Status of Ghost Condensation

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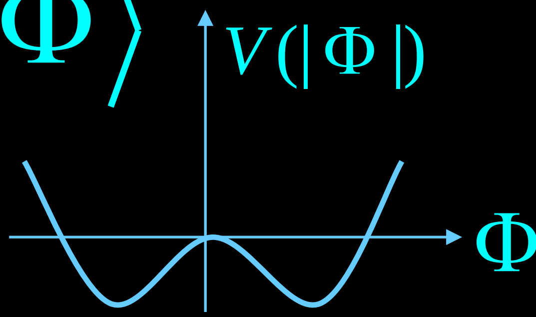
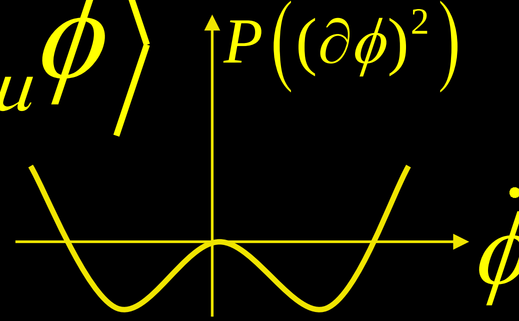
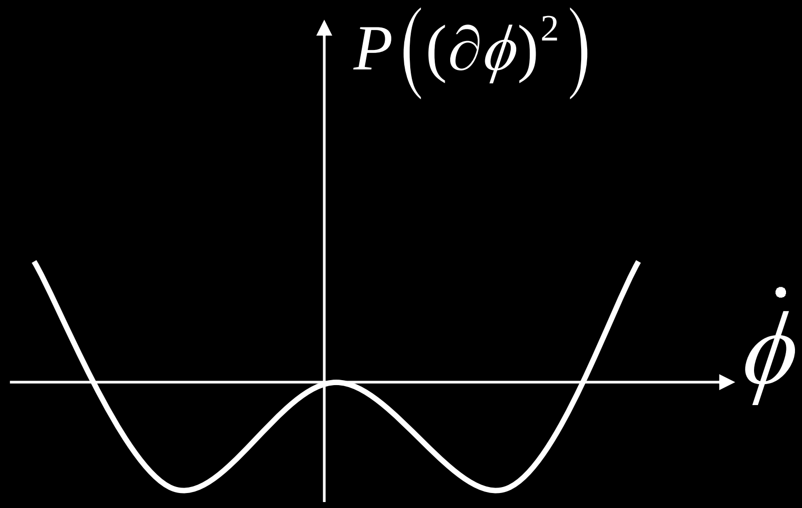
	Higgs mechanism	***Ghost condensate***
Order parameter	$\langle \Phi \rangle$ 	$\langle \partial_\mu \phi \rangle$ 
Instability	Tachyon $-\mu^2\Phi^2$	Ghost $-\dot{\phi}^2$
Condensate	V'=0, V''>0	P'=0, P''>0
Broken symmetry	Gauge symmetry	Time translational symmetry
Force to be modified	Gauge force	Gravity
New force law	Yukawa type	Newton+Oscillation

For simplicity

$$L_\varphi = P\left((\partial\phi)^2\right)$$

in FLRW background.



E.O.M.

$$\partial_t[a^3 P'\dot{\phi}] = 0 \quad \Rightarrow \quad P'\dot{\phi} \propto a^{-3} \to 0$$
$$(a \to \infty)$$

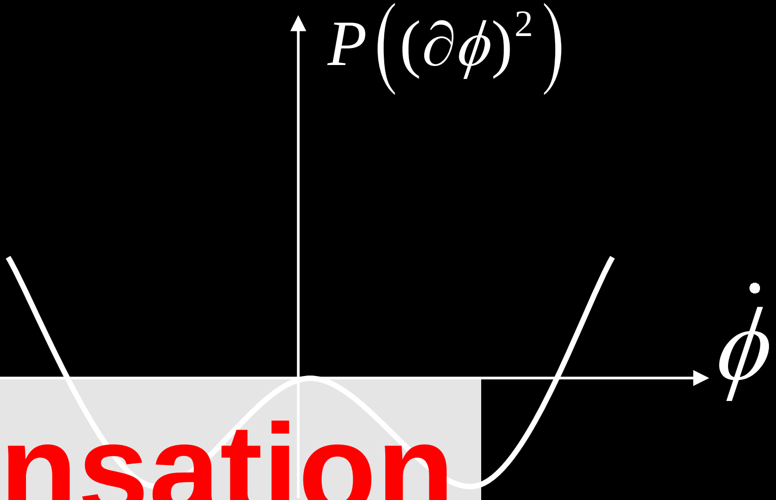
$\Rightarrow$ $\dot{\phi} = 0$
(unstable ghosty background)

or

$$P'(\dot{\phi}^2) = 0$$

For simplicity

$$L_\varphi = P\left((\partial\phi)^2\right)$$



in FLRW background.

Ghost condensation is an attractor!

E.O.M.

$$\partial_t[a^3 P'\dot{\phi}] = 0 \Longrightarrow P'\dot{\phi} \propto a^{-3} \rightarrow 0 \quad (a \rightarrow \infty)$$

$\Longrightarrow$ $\dot{\phi} = 0$ (unstable ghosty background) or $P'(\dot{\phi}^2) = 0$

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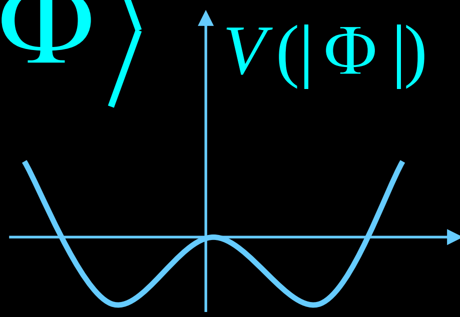
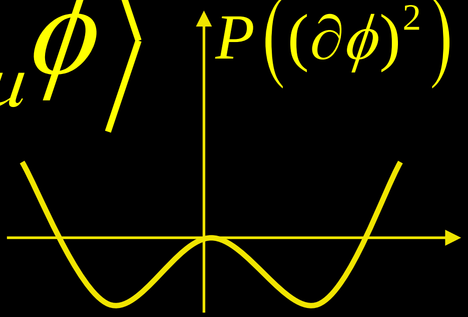
Order parameter	$\langle \Phi \rangle$ 	$\langle \partial_\mu \phi \rangle$ 
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Systematic construction of Low-energy effective theory

Backgrounds characterized by

- ✧ $\langle \partial_\mu \phi \rangle \neq 0$ and timelike

- ✧Background metric is maximally symmetric, either Minkowski or dS.

Gauge choice: $\phi(t,\vec{x}) = t.$ $\pi \equiv \delta\phi = 0$

(Unitary gauge)

Residual symmetry: $\vec{x} \to \vec{x}'(t,\vec{x})$

⇨ Write down most general action invariant under this residual symmetry.

(⇨ Action for π: undo unitary gauge!)

Start with flat background $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$

$$\delta h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$$

Under residual ξ^i

$$\delta h_{00} = 0, \delta h_{0i} = \partial_0 \xi_i, \delta h_{ij} = \partial_i \xi_j + \partial_j \xi_i$$

Action invariant under ξ^i

$$\begin{cases} (h_{00})^2 & \textbf{OK} \\ \cancel{(h_{0i})^2} & \\ K^2, K^{ij}K_{ij} & \textbf{OK} \end{cases}$$

Beginning at quadratic order, since we are assuming flat space is good background.

$$K_{ij} = \frac{1}{2}\left(\partial_0 h_{ij} - \partial_j h_{0i} - \partial_i h_{0j}\right)$$

$$\Rightarrow \quad L_{eff} = L_{EH} + M^4\left\{(h_{00})^2 - \frac{\alpha_1}{M^2}K^2 - \frac{\alpha_2}{M^2}K^{ij}K_{ij} + \cdots\right\}$$

Action for π

$$\xi^0 = \pi \quad \begin{cases} h_{00} \to h_{00} - 2\partial_0 \pi \\ K_{ij} \to K_{ij} + \partial_i \partial_j \pi \end{cases}$$

$$\Rightarrow \quad L_{eff} = L_{EH} + M^4\left\{(h_{00} - 2\dot{\pi})^2 - \frac{\alpha_1}{M^2}\left(K + \vec{\nabla}^2\pi\right)^2 - \frac{\alpha_2}{M^2}\left(K^{ij} + \vec{\nabla}^i\vec{\nabla}^j\pi\right)\left(K_{ij} + \vec{\nabla}_i\vec{\nabla}_j\pi\right) + \cdots\right\}$$

$$\left.\begin{aligned} E &\to rE \\ dt &\to r^{-1}dt \\ dx &\to r^{-1/2}dx \\ \pi &\to r^{1/4}\pi \end{aligned}\right\}$$

Make invariant ➡ $\int dt d^3x \left[\frac{1}{2}\dot{\pi}^2 - \frac{\alpha(\vec{\nabla}^2\pi)^2}{M^2} + \cdots\right]$

Leading nonlinear operator in infrared $\int dt d^3x \frac{\dot{\pi}(\nabla\pi)^2}{\tilde{M}^2}$

has scaling dimension 1/4. **(Barely) irrelevant**

⇨ Good low-E effective theory

Robust prediction

Bounds on symmetry breaking scale M

Arkani-Hamed, Cheng, Luty and Mukohyama and Wiseman, JHEP 0701:036,2007

0 100GeV 1TeV M

allowed

Jeans Instability (sun) — **ruled out**

Twinkling from Lensing (CMB) — **ruled out**

Supernova time-delay — **ruled out**

So far, there is no conflict with experiments and observations if M < 100GeV.

Holography and GSL

- Do holographic dual descriptions always exist? PROBABLY NO. e.g.) A de Sitter space is only meta-stable and a unitary holographic dual is not known.
- How about ghost condensate?
- Let's look for violation of GSL in ghost condensate, since violation of GSL would indicate absence of holographic dual. (GSL is expected to be dual to ordinary 2nd law.)
- Three proposals: (i) semi-classical heat flow; (ii) analogue of Penrose process; (iii) negative energy.
- **The generalized 2nd law holds in the presence of ghost condensate.** **(Mukohyama 2009, 2010)**

Summary so far

- Ghost condensation is the simplest Higgs phase of gravity.
- The low-E EFT is determined by the symmetry breaking pattern. No ghost in the EFT.
- Gravity is modified in IR.
- Consistent with experiments and observations if $M < 100$GeV.
- It appears easy but is actually difficult to violate the generalized 2nd law by ghost condensate.

Ghost inflation and de Sitter entropy bound

S.Jazayeri, S.Mukohyama, R.Saitou, Y.Watanabe 2016

- **Black holes & cosmology** in gravity theories are **as important as Hydrogen atoms** in quantum mechanics
- Provides **non-trivial tests** for theories of gravity e.g. black-hole entropy in string theory
- **Does the theory of ghost condensation pass those tests?**
- **Ghost condensation is known to be consistent with BH thermodynamics (Mukohyama 2009, 2010)**
- **How about de Sitter thermodynamics?**

de Sitter thermodynamics

- de Sitter (dS) spacetime is one of the three spacetimes with maximal symmetry
- dS horizon has temperature $T_H = H/(2\pi)$
- In quantum gravity, a dS space is probably unstable (e.g. KKLT, Susskind, …). So, let's consider a dS space as a part of inflation
- Friedmann equation → 1st law with entropy $S = A/(4G_N) = \pi/(G_N H^2)$

de Sitter entropy bound

Arkani-Hamed, et.al. 2007

- Slow roll inflation (non-eternal)

$$\dot{H} = -4\pi G_{\rm N}\dot{\phi}^2$$

$$S = \pi/(G_{\rm N} H^2) \qquad dN = H dt$$

$$\frac{\delta\rho}{\rho} \sim \frac{\delta a}{a} \sim H\delta t \sim H\frac{\delta\phi}{\dot{\phi}} \sim \frac{H^2}{\dot{\phi}}$$

$$\frac{dS}{dN} = \frac{8\pi^2\dot{\phi}^2}{H^4} \sim \left(\frac{\delta\rho}{\rho}\right)^{-2}$$

$|\delta\rho/\rho| \lesssim 1$ for non-eternal inflation

$$N_{\rm tot} \lesssim S_{\rm end} - S_{\rm beginning} < S_{\rm end}$$

de Sitter entropy bound

Arkani-Hamed, et.al. 2007

- Eternal inflation

$$\delta\rho/\rho \gtrsim 1 \quad \Rightarrow \quad \Delta N \gtrsim \Delta S.$$

- Fluctuation generated during eternal epoch would collapse to form BH → unobservable!

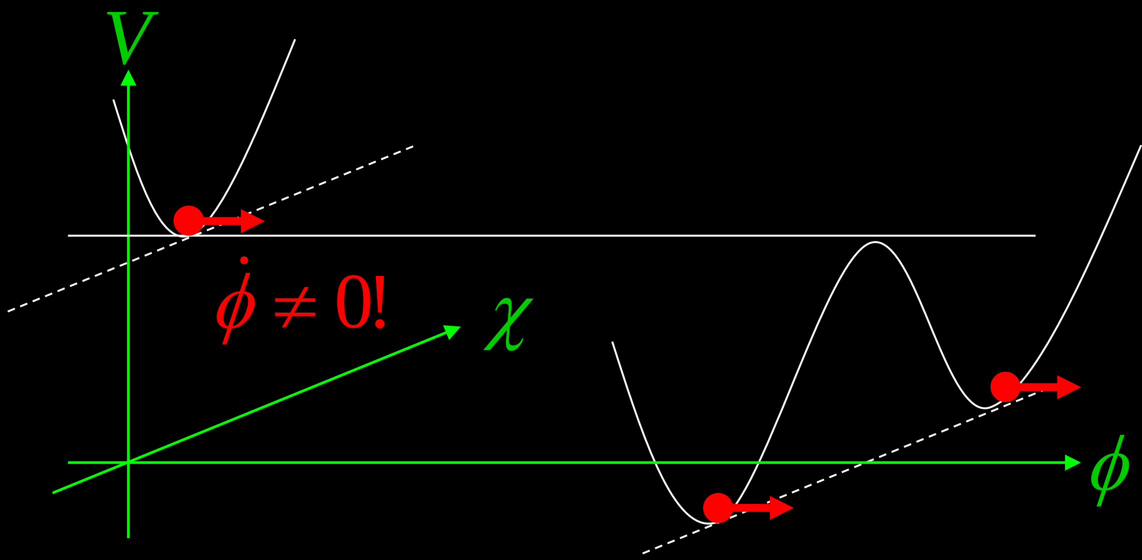
$$\Rightarrow \quad \boxed{N_{\rm obs} \lesssim S_{\rm end}}$$

- This bound holds for a large class of models of inflation

- Does ghost inflation satisfy the bound?
The answer appears to be "no" since N_{tot} can be arbitrarily large. Swampland?

Ghost inflation

Arkani-Hamed, Creminelli, Mukohyama and Zaldarriaga, JHEP 0404:001,2004



Similar to
hybrid inflation but
NOT SLOW ROLL

Scale-invariant perturbations

cf. tilted ghost inflation, Senatore (2004)

scaling dim of π

$$\frac{\delta\rho}{\rho} \sim \frac{H\delta\pi}{\dot{\phi}} \sim \left(\frac{H}{M}\right)^{5/4}$$

$$\delta\pi \sim M \cdot (H/M)^{1/4} \qquad \dot{\phi} \sim M^2$$

[compare $\frac{H}{M_{Pl}\sqrt{\varepsilon}}$]

Prediction of Large non-Gauss.

Leading non-linear interaction $\beta \dfrac{\dot{\pi}(\nabla\pi)^2}{M^2}$

non-G of $\sim \beta\left(\dfrac{H}{M}\right)^{1/4}$ ← scaling dim of op.

$\sim \beta\left(\dfrac{\delta\rho}{\rho}\right)^{1/5}$

$$\int dt d^3x \left[\frac{1}{2}\dot{\pi}^2 - \frac{\alpha(\vec{\nabla}^2\pi)^2}{M^2} + \cdots\right]$$

[Really "0.1" $\times(\delta\rho/\rho)^{1/5}$ ~ 10^{-2}. VISIBLE.
In usual inflation, non-G ~ $(\delta\rho/\rho)$ ~ 10^{-5} too small.]

⇒ $f_{NL} \sim 82\ \beta\alpha^{-4/5}$, equilateral type

Planck 2015 constraint (equilateral type)

$f_{NL} = -4 \pm 43$ (68% CL statistical) ➔ $-0.6 \leq \beta\alpha^{-4/5} \leq 0.5$

de Sitter entropy bound

Arkani-Hamed, et.al. 2007

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Lower bound on Λ?

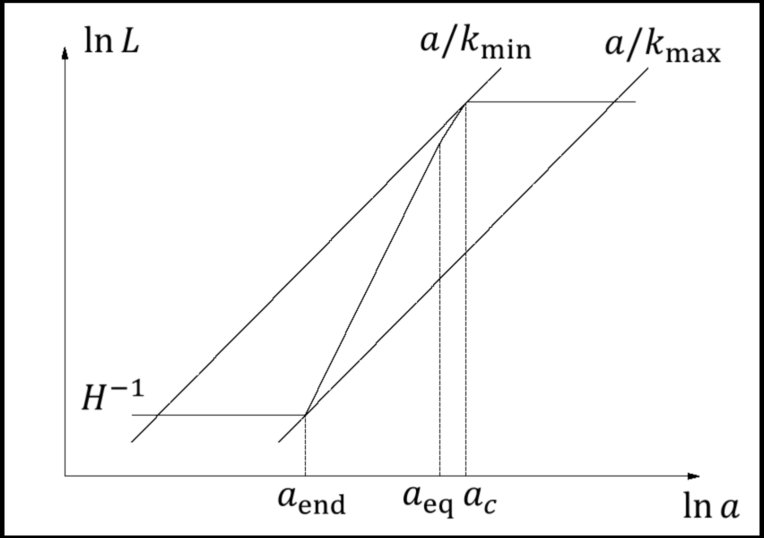
S.Jazayeri, S.Mukohyama, R.Saitou, Y.Watanabe 2016

- Tiny Λ prevents earlier inflationary modes from being observed.

$$\frac{a_{\rm end}}{a_{\rm reh}} \sim \left(\frac{\rho_{\rm reh}}{\rho_{\rm inf}}\right)^{1/3} \qquad \frac{a_{\rm reh}}{a_{\rm eq}} \sim \left(\frac{s_{\rm eq}}{s_{\rm reh}}\right)^{1/3}$$

$\ddot{a}(t = t_c) = 0$ with

$$6M_{\rm Pl}^2 \frac{\ddot{a}}{a} = -\rho_{\rm m}^{\rm eq} \left(\frac{a_{\rm eq}}{a}\right)^3 + 2\rho_\Lambda$$



- $N_{\rm obs} \sim \ln(k_{\rm max}/k_{\rm min}) \lesssim S = \pi/(G_N H^2)$

$$\Rightarrow \quad \Omega_\Lambda \gtrsim \exp\left[-10^{42}\left(\frac{M}{100\,{\rm GeV}}\right)^{-2}\right] \qquad M \lesssim 100\,{\rm GeV}$$

- In our universe, Ω_Λ=O(1) and thus the bound is well satisfied.

Summary

- Ghost condensation is **the simplest Higgs phase of gravity.**
- The low-E EFT is determined by the symmetry breaking pattern. **No ghost in the EFT.**
- Gravity is modified in IR.
- Consistent with experiments and observations if $M < 100\,\mathrm{GeV}$.
- It appears easy but is actually difficult to violate the generalized 2nd law by ghost condensate. (Mukohyama 2009, 2010)
- Ghost inflation predicts large non-Gaussianity that can be tested.
- de Sitter entropy bound appears to be violated but is actually satisfied by ghost inflation.

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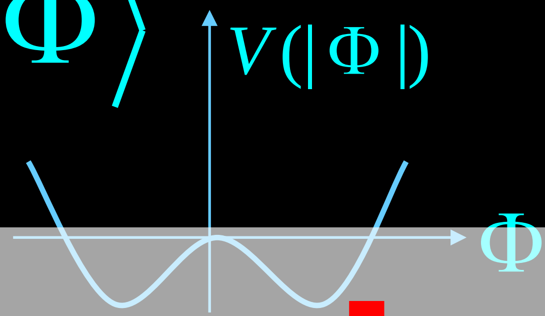
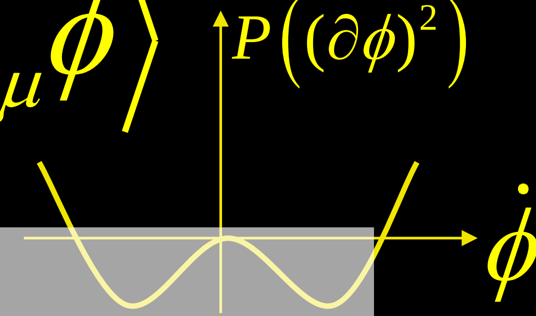
Order parameter	$\langle \Phi \rangle$ 	$\langle \partial_\mu \phi \rangle$ 
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Thank you very much!

BACKUP SLIDES

Approximate black hole solution

Mukohyama 2005

- Two time scales: $t_{BH} << t_{GC}$
- For $t_{BH} << t << t_{GC}$, a usual BH sol is a good approximation

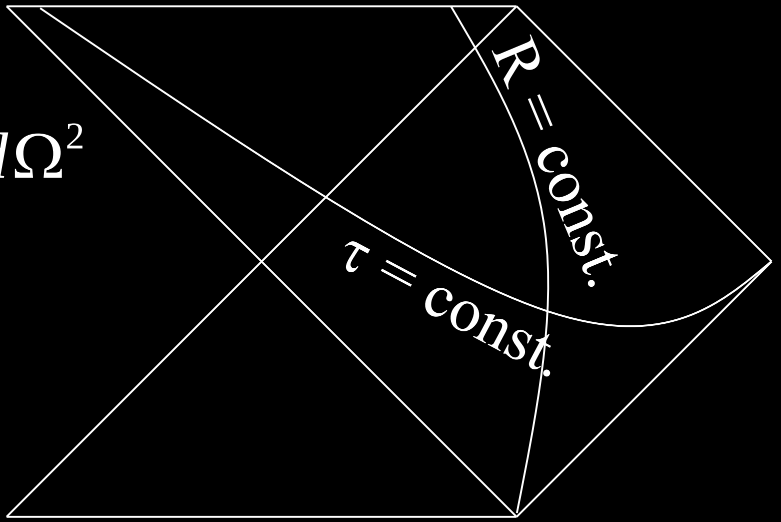
Schwarzschild metric:

$$g_{\mu\nu}dx^{\mu}dx^{\nu} = -d\tau^2 + \frac{r_g dR^2}{r(\tau,R)} + r^2(\tau,R)d\Omega^2$$

$$r(\tau,R) = \left[\frac{3}{2}\sqrt{r_g}(R-\tau)\right]^{2/3}$$

$$E = -\xi^{\mu}p_{\mu} \qquad \xi^{\mu} = \partial_{\tau} + \partial_{R}$$

$\phi = M^2\tau$ ➡ Exact sol in the absence of higher derivative terms



Accretion of ghost condensate

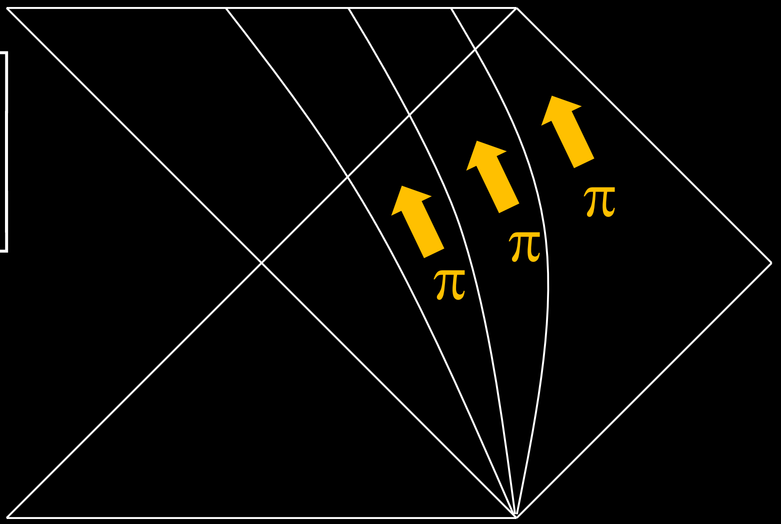
Mukohyama 2005; Cheng, Luty, Mukohyama and Thaler 2006

- A tiny tadpole due to higher derivative terms is canceled by extremely slow time-dependence.
- As a result, $\pi = \delta\phi$ starts accreting gradually.
- XTE J1118+480 (M_{bh}~$7M_{sun}$, r~$3R_{sun}$, t~240Myr or 7 Gyr) ➡ $M<10^{12}$GeV much weaker than $M<100$GeV

$$M_{bh} = M_{bh0} \times \left[1 + \frac{9\alpha M^2}{4M_{Pl}^2} \left(\frac{3M_{Pl}^2 v}{4M_{bh0}} \right)^{2/3} \right]$$

v : advanced null coordinate

α : coefficient of h.d. term



Holography and GSL

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- Let's look for violation of GSL in ghost condensate, since violation of GSL would indicate absence of holographic dual. (GSL is expected to be dual to ordinary 2^{nd} law.)
- Three proposals: (i) semi-classical heat flow; (ii) analogue of Penrose process; (iii) negative energy.

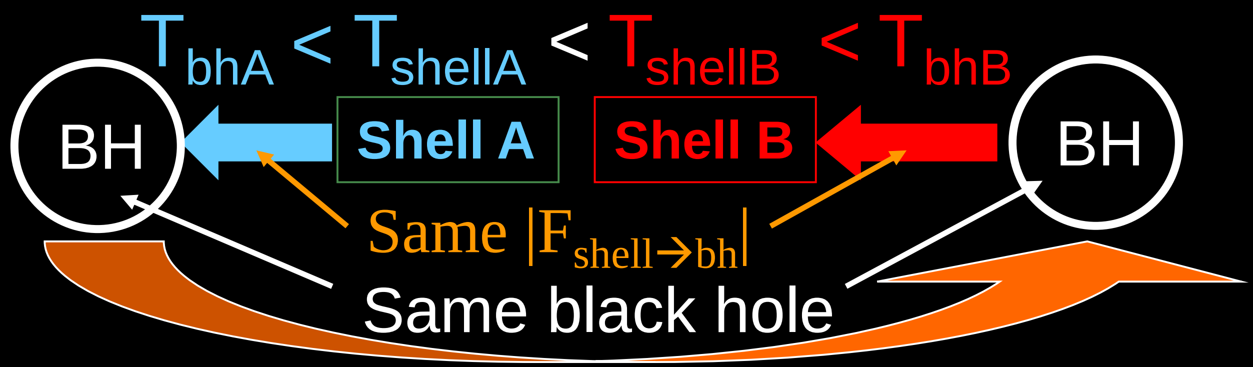
Different limits of speed

$$g_{A,B\mu\nu} = -u_\mu u_\nu + c_{A,B}^{-2}(g_{\mu\nu} + u_\mu u_\nu) \qquad u_\mu = \frac{\partial_\mu \phi}{\sqrt{-(\partial\phi)^2}}$$

- $\langle \partial_\mu \phi \rangle \sim M^2 \neq 0$ ➡ preferred direction u_μ.
- Different particles A and B may follow geodesics of different metrics $g_{A\mu\nu}$ and $g_{B\mu\nu}$.
- Lorentz breaking effects such as $|c_{A,B}^2-1|$ vanish in the limit $M^2 \to 0$ (M^2: order parameter) $c_{A,B}^2 = 1 + O(M^2/M_{Pl}^2)$.

Semi-classical heat flow

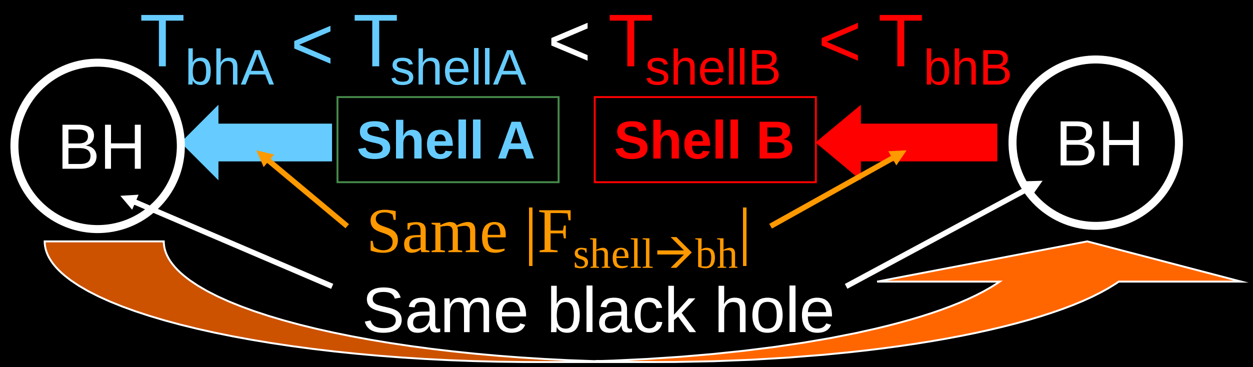
Dubovsky and Sibiryakov 2006



$dS_{shell}/dt =$
$(1/T_{shellB} - 1/T_{shellA})$
$* |F_{shell \rightarrow bh}| < 0$
$dS_{bh}/dt = 0$???

Semi-classical heat flow

Dubovsky and Sibiryakov 2006; Mukohyama 2009



$dS_{shell}/dt =$
$(1/T_{shellB} - 1/T_{shellA})$
$*|F_{shell\to bh}| < 0$
$dS_{bh}/dt = 0$???

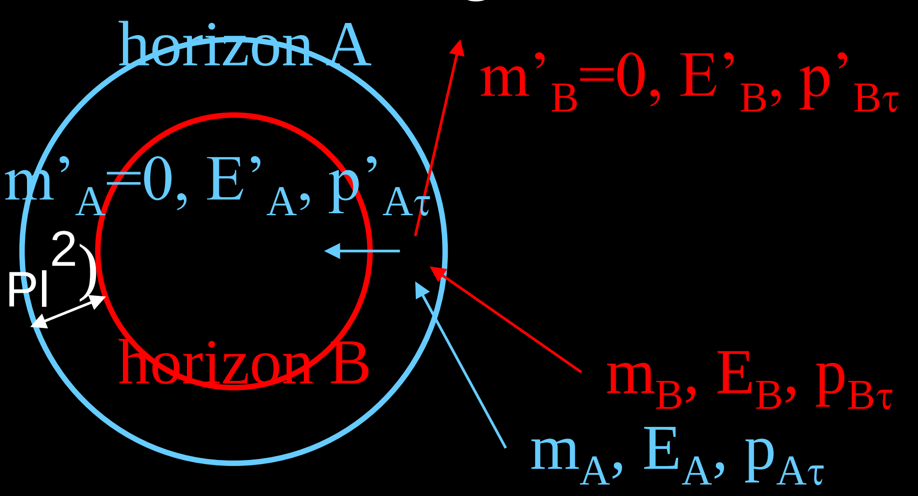
GSL not violated!

- $|T_{bhB} - T_{bhA}| / T_{bh} = O(M^2/M_{Pl}^2)$
- $|F_{shell\to bh}| / T_{bh}^2 = O(M^2/M_{Pl}^2)$
- $|dS_{shell}/dt| / T_{bh} = O(M^4/M_{Pl}^4)$
- dS_{bh}/dt due to accretion is much larger.
- $S_{tot}=S_{shell}+S_{bh}$ does increase!

Analogue of Penrose process

Elling, Foster, Jacobson, Wall 2007

$\Delta r = r_g * O(M^2/M_{Pl}^2)$

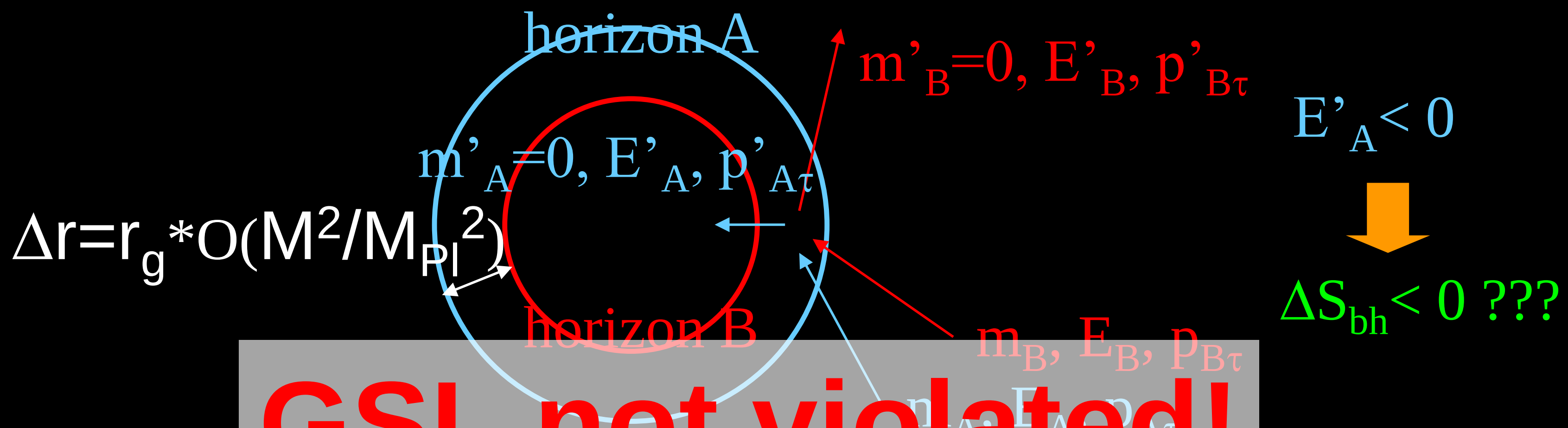


$E'_A < 0$

$\Delta S_{bh} < 0$???

Analogue of Penrose process

Elling, Foster, Jacobson, Wall 2007; Mukohyama 2009

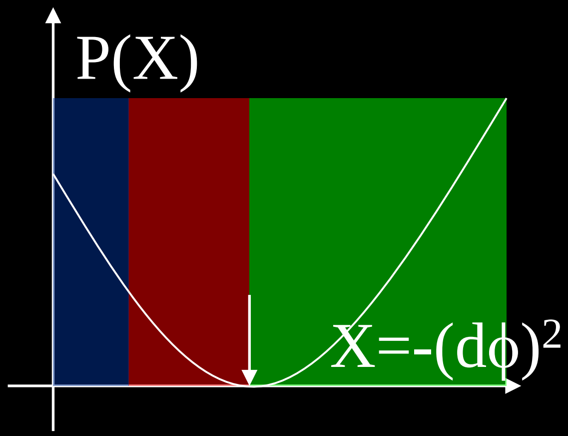


GSL not violated!

- $c_A < c_B$ ➡ horizon A outside horizon B
- $E_A+E_B=E'_A+E'_B$, $p_{A\tau}+p_{B\tau}=p'_{A\tau}+p'_{B\tau}$
- Test particle approx. ➡ $m_{A,B}/M_{Pl}^2 <<(r_g\Delta r)^{1/2}$
  ➡ $m_{A,B}^2/M_{bh}^2<<M^2/M_{Pl}^2$ ➡ $|E'_A|/M_{bh}<<M^2/M_{Pl}^2$
- This process takes time scale ~ r_g , at least.
- $\Delta M_{bh,acc}/M_{bh}$~M^2/M_{Pl}^2 ➡ $\Delta S_{bh} > 0!$

Negative energy

Arkani-Hamed, talk at PI 2006



UV (ghost) instability

IR (gradient) instability

} NEC violated

Ghost condensate

No instability

} NEC OK

It appears that S_{bh} can be decreased by sending excitation with P’<0.

Averaged NEC

Mukohyama 2009

Action

$$I = \int dx^4 \sqrt{-g} P(X) \qquad X = -\partial^\mu \phi \partial_\mu \phi$$

Stress-energy tensor

$$T_{\mu\nu} = (\rho + P) u_\mu u_\nu + P g_{\mu\nu} \qquad \rho = 2P' X - P \qquad u_\mu = \frac{\partial_\mu \phi}{\sqrt{X}}$$

EOM & shift charge

$$\nabla^\mu J_\mu = 0 \qquad J_\mu = -2P' \partial_\mu \phi \qquad Q = \int d\Sigma J_\mu u^\mu$$

In the regime of validity of EFT (|χ|<<1)

$$P = M^4 \left[p_0 + \tfrac{1}{2} p_2 \chi^2 + O(\chi^3) \right] \qquad \chi = \frac{X}{M^4} - 1$$

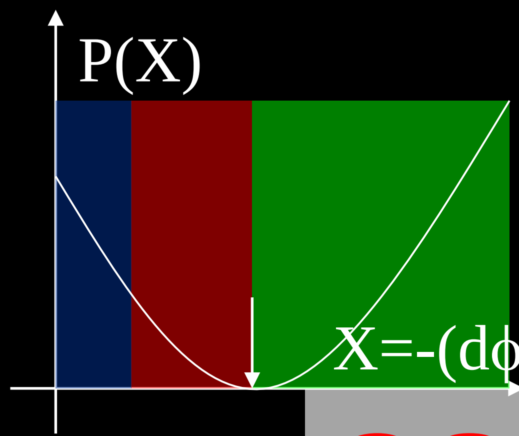
$$\rho + P - M^4 J_\mu u^\mu = M^4 \left[p_2 \chi^2 + O(\chi^3) \right]$$

Averaged NEC

$$\int d\Sigma (\rho + P) \geq M^2 Q \quad \Rightarrow \quad \int d\Sigma (\rho + P) \geq 0 \text{ for } Q \geq 0$$

Negative energy

Arkani-Hamed, talk at PI 2006; Mukohyama 2009



UV (ghost) instability
IR (gradient) instability
} NEC violated

Ghost condensate

No instability } NEC OK

It appears that S_{bh} can be decreased by sending excitation with P'<0.

GSL not violated!

- GSL in a coarse-grained sense can be protected by the averaged NEC if the shift charge is non-negative. (Negative energy is followed by larger positive energy.)
- Negative charge states are plugged by instabilities in the early universe if the shift symmetry is exact. (|P'| would be large in the early universe.)

Bounds on symmetry breaking scale M

Arkani-Hamed, Cheng, Luty and Mukohyama and Wiseman, JHEP 0701:036,2007

0 100GeV 1TeV M

allowed

Jeans Instability (sun) — **ruled out**

Twinkling from Lensing (CMB) — **ruled out**

Supernova time-delay — **ruled out**

So far, there is no conflict with experiments and observations if M < 100GeV.

Nonlinear effects cutoff Jeans Instability

Arkani-Hamedand, Cheng, Luty and Mukohyama Wiseman, JHEP 0701:036,2007.

- In the linear regime, fluctuations with $\lambda >> L_J$ ($\sim M_{pl}/M^2$) grow on a timescale $\tau \sim \lambda M_{Pl}/M$.

$$\omega^2 = \frac{\alpha k^4}{M^2} - \frac{\alpha M^2}{M_{Pl}^2} k^2 \sim -\frac{\alpha M^2}{M_{Pl}^2} k^2$$

- Nonlinear effects become important for $\pi > \pi_c$, where $\pi_c \sim \lambda^2/\tau$, or equivalently $\rho > \rho_c$, where $\rho_c \sim M^4 \pi_c/\tau \sim M^4 \lambda^2/\tau^2 \sim M^6/M_{Pl}^2$.

$$L_{eff} = M^4 \left\{ \frac{1}{2}\left[\dot{\pi} - (\nabla \pi)^2 - \Phi\right]^2 - \frac{\alpha}{2M^2}(\nabla^2 \pi)^2 \right\}$$

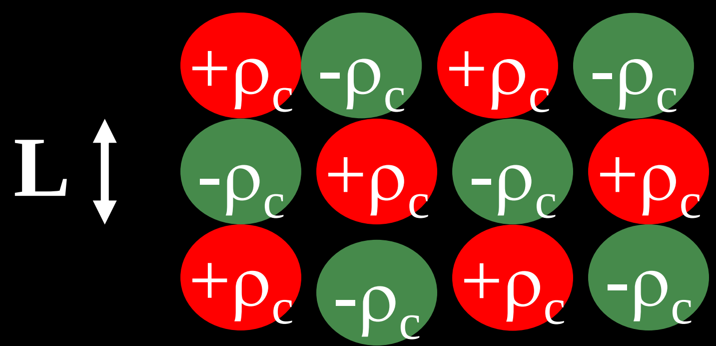
- Hereafter, we assume that nonlinear effects cutoff Jeans instability at $|\rho| \sim \rho_c$.

Twinkling from Lensing

Arkani-Hamed, Cheng, Luty and Mukohyama and Wiseman, hep-ph/0507120

- Universe is filled with $+\rho_c$ and $-\rho_c$ of the size $L_J < L < L_{max}$. ($L_J \sim M_{pl}/M^2$, L_{max} ~ $M/M_{pl}H_0$ ~ (M/TeV)*10R_{sun}.)

$\tau \sim L M_{Pl}/M < 1/H_0$



- Those patches have v~300km/s~10^{-3} relative to the CMB rest frame.

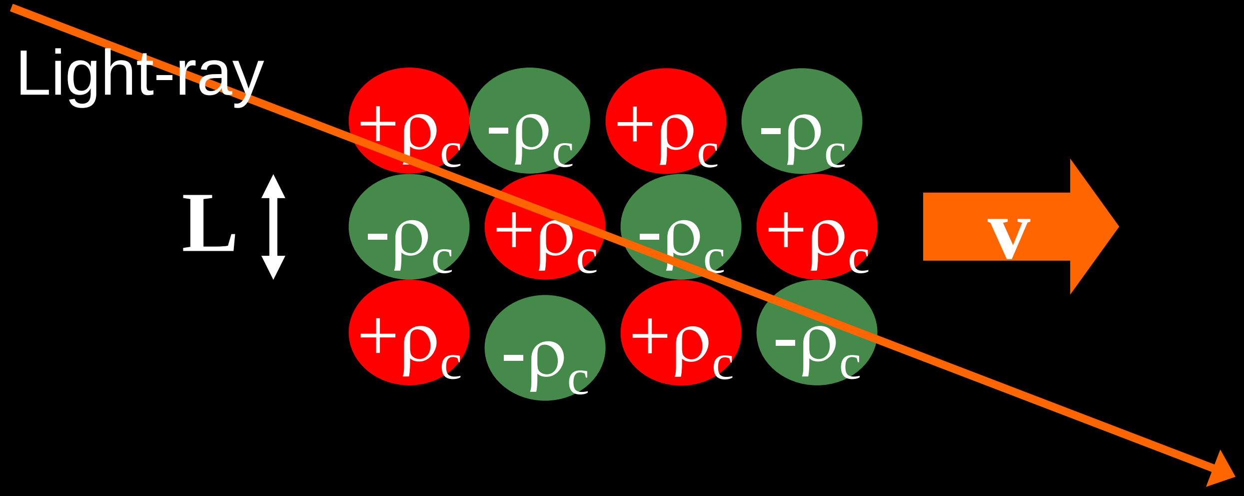
Twinkling from Lensing

- Weak gravitational lensing by each region

$$\Delta\theta_{each} \sim \frac{r_g}{b} \sim \frac{\rho_c L^3 / M_{Pl}^2}{L} \sim \frac{M^6 L^2}{M_{Pl}^4}$$

- N (~ d/L) lens events for light-ray from distance d

$$\Delta\theta_{total} \sim \Delta\theta_{each}\sqrt{N} \sim \frac{M^6 d^{1/2} L^{3/2}}{M_{Pl}^4}$$



Twinkling from Lensing

- Requiring that $\Delta\theta_{total} < 10^{-2}$ for CMB ($d \sim 1/H_0$), we obtain the bound

$$M < 100 GeV$$

- Twinkling time-scale

$$\tau \sim L_{max}/v \sim (M/100GeV) * 0.1 \text{ day}$$