Revival of classical black hole evaporation?

\- BH localized on the Randall-Sundrum II brane -

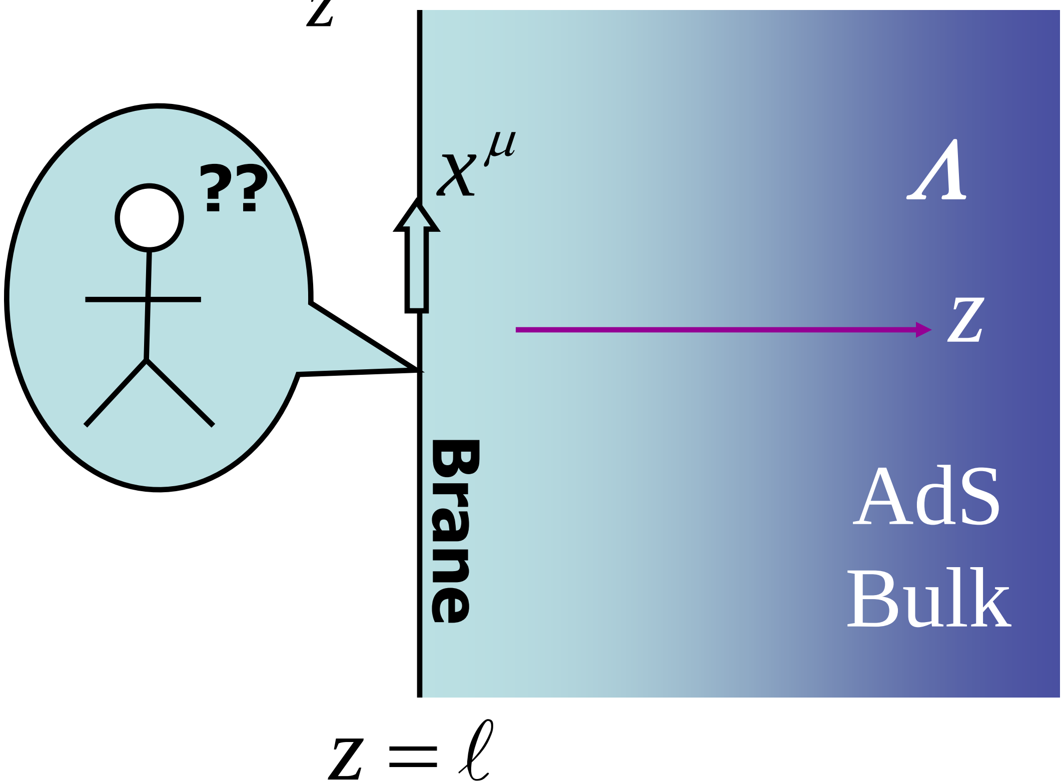
Takahiro Tanaka (Kyoto university)

work with Roberto Emparan

Infinite extra-dimension: Randall-Sundrum II model

Volume of the bulk is finite due to warped geometry although its extension is infinite.

$$ds^2 = \frac{\ell^2}{z^2}\left(dz^2 + \eta_{\mu\nu}dx^\mu dx^\nu\right)$$



ℓ : AdS curvature radius

$\Lambda = -\frac{6}{\ell^2}$ Negative cosmologicall constant

$\sigma = \frac{3}{4\pi G_5 \ell}$ Brane tension

Z_2-symmetry across the brane

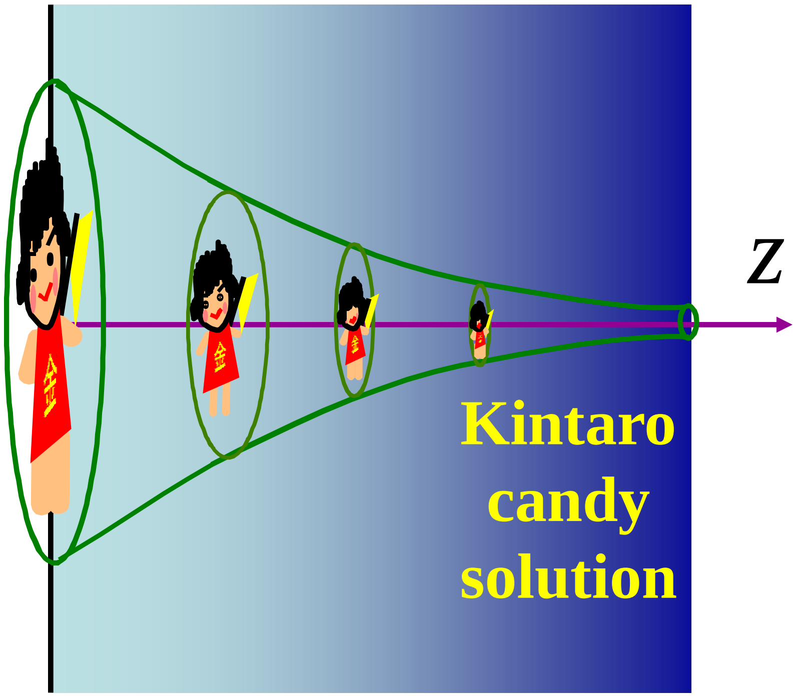
- Extension is infinite, but 4-D GR seems to be recovered!

Gravity on the brane looks like 4D GR approximately, BUT for many years Schwarzschild-like BH solution had been unknown.

Black string solution

(Chamblin, Hawking, Reall ('00))

$$ds^2 = \frac{\ell^2}{z^2}\left(dz^2 + \bar{g}^{(Sch)}_{\mu\nu} dx^\mu dx^\nu\right)$$



Metric induced on the brane $\bar{g}_{\mu\nu}(x)$ is exactly Schwarzschild solution.

However, this solution is singular.

- $C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} \propto z^4$

  behavior of zero mode

Moreover, this solution is unstable.

- *Gregory Laflamme instability*

AdS/CFT correspondence

(Maldacena ('98))
(Gubser ('01))
(Hawking, Hertog, Reall ('00))

- $Z[q]=\int d[\phi]\,\exp(-S_{CFT}[\phi,q])$

Boundary metric (q)

$$=\int d[g]\,\exp(-S_{HE}-S_{GH}+S_1+S_2+S_3)\equiv \exp(-W_{CFT}[q])$$

Counter terms ($S_1+S_2+S_3$)

$$S_{EH}=-\frac{1}{2\kappa_5^2}\int d^5x\sqrt{-g}\left({}^{(5)}R+\frac{12}{\ell^2}\right)$$

$$S_{GH}=-\frac{1}{\kappa_5^2}\int d^4x\sqrt{-q}\,K$$

$$S_1=-\frac{3}{\kappa_5^2\ell}\int d^4x\sqrt{-q}$$

$$S_2=-\frac{\ell}{4\kappa_5^2}\int d^4x\sqrt{-q}\;{}^{(4)}R$$

$$S_3=\cdots$$

- $z_0\longrightarrow 0$ limit is well defined with the counter terms

Brane position (z_0)

- $\int d[g]\,\exp(-S_{RS})=\int d[g]\,\exp(-2(S_{EH}+S_{GH})+2S_1-S_{matter})$

$$=\exp(-2S_2-S_{matter}-2(W_{CFT}+S_3))$$

brane tension ($2S_1$)

4D Einstein-Hilbert action ($2S_2$)

$z_0\Leftrightarrow$ cutoff scale parameter

Classical black hole evaporation conjecture

(T.T. ('02), Emparan et al ('02))

4D Einstein+CFT with the lowest order quantum correction

$$\left(\begin{array}{c}\text{number of} \\ \text{field of CFT}\end{array}\right) \approx \frac{\ell^2}{\kappa_4^2}$$

AdS/CFT correspondence

⟺ equivalent

Classical 5D dynamics in RS II model

4D BH with CFT ⟺ equivalent ⟺ 5D BH on brane

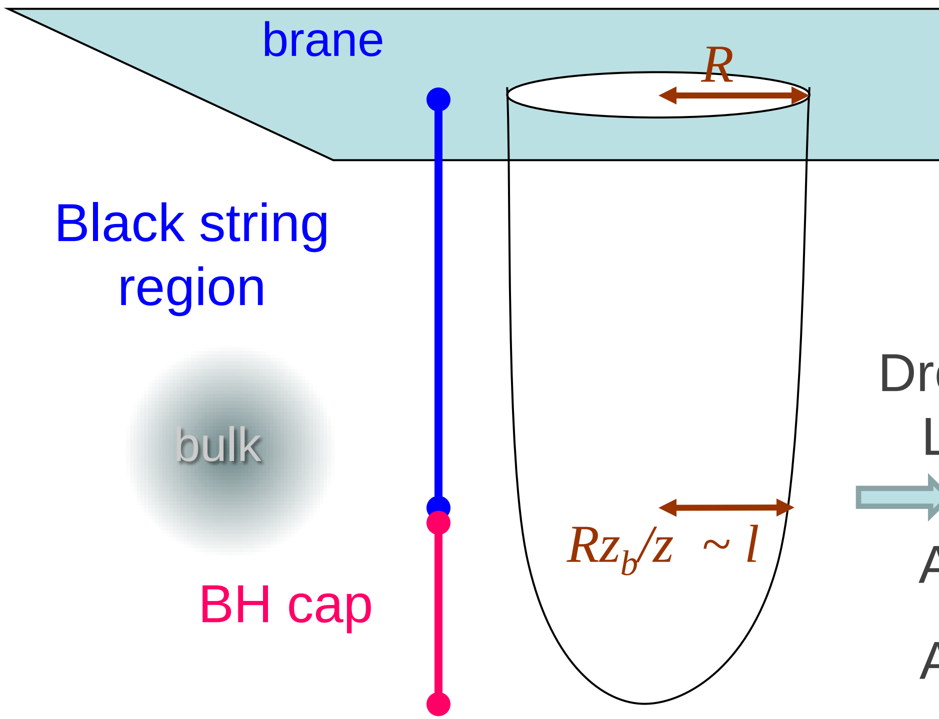
Hawking radiation in 4D Einstein+CFT picture ⟺ equivalent ⟺ Classical evaporation of 5D BH

Time scale of BH evaporation

$$\tau = \left(\frac{M}{M_{Solar}}\right)^3 \left(\frac{1\text{mm}}{\ell}\right)^2 \times 1\text{year}$$

$$\frac{\dot{M}}{M} \approx \left(\begin{array}{c}\text{Number of} \\ \text{species}\end{array}\right) \times \frac{1}{G_N^2 M^3} \approx \frac{\ell^2}{(G_N M)^3}$$

most-probable shape of a large BH



Structure near the cap region will be almost independent of the size of the black hole. ~discrete self-similarity

Assume Gregory-Laflamme instability at the cap region

⇒ Droplet escaping to the bulk

Droplet formation

Local proper time scale: l

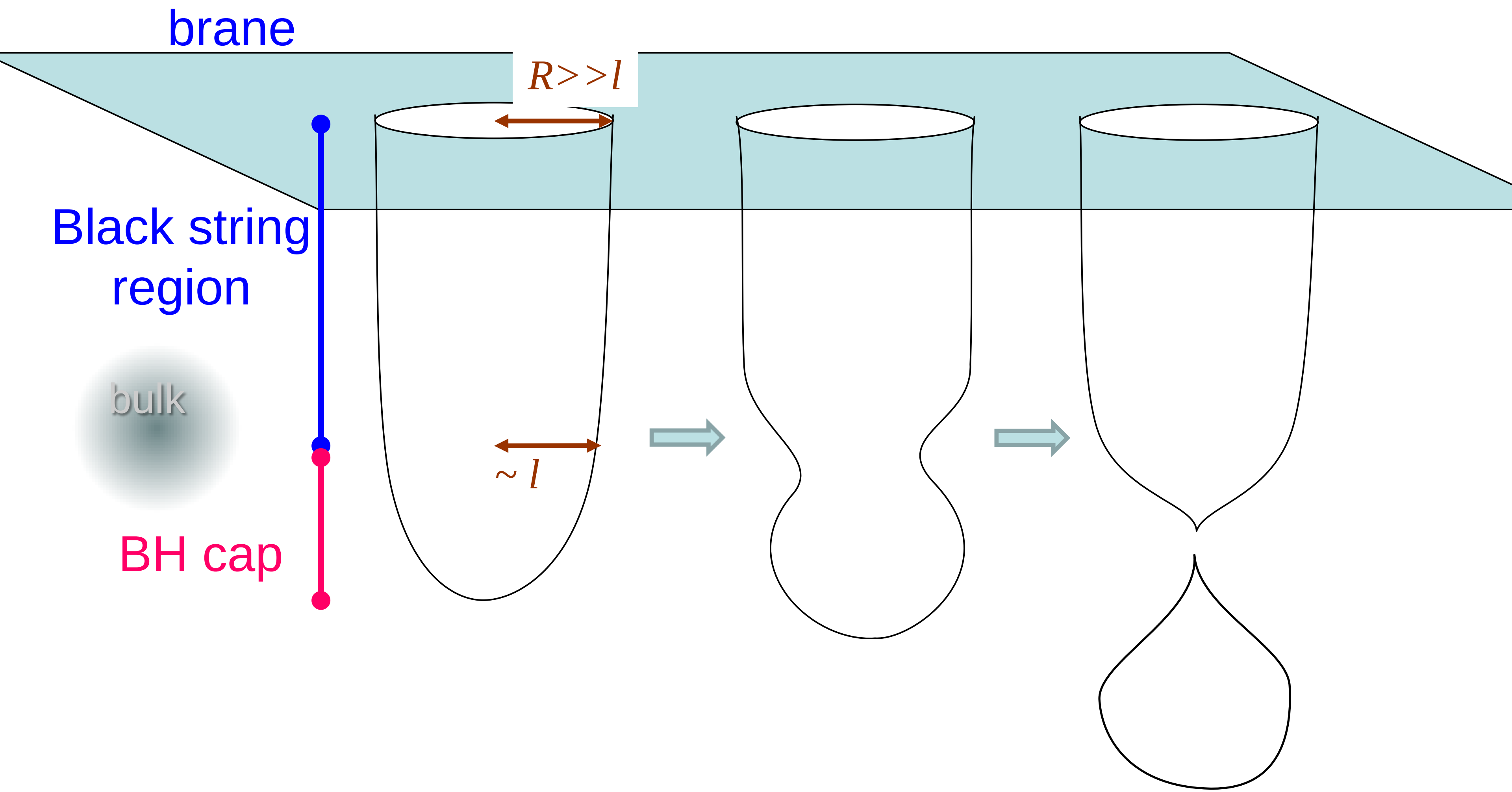
⇒ R on the brane due to redshift factor

Area of a droplet: l^3

Area of the black hole: $A \sim lR^2$

$$\frac{dA}{dt} \approx \frac{l^3}{R} \Rightarrow \frac{1}{M}\frac{dM}{dt} \approx \frac{1}{A}\frac{dA}{dt} \approx \frac{l^2}{R^3}$$

Most-probable path of BH evaporation



However, static brane-localized black hole (black droplet) was obtained numerically

Pau Figueras, James Lucietti, Toby Wiseman (2011)

Shape of the horizon in the bulk:
The boundary metric is conformal to Schwarzschild BH

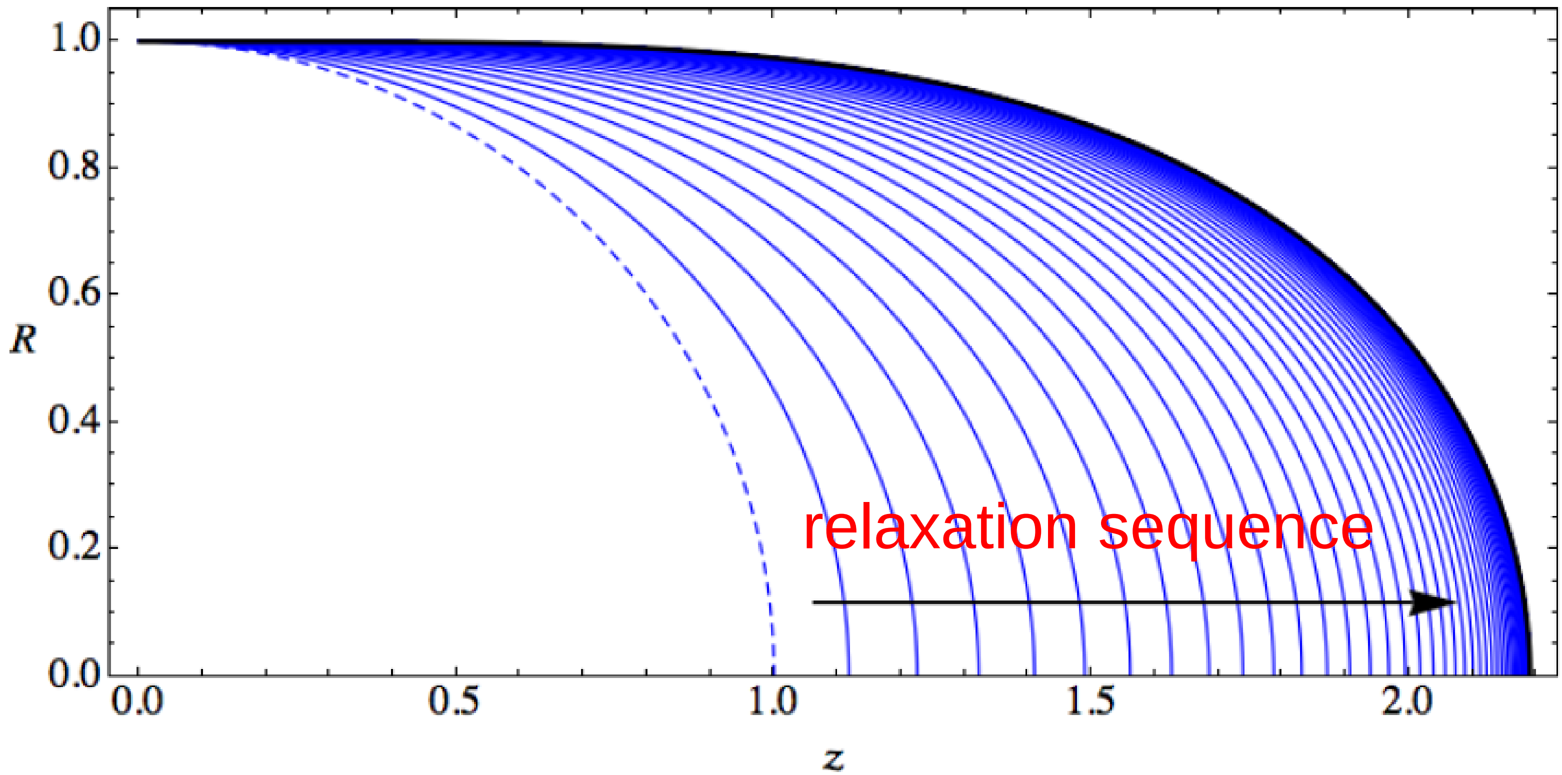
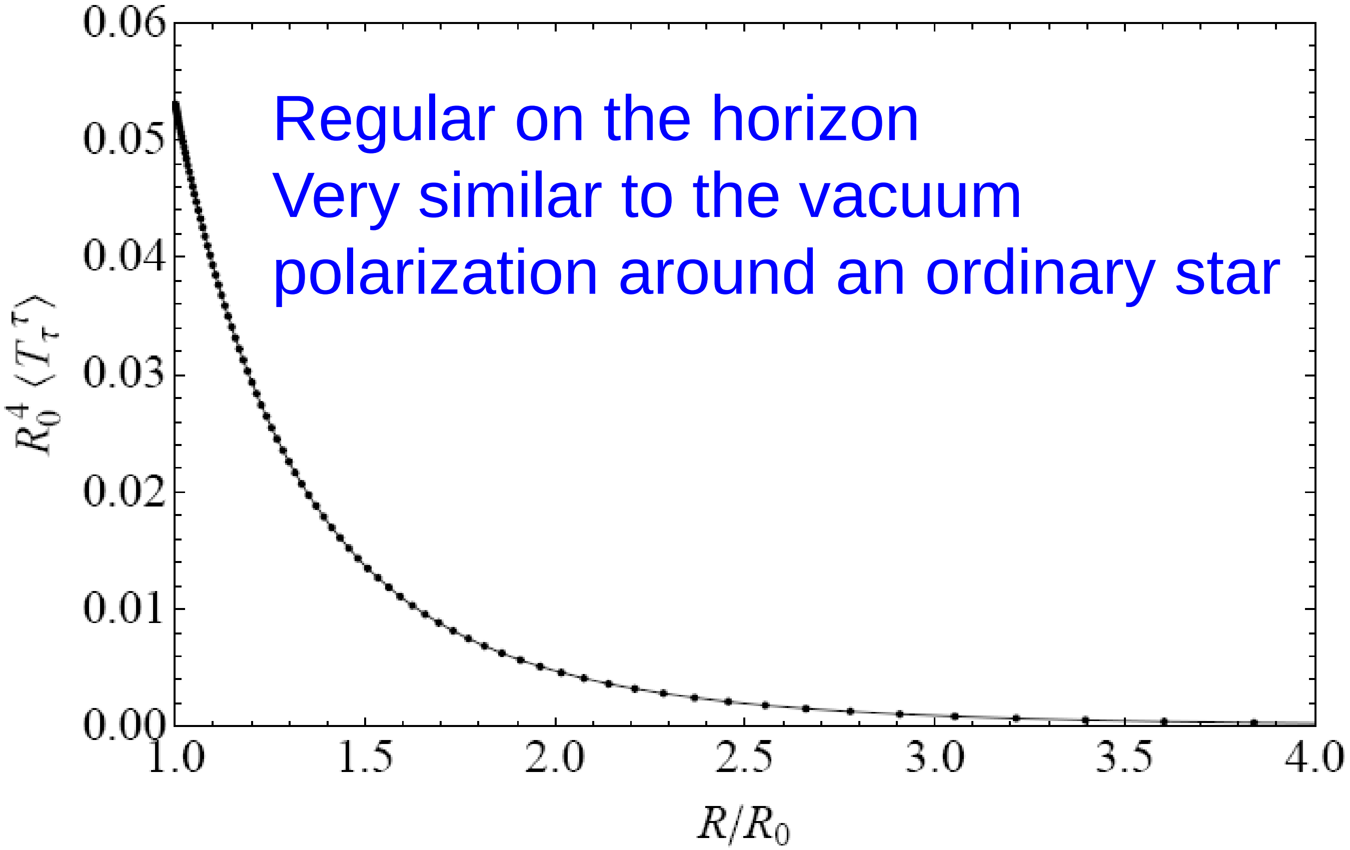


Figure 4: Embedding into hyperbolic space, $ds^2 = \frac{\ell^2}{z^2}\left(dz^2 + dR^2 + R^2 d\Omega_{(2)}^2\right)$, of the spatial cross sections of the horizon along the flow as curves $R(z)$. The dashed line corresponds to the initial data, for which the horizon is round, and the thick black line is the embedding of the horizon of the fixed point. The snapshots are drawn at intervals of λ of 0.05.

CFT energy density profile on the AdS boundary



An apocalypse

For analytic perturbative approach,
we need some small parameter.

In large D limit, $1/D$ can play
the role of the small parameter.

Things are simplified a lot
because gravity is effectively short-ranged.


$$\phi \approx \frac{1}{r} \quad \Rightarrow \quad \phi \approx \frac{1}{r^{D-3}}$$ In D dimensions

A cue

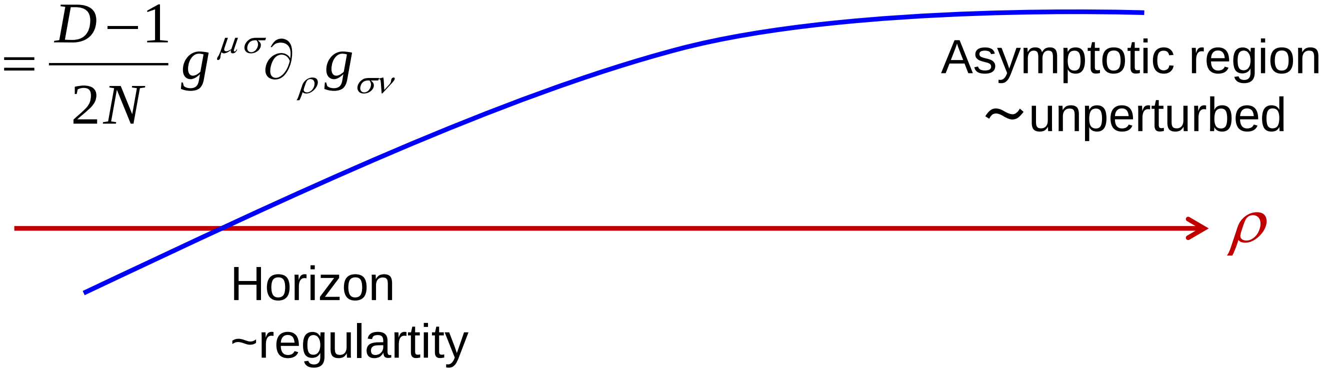
"We may easily find the black hole attached
to the AdS boundary in the large D limit."

Gradient expansion

$$ds^2 = N^2 \frac{d\rho^2}{(D-1)^2} + g_{\mu\nu} dx^\mu dx^\nu$$

$$\frac{D-1}{N} \partial_\rho K_\mu^\nu + K K_\mu^\nu = R_\mu^\nu + \delta_\mu^\nu \frac{D-1}{\ell^2} - \frac{1}{N} \nabla^\mu \nabla_\nu N$$

$$K_\mu^\nu = \frac{D-1}{2N} g^{\mu\sigma} \partial_\rho g_{\sigma\nu}$$



“Hamiltonian” constraint

$$K^2 - K_\nu^\mu K_\mu^\nu = R + \frac{(D-1)(D-2)}{\ell^2}$$ = automatic from the horizon regularity

“momentum” constraint

$$\nabla_\nu K_\mu^\nu - \nabla_\mu K = 0$$ ⇒ constancy of the surface gravity

Effective equation of motion

～how to embed horizon in a given background spacetime

(Emparan, Shiromizu, Suzuki, Tanabe, Tanaka, JHEP1506 (2015) 159)

For large D, the perturbation far from the horizon rapidly decays.

⇒ Problem is how to embed the horizon surface in a given spacetime.

AdS space in Poincare chart:

$$ds^2 = \frac{\ell^2}{z^2}\left(-dt^2 + dr^2 + r^2 d\Omega_{n+1} + dz^2\right)$$

Leading order in large n expansion.

$$4\tilde{\kappa}^2 = \frac{\ell^2}{z^2 r^2}\left(1 - \gamma^{zz} r_{,z}^2 + \frac{r^2}{\ell^2}\right) \qquad \gamma^{zz} = \frac{z^2}{\ell^2\left(1 + r_{,z}^2\right)}$$

```
In[1]:= zI = 1.1;
        f[x_, z_] := - zI^2 / z / (1 - zI^2 Exp[2 x])
           (1 - Sqrt[z^2 / zI^2 (1 - Exp[-2 x] / zI^2) + Exp[-2 x] / zI^2]);

In[3]:= xi = 5;
        ΔzI = - 0.1;
        zi = zI + ΔzI Exp[1 / zI^2 Exp[-2 xi]];
        xstep = - 0.001;

In[7]:= ζ = zi; ξ = xi;
        list = {{xi, zi}};
        While[ζ > 0.01,
         ζ1 = xstep f[ξ, ζ];
         ζ2 = xstep f[ξ + xstep / 2, ζ + ζ1 / 2];
         ζ3 = xstep f[ξ + xstep / 2, ζ + ζ2 / 2];
         ζ4 = xstep f[ξ + xstep, ζ + ζ3];
         ζ = ζ + (ζ1 + 2 ζ2 + 2 ζ3 + ζ4) / 6.;
         AppendTo[list, {ξ, ζ}];
         ξ = ξ + xstep]

In[11]:= ListPlot[list, PlotJoined → True, PlotRange → {{-1, 2}, {0, 2}}]

Out[11]=
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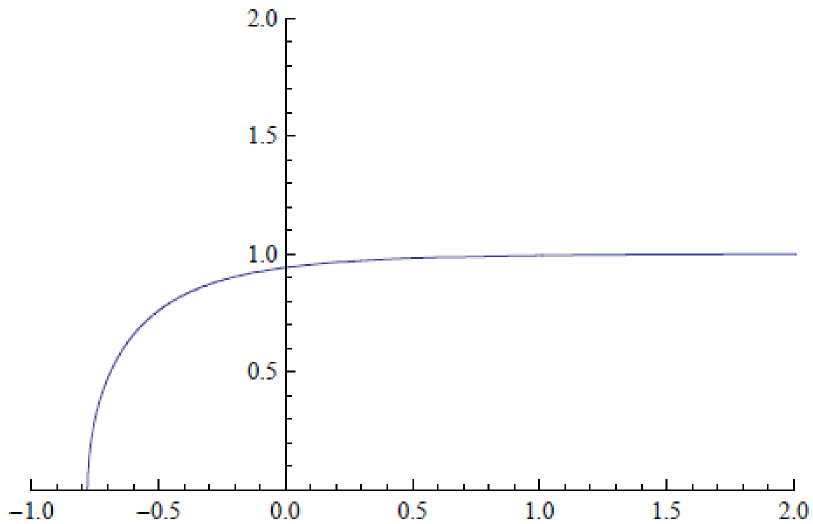

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Perturbation equation around the black string should be second order,
but the linearization of the above equation is first order.

$$\Rightarrow \quad \partial_z \delta r - z\delta r = 0$$

Expected form of solutions from linear perturbation of black string:

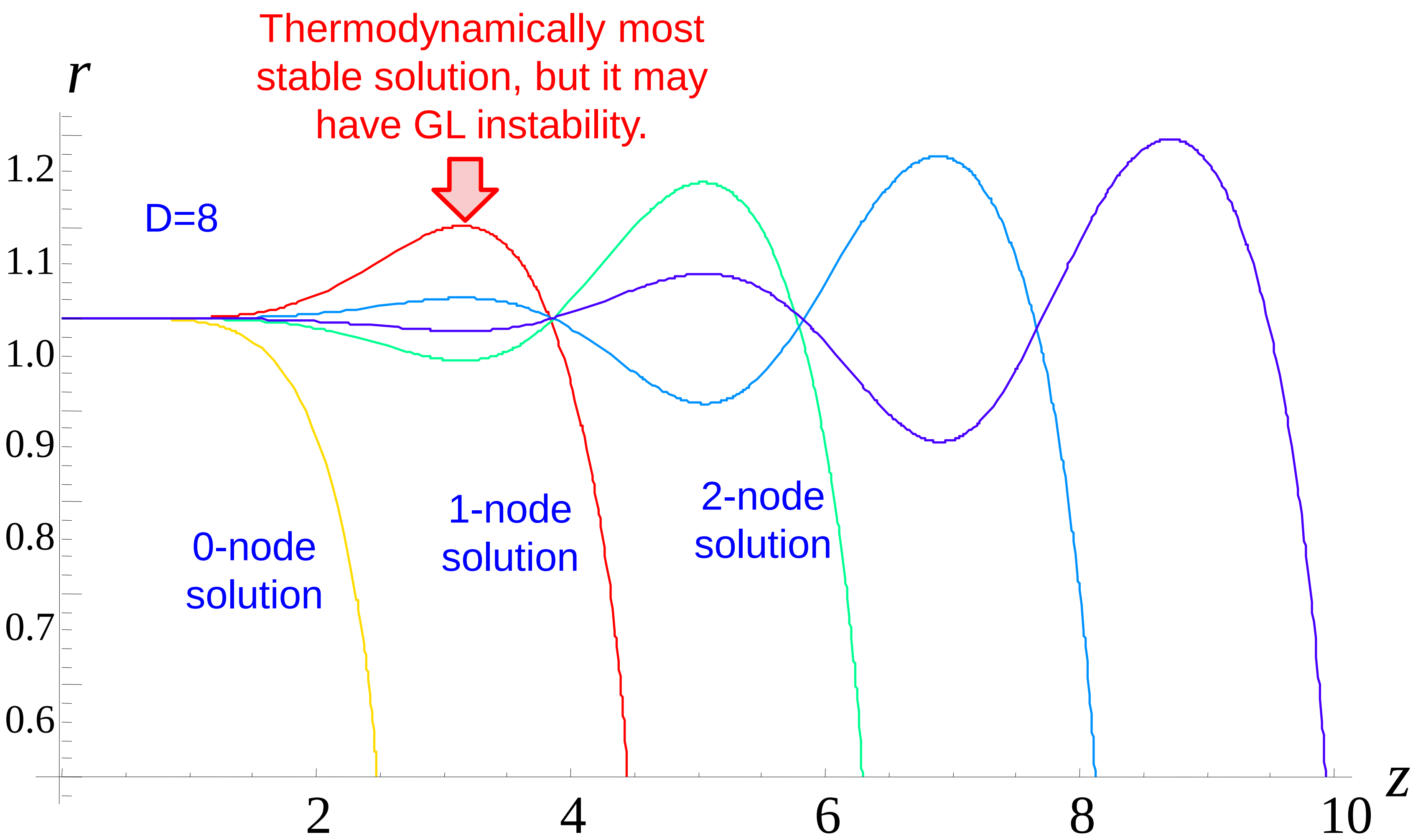
$$h_{\mu\nu} = \underline{\left(z^0 + \cdots\right)} + \underline{z^{D-1}\left(1 + \cdots\right)}$$

Deformation of boundary metric

Holographic $T_{\mu\nu}$

We'd like to choose vanishing deformation of boundary metric as physical boundary condition.

A series of Brane localized BH solutions



Comparison of the areas of black objects:

$A_{[0]}<A_{[2]}<A_{[4]}<\cdots<A_{[BS]}<\cdots<A_{[3]}<A_{[1]}$

Only gradient expansion

$1/D$ corrections must be maintained to recover the second term of

$$h_{\mu\nu} = \left(z^0 + \cdots\right) + z^{D-1}\left(1 + \cdots\right)$$

⇒ Problem is how to truncate the equation consistently.

We keep higher order $1/D$ terms in the computation of $R_{\mu\nu}$, but we neglect higher order in gradient expansion.

Later we check the neglected higher order terms are small.

$$4\tilde{\kappa}^2 = \frac{\ell^2}{z^2 r^2}\left(1 - \gamma^{zz} r_{,z}^2 + \frac{r^2}{\ell^2} + \frac{1}{D-4}\left(\frac{5}{\ell^2} - \frac{\sqrt{\gamma}\partial_z \sqrt{\gamma}\partial_z r}{r} + \frac{6\gamma\,\partial_z r}{zr}\right)\right)$$

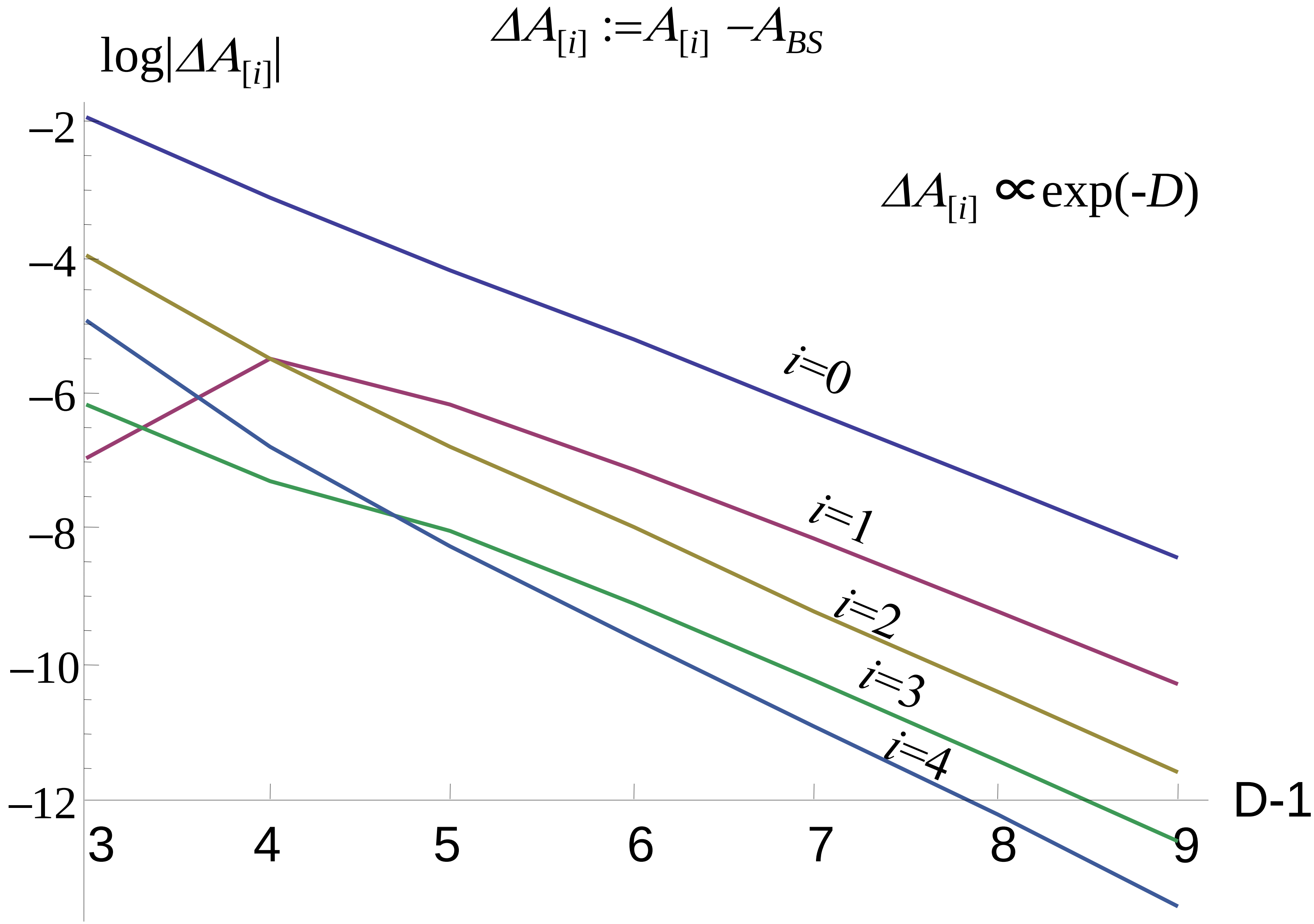
The linearization of the above equation:

$$-\partial_z^2 \delta r + \left(D - \frac{3}{2}\right)\partial_z \delta r - (D-4) z\,\delta r = 0$$

Asymptotic form of solutions

$$\delta r = \left(z^0 + \cdots\right) + z^{D-5/2}\left(1 + \cdots\right)$$

We can choose vanishing deformation of boundary metric as physical boundary condition.



$\Delta A_{[i]} := A_{[i]} - A_{BS}$
$\log|\Delta A_{[i]}|$
$\Delta A_{[i]} \propto \exp(-D)$
$i=0$
$i=1$
$i=2$
$i=3$
$i=4$
−2
−4
−6
−8
−10
−12
3
4
5
6
7
8
9
D-1

Summary

If the BH evaporation is accelerated, we have a chance to observe it. Such an acceleration was expected in Randall-Sundrum braneworld setup as classical BH evaporation.

However, a static brane-localized BH solution was found, which made many people to think that strongly interacting conformal field may not have Hawking radiation.

We revisited this issue using large D expansion.

We derived a master equation which reproduces static spherical black hole, black string and its deformation.

We obtained a sequence of brane localized BH solutions and found that thermodynamically the 1-node solution is most stable, while 0-node solution is most unstable.

This suggests that the classical BH evaporation scenario may revive.