Aspects of Dark Energy Eschatology: Big Crunch, Big Rip and Parametrizations

L. Perivolaropoulos
http://leandros.physics.uoi.gr
Department of Physics
University of Ioannina, Greece

Collaborators:
I. Antoniou (U. of Ioannina)
A. Lykkas (U. of Ioannina)
S. Nesseris (IFT, U. of Madrid, Spain)
G. Pantazis (U. of Ioannina)

3rd Korea-Japan Workshop on Dark Energy
KASI, Daejeon, Korea

Possible Ends of the Cosmos with Dark energy

The evolution of the Dark energy equation of state w(z) determines the type of the final state of a flat Universe.

Three main possible classes of evolution:

V < 0 : Big Crunch Singularity (Quintessence or Scalar Tensor Quintessence with Negative potential)

G. Felder et. al. Phys.Rev. D66 (2002) 023507
R. Kallosh et al, JCAP 0310 (2003) 015

$w_{de}<-1$: Big Rip singularity (ghosts, scalar tensor quintessence)

R. Calwell, M. Kamionkwski and N. Weinberg. Phys.Rev. D66 (2002) 023507

$-1 \leq w_{de} < -1/3$: deSitter/Static island bound system-universe of Black Holes and Radiation (Quintessence, ΛCDM)

F. Adams et. al.. Int.J.Mod.Phys. D12 (2003) 1743-1750
T. Chiba, R. Takahashi and N. Sugiyama, Class.Quant.Grav. 22 (2005) 3745-3758
L. Krauss and R. Scherrer, Int.J.Mod.Phys. D17 (2008) 685-690

Questions to be addressed

Q1: How generic is the Big Crunch singularity in the presence of potentials with negative range in scalar tensor quintessence?

LP, Phys.Rev. D71 (2005) 063503
A. Lykkas, LP, Phys.Rev. D93 (2016) 4, 043513, arXiv:1511.08732

Q2: How and when does a strongly bound system dissociate before the Big Rip? What new effects emerge beyond the Newtonian approximation?

S. Nesseris, LP, Phys.Rev. D70 (2004) 123529 , astro-ph/0410309,
I. Antoniou, LP arXiv:1603.02569

Q3: How can we avoid misleading conclusions when using cosmological data to fit w(z) and predict its future evolution using parameterizations?
Is the CPL parameterization adequate to describe a possible non-trivial evolution of w(z)?

G. Pantazis, S. Nesseris, LP, arXiv:1603.02164

Linear Negative Potential in GR

Quintessence or Phantom scalar field Lagrangian

$$\mathcal{L} = \pm\frac{1}{2}\dot{\phi}^2 - V(\phi)$$

Equation of state parameter

$$w = \frac{p}{\rho} = \frac{\pm\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\pm\frac{1}{2}\dot{\phi}^2 + V(\phi)}$$

Linear potential:

$$V(\phi) = \mp s\ \phi$$

Dynamical equations:

$$\frac{\ddot{a}}{a} = \mp\frac{1}{3M_p^2}(\dot{\phi}^2 + s\ \phi) - \frac{\Omega_{0m} H_0^2}{2a^3}$$

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} - s = 0$$

Cosmological Evolution

Rescaling:

$$H_0 t \rightarrow t$$
$$\frac{\phi}{\sqrt{3}M_p} \rightarrow \phi$$
$$\frac{s}{\sqrt{3}M_p H_0^2} \rightarrow s$$

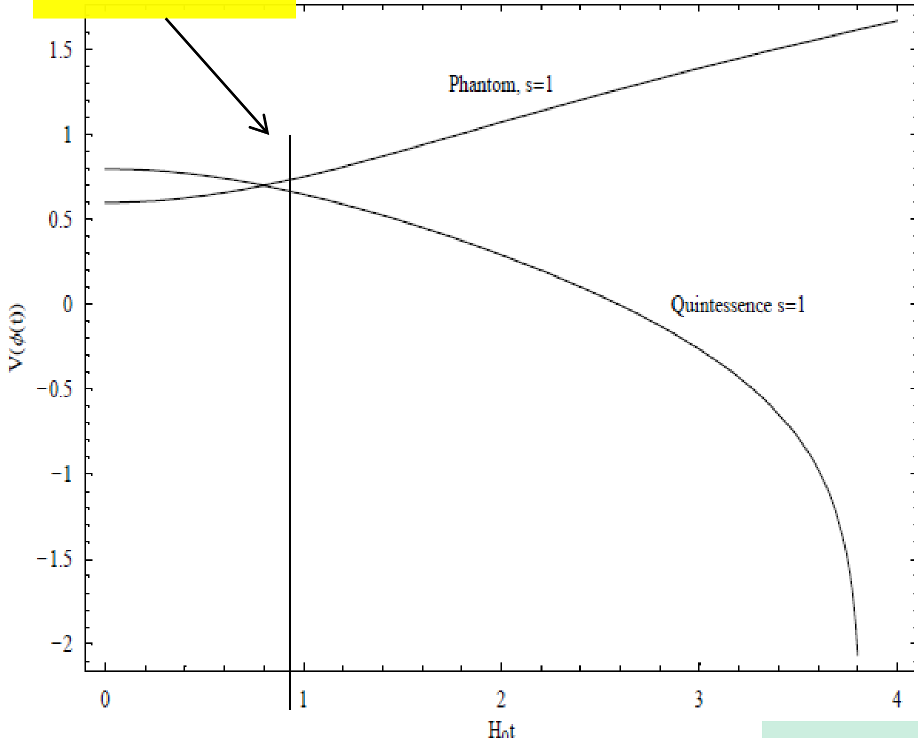
Initial Conditions:

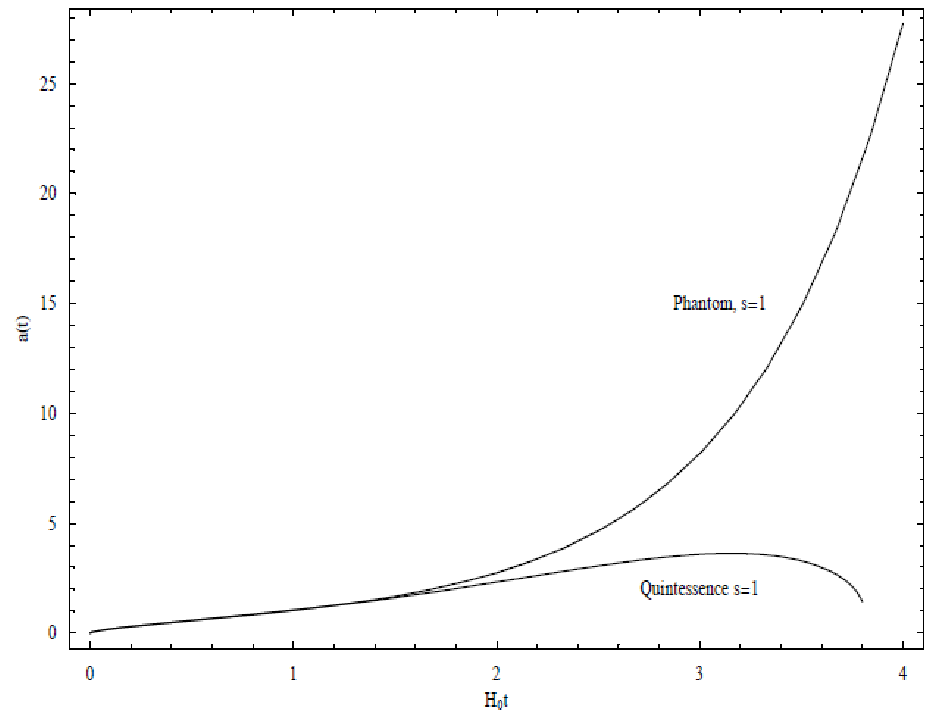
$$a(t_i) = (\frac{9\Omega_{0m}}{4})^{1/3}\, t_i^{2/3}$$
$$\dot{\phi}(t_i) = 0$$
$$\phi(t_i) = \phi_i$$

$$\Omega_{0m} = 0.3$$

$$\Omega_{0\phi} = \pm\dot{\phi}^2(t_0) + V(\phi(t_0)) = 1 - \Omega_{0m}$$

Present time



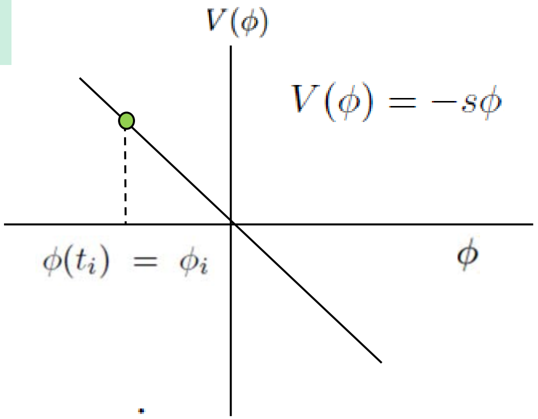


LP, Phys.Rev. D71 (2005) 063503

Scalar-Tensor Quintessence

Scalar field Lagrangian

A. Lykkas, LP, Phys.Rev. D93 (2016) 4, 043513, arXiv:1511.08732



$$\mathcal{L} = \frac{F(\phi)}{2} R - \frac{1}{2} g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi - V(\phi) + \mathcal{L}_{\rm m}[\psi_{\rm m}; g_{\mu\nu}]$$

Linear non-minimal coupling: $F(\phi) = 1 - \lambda\phi$

Dynamical equations:

$$3FH^2 = \rho_m + \frac{\dot\phi^2}{2} + V - 3H\dot F$$

$$\ddot\phi + 3\frac{\dot a}{a}\dot\phi - 3F_\phi\left(\frac{\ddot a}{a} + \frac{\dot a^2}{a^2}\right) + V_\phi = 0.$$

$$-2F\left(\frac{\ddot a}{a} - \frac{\dot a^2}{a^2}\right) = \rho_m + \dot\phi^2 + \ddot F - H\dot F$$

Cosmic densities:

$$H = \bar H H_0,\ t = \bar t / H_0,\ V = \bar V H_0^2$$

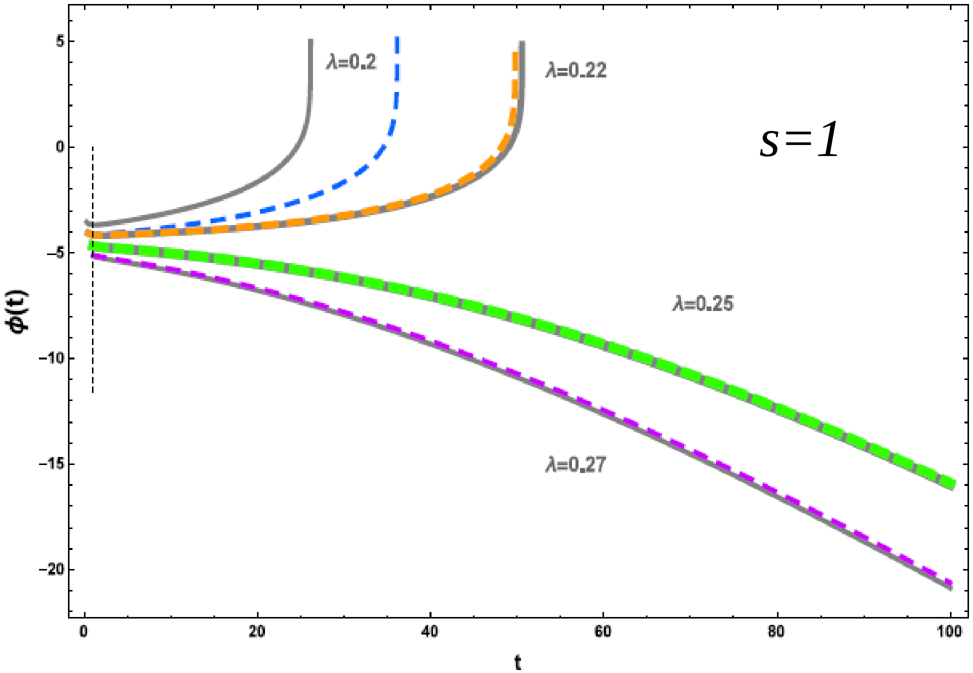
$$\rho_m = \bar\rho_m H_0^2$$

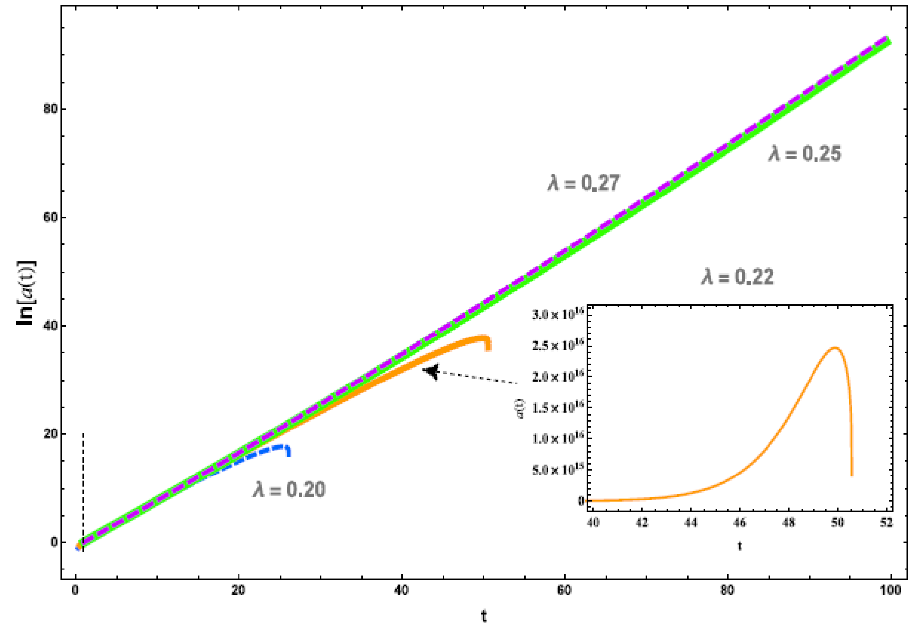
$$\Omega_m = \frac{\bar\rho_m}{3F\bar H^2} \Rightarrow \Omega_{0m} = \frac{\bar\rho_{0m}}{3F_0}$$

$$\Omega_\phi = 1 - \Omega_m = \frac{1}{3F\bar H^2}\left(\frac{\dot\phi^2}{2} + \bar V\right) - \frac{\dot F}{\bar H F} \Rightarrow$$

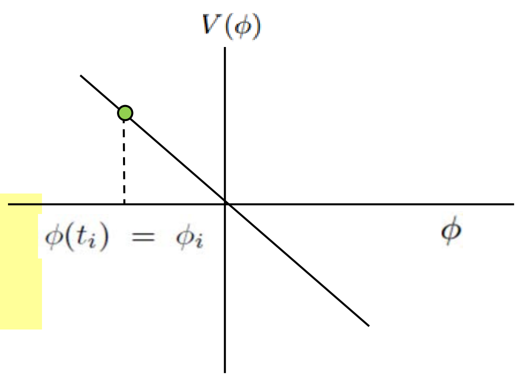
$$\Omega_{0\phi} = \frac{1}{3F_0}\left(\frac{\dot\phi_0^2}{2} + \bar V_0\right) - \frac{\dot F_0}{F_0}$$

Cosmological Evolution





From dynamical equations it may be shown that:



Force terms pushing up the scalar potential *(φ<0)*

$$\ddot{\phi}\left(1+\frac{3\lambda^2}{2F}\right)+3H\left(1+\frac{3\lambda^2}{2F}\right)\dot{\phi}-\frac{\lambda}{2F}\dot{\phi}^2 = s+\boxed{\frac{2\lambda s\phi}{F}-\lambda\frac{\Omega_{0m}}{2a^3}\frac{F_0}{F}}$$

Equation of State Parameter

The dynamical equations may also be written as:

$$3F_0H^2 = \rho_{DE} + \rho_m$$

$$-2F_0\dot{H} = \rho_{DE} + p_{DE} + \rho_m$$

where:

$$\rho_{DE} = \frac{1}{2}\dot{\phi}^2 + V - 3H^2\left(F - F_0\right) - 3H\dot{F}$$

$$p_{DE} = \frac{1}{2}\dot{\phi}^2 - V + \ddot{F} + 2H\dot{F} + \left(2\dot{H} + 3H^2\right)\left(F - F_0\right)$$

Energy conservation is applicable in this format:

$$\dot{\rho}_{DE} + 3H\left(\rho_{DE} + p_{DE}\right) = 0,$$

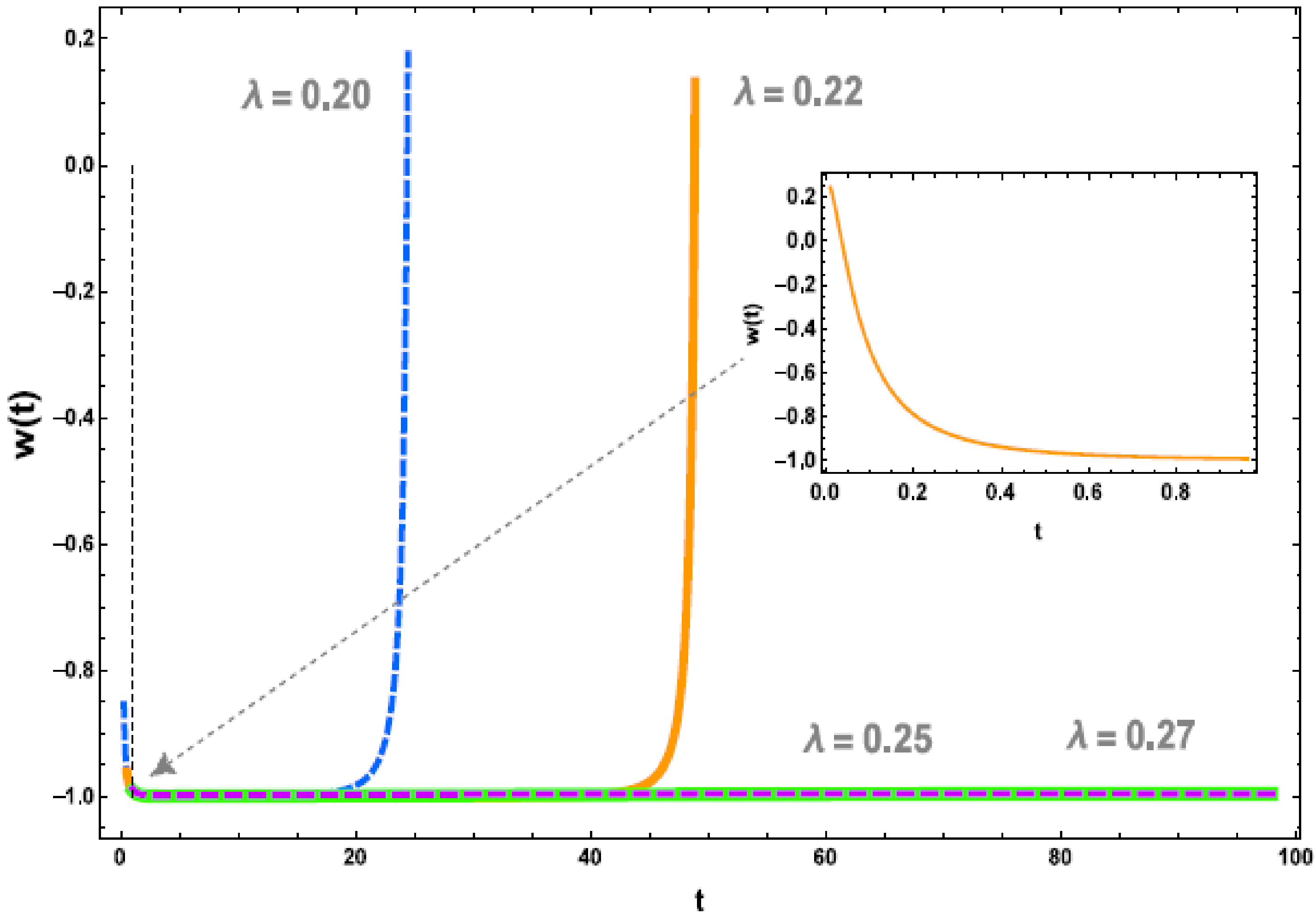
$$w_{DE} = \frac{p_{DE}}{\rho_{DE}} \quad \Rightarrow \quad w_{DE} = -1 + \frac{\dot{\phi}^2 + \ddot{F} - H\dot{F} - 2\dot{H}\left(F_0 - F\right)}{\frac{1}{2}\dot{\phi}^2 + V - 3H^2\left(F - F_0\right) - 3H\dot{F}}$$

$$\Downarrow$$

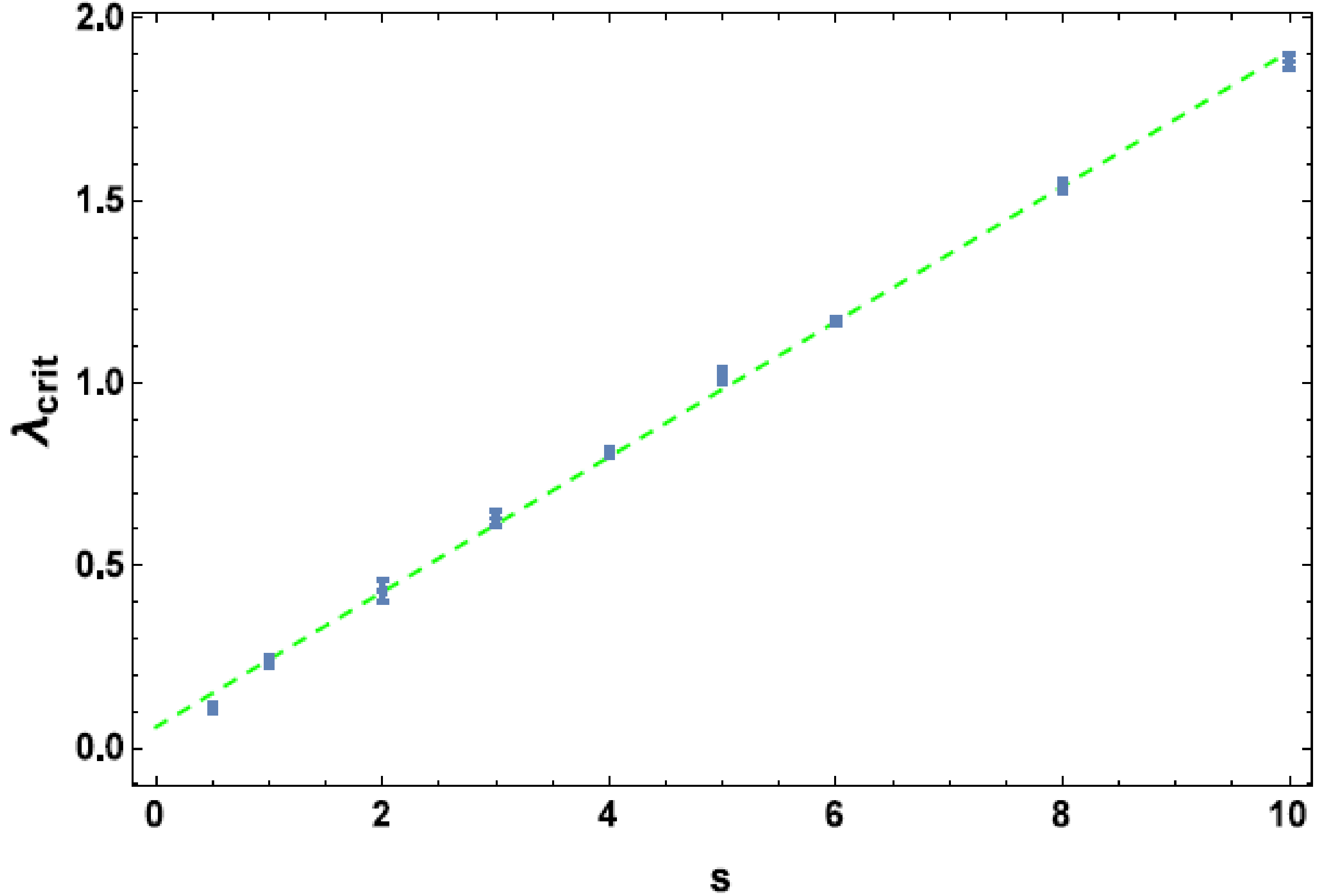
$$w_{DE} = -\frac{3H^2(z) - (1+z)\left(\mathrm{d}H^2(z)/\mathrm{dz}\right)}{3H^2(z) - 3\,\Omega_{0,m}\left(1+z\right)^3}$$

Equation of State Parameter

The evolution of w(z) above and below the critical value of λ

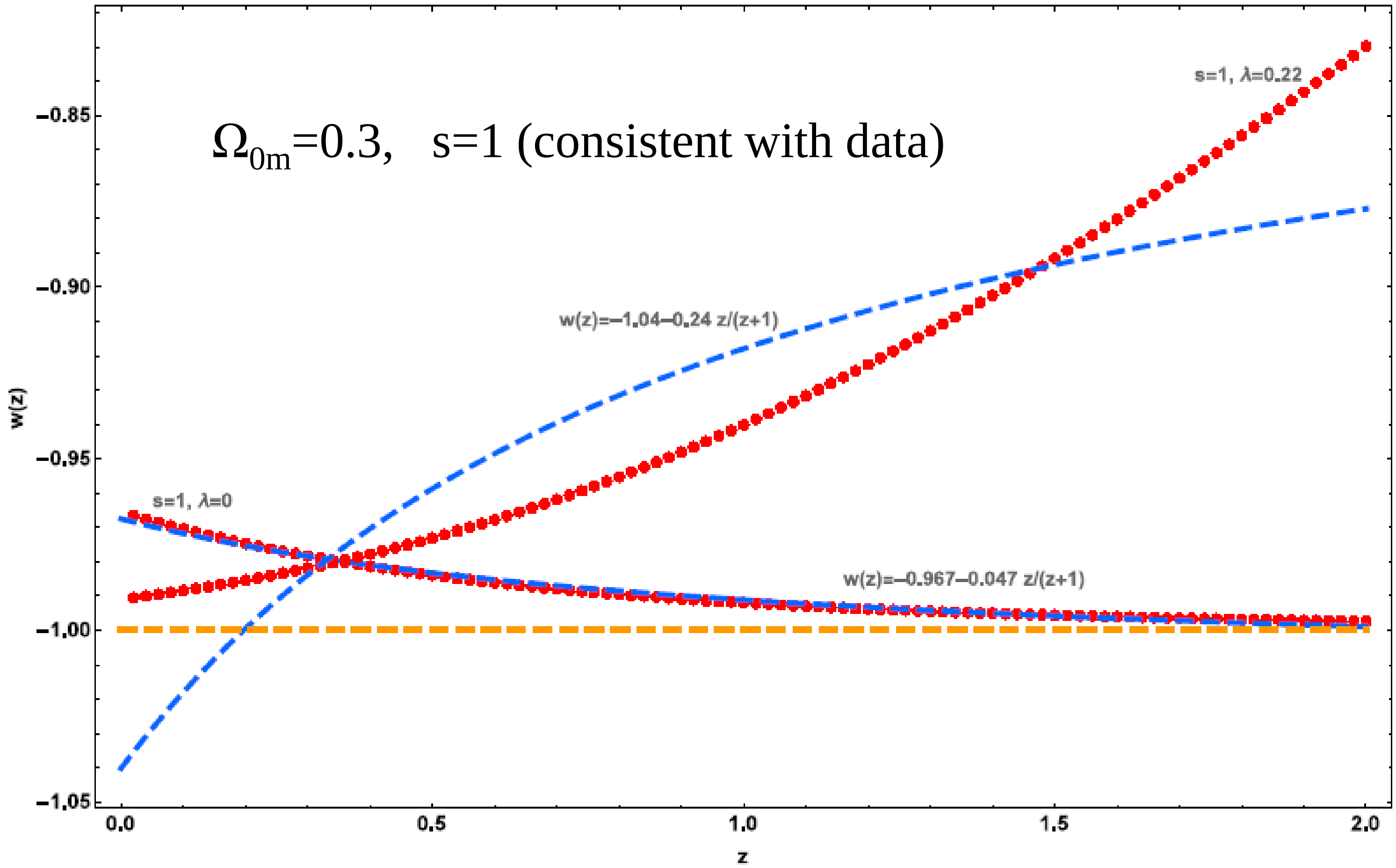


The Critical Values of λ



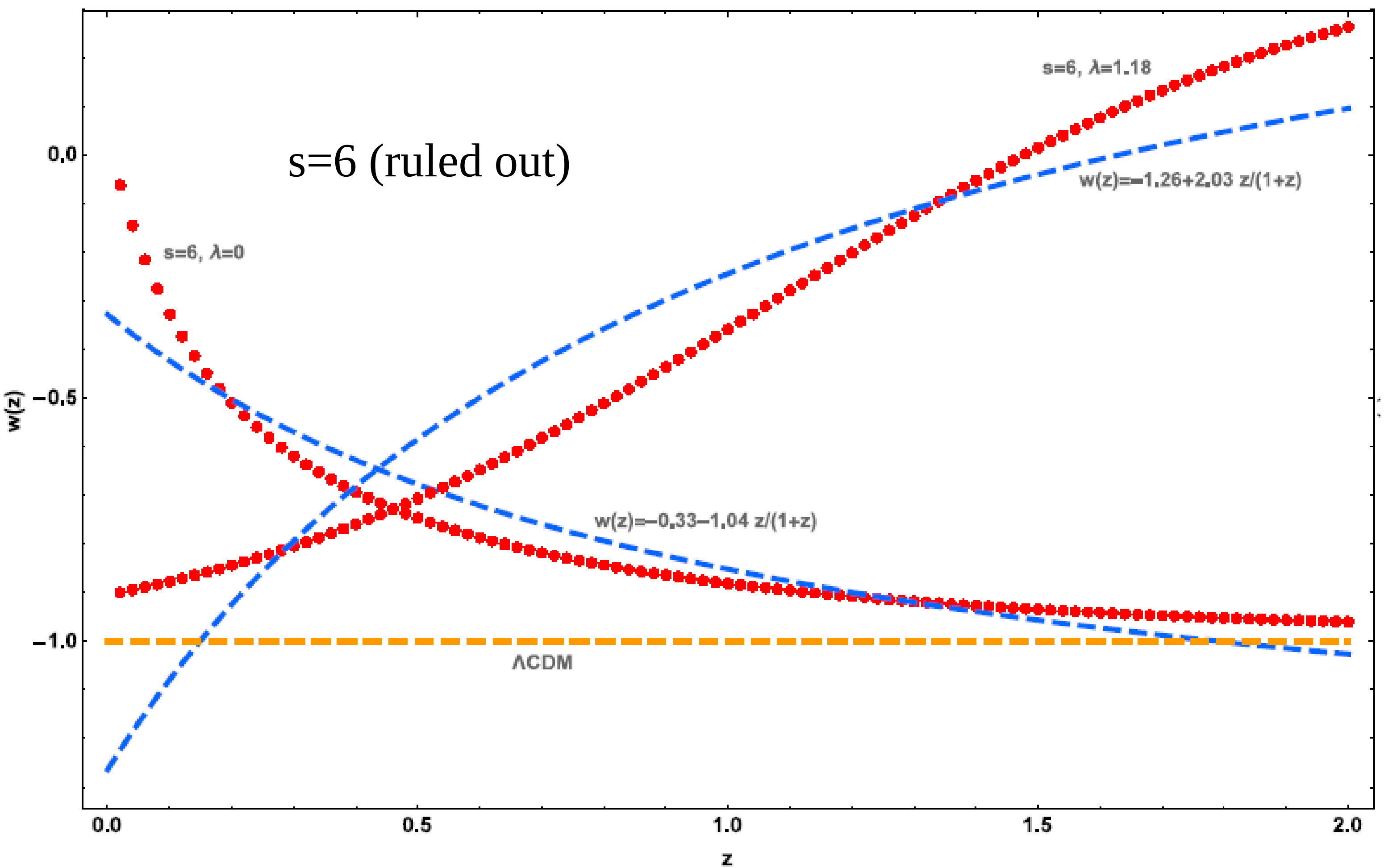
$$\ddot{\phi}\left(1+\frac{3\lambda^2}{2F}\right)+3H\left(1+\frac{3\lambda^2}{2F}\right)\dot{\phi}-\frac{\lambda}{2F}\dot{\phi}^2 = s+\frac{2\lambda s\phi}{F}-\lambda\frac{\Omega_{0m}}{2a^3}\frac{F_0}{F}$$

Equation of State vs Redshift



$$\ddot{\phi}\left(1+\frac{3\lambda^2}{2F}\right)+3H\left(1+\frac{3\lambda^2}{2F}\right)\dot{\phi}-\frac{\lambda}{2F}\dot{\phi}^2 \quad = s+\frac{2\lambda s\phi}{F}-\lambda\frac{\Omega_{0m}}{2a^3}\frac{F_0}{F}$$

Equation of State: Constraints



Result for Question 1

Q1: How generic is the Big Crunch singularity in the presence of potentials with negative range in scalar tensor quintessence?

A1: The Big Crunch singularity in the presence of potentials with negative range in scalar tensor quintessence is not generic and can be avoided in the presence of a non-minimal coupling to gravity which for values larger than a critical value can reverse the scalar field dynamical evolution.

Q2: How does a strongly bound system dissociate before the Big Rip? What new effects emerge beyond the Newtonian approximation?

Big Rip: The dissociation of bound systems.

Phantom cosmological background (w<-1):

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}[\rho_m + \rho_x] = H_0^2[\Omega_m^0(\frac{a_0}{a})^3 + \Omega_x^0(\frac{a_0}{a})^{3(1+w)}]$$

Scale factor solution:

$$a(t) = \frac{a(t_m)}{[-w + (1+w)\frac{t}{t_m}]^{-\frac{2}{3(1+w)}}}, \quad t > t_m \qquad 1 + z_m = \frac{a_0}{a_m} = [\frac{-(3w+1)\Omega_x^0}{\Omega_m^0}]^{-\frac{1}{3w}}$$

Big rip singularity:

$$t_* = \frac{w}{1+w} t_m > 0$$

Point mass metric (Newtonian approximation):

$$ds^2 = (1 - \frac{2Gm}{c^2 a(t)\rho}) d(ct)^2 - a(t)^2 (d\rho^2 + \rho^2 (d\theta^2 + \sin^2\theta d\varphi^2))$$

S. Nesseris, LP, **Phys.Rev. D70 (2004) 123529 , astro-ph/0410309,**

Geodesics (Newtonian)

Point mass metric (Newtonian approximation):

$$ds^2 = (1 - \frac{2Gm}{c^2 a(t)\rho}) d(ct)^2 - a(t)^2 (d\rho^2 + \rho^2 (d\theta^2 + \sin^2\theta d\varphi^2))$$

Geodesics (Newtonian approximation):

$$\ddot{r} - \frac{\ddot{a}}{a} r + \frac{Gm}{r^2} - r\dot{\varphi}^2 = 0$$

$$r^2 \dot{\varphi} = L$$

$$\Rightarrow \quad \ddot{r} = \frac{\ddot{a}}{a} r + \frac{L^2}{r^3} - \frac{Gm}{r^2}$$

Stable circular orbit (no expansion):

$$\dot{\varphi}(t_0)^2 = \omega_0^2 \equiv (\frac{2\pi}{T})^2 = \frac{Gm}{r_0^3}$$

New units:

$$t_m = t_0 \qquad \bar{r} \equiv \frac{r}{r_0}, \; \bar{\omega}_0 \equiv \omega_0 t_0 \qquad \bar{t} \equiv \frac{t}{t_0}$$

Rescaled Newtonian Geodesics:

$$\ddot{\bar{r}} + \frac{\bar{\omega}_0^2}{\bar{r}^2}(1 - \frac{1}{\bar{r}}) + \frac{2}{9} \frac{(1+3w)\bar{r}}{(-w + (1+w)\bar{t})^2} = 0$$

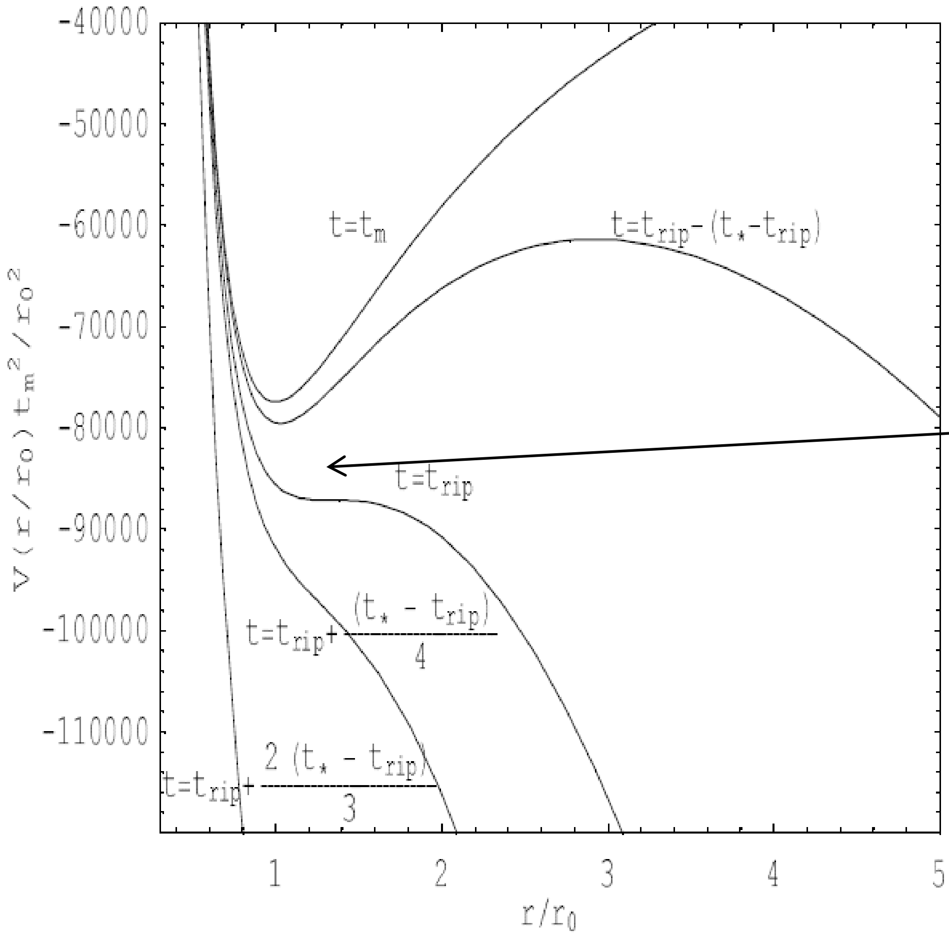
Effective Potential

Effective Force:

$$F_{eff} = -\frac{\overline{\omega}_0^2}{\overline{r}^2}(1 - \frac{1}{\overline{r}}) - \frac{2}{9}\frac{(1+3w)\overline{r}}{(-w+(1+w)\overline{t})^2}$$

Effective potential:

$$V_{eff} = -\frac{\overline{\omega}_0^2}{\overline{r}} + \frac{\overline{\omega}_0^2}{2\overline{r}^2} - \frac{1}{2}\lambda(\overline{t})^2\overline{r}^2$$



Potential minimum disappears at time t_{rip}

$$t_* - t_{rip} = \frac{16\sqrt{3}}{9}\frac{T\sqrt{2|1+3w|}}{6\pi|1+w|}$$

t_{rip} (bound system dissociation)

Early estimates based on force balance:

$$t_* - t_{rip} = \frac{T\sqrt{2|1+3w|}}{6\pi|1+w|}$$

McVittie Metric

G. C. McVittie, Mon. Not. Roy. Astron. Soc. 93,325 (1933)

Solution of Einstein Eq. Interpolating between Schwarzschild – FRW:

$$ds^2 = -(f - \frac{r^2 H^2}{c^2})d(ct)^2 - 2rHf^{-1/2}dtdr + f^{-1}dr^2 + r^2 d\Omega^2$$

$$f = f(r) = 1 - 2Gm/(c^2 r) \qquad r = a(t)\rho$$

$$\ddot{r} = rf^{1/2}H'\dot{t}^2 + (1 - \frac{3Gm}{c^2 r})\frac{L^2}{r^3} - \frac{Gm}{r^2} + rH^2$$

Geodesics:

$$\ddot{t} = -(1 - \frac{3Gm}{rc^2})f^{-1/2}H\dot{t}^2 - \frac{2Gm}{r^2}f^{-1}\dot{t}\dot{r} + f^{-1/2}H$$

$$\chi\dot{t}^2 + 2\frac{\alpha\dot{t}\dot{r}}{c} - \frac{f^{-1}\dot{r}^2}{c^2} - \frac{L^2}{c^2 r^2} = 1$$

B. C. Nolan, Class. Quant. Grav. 31, no. 23, 235008 (2014)

$$\chi(t,r) = f - \frac{r^2 H^2}{c^2}, \qquad \alpha(t,r) = \frac{rf^{-1/2}H}{c}$$

Rescaled Geodesics

Dimensionless quantities: $\bar{t} \equiv t/t_0, \ \bar{\tau} \equiv \tau/t_0$

$$\bar{r} \equiv r/r_0, \ \bar{m} \equiv Gm/r_0c^2, \ \bar{H} \equiv Ht_0, \ \bar{\omega}_0 \equiv \omega_0 t_0$$

Rescaled Geodesics:

$$\ddot{\bar{r}} = \bar{r} f^{1/2} \bar{H}' \dot{\bar{t}}^2 + (1 - \frac{3\bar{m}}{\bar{r}})\frac{\bar{\omega}_0^2}{\bar{r}^3} - \frac{\bar{m}}{\bar{r}^2}(\frac{ct_0}{r_0})^2 + \bar{r}\bar{H}^2$$

$$[f - (\frac{r_0}{ct_0})^2 \bar{r}^2 \bar{H}^2]\dot{\bar{t}}^2 + 2(\frac{r_0}{ct_0})^2 \bar{r}\bar{H} f^{-1/2}\dot{\bar{t}}\dot{\bar{r}} - \frac{\dot{\bar{r}}^2}{f}(\frac{r_0}{ct_0})^2 - \frac{\bar{\omega}_0^2}{\bar{r}^2}(\frac{r_0}{ct_0})^2 = 1$$

Effective Radial Force: $F_{eff} = (1 - \frac{3\bar{m}}{\bar{r}})\frac{\bar{\omega}_0^2}{\bar{r}^3} - \frac{\bar{m}}{\bar{r}^2}(\frac{ct_0}{r_0})^2 \qquad (H = 0)$

Scales for Rescaling: $\bar{\omega}_0 = \frac{ct_0}{r_0}\sqrt{\frac{\bar{m}}{1 - 3\bar{m}}} \qquad t_0 = t_m$

Rescaled Geodesics

Scales for Rescaling:

$$\bar{\omega}_0 = \frac{ct_0}{r_0}\sqrt{\frac{\bar{m}}{1-3\bar{m}}} \qquad t_0 = t_m$$

I. Antoniou, LP arXiv:1603.02569

Rescaled Geodesics:

$$\ddot{\bar{r}} = \bar{r} f^{1/2} \bar{H}' \dot{\bar{t}}^2 + (1 - \frac{3\bar{m}}{\bar{r}})\frac{\bar{\omega}_0^2}{\bar{r}^3} - \frac{(1-3\bar{m})\bar{\omega}_0^2}{\bar{r}^2} + \bar{r}\bar{H}^2$$

$$[f - \frac{\bar{r}^2 \bar{m} \bar{H}^2}{\bar{\omega}_0^2(1-3\bar{m})}]\dot{\bar{t}}^2 + \frac{2\bar{m}}{\bar{\omega}_0^2(1-3\bar{m})}\bar{r}\bar{H} f^{-1/2}\dot{\bar{t}}\dot{\bar{r}} - \frac{\dot{\bar{r}}^2}{f\bar{\omega}_0^2}\frac{\bar{m}}{1-3\bar{m}} - \frac{\bar{m}}{\bar{r}^2(1-3\bar{m})} = 1$$

Newtonian limit:

$$\bar{m} \equiv \frac{Gm}{c^2 r_0} \to 0 \qquad f = 1 - \frac{2Gm}{c^2 r} \to 1$$

Effective Potential

Rescaled Geodesics:

$$\ddot{\bar{r}} = \bar{r} f^{1/2} \bar{H}' \dot{\bar{t}}^2 + (1 - \frac{3\bar{m}}{\bar{r}}) \frac{\bar{\omega}_0^2}{\bar{r}^3} - \frac{(1-3\bar{m})\bar{\omega}_0^2}{\bar{r}^2} + \bar{r}\bar{H}^2$$

$$[f - \frac{\bar{r}^2 \bar{m} \bar{H}^2}{\bar{\omega}_0^2 (1-3\bar{m})}] \dot{\bar{t}}^2 + \frac{2\bar{m}}{\bar{\omega}_0^2(1-3\bar{m})} \bar{r}\bar{H} f^{-1/2} \dot{\bar{t}}\dot{\bar{r}} - \frac{\dot{\bar{r}}^2}{f\bar{\omega}_0^2} \frac{\bar{m}}{1-3\bar{m}} - \frac{\bar{m}}{\bar{r}^2(1-3\bar{m})} = 1$$

Effective force ($\dot{\bar{r}} \to 0$):

$$F_{eff} = \bar{r} f^{1/2} \bar{H}' [\frac{1 + \frac{\bar{m}}{\bar{r}^2(1-3\bar{m})}}{f - \frac{\bar{r}^2 \bar{H}^2 \bar{m}}{\bar{\omega}_0^2(1-3\bar{m})}}] + (1 - \frac{3\bar{m}}{\bar{r}}) \frac{\bar{\omega}_0^2}{\bar{r}^3} - \frac{(1-3\bar{m})\bar{\omega}_0^2}{\bar{r}^2} + \bar{r}\bar{H}^2$$

Effective Potential:

$$V_{eff}(\bar{r}) = -\int_1^r F_{eff}(\bar{r}')d\bar{r}'$$

Innermost Stable Orbit

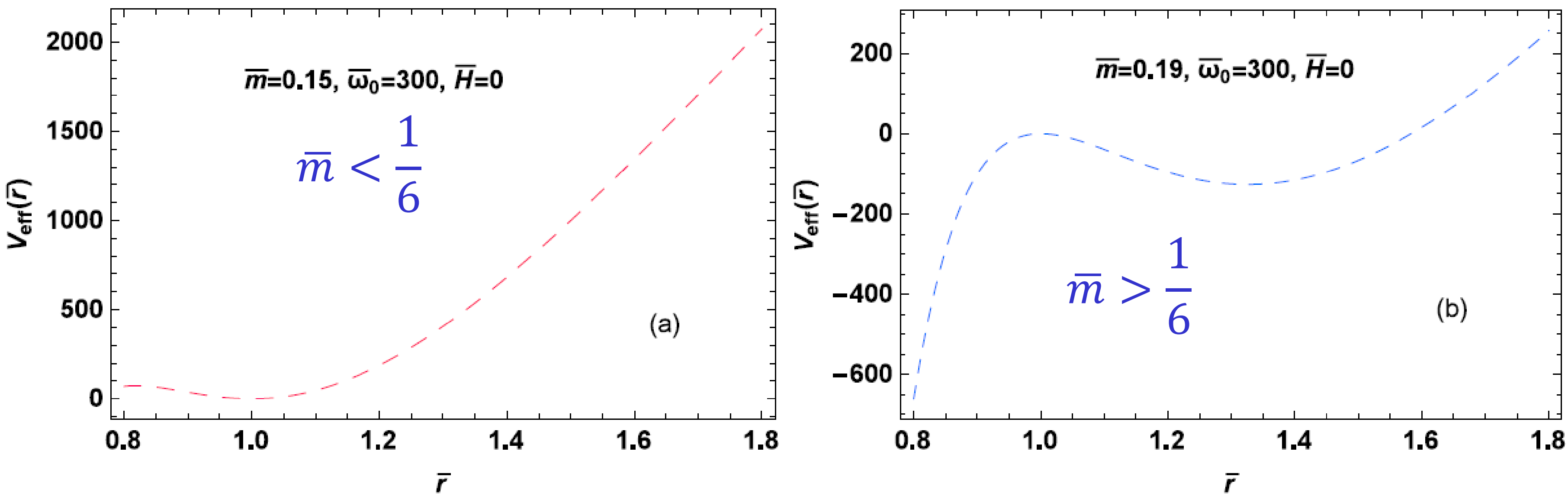
Ignoring expansion, the radial geodesic becomes:

$$\ddot{\bar{r}} = \left(1 - \frac{3\bar{m}}{\bar{r}}\right)\frac{\bar{\omega}_0^2}{\bar{r}^3} - \frac{(1-3\bar{m})\bar{\omega}_0^2}{\bar{r}^2}$$

Stationary points:

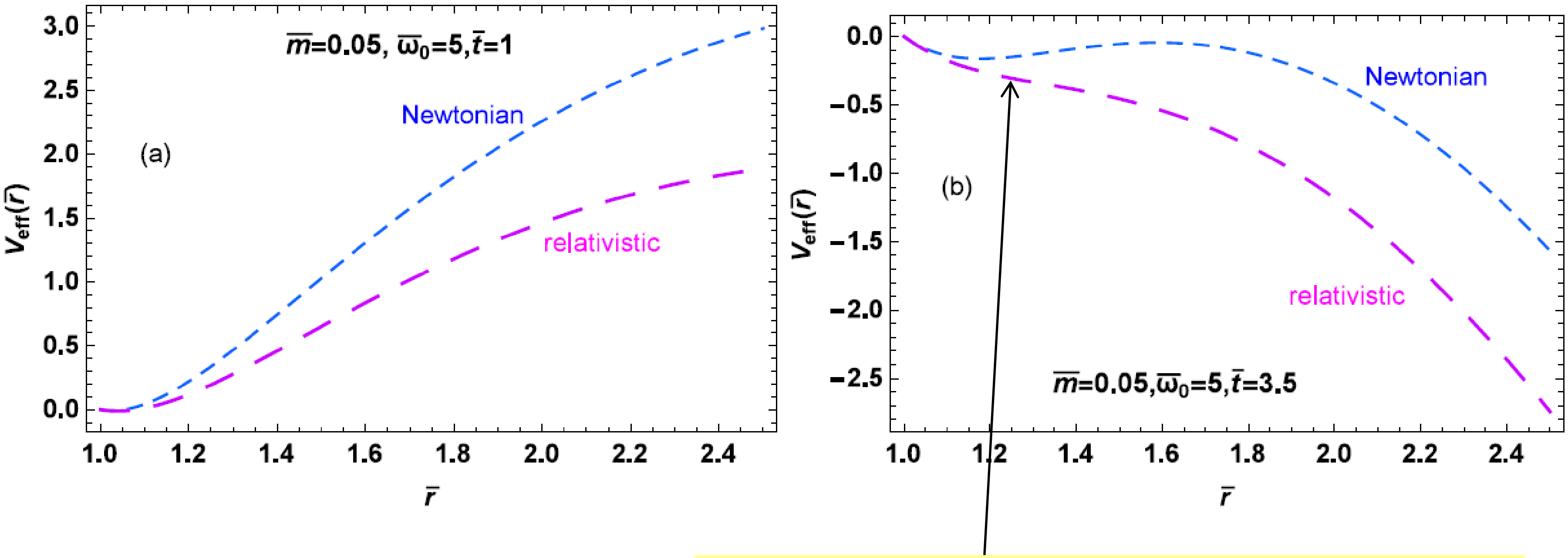
$$\bar{r} = 1, \qquad \bar{r} = \frac{3\bar{m}}{1-3\bar{m}}$$

Stable for $r_0 > 6Gmc^2$ ie $\bar{m} < \frac{1}{6}$



Relativistic Rip

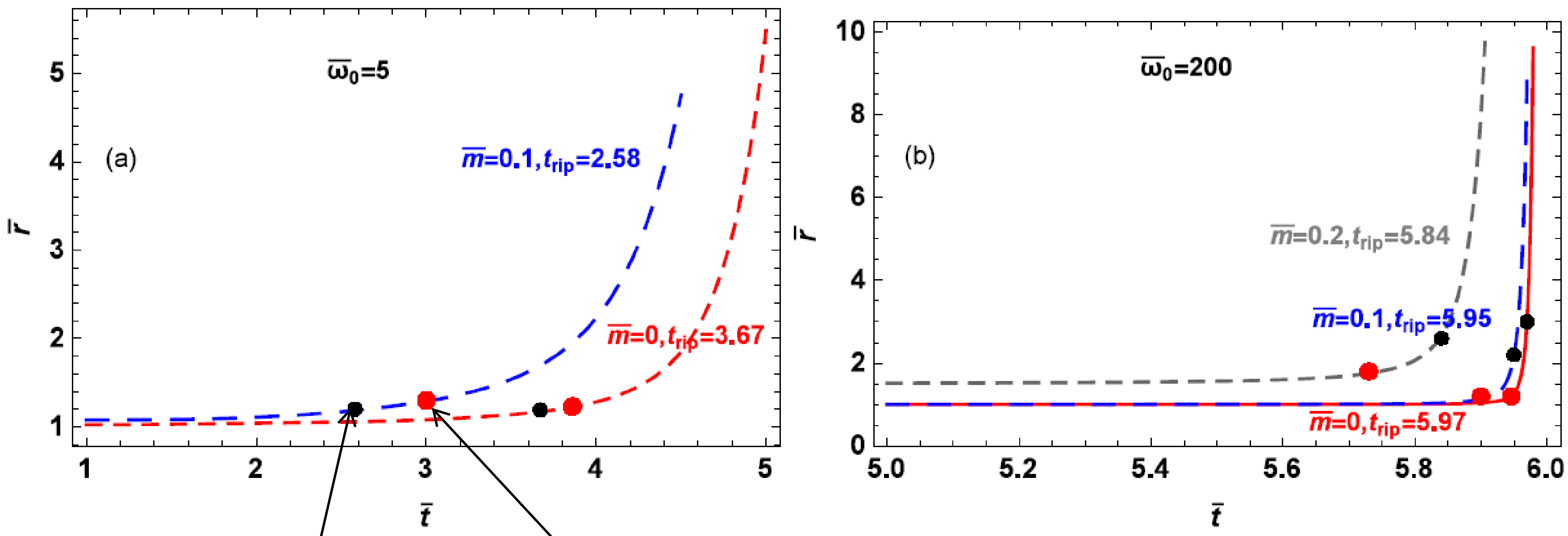
The Rip of a bound system occurs later in the Newtonian approximation
(for concreteness set w=-1.2)



The bound system is dissociated in the relativistic treatment but remains bound in the Newtonian approximation

The form of the Geodesics

Numerical solution of full system (Initial conditions at $\bar{t} = 1$, $\bar{r}$ at potential minimum, $\dot{\bar{r}} = 0$):



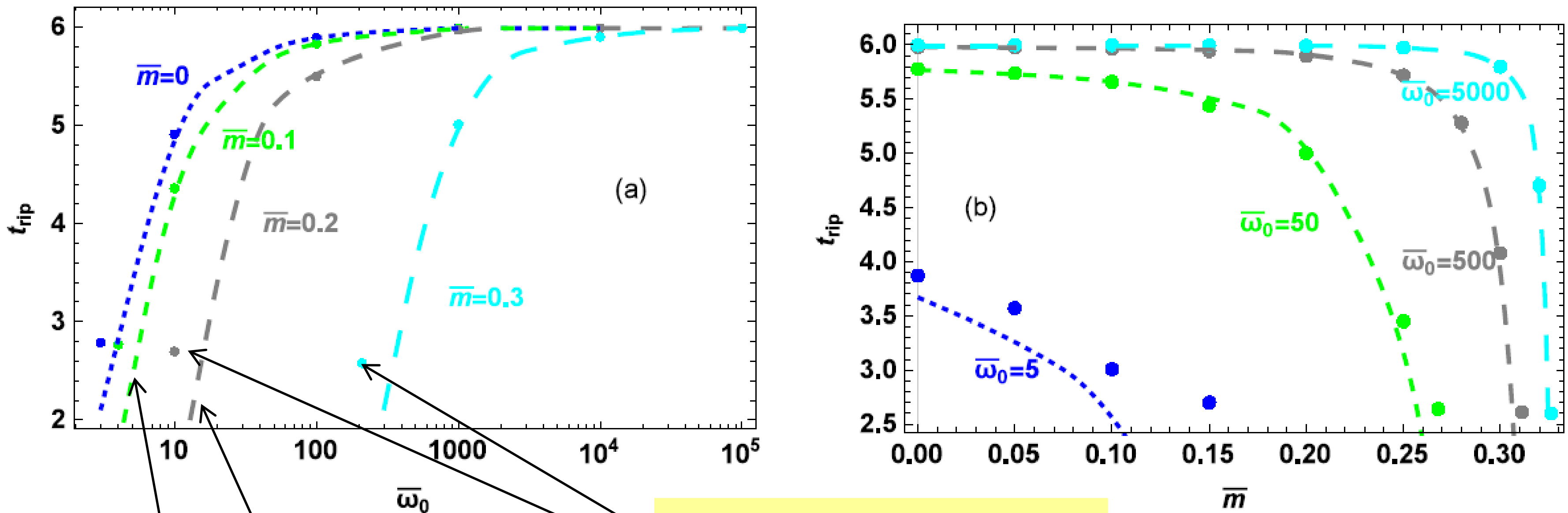
Potential minimum disappears

Radius increases 20% compared to initial equilibrium

I. Antoniou, LP arXiv:1603.02569

The dissociation time t_{rip}

The dissociation occurs earlier in the context of the relativistic approach:



Potential minimum disappears

Radius increases 20% compared to initial equilibrium

System	Mass($M_\odot$)	Size(Mpc)	$\bar{\omega}_0$	$\bar{m}$	$\Delta\bar{t}_{rip}$
Solar System	1.0	2.3×10^{-9}	3.5×10^{6}	2.1×10^{-11}	$< 10^{-8}$
Milky Way Galaxy	1.0×10^{12}	1.7×10^{-2}	1.8×10^{2}	2.9×10^{-6}	2.4×10^{-7}
Typical Cluster	1.0×10^{15}	1.0	12	4.9×10^{-5}	5.9×10^{-5}
Accretion Disk (neutron star)	1.5	3.3×10^{-19}	4.3×10^{21}	0.22	$< 10^{-8}$
Hypothetical Large Massive	3.0×10^{20}	1.0×10^{2}	9.1	0.15	0.93

Result for Question 2

Q2: How does a strongly bound system dissociate before the Big Rip? What new effects emerge beyond the Newtonian approximation?

A2: In a Big Rip cosmology, a bound system dissociates earlier than anticipated in the context of the Newtonian approximation. The difference becomes appreciable only for hypothetical cosmologically large strongly bound systems.

Q3: How can we avoid misleading conclusions when using cosmological data to fit w(z) and predict its future evolution? Is the CPL parameterization adequate to describe a possible non-trivial evolution of w(z)?

G. Pantazis, S. Nesseris, LP,
arXiv:1603.02164

Thawing Quintessence

Quintessence Field Evolution:

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0,$$

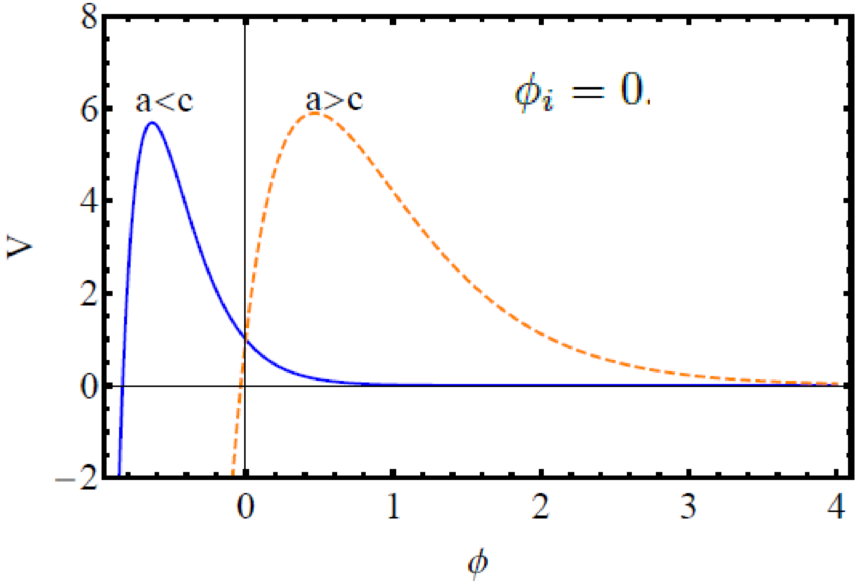
Equation of State:

$$w \equiv \frac{p_\phi}{\rho_\phi} = \frac{\dot{\phi}^2/2 - V(\phi)}{\dot{\phi}^2/2 + V(\phi)}$$

Quintessence Potential:

$$V(\phi) = V_i \, e^{-c\phi} \left[1 + \alpha\phi\right]$$

T. Clemson and A. Liddle
M.N.R.A.S. 395 (2009) 1585-1590



Dynamical System:

$$x \equiv \frac{\dot{\phi}}{\sqrt{6}H}, \quad y \equiv \frac{\sqrt{V}}{\sqrt{3}H}, \quad \lambda \equiv -\frac{V_\phi}{V}, \quad \Gamma \equiv \frac{V V_{\phi\phi}}{V_\phi^2},$$

$$\frac{dx}{dN} = -3x + \sqrt{\frac{3}{2}}\lambda y^2 + \frac{3}{2}x\left(1 + x^2 - y^2\right),$$

$$\frac{dy}{dN} = -\sqrt{\frac{3}{2}}\lambda x y + \frac{3}{2}y\left(1 + x^2 - y^2\right),$$

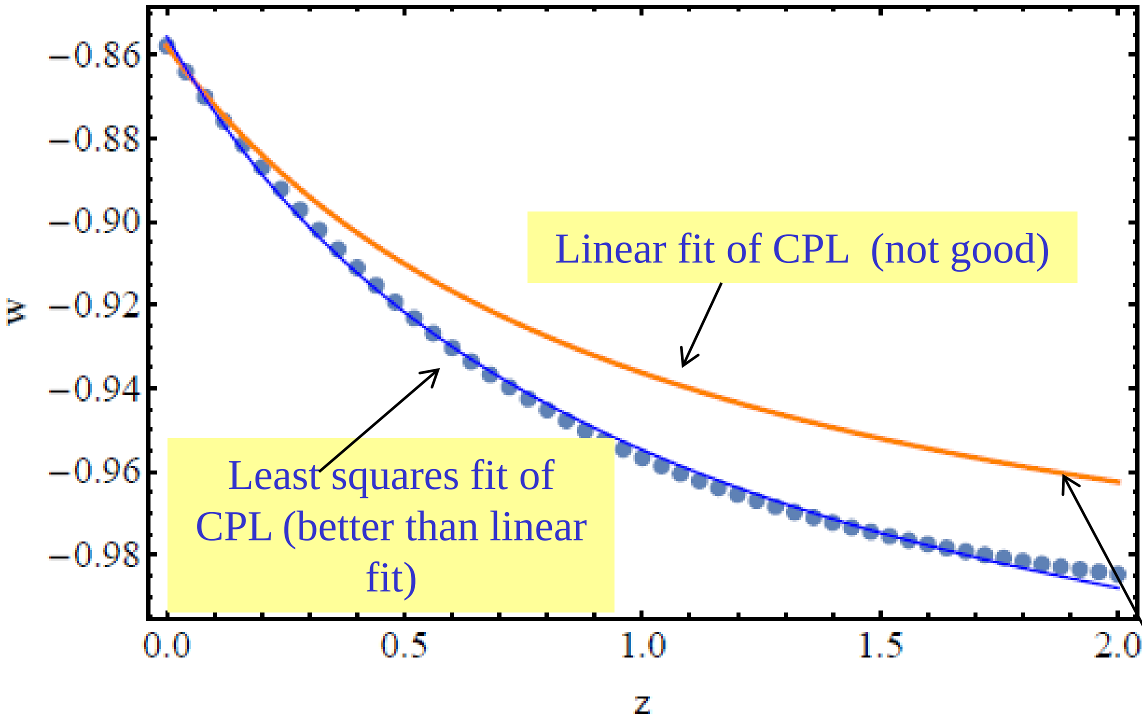
$$\frac{d\lambda}{dN} = -\sqrt{6}\lambda^2(\Gamma - 1)x,$$

General Potential:
Possible future
Crossing to V<0

$$\Omega_\phi = x^2 + y^2 \qquad w(N) = \frac{x^2(N) - y^2(N)}{x^2(N) + y^2(N)}$$

Equation of State

Thawing behavior: Early time freeze due to cosmic friction, late time thawing due to reduced friction



$$w(N) = \frac{x^2(N) - y^2(N)}{x^2(N) + y^2(N)}$$

$$a = \frac{1}{1+z} = e^{N-N_0}$$

$$N = N_0 \longrightarrow \Omega_\phi = 0.75$$

T. Clemson and A. Liddle
M.N.R.A.S. 395 (2009) 1585-1590

Fit by CPL:

$$w(N) = w_0 + w_a(1 - e^{N-N_0})$$

$$w(N) = \frac{x^2(N) - y^2(N)}{x^2(N) + y^2(N)}$$

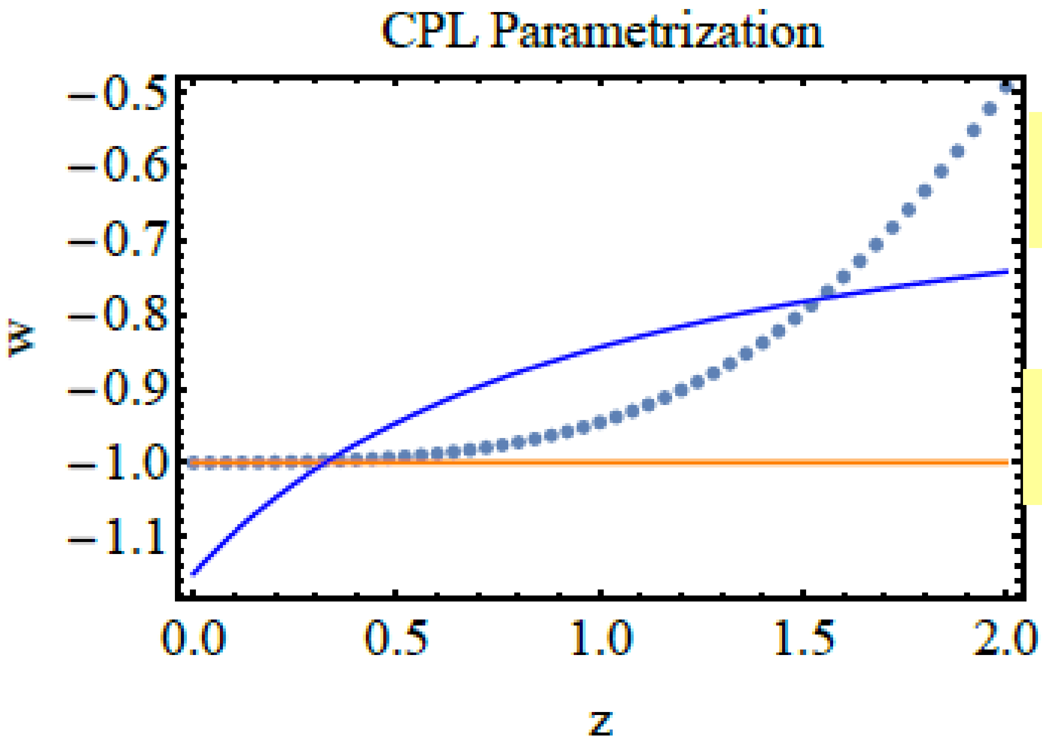
Linear expansion of numerical result

$$w_0^{\text{lin.}} = \frac{x_0 - y_0}{x_0 + y_0} \qquad w_a^{\text{lin.}} = -\left.\frac{dw}{da}\right|_{a\to 1}$$

$$w_a^{\text{lin.}} = \frac{4x_0 y_0}{(x_0^2 + y_0^2)^2}\left(3x_0 y_0 - \lambda_0\sqrt{\frac{3}{2}}y_0^3 - \lambda_0\sqrt{\frac{3}{2}}x_0^2 y_0\right)$$

M. Chevallier and D. Polarski, Int. J. Mod. Phys. D 10, 213 (2001), E.V. Linder, Phys. Rev. Lett. 90, 091301 (2003).

Freezing Quintessence



Early times: Kinetic term significant due to large potential slope.

Late times: Kinetic term small due to diminishing potential slope.

The CPL parametrization is not efficient in fitting freezing quintessence (convex increasing w(z)) because it is generically concave when increasing.

All two parameter parametrizations which can reduce to ΛCDM (w=-1) and have a linear term in the expansion around a=1 have reduced efficiency in fitting freezing models.

Quality of Fit Measure

Q: How can we quantify the quality of fit of a parametrization to a given physical model?

The measure q is defined as:

$$q = \int_0^{z_{\max}} \frac{\left|w_{\rm num}(z) - w^{\rm bf}_{\rm par.}(z)\right|}{z_{\max}} \, dz.$$

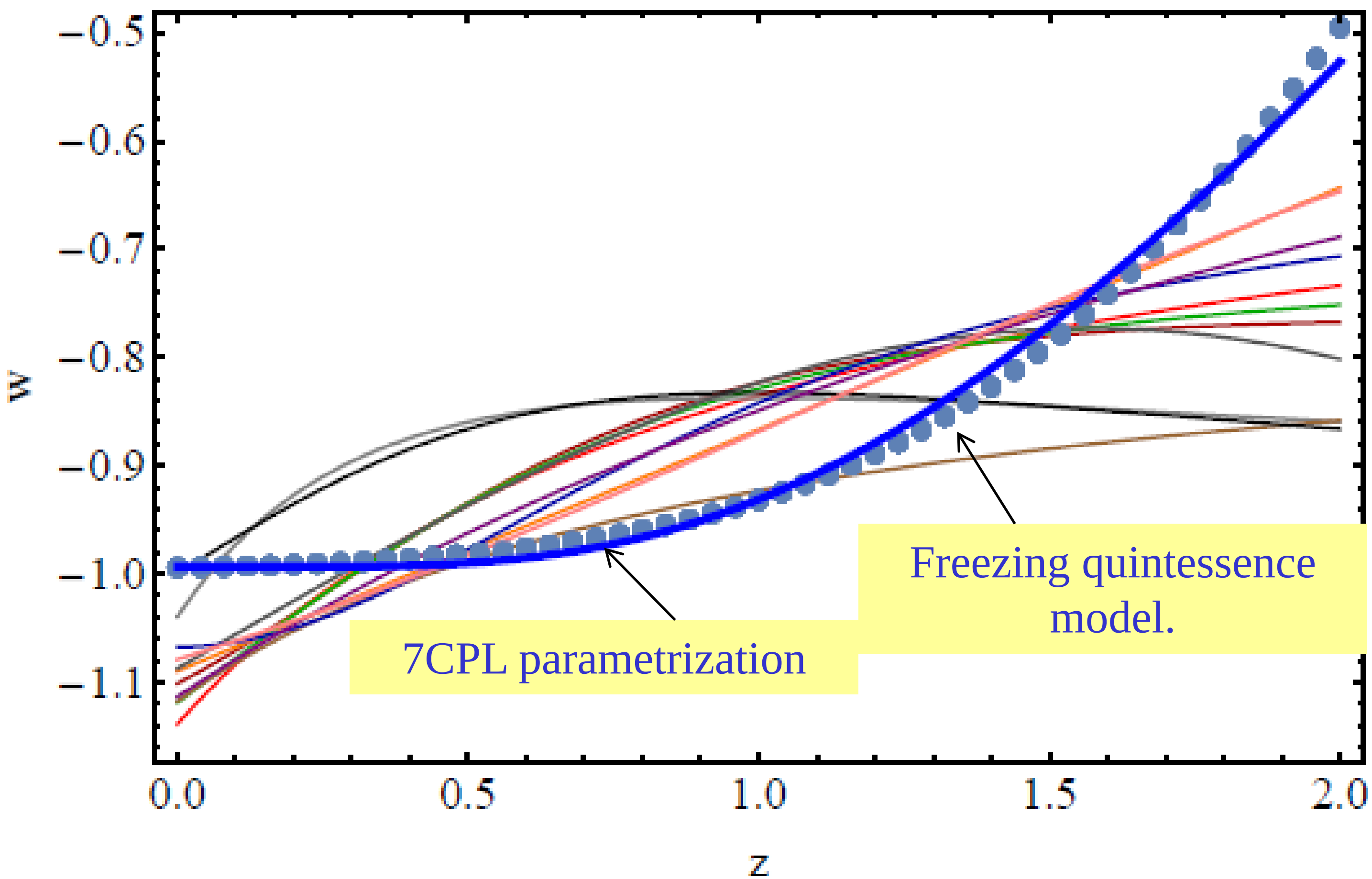
Alternative parametrizations:

$$w_{\rm nCPL}(z) = w_0 + w_a (1-a)^n = w_0 + w_a \left(\frac{z}{1+z}\right)^n$$

Name	Parametrization	Type	$q_{\rm th.} \times 10^{-4}$	$q_{\rm fr.} \times 10^{-4}$
Sqrt	$w(z) = w_0 + w_a \frac{z}{\sqrt{1+z^2}}$	Thawing	5.2	787.0
CPL [33, 34]	$w(z) = w_0 + w_a \frac{z}{1+z}$	Thawing	12.8	749.0
BA [75]	$w(z) = w_0 + w_a \frac{z(1+z)}{1+z^2}$	Thawing	21.2	817.4
Sine [69]	$w(z) = w_0 + w_a \sin(z)$	Thawing	36.5	826.0
Logarithmic [71]	$w(z) = w_0 + w_a \ln(1+z)$	Thawing	52.3	628.5
MZ Model 1 [61]	$w(z) = w_0 + w_a \left(\frac{\ln(2+z)}{1+z} - \ln 2\right)$	Thawing	55.1	620.7
FSLL Model 2 [57]	$w(z) = w_0 + w_a \frac{z^2}{1+z^2}$	Thawing	58.9	643.2
Linear [60]	$w(z) = w_0 + w_a z$	Thawing	92.6	494.6
MZ Model 2 [61]	$w(z) = w_0 + w_a \left(\frac{\sin(1+z)}{1+z} - \sin(1)\right)$	Thawing	97.1	485.4
FSLL Model 1 [57]	$w(z) = w_0 + w_a \frac{z}{1+z^2}$	Thawing	170.1	1085.4
JBP [56]	$w(z) = w_0 + w_a \frac{z}{(1+z)^2}$	Thawing	180.2	1087.5
7CPL	$w(z) = w_0 + w_a \left(\frac{z}{1+z}\right)^7$	Freezing	187.2	84.0

G. Pantazis, S. Nesseris, LP,
arXiv:1603.02164

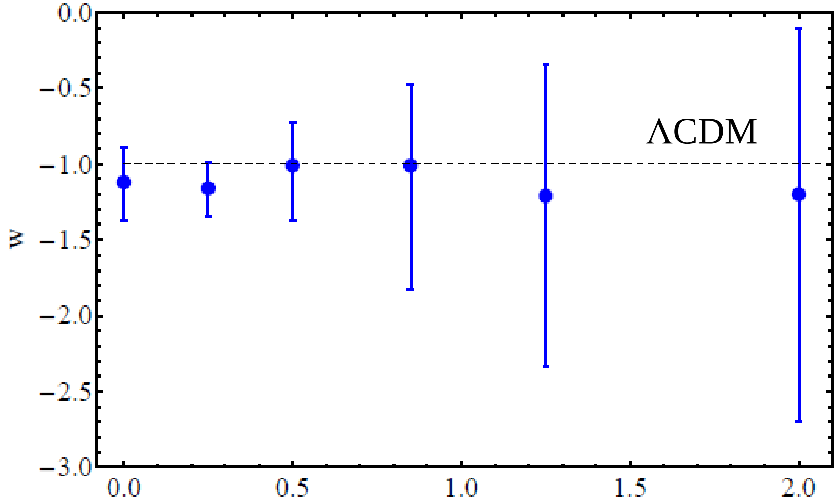
Comparison of Parametrizations



Least squares best fit of various proposed parametrizations to a freezing quintessence model.

Fitting Simulated Data

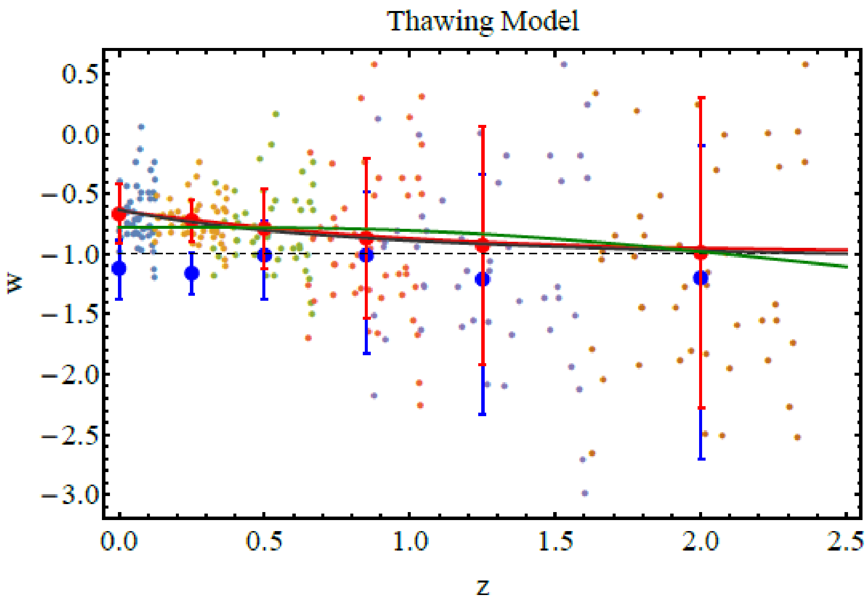
Observational Constraints on w(z) in redshift bins

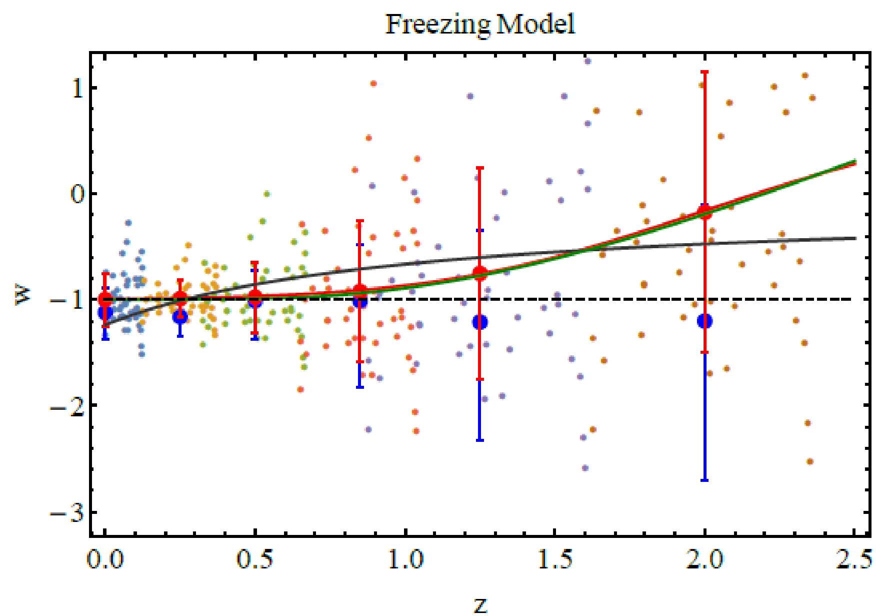


N. Said, C. Baccigalupi et. al.
Phys.Rev. D88 (2013) 043515

$$w_{\mathrm{nCPL}}(z) = w_0 + w_a(1-a)^n = w_0 + w_a\left(\frac{z}{1+z}\right)^n$$

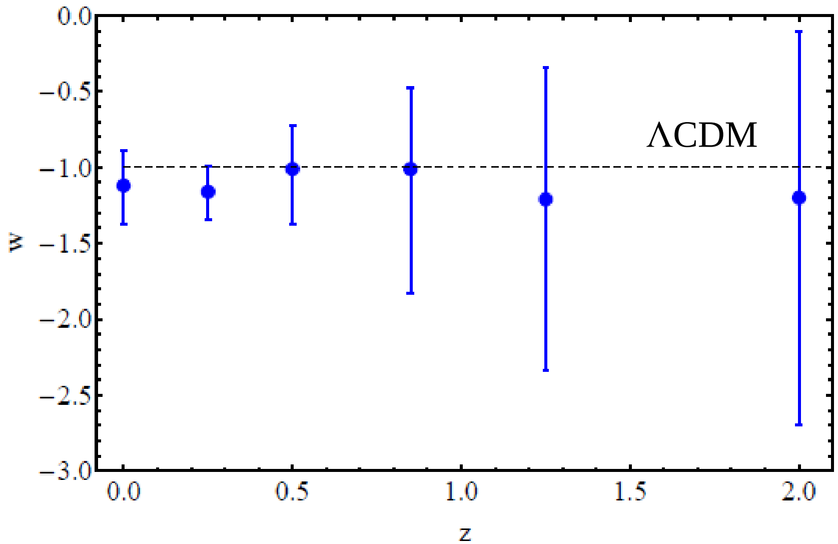
Simulated data for freezing and thawing underlying model.





Fitting Simulated Data

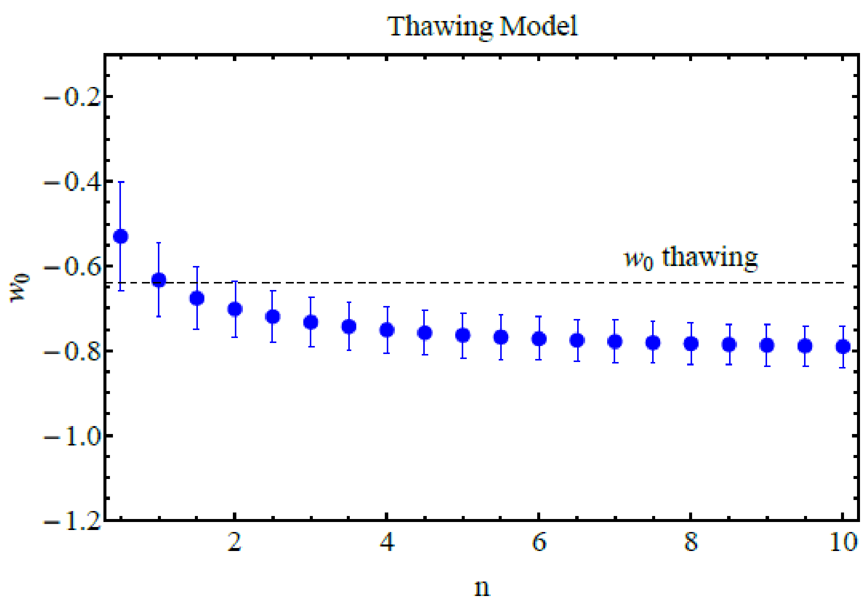
Observational Constraints on w(z) in redshift bins

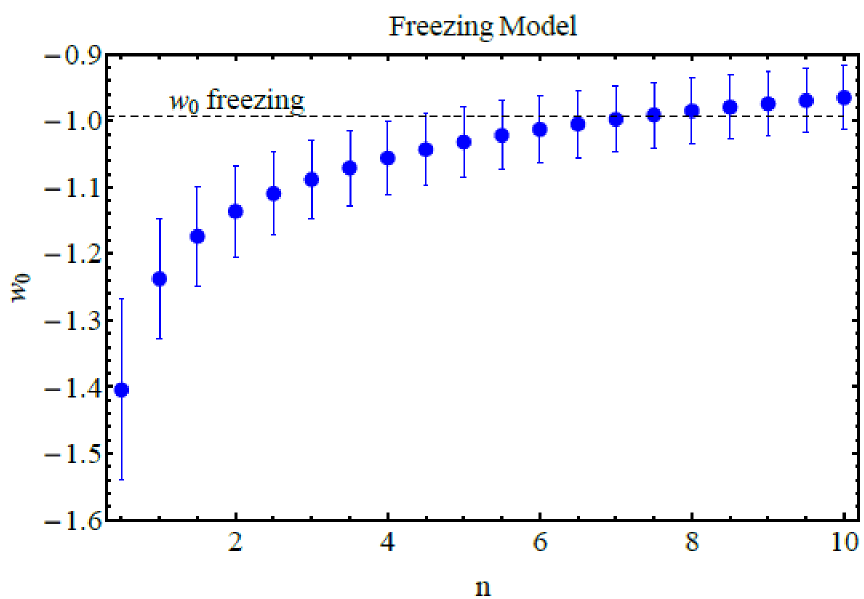


N. Said, C. Baccigalupi et. al.
Phys.Rev. D88 (2013) 043515

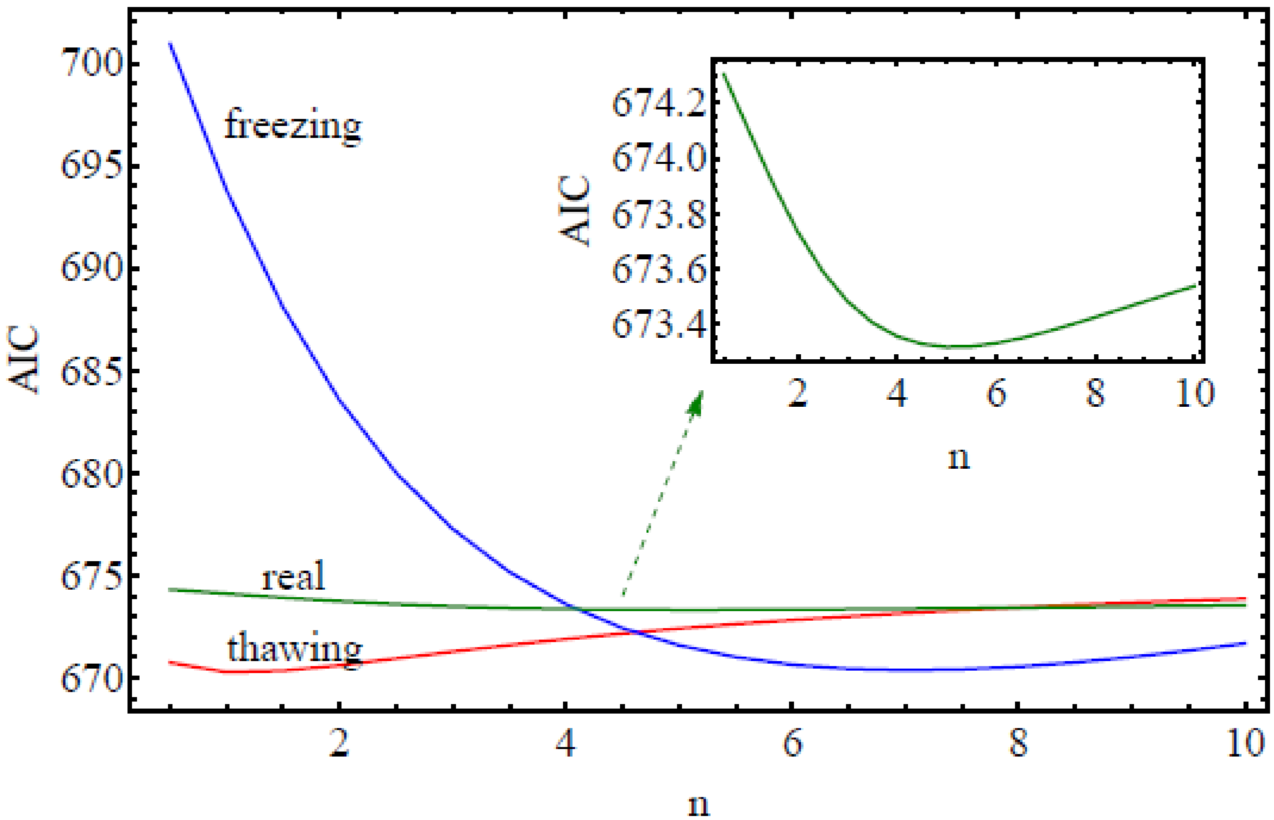
$$w_{\mathrm{nCPL}}(z) = w_0 + w_a(1-a)^n = w_0 + w_a\left(\frac{z}{1+z}\right)^n$$

Simulated data for freezing and thawing underlying model. Best fit value of w_0 fit with nCPL as a function of n.





AIC and Quality of Fit



$$\mathrm{AIC} = 2k - 2\ln L,$$

$$\mathrm{AIC}=2+\chi^2$$

Result for Question 3

Q3: How can we avoid misleading conclusions when using cosmological data to fit w(z) and predict its future evolution? In the CPL parameterization adequate to describe a possible non-trivial evolution of w(z)?

A3: Two parameter dark energy parametrizations may be divided in two classes depending on their convexity properties: Thawing and Freezing parametrizations. Fitting cosmological data with a parametrization that is not suitable for the underlying cosmological model may lead to misleading conclusions including an incorrect value for w(z=0) and/or incorrect value for the slope of w(z) at a statistically significant level.