General Pole Inflation

Hilltop, Natural, and Chaotic Inflationary Attractors

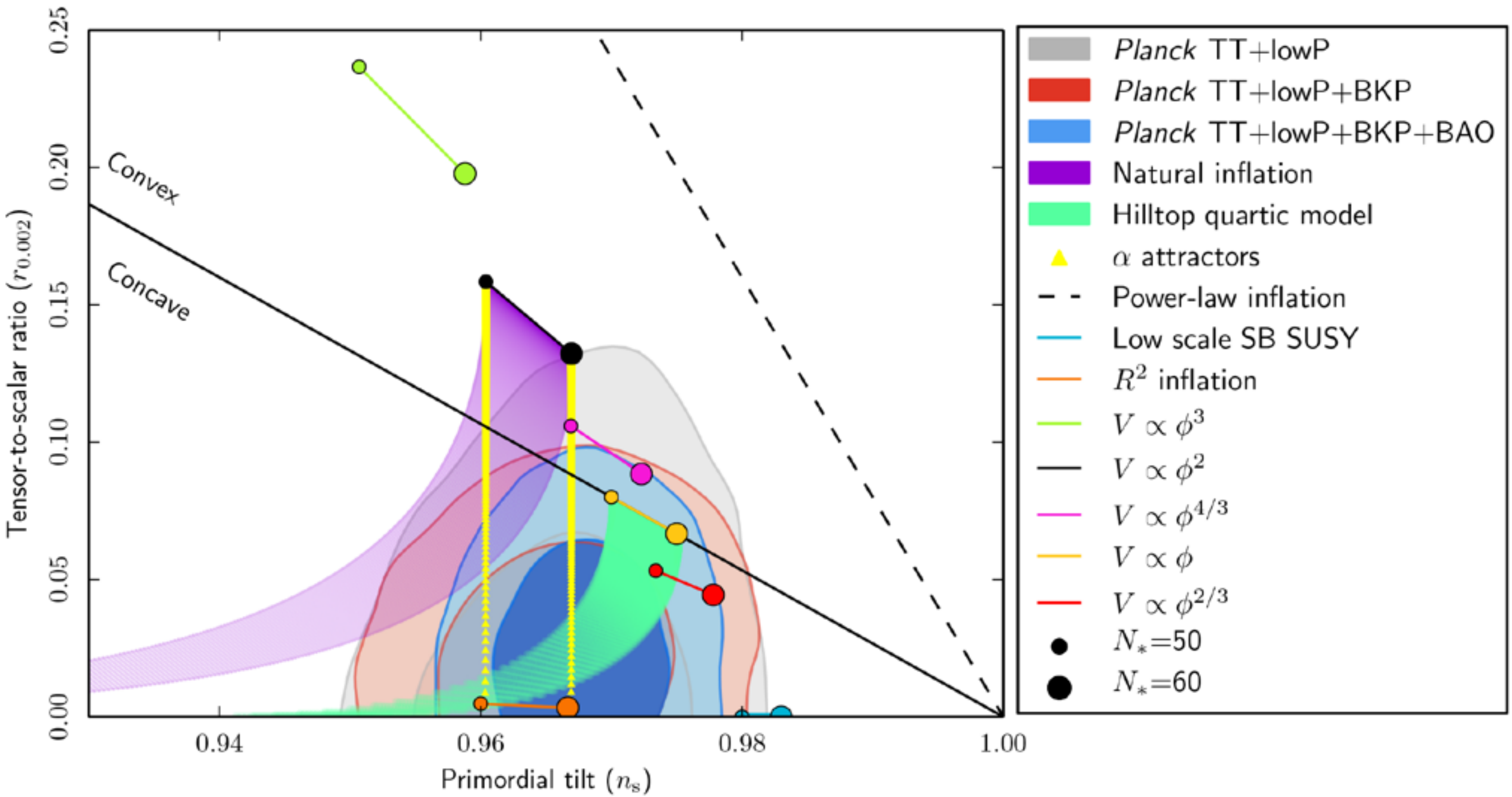
Takahiro Terada

(APCTP)

TT, arXiv:1602.07867 [hep-th]

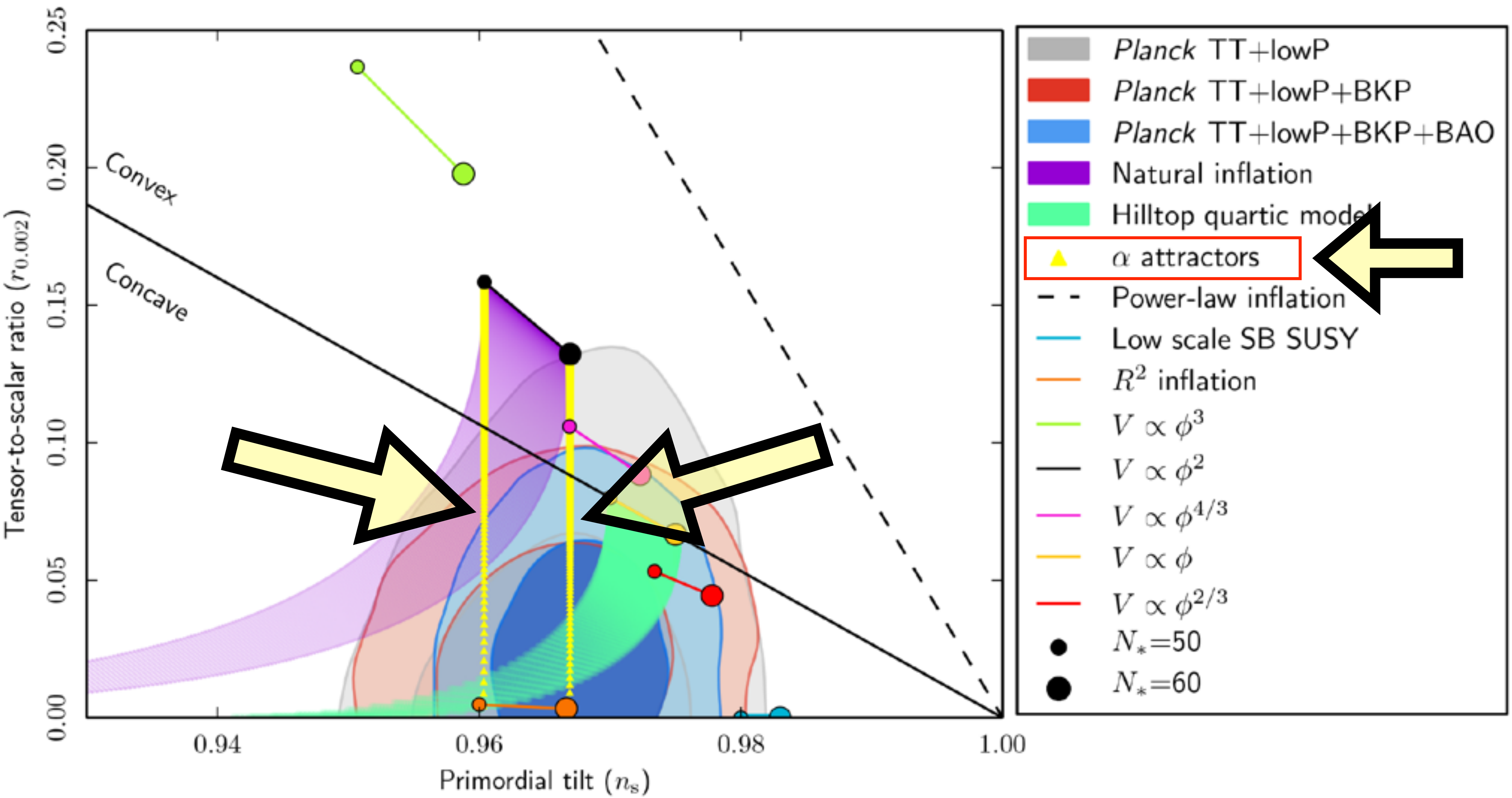
3rd Korea-Japan Workshop on Dark Energy, KASI, April 6

[Ade *et al.* (Planck collaboration), 1502.02114]



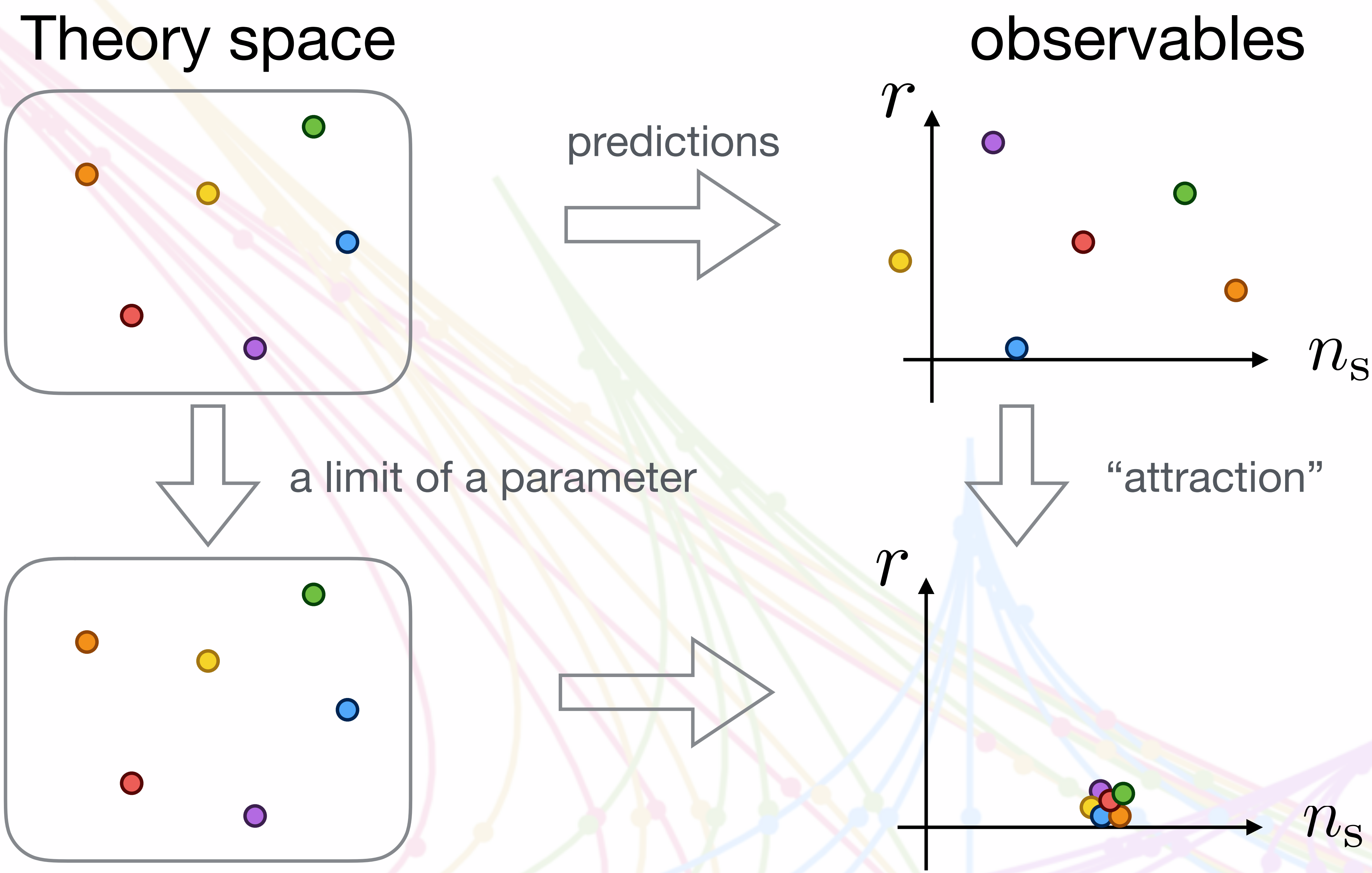
Introduction

[Ade *et al.* (Planck collaboration), 1502.02114]



Introduction

Inflationary Attractor Models



Ex.) α-attractor

$$(\sqrt{-g})^{-1}\mathcal{L} = -\frac{1}{2}R - \frac{1}{2}K_{\rm E}(\phi)(\partial_\mu\phi)^2 - V_{\rm E}(\phi)$$

Ex.) α-attractor

$$(\sqrt{-g})^{-1}\mathcal{L} = -\frac{1}{2}R - \frac{1}{2}K_{\rm E}(\phi)(\partial_\mu\phi)^2 - V_{\rm E}(\phi)$$

where

$$K_{\rm E}(\phi) = \frac{\alpha}{(1-\phi^2/6)^2}$$

$$V_{\rm E} = \alpha f(\phi/\sqrt{6})^2$$

Ex.) α-attractor

$$(\sqrt{-g})^{-1}\mathcal{L} = -\frac{1}{2}R - \frac{1}{2}K_{\mathrm{E}}(\phi)(\partial_\mu\phi)^2 - V_{\mathrm{E}}(\phi)$$

where

$$K_{\mathrm{E}}(\phi) = \frac{\alpha}{(1-\phi^2/6)^2}$$

$$V_{\mathrm{E}} = \alpha f(\phi/\sqrt{6})^2$$

Universal predictions

$$n_{\mathrm{s}} = 1 - \frac{2}{N}$$

$$r = \frac{12\alpha}{N^2}$$

Ex.) α-attractor

$$(\sqrt{-g})^{-1}\mathcal{L} = -\frac{1}{2}R - \frac{1}{2}K_{\rm E}(\phi)(\partial_\mu\phi)^2 - V_{\rm E}(\phi)$$

[Kallosh, Linde, Roest, 1311.0472]

where

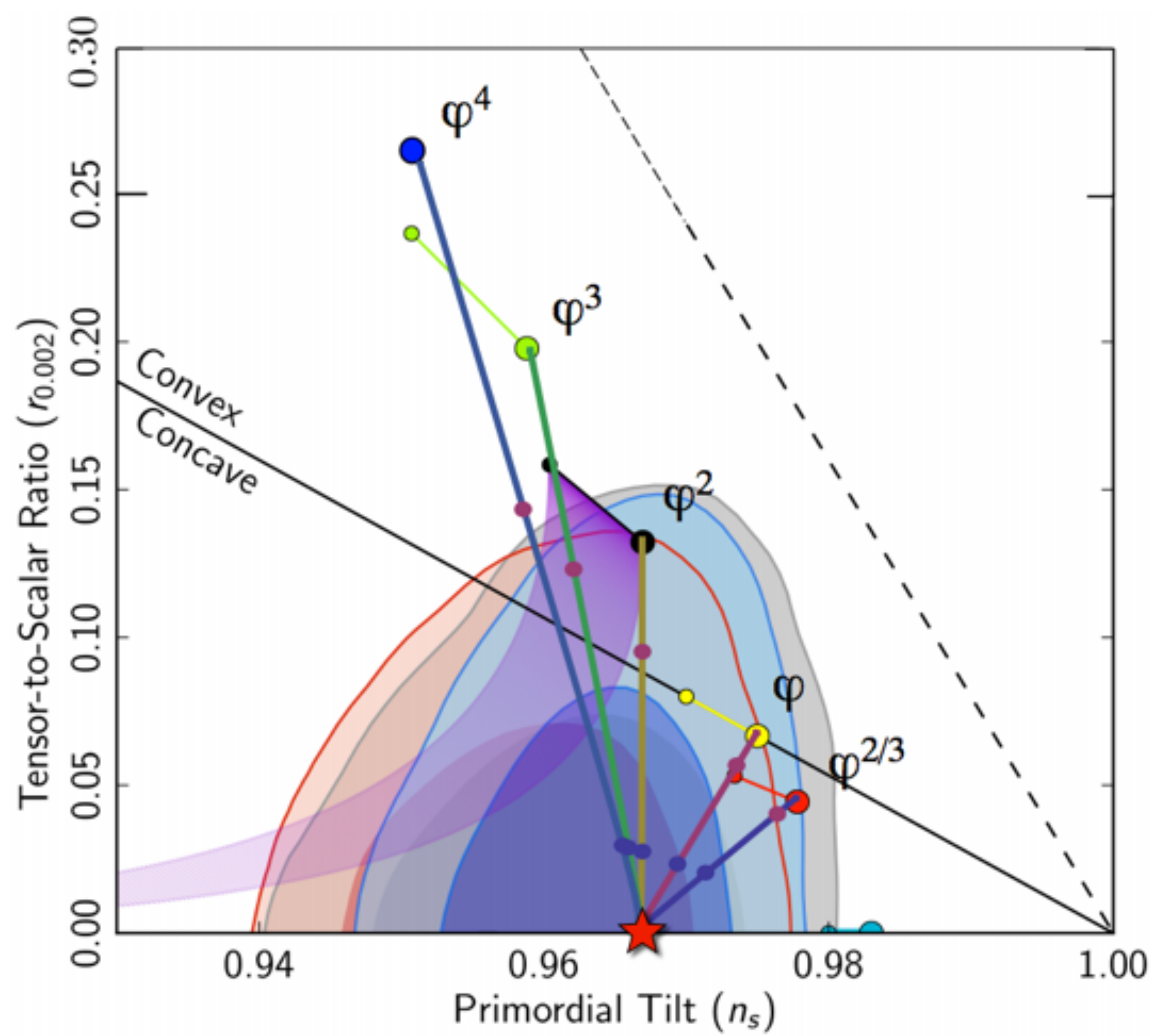
$$K_{\rm E}(\phi) = \frac{\alpha}{(1-\phi^2/6)^2}$$

$$V_{\rm E} = \alpha f(\phi/\sqrt{6})^2$$

Universal predictions

$$n_{\rm s} = 1 - \frac{2}{N}$$

$$r = \frac{12\alpha}{N^2}$$



Ex.) α-attractor

$$(\sqrt{-g})^{-1}\mathcal{L} = -\frac{1}{2}R - \frac{1}{2}K_{\mathrm{E}}(\phi)(\partial_\mu\phi)^2 - V_{\mathrm{E}}(\phi)$$

[Kallosh, Linde, 1502.07733]

where

$$K_{\mathrm{E}}(\phi) = \frac{\alpha}{(1-\phi^2/6)^2}$$

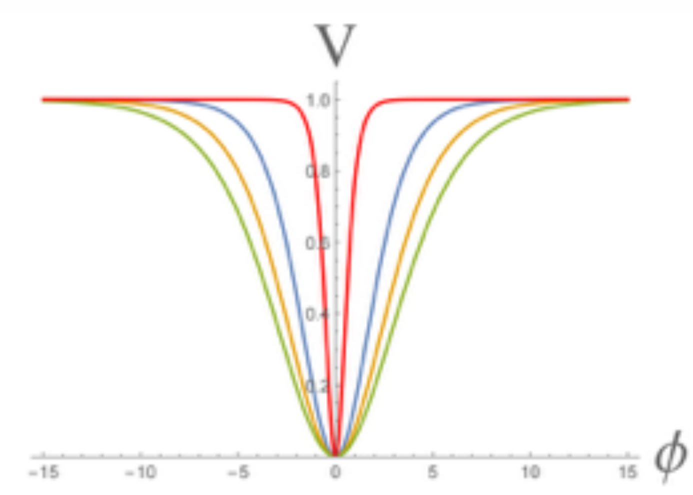
$$V_{\mathrm{E}} = \alpha f(\phi/\sqrt{6})^2$$

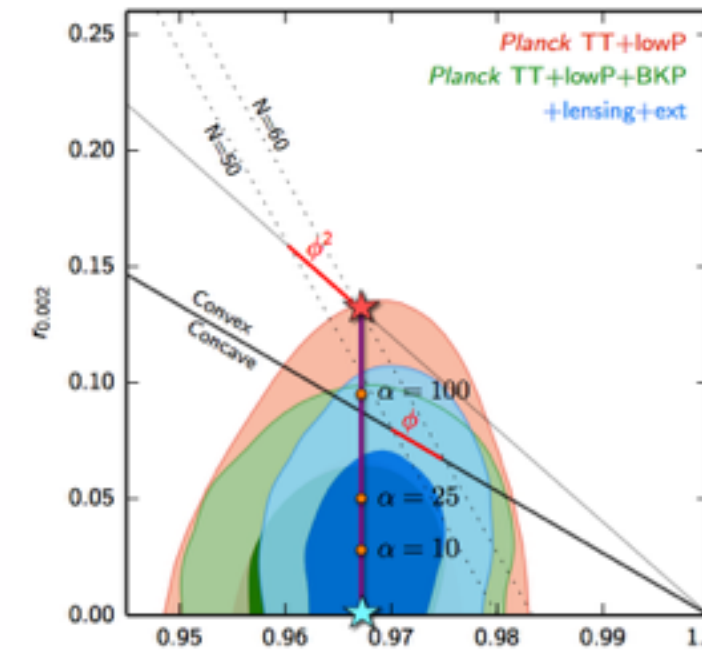
Universal predictions

$$n_{\mathrm{s}} = 1 - \frac{2}{N}$$

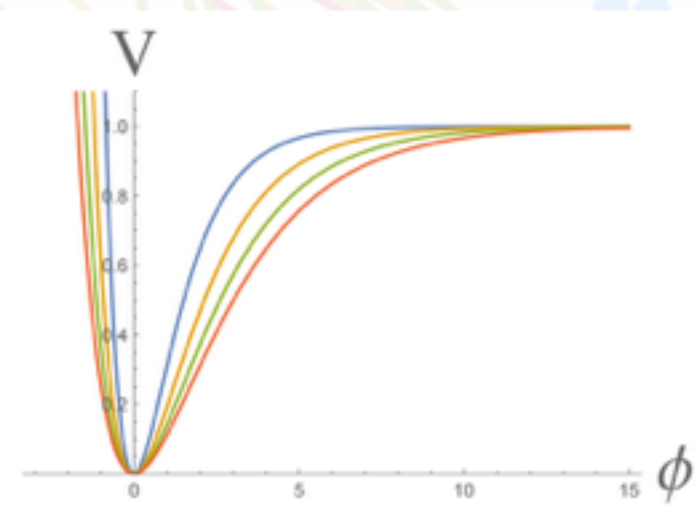
$$r = \frac{12\alpha}{N^2}$$

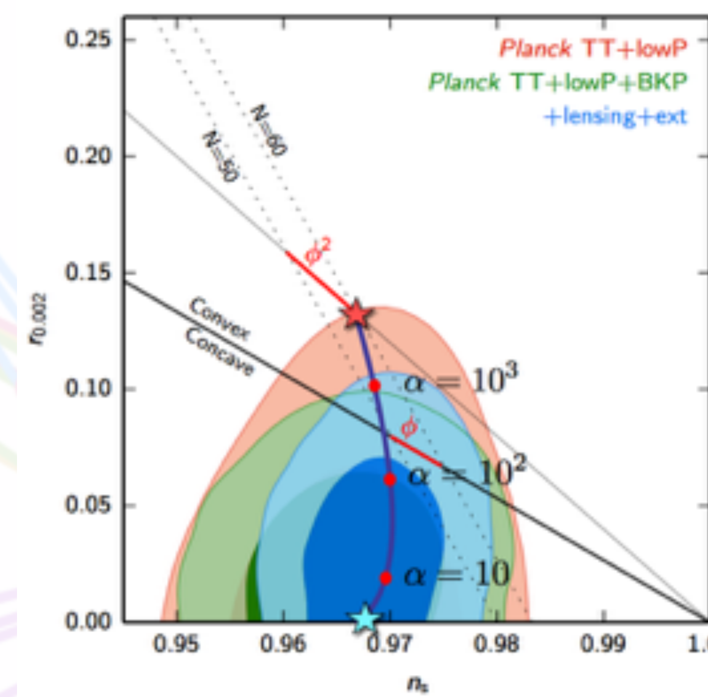
T-model





E-model





Ex.) Universal Attractor

$$(\sqrt{-g})^{-1}\mathcal{L} = -\frac{1}{2}\Omega(\phi)R - \frac{1}{2}K_{\mathrm{J}}(\phi)(\partial_\mu\phi)^2 - V_{\mathrm{J}}(\phi)$$

Ex.) Universal Attractor

$$(\sqrt{-g})^{-1}\mathcal{L} = -\frac{1}{2}\Omega(\phi)R - \frac{1}{2}K_{\rm J}(\phi)(\partial_\mu\phi)^2 - V_{\rm J}(\phi)$$

where

$$\Omega(\phi) = 1 + \xi f(\phi)$$

$$K_{\rm J}(\phi) = 1$$

$$V_{\rm J}(\phi) = \frac{\lambda}{\xi^2}\left(\Omega(\phi) - 1\right)^2 = \lambda f(\phi)^2$$

Ex.) Universal Attractor

$$(\sqrt{-g})^{-1}\mathcal{L} = -\frac{1}{2}\Omega(\phi)R - \frac{1}{2}K_{\rm J}(\phi)(\partial_\mu\phi)^2 - V_{\rm J}(\phi)$$

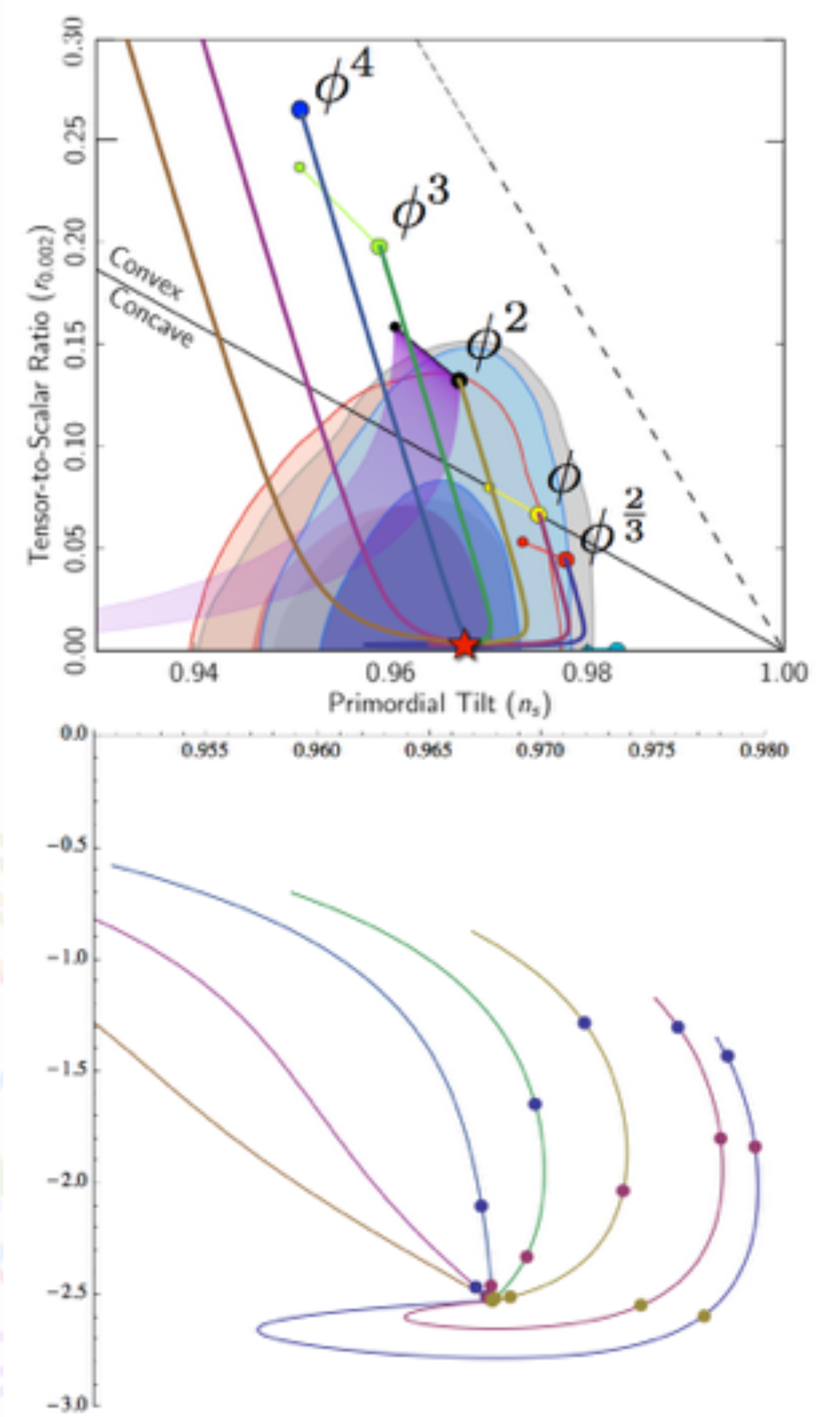
where

$$\Omega(\phi) = 1 + \xi f(\phi)$$

$$K_{\rm J}(\phi) = 1$$

$$V_{\rm J}(\phi) = \frac{\lambda}{\xi^2}\left(\Omega(\phi) - 1\right)^2 = \lambda f(\phi)^2$$

[Kallosh, Linde, Roest, 1310.3950]



Ex.) Universal Attractor

$$(\sqrt{-g})^{-1}\mathcal{L} = -\frac{1}{2}\Omega(\phi)R - \frac{1}{2}K_{\rm J}(\phi)(\partial_\mu\phi)^2 - V_{\rm J}(\phi)$$

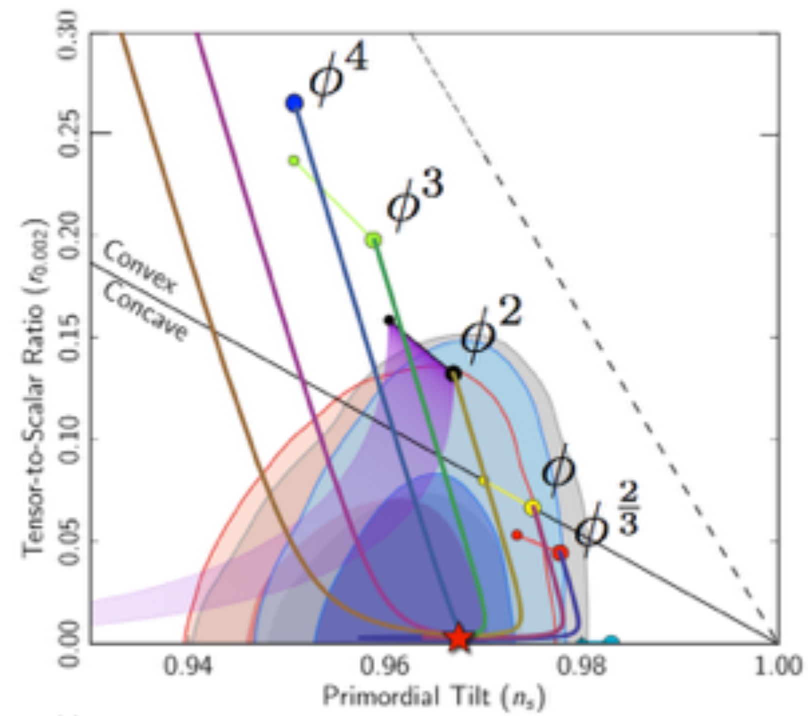
where

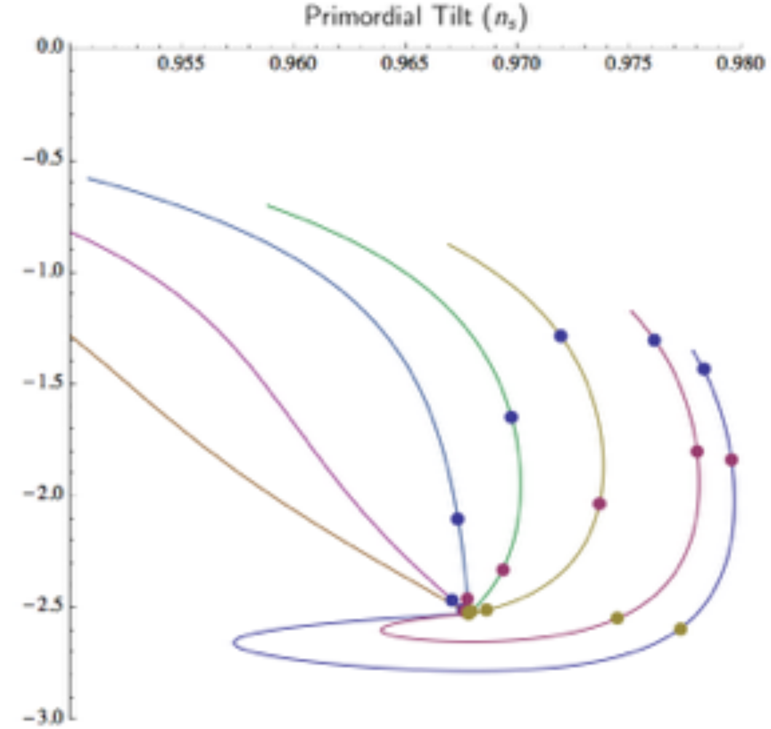
$$\Omega(\phi) = 1 + \xi f(\phi)$$

$$K_{\rm J}(\phi) = 1$$

$$V_{\rm J}(\phi) = \frac{\lambda}{\xi^2}\left(\Omega(\phi) - 1\right)^2 = \lambda f(\phi)^2$$

[Kallosh, Linde, Roest, 1310.3950]





Ex.) Induced inflation

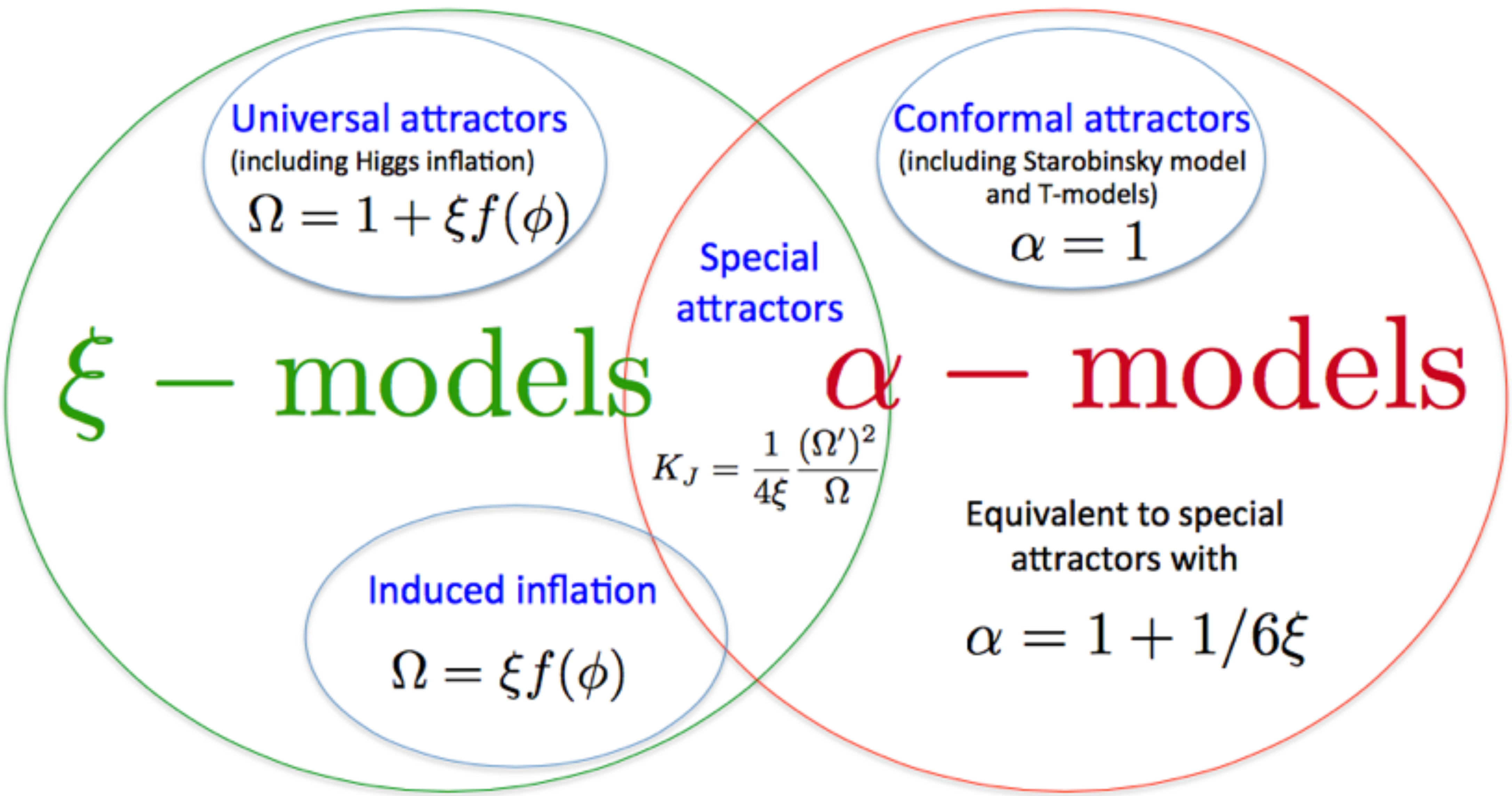
$$\Omega(\phi) = \xi f(\phi)$$
$$K_{\rm J}(\phi) = 1$$
$$V_{\rm J}(\phi) = \frac{\lambda}{\xi^2}\left(\Omega(\phi) - 1\right)^2$$

Unitarity property is improved.

Unity of cosmological attractors

$$(\sqrt{-g})^{-1}\mathcal{L} = -\frac{1}{2}\Omega(\phi)R - \frac{1}{2}K_{\rm J}(\phi)(\partial_\mu\phi)^2 - V_{\rm J}(\phi)$$

[Galante, Kallosh, Linde, Roest, 1412.3797]

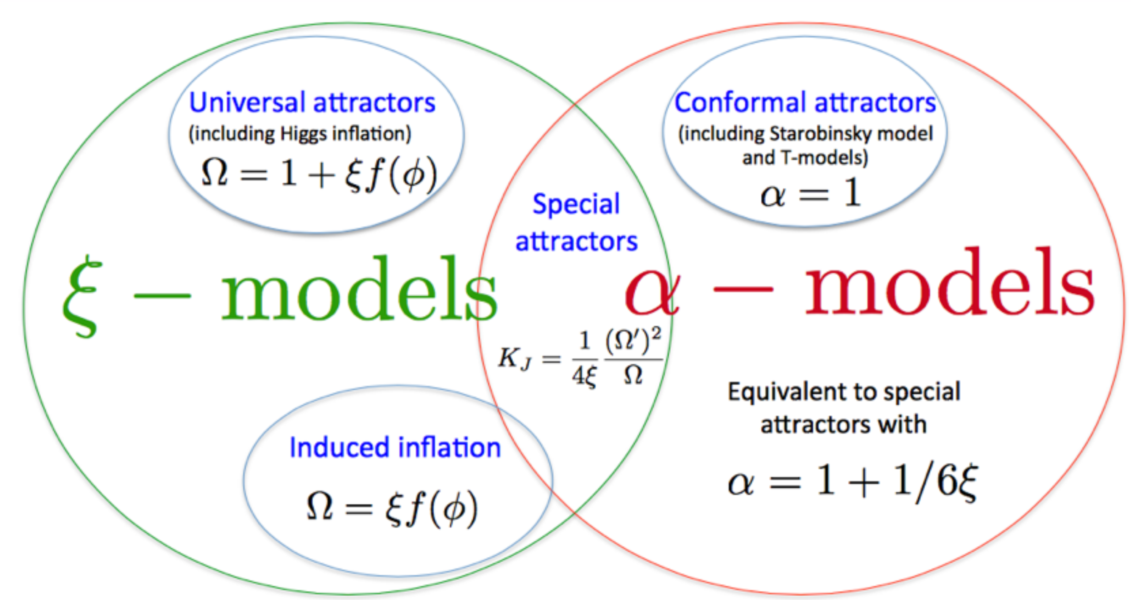


Unity of cosmological attractors

$$(\sqrt{-g})^{-1}\mathcal{L} = -\frac{1}{2}R - \frac{1}{2}K_{\mathrm{E}}(\varphi)(\partial_\mu\varphi)^2 - V_{\mathrm{E}}(\varphi)$$

[Galante, Kallosh, Linde, Roest, 1412.3797]

$$K_{\mathrm{E}}(\phi) \simeq \frac{3\alpha/2}{(\phi-\phi_0)^2}$$



2nd order pole(s) in K_{E} !

Inflation occurs near the pole.

Canonical normalization makes the potential flat.

Pole inflation

$$(\sqrt{-g})^{-1}\mathcal{L} = -\frac{1}{2}R - \frac{1}{2}K_{\mathrm{E}}(\varphi)(\partial_\mu\varphi)^2 - V_{\mathrm{E}}(\varphi)$$

More generally,

$$K_{\mathrm{E}}(\varphi) = \frac{a_p}{\varphi^p}$$

$$V_{\mathrm{E}}(\varphi) = V_0\left(1 - \varphi + \mathcal{O}\left(\varphi^2\right)\right)$$

[Galante, Kallosh, Linde, Roest, 1412.3797]

[Broy, Galante, Roest, Westphal, 1507.02277]

Pole inflation

$$(\sqrt{-g})^{-1}\mathcal{L} = -\frac{1}{2}R - \frac{1}{2}K_{\mathrm{E}}(\varphi)(\partial_\mu\varphi)^2 - V_{\mathrm{E}}(\varphi)$$

More generally,

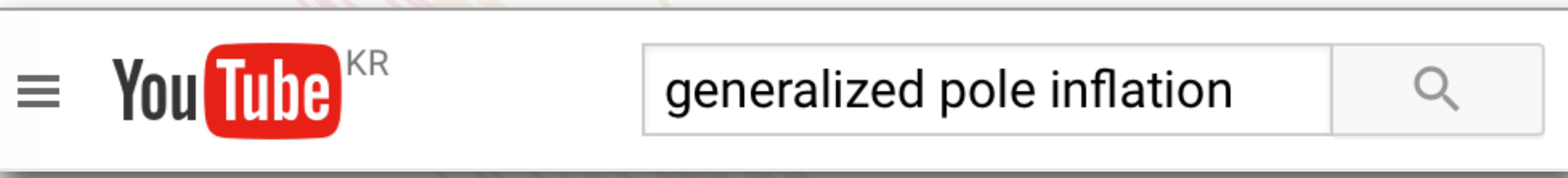
$$K_{\mathrm{E}}(\varphi) = \frac{a_p}{\varphi^p}$$

$$V_{\mathrm{E}}(\varphi) = V_0\left(1 - \varphi + \mathcal{O}\left(\varphi^2\right)\right)$$

$$\Rightarrow \quad n_{\mathrm{s}} = 1 - \frac{p}{(p-1)N} \qquad r = \frac{8}{a_p}\left(\frac{a_p}{(p-1)N}\right)^{\frac{p}{p-1}}$$

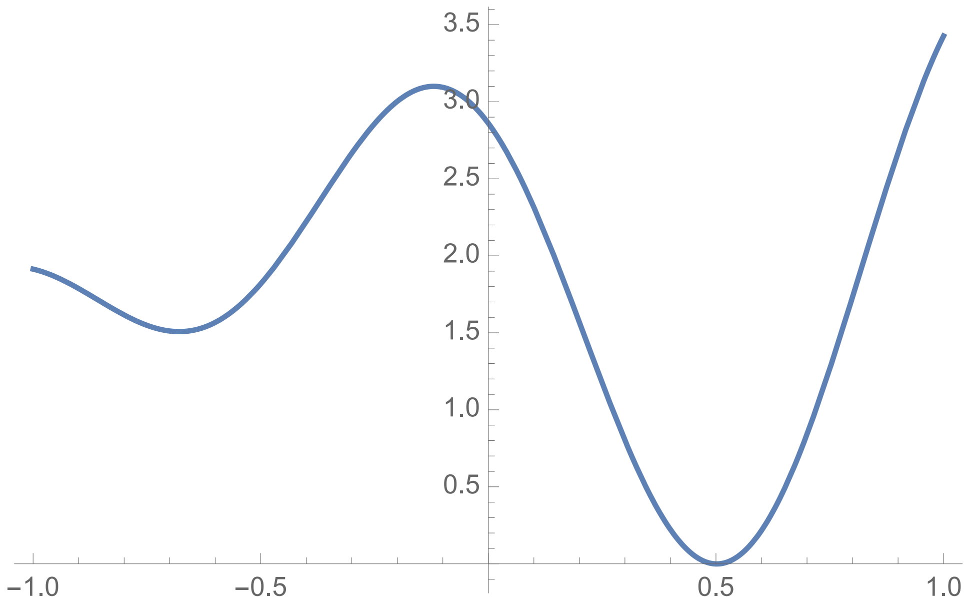
[Galante, Kallosh, Linde, Roest, 1412.3797]

[Broy, Galante, Roest, Westphal, 1507.02277]

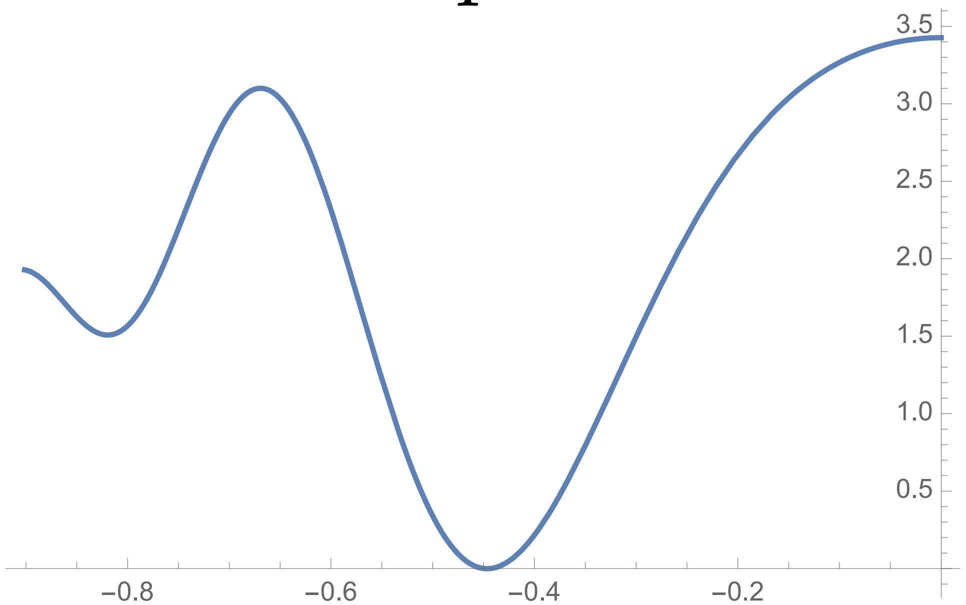

KR
generalized pole inflation

Change of potential shape

The original potential

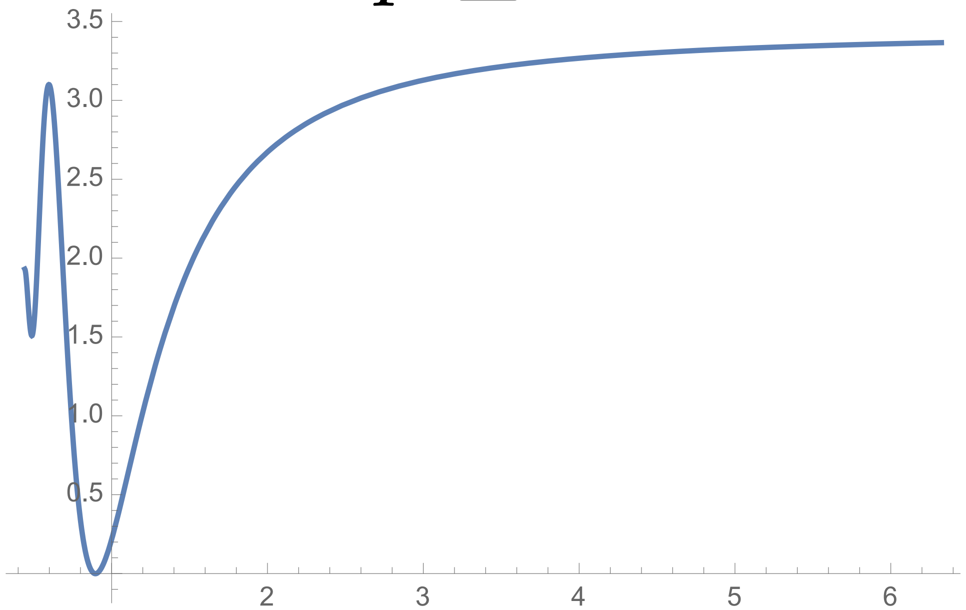


$0 < p < 2$



"hilltop"

$p \geq 2$



"inverse-hilltop"

Attractor behaviors

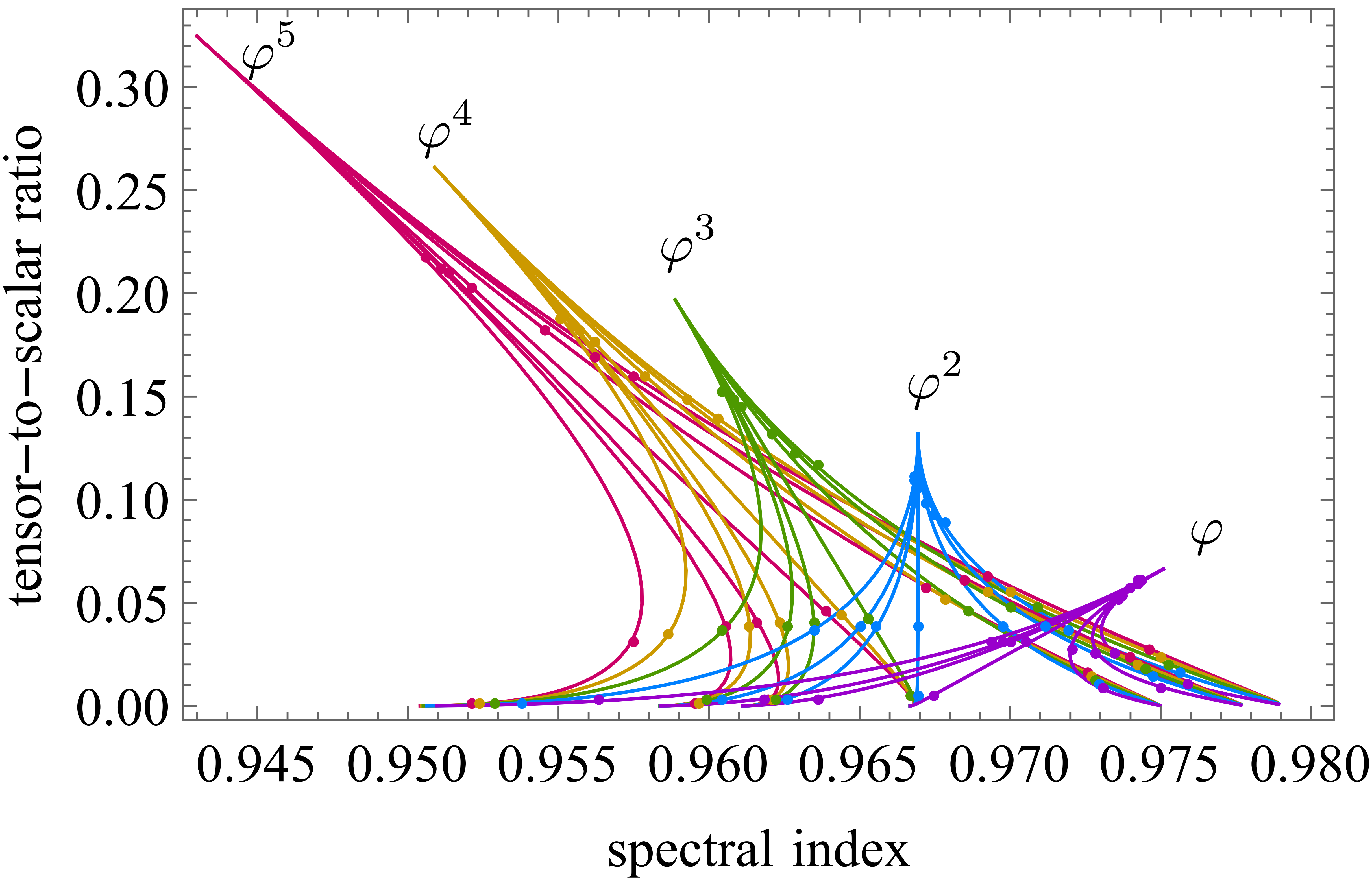
Consider an example,

$$\left(\sqrt{-g}\right)^{-1}\mathcal{L} = -\frac{a_p}{2(1-\widetilde{\varphi}^2)^p}\partial^\mu\widetilde{\varphi}\partial_\mu\widetilde{\varphi} - \lambda_m\widetilde{\varphi}^m.$$

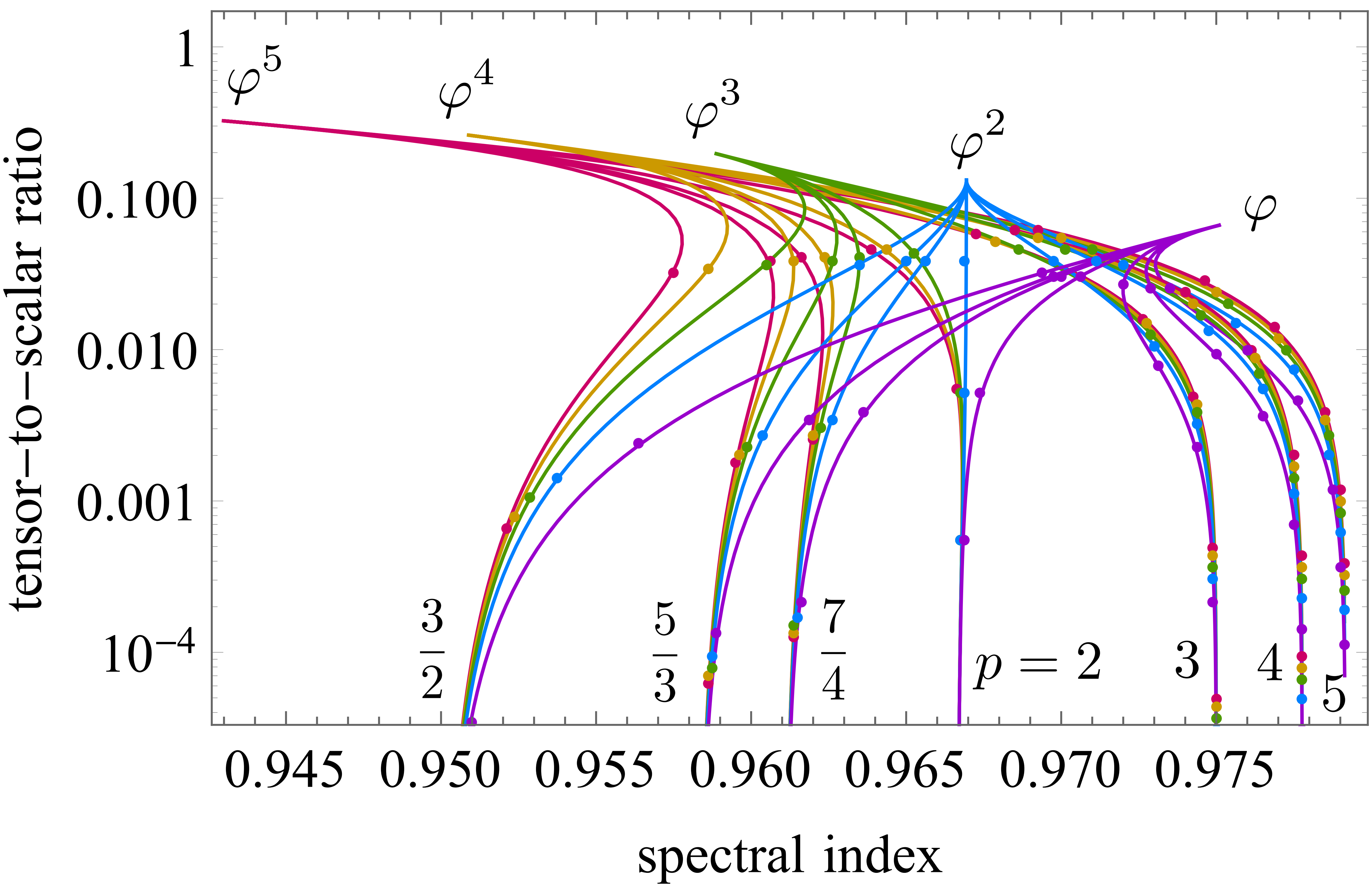
(inspired by the alpha-attractor Lagrangian)

$$-K_{\Phi\bar{\Phi}}\partial^\mu\bar{\Phi}\partial_\mu\Phi = -\frac{3\alpha}{\left(1-|\Phi|^2\right)^2}\partial^\mu\bar{\Phi}\partial_\mu\Phi$$

Attractor behaviors



Attractor behaviors



A closer look

canonical field

$$\phi = \begin{cases} \frac{2\sqrt{a_p}}{p-2}\varphi^{-\frac{p-2}{2}} & (p \neq 2), \\ -\sqrt{a_p}\log\varphi & (p = 2), \end{cases}$$

canonical potential

$$V = \begin{cases} V_0\left(1 - \left(\frac{p-2}{2\sqrt{a_p}}\phi\right)^{-\frac{2}{p-2}} + \cdots\right) & (p \neq 2), \\ V_0\left(1 - e^{-\phi/\sqrt{a_p}} + \cdots\right) & (p = 2), \end{cases}$$

e-folding number

$$N = \begin{cases} \frac{a_p}{p-1}\left(\frac{1}{\varphi_N^{p-1}} - \frac{1}{\varphi_{\rm end}^{p-1}}\right) & (p \neq 1), \\ a_p \log\left(\frac{\varphi_{\rm end}}{\varphi_N}\right) & (p = 1). \end{cases}$$

observables

$$n_{\rm s} = 1 - \frac{p}{(p-1)N} \qquad r = \frac{8}{a_p}\left(\frac{a_p}{(p-1)N}\right)^{\frac{p}{p-1}} \qquad (p \neq 1)$$

$$n_{\rm s} = 1 - \frac{1}{a_1} \qquad r = 16\left(\frac{\varphi_{\rm end}}{2a_1}\right)e^{-N/a_1} \qquad (p = 1)$$

p=1 pole inflation

Consider the example with p=1,

$$\left(\sqrt{-g}\right)^{-1}\mathcal{L} = -\frac{a_p}{2(1-\widetilde{\varphi}^2)^p}\partial^\mu\widetilde{\varphi}\partial_\mu\widetilde{\varphi} - \lambda_m\widetilde{\varphi}^m.$$

p=1 pole inflation

Consider the example with p=1,

$$\left(\sqrt{-g}\right)^{-1}\mathcal{L} = -\frac{a_p}{2(1-\widetilde{\varphi}^2)^p}\partial^\mu\widetilde{\varphi}\partial_\mu\widetilde{\varphi} - \lambda_m\widetilde{\varphi}^m.$$

Upon canonical normalization,

$$\left(\sqrt{-g}\right)^{-1}\mathcal{L} = -\frac{1}{2}\partial^\mu\phi\partial_\mu\phi - \lambda_m\sin^m\left(\frac{\phi}{\sqrt{a_1}}\right).$$

p=1 pole inflation

Consider the example with p=1,

$$\left(\sqrt{-g}\right)^{-1}\mathcal{L} = -\frac{a_p}{2(1-\widetilde{\varphi}^2)^p}\partial^\mu\widetilde{\varphi}\partial_\mu\widetilde{\varphi} - \lambda_m\widetilde{\varphi}^m.$$

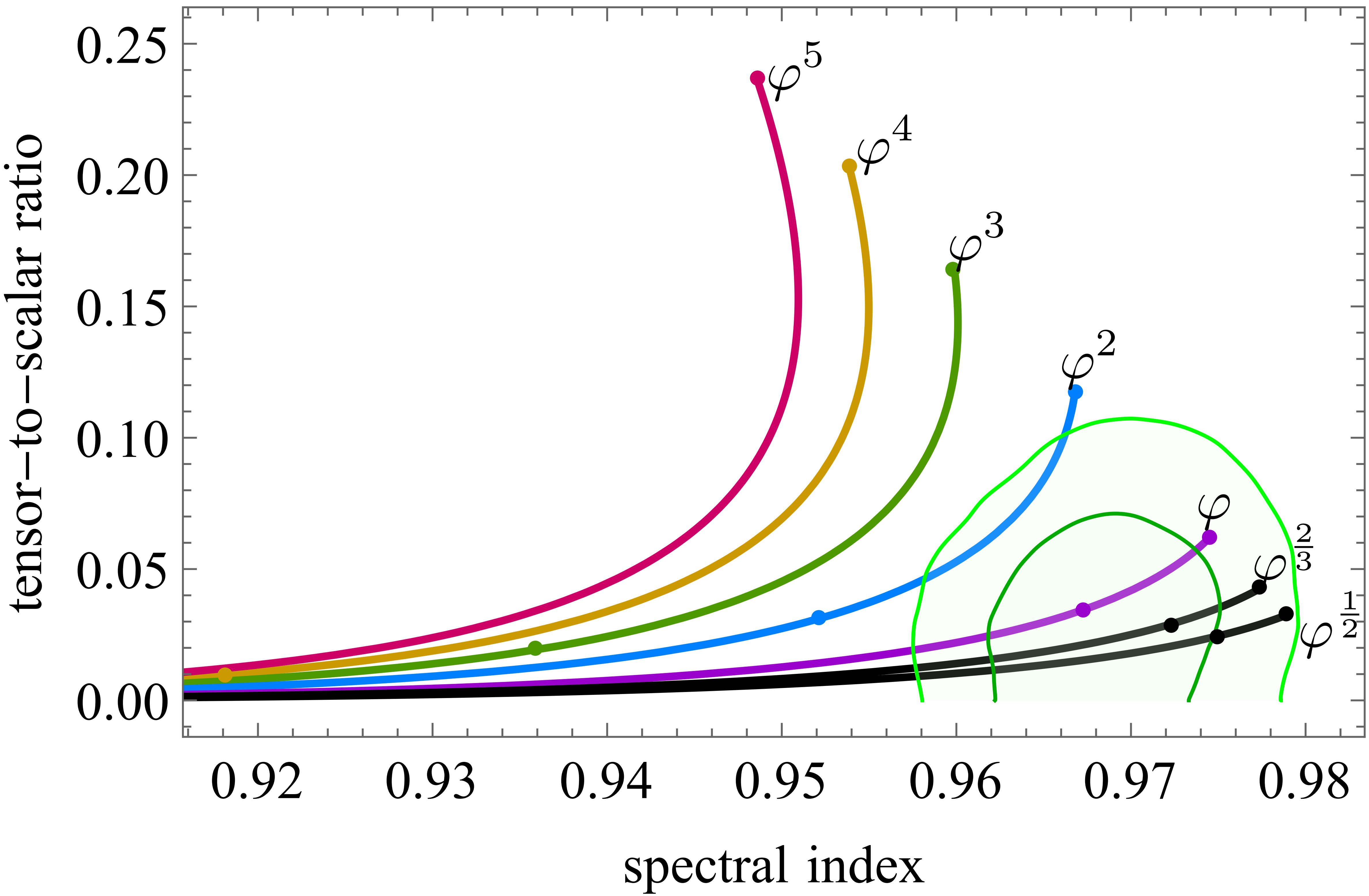
Upon canonical normalization,

$$\left(\sqrt{-g}\right)^{-1}\mathcal{L} = -\frac{1}{2}\partial^\mu\phi\partial_\mu\phi - \lambda_m\sin^m\left(\frac{\phi}{\sqrt{a_1}}\right).$$

Generalization of **natural inflation** potential (m=2).

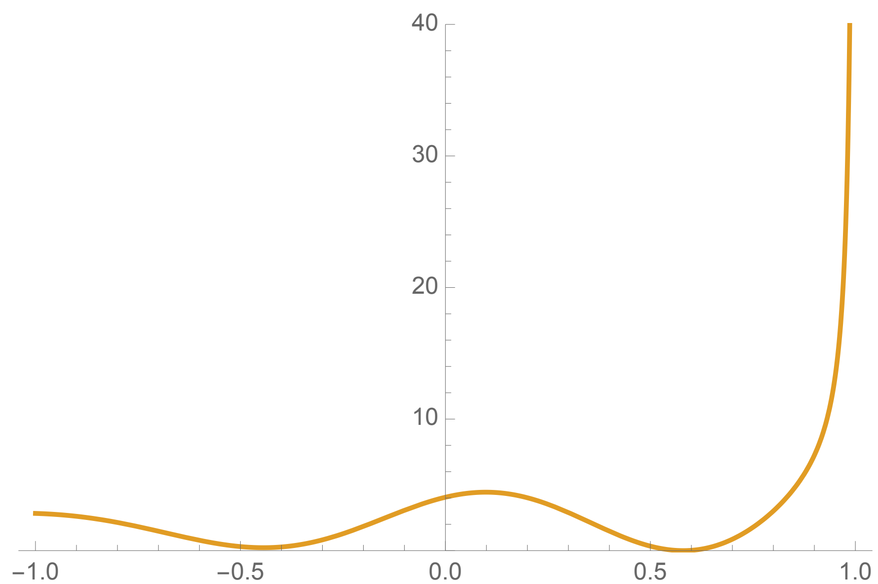
$$V = V_0(1-\cos(\phi/f)) = 2V_0\sin^2(\phi/2f)$$

p=1 pole inflation

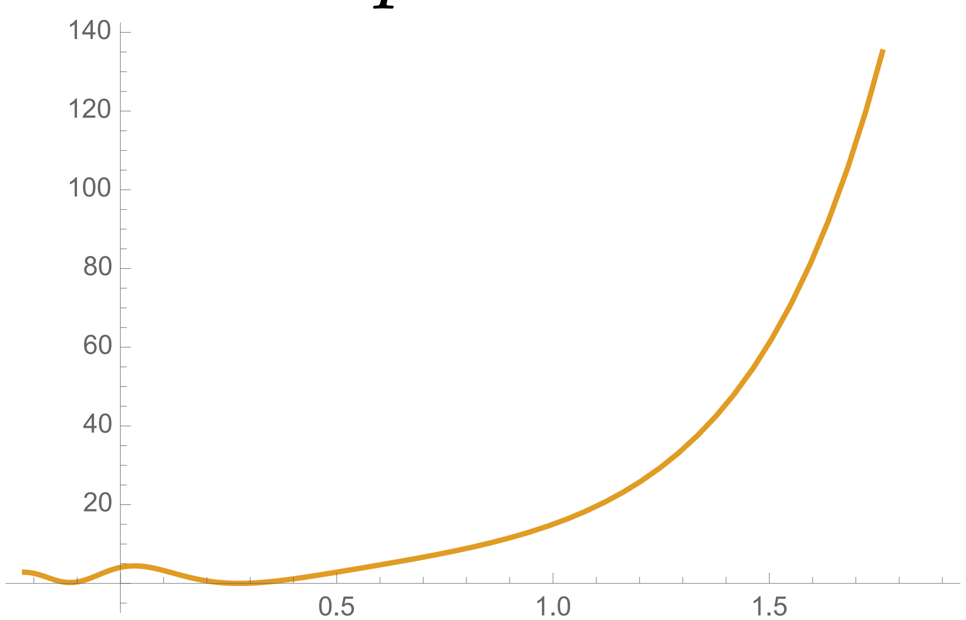


Inflation with a singular potential

The original *diverging* potential

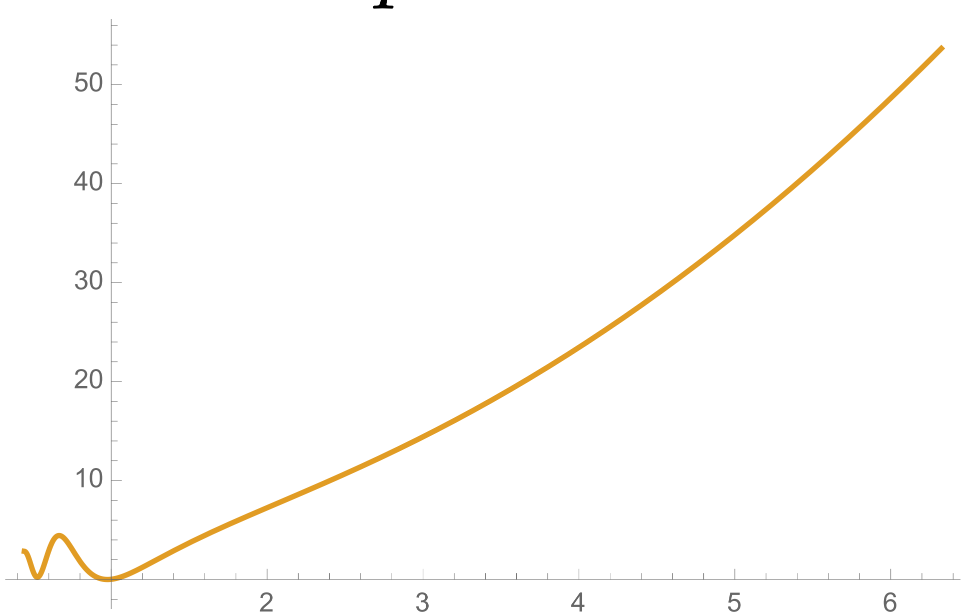


$p = 2$



“power-law”

$p > 2$



“chaotic”

Inflation with a singular potential

See also [Rinaldi, L. Vanzo, S. Zerbini, and G. Venturi, 1505.03386]

$$\left(\sqrt{-g}\right)^{-1}\mathcal{L} = -\frac{a_p}{2\varphi^p}\partial^\mu\varphi\partial_\mu\varphi - \frac{C}{\varphi^s}\left(1+\mathcal{O}(\varphi)\right)$$

Inflation with a singular potential

See also [Rinaldi, L. Vanzo, S. Zerbini, and G. Venturi, 1505.03386]

$$\left(\sqrt{-g}\right)^{-1}\mathcal{L} = -\frac{a_p}{2\varphi^p}\partial^\mu\varphi\partial_\mu\varphi - \frac{C}{\varphi^s}\left(1+\mathcal{O}(\varphi)\right)$$

$$V = \begin{cases} C\left(\frac{p-2}{2\sqrt{a_p}}\phi\right)^{\frac{2s}{p-2}} + \cdots & (p\neq 2), \\ Ce^{s\phi/\sqrt{a_p}} + \cdots & (p=2). \end{cases}$$

Potentials for monomial **chaotic** and **power-law** inflation

Inflation with a singular potential

See also [Rinaldi, L. Vanzo, S. Zerbini, and G. Venturi, 1505.03386]

$$\left(\sqrt{-g}\right)^{-1}\mathcal{L} = -\frac{a_p}{2\varphi^p}\partial^\mu\varphi\partial_\mu\varphi - \frac{C}{\varphi^s}\left(1+\mathcal{O}(\varphi)\right)$$

$$V = \begin{cases} C\left(\frac{p-2}{2\sqrt{a_p}}\phi\right)^{\frac{2s}{p-2}} + \cdots & (p \neq 2), \\ Ce^{s\phi/\sqrt{a_p}} + \cdots & (p = 2). \end{cases}$$

Potentials for monomial **chaotic** and **power-law** inflation

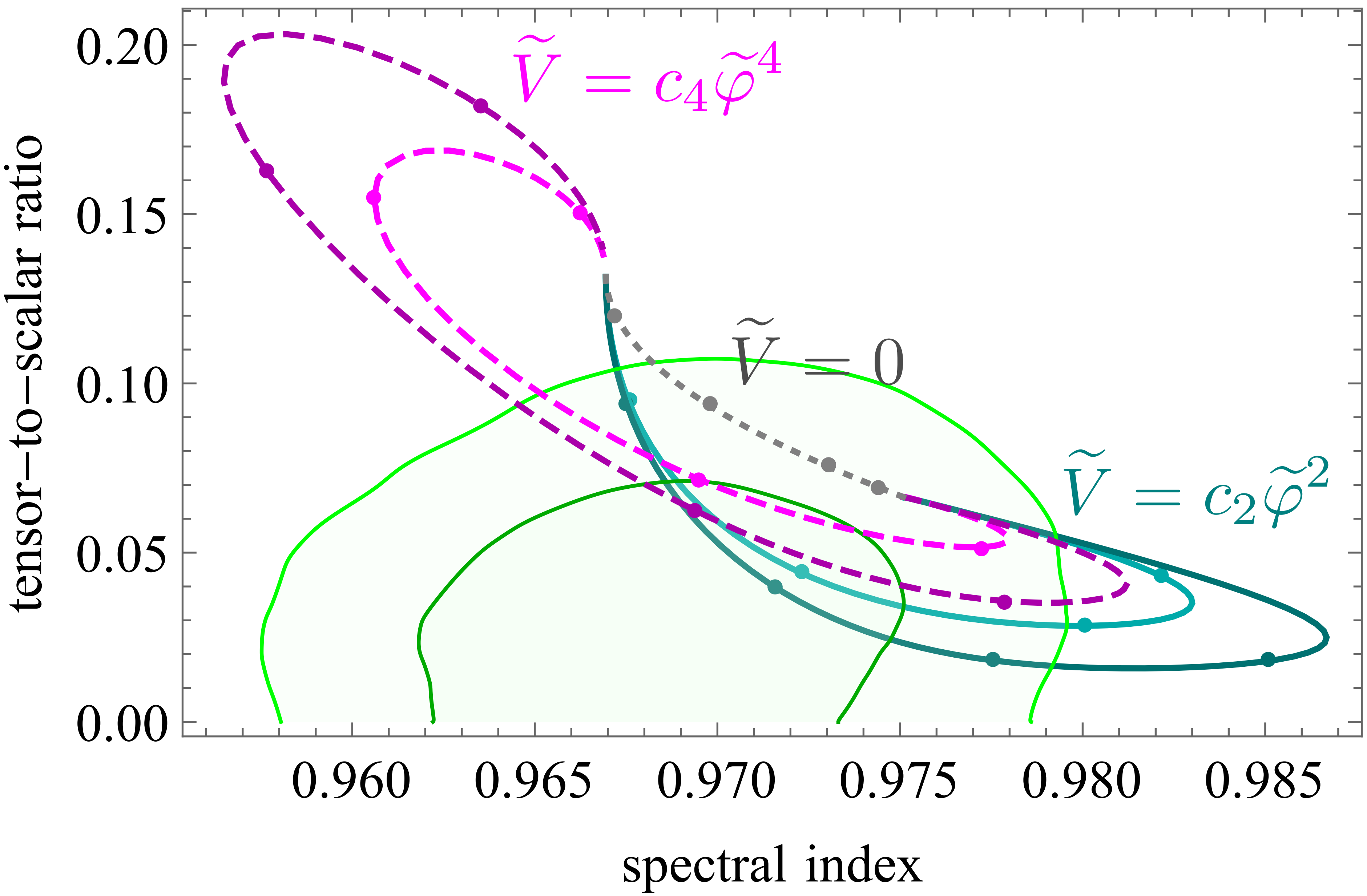
$$n_\mathrm{s} = 1 - \frac{p+s-2}{(p-2)N} \qquad r = \frac{8s}{(p-2)N}$$

Inflation with a singular potential

$$\left(\sqrt{-g}\right)^{-1}\mathcal{L} = -\frac{1}{2}\frac{a_4}{(1-\widetilde{\varphi}^2)^4}\partial^\mu\widetilde{\varphi}\partial_\mu\widetilde{\varphi} - C\left(\frac{2\widetilde{\varphi}^2}{1-\widetilde{\varphi}^2} + \widetilde{V}(\widetilde{\varphi})\right)$$

Inflation with a singular potential

$$\left(\sqrt{-g}\right)^{-1}\mathcal{L} = -\frac{1}{2}\frac{a_4}{(1-\widetilde{\varphi}^2)^4}\partial^\mu\widetilde{\varphi}\partial_\mu\widetilde{\varphi} - C\left(\frac{2\widetilde{\varphi}^2}{1-\widetilde{\varphi}^2} + \widetilde{V}(\widetilde{\varphi})\right)$$



Summary

For more details, see [TT, arXiv:1602.07867].

	$p=1$	$1<p<2$	$p=2$	$2<p$
non-singular potential	2nd order hilltop generalization of natural inflation	hilltop	alpha-attractor xi-attractor Starobinsky model Higgs inflation	inverse-hilltop
singular potential	run-away	run-away	power-law inflation (exponential potential)	chaotic (monomial/ polynomial)

Conclusions

For more details, see [TT, arXiv:1602.07867].

Conclusions

For more details, see [TT, arXiv:1602.07867].

- We have widened the notion of pole inflation and inflationary attractors.

Conclusions

For more details, see [TT, arXiv:1602.07867].

- We have widened the notion of pole inflation and inflationary attractors.
- They include various shapes of canonical potential. It almost covers all categories of "universality classes of inflation". [Mukhanov, 1303.3925] [Roest, 1309.1285] [Binetruy, Kiritsis, Mabillard, Pieroni, Rosset, 1407.0820]

Conclusions

For more details, see [TT, arXiv:1602.07867].

- We have widened the notion of pole inflation and inflationary attractors.
- They include various shapes of canonical potential. It almost covers all categories of "universality classes of inflation". [Mukhanov, 1303.3925] [Roest, 1309.1285] [Binetruy, Kiritsis, Mabillard, Pieroni, Rosset, 1407.0820]
- UV embedding and formulation in Jordan frames will be studied in future.

Effects of additional poles (p>1)

$$\left(\sqrt{-g}\right)^{-1}\mathcal{L} = -\frac{1}{2}\left(\frac{a_p}{\varphi^p}+\frac{a_q}{\varphi^q}\right)\partial^\mu\varphi\partial_\mu\varphi - V_0\left(\frac{b_t}{\varphi^t}+1-\varphi+\mathcal{O}(\varphi^2)\right)$$

$$n_{\mathrm{s}} = n_{\mathrm{s,\,0}} + \delta_{\mathrm{kin}} n_{\mathrm{s}} + \delta_{\mathrm{pot}} n_{\mathrm{s}}$$

$$r = r_0 + \delta_{\mathrm{kin}} r + \delta_{\mathrm{pot}} r$$

$$n_{\mathrm{s},0} = 1 - \frac{p}{(p-1)N}$$

$$r_0 = \frac{8}{a_p}\left(\frac{a_p}{(p-1)N}\right)^{\frac{p}{p-1}}$$

$$\delta_{\mathrm{kin}} n_{\mathrm{s}} = -\frac{(q-p)(q-p-1)a_q}{(q-1)a_p^2}\left(\frac{p-1}{a_p}N\right)^{\frac{q-2p+1}{p-1}}$$

$$\delta_{\mathrm{kin}} r = -\frac{8(q-p-1)a_q}{(q-1)a_p^2}\left(\frac{p-1}{a_p}N\right)^{\frac{q-2p}{p-1}}$$

$$\delta_{\mathrm{pot}} n_{\mathrm{s}} = \frac{t(t+1)(p+2t)b_t}{(p+t)a_p}\left(\frac{p-1}{a_p}N\right)^{\frac{t-p+2}{p-1}}$$

$$\delta_{\mathrm{pot}} r = \frac{8t(p+2t)b_t}{(p+t)a_p}\left(\frac{p-1}{a_p}N\right)^{\frac{t-p+1}{p-1}}$$

Effects of additional poles (p=1)

$$\left(\sqrt{-g}\right)^{-1}\mathcal{L} = -\frac{1}{2}\left(\frac{a_p}{\varphi^p}+\frac{a_q}{\varphi^q}\right)\partial^\mu\varphi\partial_\mu\varphi - V_0\left(\frac{b_t}{\varphi^t}+1-\varphi+\mathcal{O}(\varphi^2)\right)$$

$$n_\mathrm{s} = n_{\mathrm{s},\,0} + \delta_\mathrm{kin} n_\mathrm{s} + \delta_\mathrm{pot} n_\mathrm{s} \qquad r = r_0 + \delta_\mathrm{kin} r + \delta_\mathrm{pot} r$$

$$n_{\mathrm{s},0} = 1 - \frac{1}{a_1} \qquad r_0 = 16\left(\frac{\varphi_\mathrm{end}}{2a_1}\right)e^{-N/a_1}$$

$$\delta_\mathrm{kin} n_\mathrm{s} = -\frac{(q-2)a_q}{a_{p=1}^2}\left(\frac{e^{N/a_{p=1}}}{\varphi_\mathrm{end}}\right)^{q-1} \qquad \delta_\mathrm{kin} r = -\frac{8(q-2)a_q}{(q-1)a_{p=1}^2}\left(\frac{e^{N/a_{p=1}}}{\varphi_\mathrm{end}}\right)^{q-2}$$

$$\delta_\mathrm{pot} n_\mathrm{s} = \frac{t(2t+1)b_t}{a_{p=1}}\left(\frac{e^{N/a_{p=1}}}{\varphi_\mathrm{end}}\right)^{t+1} \qquad \delta_\mathrm{pot} r = \frac{8t(2t+1)b_t}{(t+1)a_{p=1}}\left(\frac{e^{N/a_{p=1}}}{\varphi_\mathrm{end}}\right)^{t}$$

Effects of additional poles

$$\left(\sqrt{-g}\right)^{-1}\mathcal{L} = -\frac{1}{2}\left(\frac{a_p}{\varphi^p}+\frac{a_q}{\varphi^q}\right)\partial^\mu\varphi\partial_\mu\varphi - \frac{C}{\varphi^s}\left(\frac{b_t}{\varphi^t}+1+\mathcal{O}(\varphi)\right)$$

$$n_{\mathrm{s}} = n_{\mathrm{s},\,0} + \delta_{\mathrm{kin}} n_{\mathrm{s}} + \delta_{\mathrm{pot}} n_{\mathrm{s}} \qquad r = r_0 + \delta_{\mathrm{kin}} r + \delta_{\mathrm{pot}} r$$

$$n_{\mathrm{s},0} = 1 - \frac{p+s-2}{(p-2)N} \qquad r_0 = \frac{8s}{(p-2)N}$$

$$\delta_{\mathrm{kin}} n_{\mathrm{s}} = -\frac{s(q-p)(q-p-s)a_q}{(q-2)a_p^2}\left(\frac{s(p-2)}{a_p}N\right)^{\frac{q-2p+2}{p-2}} \qquad \delta_{\mathrm{kin}} r = -\frac{8s^2(q-p)a_q}{(q-2)a_p^2}\left(\frac{s(p-2)}{a_p}N\right)^{\frac{q-2p+2}{p-2}}$$

$$\delta_{\mathrm{pot}} n_{\mathrm{s}} = \frac{t(t-s)(p+2t-2)b_t}{(p+t-2)a_p}\left(\frac{s(p-2)}{a_p}N\right)^{\frac{t-p+2}{p-2}} \qquad \delta_{\mathrm{pot}} r = \frac{8st(p+2t-2)b_t}{(p+t-2)a_p}\left(\frac{s(p-2)}{a_p}N\right)^{\frac{t-p+2}{p-2}}$$