**

    In the absence of L_5, the conical singularity arises at the center of a spherically symmetric body with the divergence of the Ricci scalar <u>if the deviation from Horndeski theories is non-zero at the origin</u>. For the model without conical singularities, the screening mechanism works sufficiently.

    De Felice, RK and Tsujikawa PRD (2015).

2. Conical singularities in the absence of L_5

- **Solutions around the origin**

$$S = \int d^4x\sqrt{-g}\sum_{i=2}^{4} L_i + \int d^4x\sqrt{-g}L_m(g_{\mu\nu}, \Psi_m)\,.$$

$$ds^2 = -e^{2\Psi(r)}dt^2 + e^{2\Phi(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\,,$$

In order to derive solutions around the origin, we employ the following expansion:

$$\Psi(r) = \Psi_0 + \sum_{i=2}^{\infty}\Psi_i r^i\,, \quad \Phi(r) = \Phi_0 + \sum_{i=2}^{\infty}\Phi_i r^i\,, \quad \phi(r) = \phi_0 + \sum_{i=2}^{\infty}\phi_i r^i\,.$$

that satisfy regularity conditions: $\Psi'(0) = \Phi'(0) = \phi'(0) = 0.$

2. Conical singularities in the absence of L_5

- **Solutions around the origin**
  Solving equations of motion recursively, we obtain the following solutions

$$\Psi(r) = \Psi_0 + \frac{2A_2 - 2\rho_m + 3\rho_c e^{-\Psi_0}}{24B_4} r^2 + \dots ,$$

$$\Phi(r) = -\frac{\ln(1+\alpha_{\rm H})}{2} + \frac{\rho_m - A_2}{12B_4} r^2 + \dots$$

$$\phi(r) = \phi_0 \, .$$

A_2 : corresponding to the cosmological constant

ρ_m , ρ_c : $P_m' + \Psi'(\rho_m + P_m) = 0$

$\rightarrow P_m = -\rho_m + \rho_c e^{-\Psi}$,

ρ_c : an integration constant

$$\alpha_{\rm H} \equiv (2XB_{4,X} - B_4 - A_4)/A_4$$

This quantity represents the deviation from Horndeski theories in which $\alpha_{\rm H} = 0$.

2. Conical singularities in the absence of L_5

- **Conical singularity**
  By using the solutions, the three-dimensional spatial line-element reduces to

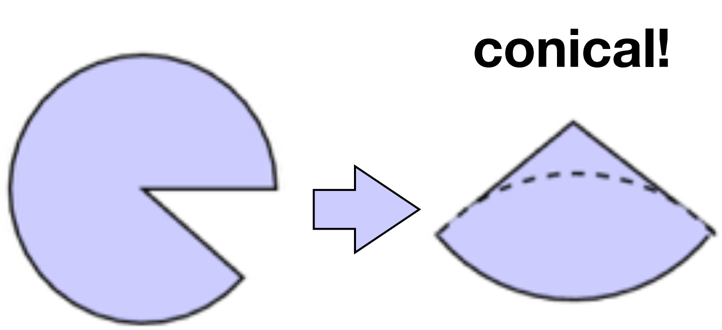
$$ds^2_{(3)} = (1+\alpha_{\rm H})^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2)\,,$$

$$ds^2_{(2)} = d\hat{r}^2 + \hat{r}^2 d\hat{\varphi}^2$$

$\hat{r} = r/\sqrt{1+\alpha_{\rm H}}\,,\ \hat{\varphi} = \sqrt{1+\alpha_{\rm H}}\varphi$

$\theta = \pi/2$ plane

- For the case $\alpha_{\rm H} \neq 0$
  The angle $\hat{\varphi}$ is not restricted between 0 and 2π .
  e.g., $-1 < \alpha_{\rm H} < 0$



- Divergence of the Ricci scalar

$$R = -\frac{2\alpha_{\rm H}}{r^2} + \mathcal{O}(r^0)$$

As long as $\alpha_{\rm H} \neq 0$, the Ricci scalar diverges at the origin!!

2. Conical singularities in the absence of L_5

- **Conditions to avoid conical singularity:** $\lim_{r\to 0}\alpha_{\rm H}=0$

  e.g., covariant Galileon (Horndeski domain: $\alpha_{\rm H}=0$)

$$A_4=-M_{\rm pl}^2/2+3c_4X^2\,,\qquad B_4=M_{\rm pl}^2/2+c_4X^2\,.$$

  we generalize these functional form as follows

$$A_4=-M_{\rm pl}^2F_1(\phi)/2+f_1(X)\,,\;B_4=M_{\rm pl}^2F_2(\phi)/2+f_2(X)\,.$$

$$\alpha_{\rm H}=\frac{1}{A_4}\left[\frac{M_{\rm pl}^2}{2}(F_1-F_2)-(f_1+f_2-2Xf_{2,X})\right]$$

We have $\phi=\phi_0$ at the origin.
In order to avoid conical singularities

$$F_1=F_2$$

Regularity condition: $\phi'(0)=0\,.$

As long as f_1 , f_2 are positive power law functions, these terms vanish.

3. Vainshtein mechanism in the absence of L_5

- **An example of models without conical singularities**

$$A_2 = -\frac{1}{2}X\,, \quad E_3 = 0\,, \quad A_4 = -\frac{1}{2}M_{\rm pl}^2 F_1(\phi) + f_1(X)\,, \quad B_4 = \frac{1}{2}M_{\rm pl}^2 F_2(\phi) + f_2(X)\,,$$

with $F_1(\phi) = F_2(\phi) = e^{-2q\phi/M_{\rm pl}}\,, \quad f_1(X) = a_4 X^2\,, \quad f_2(X) = b_4 X^2\,.$

(Horndeski condition: $a_4 = 3b_4$)

- **Field profile around the origin**
  In the following we employ the weak gravity approximation. For a compact body whose density approaches a constant ρ_m around the origin, the field equation reads

$$\phi' = cr \quad \text{with} \quad c + 8(a_4 - b_4)c^3 = \frac{q\rho_m}{3M_{\rm pl}}\,.$$

$$\alpha_{\rm H} = \frac{(a_4 - 3b_4)X^2}{M_{\rm pl}^2 F_1(\phi)/2 - a_4 X^2} \to 0$$

No conical singularities!!

3. Vainshtein mechanism in the absence of L_5

- **Schematic view of the field profile outside a compact body**

r_s :radius of a compact body

r_V :Vainshtein radius

Nonlinear terms are dominant

$$\phi'(r) = \frac{qM_{\rm pl} r_g}{r_V^2} \,,$$

$$\Phi \simeq \frac{r_g}{2r}\left[1 - 2q^2\left(\frac{r}{r_V}\right)^2 + ...\right] \,,$$

$$\Psi \simeq -\frac{r_g}{2r}\left[1 - 2q^2\left(\frac{r}{r_V}\right)^2 + ...\right] \,.$$

In this regime, the screening mechanism works sufficiently.

Linear terms are dominant

$$\phi'(r) = \frac{qM_{\rm pl} r_g}{r^2}$$

In this regime, the gravitational low is subject to change. Comparing linear terms and non-linear terms in EOM by using the above solution, the latter start to dominate over the former around

$$r_V = (|q| M_{\rm pl} r_g)^{1/3}/M$$

$$M \equiv (24|a_4 - b_4|)^{-1/6}$$

4. Effect of L_5 on the conical singularity

- **Functional forms**
  For the purpose of studying the singularity problem around the center of a compact body, we consider the function

$$G_5(\phi, X) = \sum_{n=0} g_{n/2}(\phi) X^{n/2}$$

$$A_5 = \; -X^{5/2} F_5 - \sum_{n=1} \frac{n}{6} g_{n/2}(\phi) X^{(n+1)/2} \,,$$

$$B_5 = \mathcal{B} + \sum_{n=1} \frac{n}{n+1} g_{n/2}(\phi) X^{(n+1)/2} \,.$$

For $F_5 = 0$, in EOMs, all the terms originated from L_5 just vanish at $r = 0$ and it dose not affect to the conical singularity arising from the deviation of L_4 from Horndeski.

4. Effect of L_5 on the conical singularity

- $F_5 \neq 0$
  As long as A_5 does not contain negative powers of X^m, the action remains finite.

$$A_5 = -X^{5/2}F_5 - \sum_{n=1} \frac{n}{6} g_{n/2}(\phi) X^{(n+1)/2} \,,$$

$$\Rightarrow \quad A_5 = a_0(\phi) + a_1(\phi)X^{1/2} - \sum_{n=1} \frac{n}{6} g_{n/2}(\phi) X^{(n+1)/2} \,.$$

Specific terms arising from the deviation of L_5 from Horndeski theories.

In order to derive solutions around the origin, we employ the following expansion:

$$\Phi(r) = \Phi_0 + \sum_{i=1}^{\infty} \Phi_i r^i \,, \quad \Psi(r) = \Psi_0 + \sum_{i=1}^{\infty} \Psi_i r^i \,, \quad \phi(r) = \phi_0 + \sum_{i=2}^{\infty} \phi_i r^i \,,$$

4. Effect of L_5 on the conical singularity

- **Solutions around the origin**

$$\Phi(r) = \Phi_0 + \frac{1}{6}\frac{(1+\alpha_{\rm H} - e^{-2\Phi_0})e^{3\Phi_0}G_4}{a_0(1+\alpha_{\rm H})} r + \mathcal{O}(r^2)\,,$$

$$\Psi(r) = \Psi_0 - \frac{1}{6}\frac{(1+\alpha_{\rm H} - e^{-2\Phi_0})e^{3\Phi_0}G_4}{a_0(1+\alpha_{\rm H})} r + \mathcal{O}(r^2)\,,$$

$$\Rightarrow \quad R = \frac{2(1-e^{-2\Phi_0})}{r^2} + \frac{2(1-e^{-2\Phi_0}+\alpha_{\rm H})e^{3\Phi_0}G_4}{a_0(1+\alpha_{\rm H})r} + \mathcal{O}(r^0)\,.$$

Demanding the usual boundary condition $\Phi_0 = 0$, the first term vanishes. Using this boundary condition, a condition to avoid the divergence of the second term at the origin reduces to

$$\boxed{\alpha_{\rm H} = (2XB_{4,X} - B_4 - A_4)/A_4 = 0}$$

at the origin. This condition is the same as the one in the absence of L_5.

5. Screening mechanism in the presence of L_5

- **Concrete model without conical singularities**

$$G_2 = -\frac{1}{2}X\,, \quad G_3 = 0\,, \quad G_4 = \frac{1}{2}M_{\rm pl}^2 F + \frac{c_4}{M^6}X^2\,,$$
$$G_5 = \frac{c_5}{M^6}X^2\,, \quad F_4 = \frac{d_4}{M^6}\,, \quad F_5 = \frac{d_5}{M^9}\,. \quad \left(F = e^{-2q\phi/M_{\rm pl}}\right)$$

$$\Downarrow$$

$$A_2 = -\frac{1}{2}X\,, \quad A_3 = M_{\rm pl}^2 F_{,\phi}\sqrt{X}\,,$$
$$A_4 = -\frac{1}{2}M_{\rm pl}^2 F + \frac{3c_4 - d_4}{M^6}X^2\,, \quad B_4 = \frac{1}{2}M_{\rm pl}^2 F + \frac{c_4}{M^6}X^2\,,$$
$$A_5 = -\frac{2c_5 + 3d_5}{3M^9}X^{5/2}\,, \quad B_5 = \frac{4c_5}{5M^9}X^{5/2}\,.$$

5. Screening mechanism in the presence of L_5

- **Field equation**

  Under the weak gravity approximation, the field equation reduces to

$$\frac{1}{r^2}\left(r^2\phi'\right)' \simeq \mu_1\rho_m + \mu_2 \,,$$

with

$$\mu_1 = -\frac{1}{2\beta r}\frac{(qM_{\rm pl}F-\beta\phi')r^2+(8c_5+15d_5)\phi'^4/M^9}{M_{\rm pl}^2F-2(3c_4-d_4)\phi'^4/M^6}\,,$$

$$\mu_2 = -\frac{24(2c_4-d_4)}{\beta}\frac{\phi'^3}{M^6r^2}\,,\quad \beta = -\frac{r}{2}\left[1+\frac{24(2c_4-d_4)\phi'^2}{M^6r^2}\right]\,.$$

The qualitative behavior of the field depends on the value of $s_5 \equiv 8c_5+15d_5$. For $s_5=0$ the terms originated from L_5 disappear and the field profile reduces to the same as that in the absence of L_5 .

5. Screening mechanism in the presence of L_5

- **Solutions inside a compact body**

For concreteness, we consider the density distribution $\rho_m(r) = \rho_c\, e^{-r^2/r_t^2}$ and use the following dimensionless quantities in the following:

$$x = \frac{r}{r_s}\,, \quad y = \frac{M_{\rm pl}\phi'^3(r)}{M^6\rho_c r_s^3}\,, \quad z = \frac{\phi}{M_{\rm pl}}\,, \quad \lambda_1 = \left(\frac{\rho_c r_s^2}{M_{\rm pl}^2}\right)^{1/3}\,, \quad \lambda_2 = \left(\frac{M^3 r_s^2}{M_{\rm pl}}\right)^{1/3}\,, \quad \xi_t = \frac{r_t}{r_s}\,.$$

Inside a compact body, the field equation reduces to

$$\frac{dy}{dx} \simeq \frac{1}{8\lambda_2 s_4 F}\left(qF\lambda_2 x^2 + \lambda_1^4 s_5 y^{4/3}\right) e^{-x^2/\xi_t^2}\,,$$

around the origin, the first term gives the dominant contribution and we obtain

$$y(x) = \frac{q}{24 s_4} x^3\,.$$

However, for larger x, the second term originated from L_5 starts to manifest itself, and the solution changes depending on the sign of s_5 .

5. Screening mechanism in the presence of L_5

- **Solutions inside a compact body**
  For concreteness, we consider the density distribution $\rho_m(r) = \rho_c\, e^{-r^2/r_t^2}$ and use the following dimensionless quantities in the following:

$$x = \frac{r}{r_s}\,, \quad y = \frac{M_{\rm pl}\phi'^3(r)}{M^6\rho_c r_s^3}\,, \quad z = \frac{\phi}{M_{\rm pl}}\,, \quad \lambda_1 = \left(\frac{\rho_c r_s^2}{M_{\rm pl}^2}\right)^{1/3}\,, \quad \lambda_2 = \left(\frac{M^3 r_s^2}{M_{\rm pl}}\right)^{1/3}\,, \quad \xi_t = \frac{r_t}{r_s}\,.$$

1) $s_5 > 0$

$$y(x) \simeq \left[\left(\frac{24 s_4}{q x_2^3}\right)^{1/3} - \frac{\sqrt{\pi}\lambda_1^4 s_5}{48\lambda_2 s_4 F}\xi_t {\rm erf}\left(\frac{x}{\xi_t}\right)\right]^{-3} \qquad \left(x_2 \equiv \sqrt{\frac{F\lambda_2}{|s_5|\lambda_1^4}\frac{(24|s_4|)^{2/3}}{|q|^{1/6}}}\right)$$

where ${\rm erf}(x) = (2/\sqrt{\pi})\int_0^x e^{-s^2}ds$ increase toward 1 for larger x. If the condition

$$s_5 < s_5^{\rm max} \equiv \frac{192}{\pi}\left(\frac{3s_4^4}{q}\right)^{1/3}\frac{\lambda_2}{\lambda_1^4}\frac{F}{\xi_t^2}$$

For the Sun: $s_5 < \mathcal{O}(10^{-5})$

is satisfied, the field dose not diverge inside a compact body and the field profile become similar to the one in the absence of L_5 outside a compact body.

5. Screening mechanism in the presence of L_5

- **Solutions inside a compact body**
  For concreteness, we consider the density distribution $\rho_m(r) = \rho_c\, e^{-r^2/r_t^2}$ and use the following dimensionless quantities in the following:

$$x = \frac{r}{r_s}\,, \quad y = \frac{M_{\rm pl}\phi'^3(r)}{M^6\rho_c r_s^3}\,, \quad z = \frac{\phi}{M_{\rm pl}}\,, \quad \lambda_1 = \left(\frac{\rho_c r_s^2}{M_{\rm pl}^2}\right)^{1/3}\,, \quad \lambda_2 = \left(\frac{M^3 r_s^2}{M_{\rm pl}}\right)^{1/3}\,, \quad \xi_t = \frac{r_t}{r_s}\,.$$

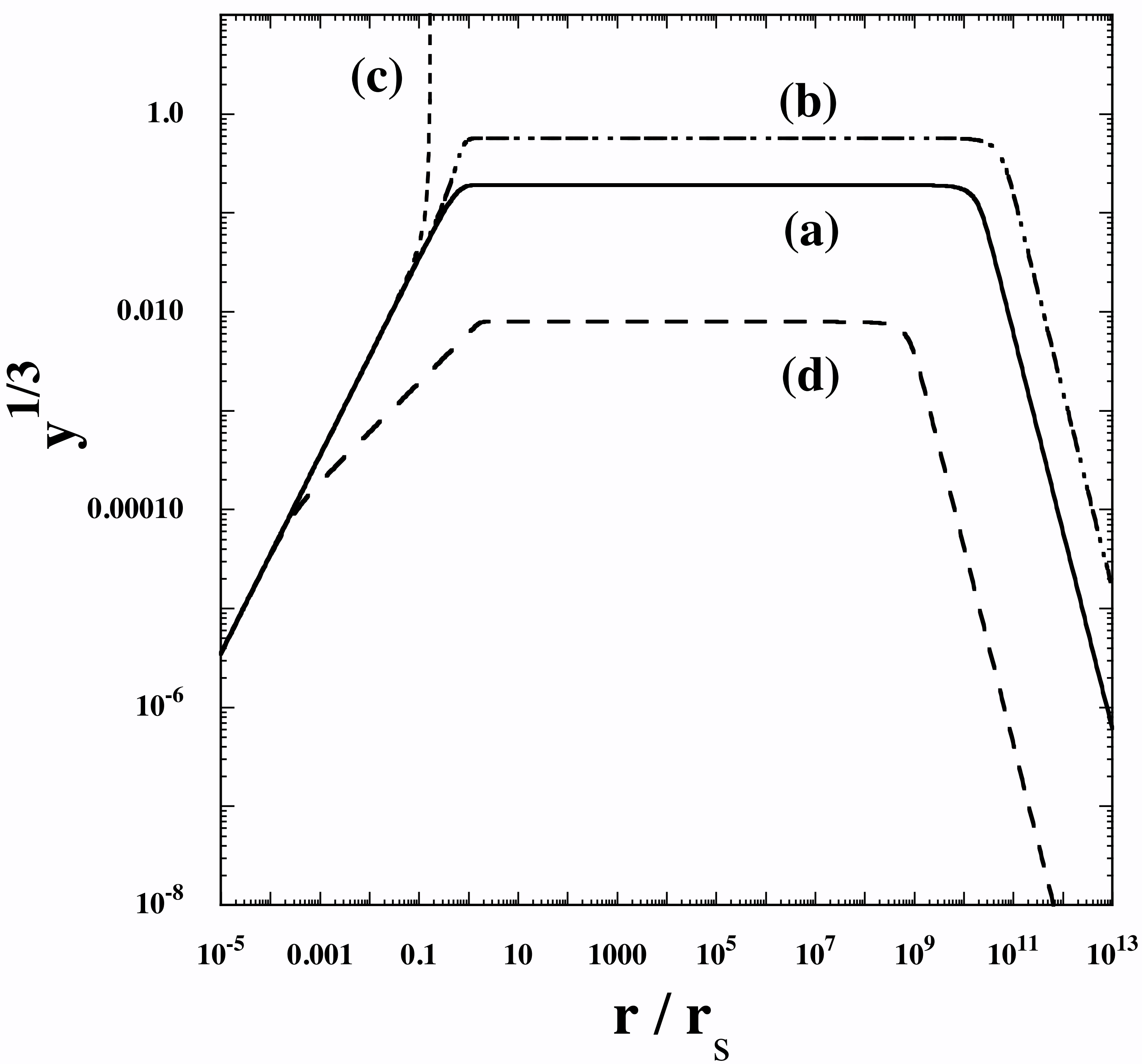
2) $s_5 < 0$

$$y(x) = \left(\frac{qF\lambda_2}{\lambda_1^4|s_5|}\right)^{3/4} x^{3/2}\,.$$

In this case, the field dose not possess the divergent behavior.
The qualitative behavior of the field outside a compact body is similar to the one in the absence of L_5 .

5. Screening mechanism in the presence of L_5

- **Numerical results**



$(a)\ s_5 = 0$

$(b)\ 0 < s_5 < s_5^{\max}$

$(c)\ s_5^{\max} < s_5$

$(d)\ s_5 < 0$

6. Conclusions

- In full GLPV theories, we showed that the GLPV Lagrangian L_5 dose not modify the divergent behavior of the Ricci scalar induced by $\alpha_{\rm H}$, i.e., the deviation of L_4 from Horndeski theories is essential for the divergent behavior.

- We derived spherically symmetric solutions around the origin in full GLPV theories, and showed that, as long as $\lim_{r\to 0}\alpha_{\rm H}=0$ is satisfied, the Ricci curvature can remain finite at the origin even in the presence of L_5 beyond the Horndeski domain.

- For the concrete model without conical singularities, we derived the field profile and gravitational potentials inside/outside a compact body under the weak gravity approximation.

- We find that there is one specific model of GLPV theories in which the effect of L_5 vanishes in the equations of motion. We also show that, depending on the sign of a L_5-dependent term in the field equation, the model can be compatible with solar-system constraints under the Vainshtein mechanism or it is plagued by the problem of a divergence of the field derivative in high-density regions.