5th—9th Sep. 2016
CoSKASI-ICG-NAOC-YITP workshop

UV problem in PT & EFT approach

Atsushi Taruya

Basic eqs. for perturbation theory

Starting point

single-stream approximation of collisionless Boltzmann eq.

Phase-space distribution function

$$f(\boldsymbol{x},\, \boldsymbol{v};\, t) \longrightarrow \overline{\rho}(t)\, \{1+\delta(\boldsymbol{x};\, t)\}\, \delta_{\rm D}\left(\boldsymbol{v}-\boldsymbol{v}(\boldsymbol{x};\, t)\right)$$

<u>Basic eqs.</u>

$$\frac{\partial \delta}{\partial t}+\frac{1}{a}\vec{\nabla}\cdot\left[(1+\delta)\vec{v}\right]=0$$

$$\frac{\partial \vec{v}}{\partial t}+\frac{\dot{a}}{a}\vec{v}+\frac{1}{a}(\vec{v}\cdot\vec{\nabla})\vec{v}=-\frac{1}{a}\vec{\nabla}\Phi$$

$$\frac{1}{a^2}\nabla^2\Phi=4\pi G\,\overline{\rho}_{\rm m}\,\delta$$

<u>PT expansion</u>

Assuming the irrotational flow

$$\delta=\delta^{(1)}+\delta^{(2)}+\cdots$$

$$\theta=\theta^{(1)}+\theta^{(2)}+\cdots$$

$$\theta\equiv\frac{\nabla\cdot\vec{v}}{a\,H}$$

PT kernels

initial density field

$$\delta^{(n)}(\boldsymbol{k};t) = \int \frac{d^3\boldsymbol{k}_1 \cdots d^3\boldsymbol{k}_n}{(2\pi)^{3(n-1)}} \delta_{\rm D}(\boldsymbol{k}-\boldsymbol{k}_{12\cdots n}) F_n(\boldsymbol{k}_1,\cdots,\boldsymbol{k}_n;t)\,\delta_0(\boldsymbol{k}_1)\cdots\delta_0(\boldsymbol{k}_n),$$

$$\theta^{(n)}(\boldsymbol{k};t) = \int \frac{d^3\boldsymbol{k}_1 \cdots d^3\boldsymbol{k}_n}{(2\pi)^{3(n-1)}} \delta_{\rm D}(\boldsymbol{k}-\boldsymbol{k}_{12\cdots n}) G_n(\boldsymbol{k}_1,\cdots,\boldsymbol{k}_n;t)\,\delta_0(\boldsymbol{k}_1)\cdots\delta_0(\boldsymbol{k}_n),$$

EdS approximation:

D_+ : linear growth factor

$f = \frac{d\ln D_+}{d\ln a}$: growth rate

$$F_n \longrightarrow [D_+(t)]^n\,\tilde{F}_n(\boldsymbol{k}_1,\cdots,\boldsymbol{k}_n)$$

$$G_n \longrightarrow -f(t)\,[D_+(t)]^n\,\tilde{G}_n(\boldsymbol{k}_1,\cdots,\boldsymbol{k}_n)$$

Kernels $(\tilde{F}_n,\ \tilde{G}_n)$ are derived from recursion relations

Goroff et al. ('86)

→ used to compute power spectrum, bispectrum, ….

Power spectrum

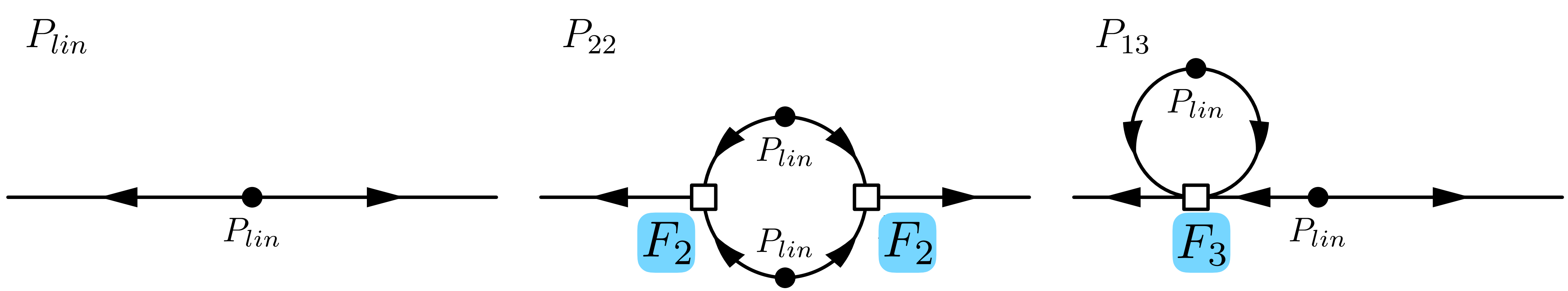
$$P(k) = \underbrace{P_{\rm lin}(k;t)}_{\text{Linear}} + \underbrace{P_{13}(k;t) + P_{22}(k;t)}_{\text{1-loop}} + \cdots$$

$$P_{\rm lin}(k;t) = [D_+(t)]^2\, P_0(k)$$

$$P_{22}(k) \;=\; 2\int_{\boldsymbol{q}} P_{lin}(q)P_{lin}(|\boldsymbol{k}-\boldsymbol{q}|)\, F_2^2(\boldsymbol{q},\boldsymbol{k}-\boldsymbol{q}),$$

$$P_{13}(k) \;=\; 6P_{lin}(k)\int_{\boldsymbol{q}} P_{lin}(q)\, F_3(\boldsymbol{k},\boldsymbol{q},-\boldsymbol{q}),$$

Diagrams



Asymptotic properties

For fixed total sum k, Goroff et al. ('86)

$$\lim_{q\to\infty} F_n(\boldsymbol{k}_1, \ldots, \boldsymbol{k}_{n-2}, \boldsymbol{q}, -\boldsymbol{q}) \propto \frac{k^2}{q^2}$$

$$\Rightarrow \quad \lim_{q\to\infty} F_2(\boldsymbol{q}, \boldsymbol{k}-\boldsymbol{q}) \propto \frac{k^2}{q^2} \qquad \lim_{q\to\infty} F_3(\boldsymbol{k}, \boldsymbol{q}, -\boldsymbol{q}) \propto \frac{k^2}{q^2}$$

Low-k behavior of 1-loop corrections:

high-q limit

$$P_{22}(k)\Big|_{q\to\infty} \propto k^4 \int dq\, q^2 \frac{P_{lin}^2(q)}{q^4}$$

$$P_{13}(k)\Big|_{q\to\infty} \propto P_{lin}(k)\, k^2 \int dq\, q^2 \frac{P_{lin}(q)}{q^2}$$

P_{13} becomes dominant at k<<1 and scales as k^2

UV sensitive terms

For higher-loops,

$P_{15},\ P_{17},\ P_{19},\ \cdots$ become dominant at low-k and scale as k^2

$$\Rightarrow\quad P_{n\text{-loop}}(k)\ \sim\ P_{1(2n+1)}(k)$$

$$P_{1(2n+1)}(k) = 2\cdot(2n+1)!!\ P_{\rm lin}(k)$$
$$\times\int d^3\boldsymbol{q}_1\cdots d^3\boldsymbol{q}_n\,F_{2n+1}(\boldsymbol{k},\boldsymbol{q}_1,-\boldsymbol{q}_1,\cdots,\boldsymbol{q}_n,-\boldsymbol{q}_n)\,P_{\rm lin}(q_1)\cdots\times\ P_{\rm lin}(q_n)$$

logarithmically divergent (q>>1)

$$\overset{k\ll q_i}{\propto}\quad k^2\,P_{\rm lin}(k)\int\frac{dq}{2\pi^2}\,P_{\rm lin}(q)\,[\sigma_L(q)]^{2(n-1)}\,;\quad[\sigma_L(q)]^2\equiv\int_0^q\frac{dq'\,q'^2}{2\pi^2}\,P_{\rm lin}(q')$$

getting sensitive to large-q contribution
for higher loop (n↗)

Blas et al. ('14)

Loop corrections at z=0

Blas et al. JCAP 01 ('14) 010

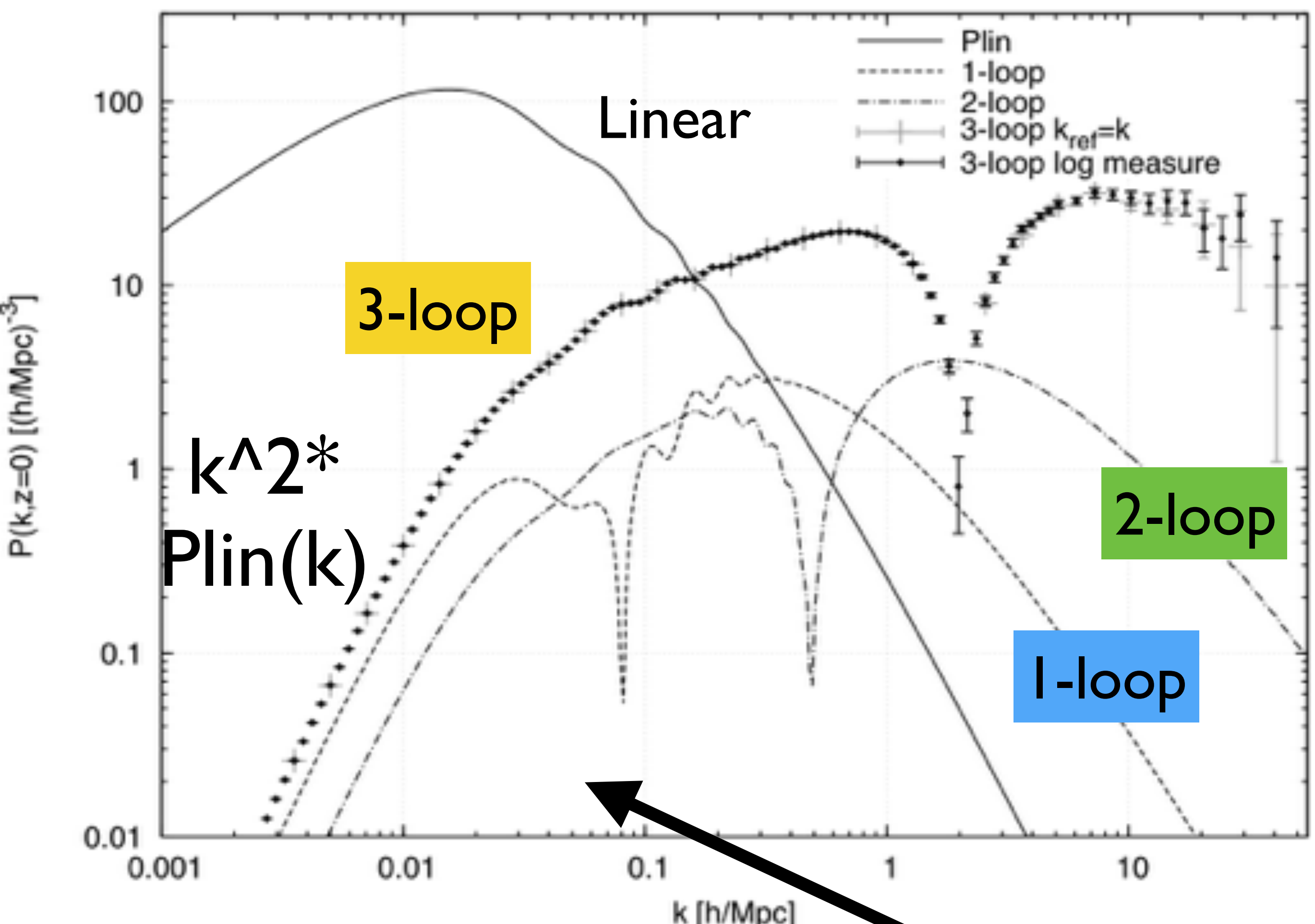


Figure 1: One, two and three-loop contributions to the equal-time power spectrum obtained from a numerical Monte Carlo integration within standard perturbation theory at $z = 0$. The linear power spectrum is obtained from the initial power spectrum from CAMB [20] using the ΛCDM model with WMAP5 parameters. For the three-loop order, the error bars show an estimate for the numerical error obtained by multiplying the error output of the CUBA routine Suave by a factor of two. The relative error is ≤ 0.002 for $k \leq 0.55\ h/\text{Mpc}$. The black diamonds and grey crosses correspond to two different parametrizations of the absolute loop momenta (see App. A).

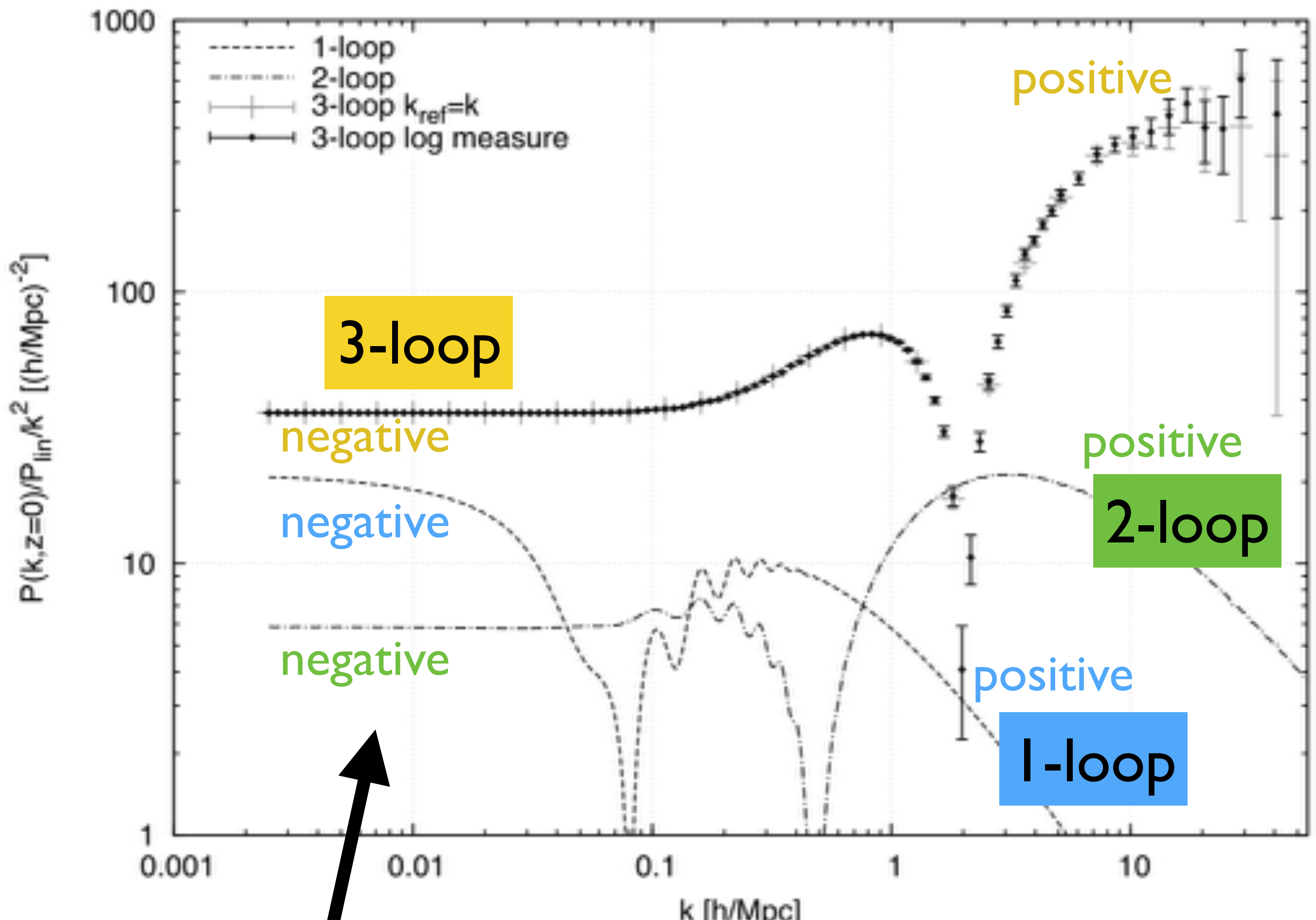
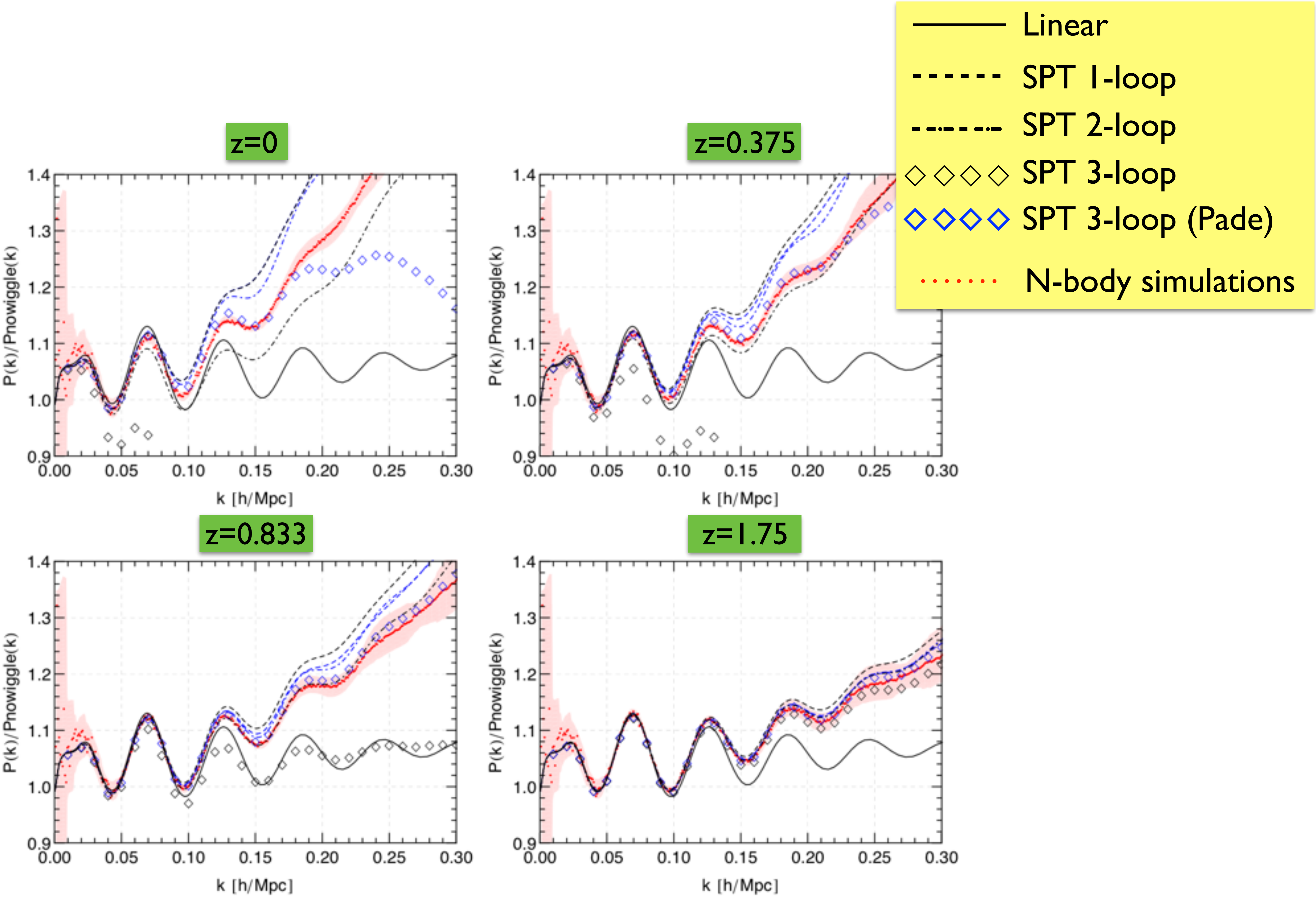


Figure 4: Ratio $P_{L-loop}(k, z = 0)/P_{lin}(k, z = 0)/k^2$ for the one- two- and three-loop contributions (line styles as in Fig. 1).

P13, P15, P17 give a major contribution



Blas et al. JCAP 01 ('14) 010

Mitigating UV sensitivity

UV sensitivity is not a real physical effect

→ needs to be cured for an improved prediction

EFT approach — add counter terms to mitigate UV sensitivity

For P(k) at 1-loop order,

free parameter

counter term to be added : $-c_s^2 k^2 P_{\rm lin}(k)$

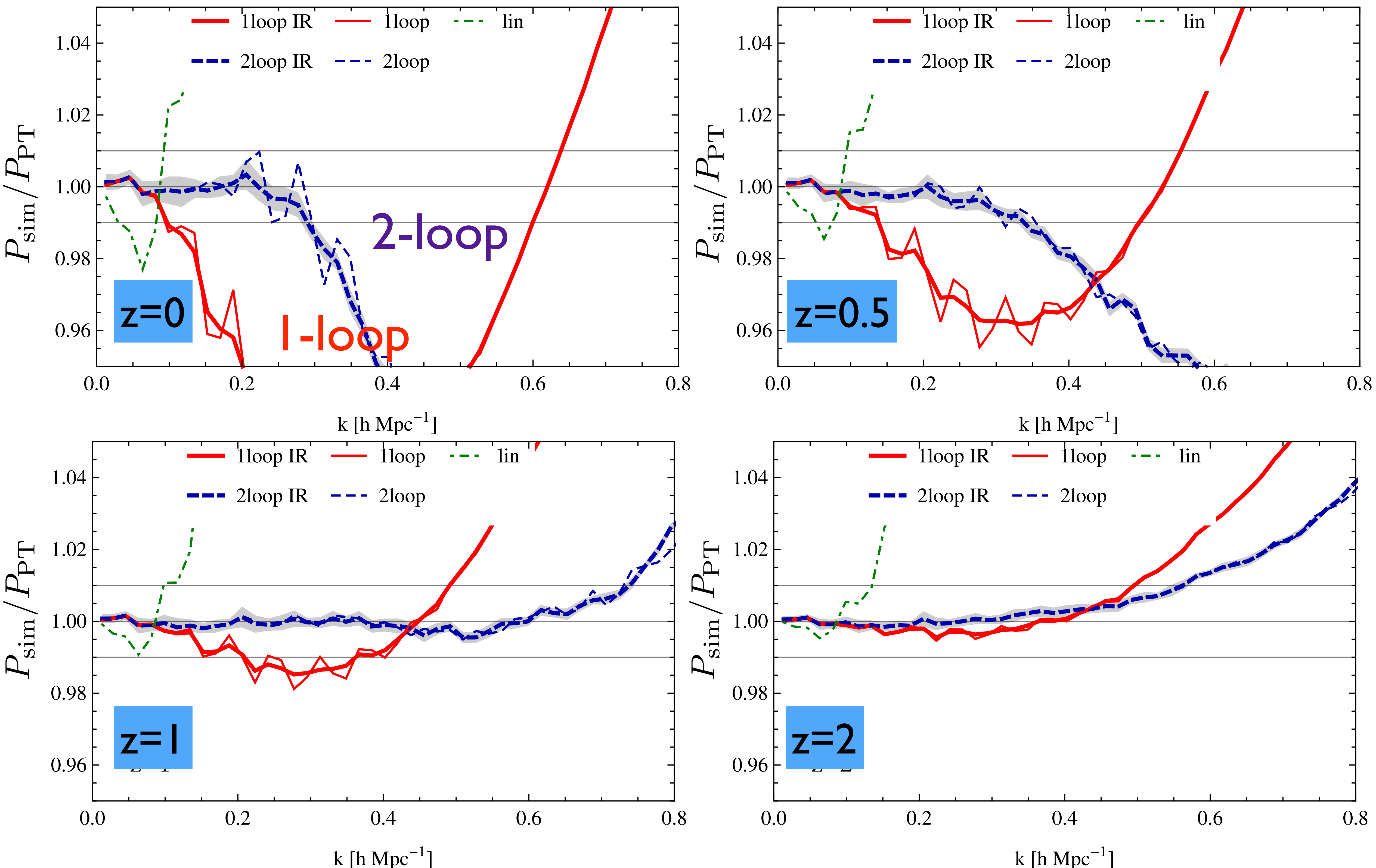
This corresponds to adding $-c_s^2 \nabla\, \delta$ at RHS of Euler eq.

effective pressure → c_s : *'sound velocity'*

→ may be interpreted as an outcome of small-scale physics

Power spectrum in EFT

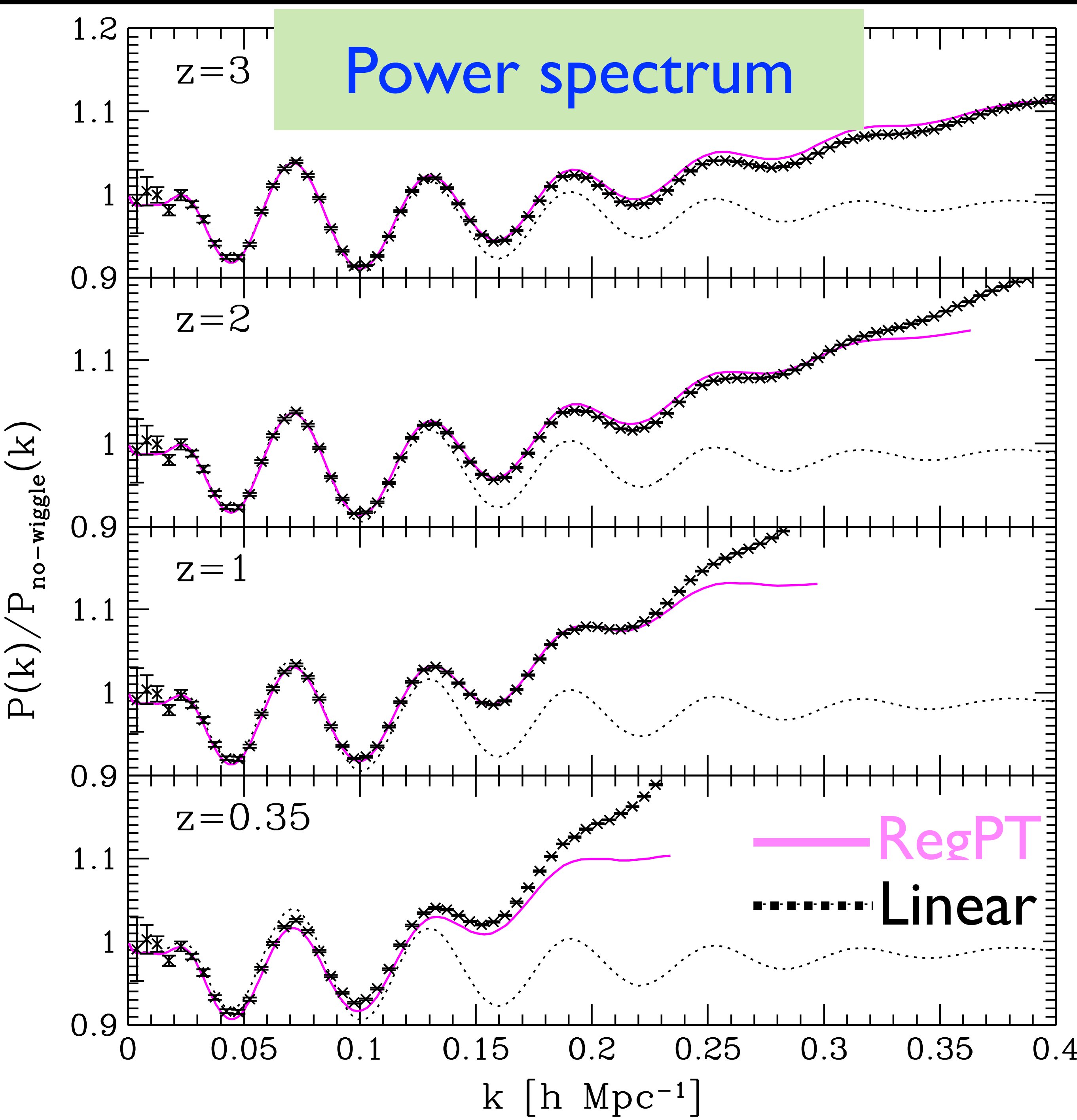
Baldauf et al. ('15b)

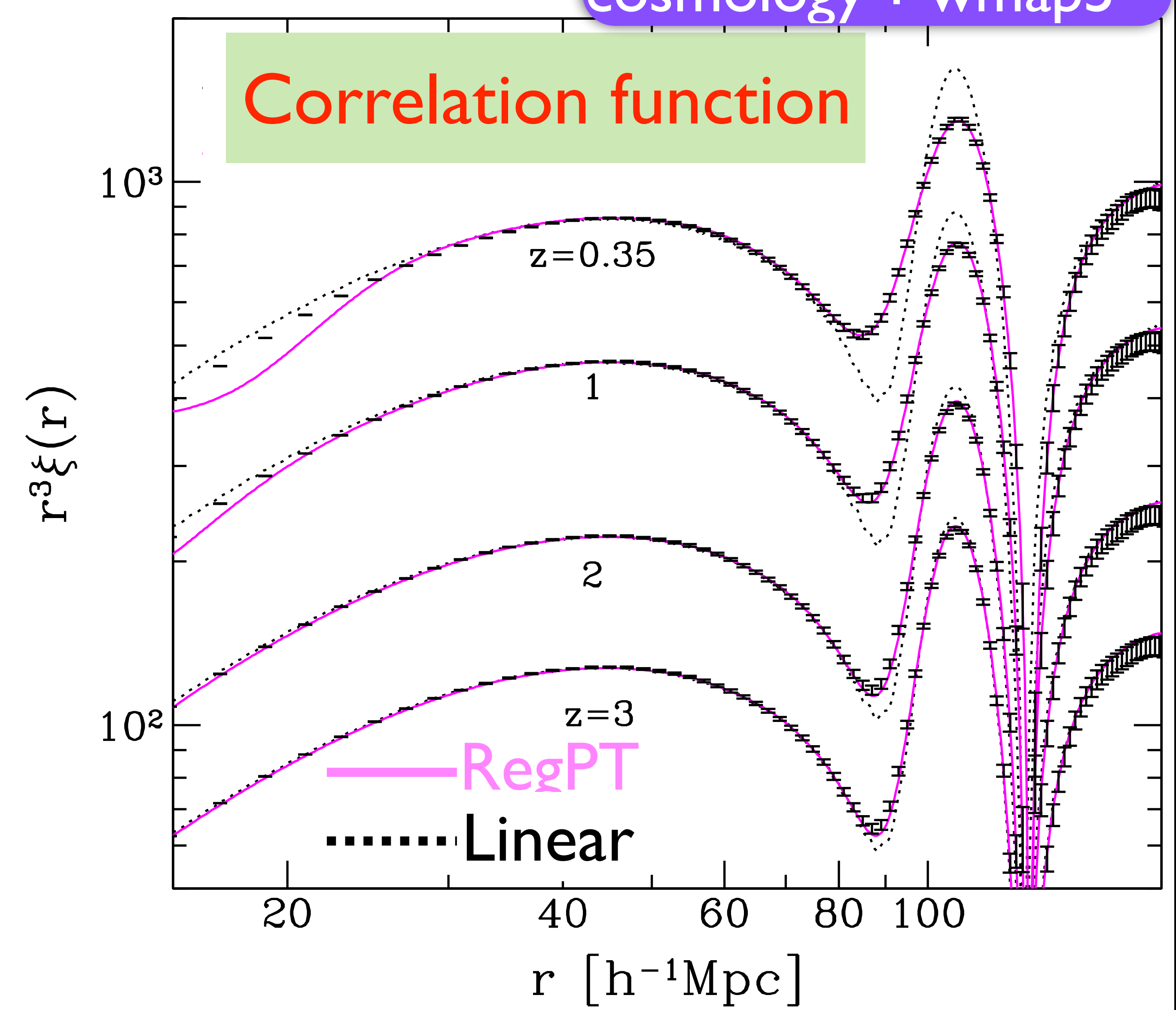


Performance of improved PT

RegPT resumed PT code (http://ascl.net/1404.012)
including *next-to-next-to-leading order*

$L_{\rm box} = 2,048\, h^{-1}\,{\rm Mpc}$
# of particles : $1,024^3$
# of runs : 60
cosmology : wmap5





AT, Bernardeau, Nishimichi & Codis ('12)

Comments/Complaints

- The size of each counter term is unknown, and it needs to be calibrated with N-body simulations

  e.g., $c_s \sim 1\,h^{-1}\,\mathrm{Mpc}$ (but, it generally depends on time & cosmology)

- At 2-loop order, counter terms for sub-leading corrections also need to be considered, increasing # of free parameters

- For bispectrum at 1-loop order, we generally need <u>3 types of counter terms</u>, in addition to the one introduced in P(k)

(Baldauf et al. '15a)

Physical origin or meaning of each counter term is unclear

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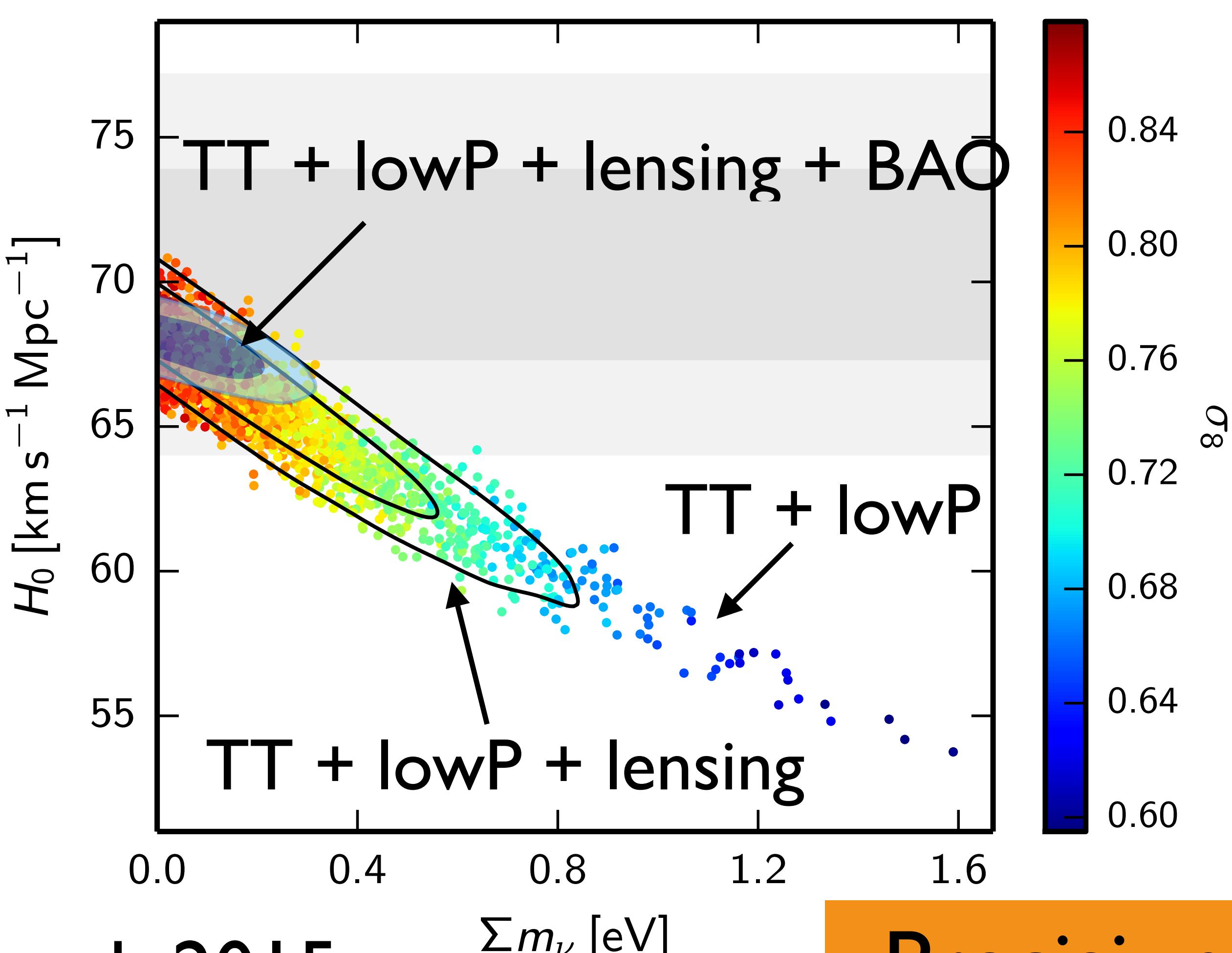
Fluid description of massive neutrinos and its limitation

Atsushi Taruya

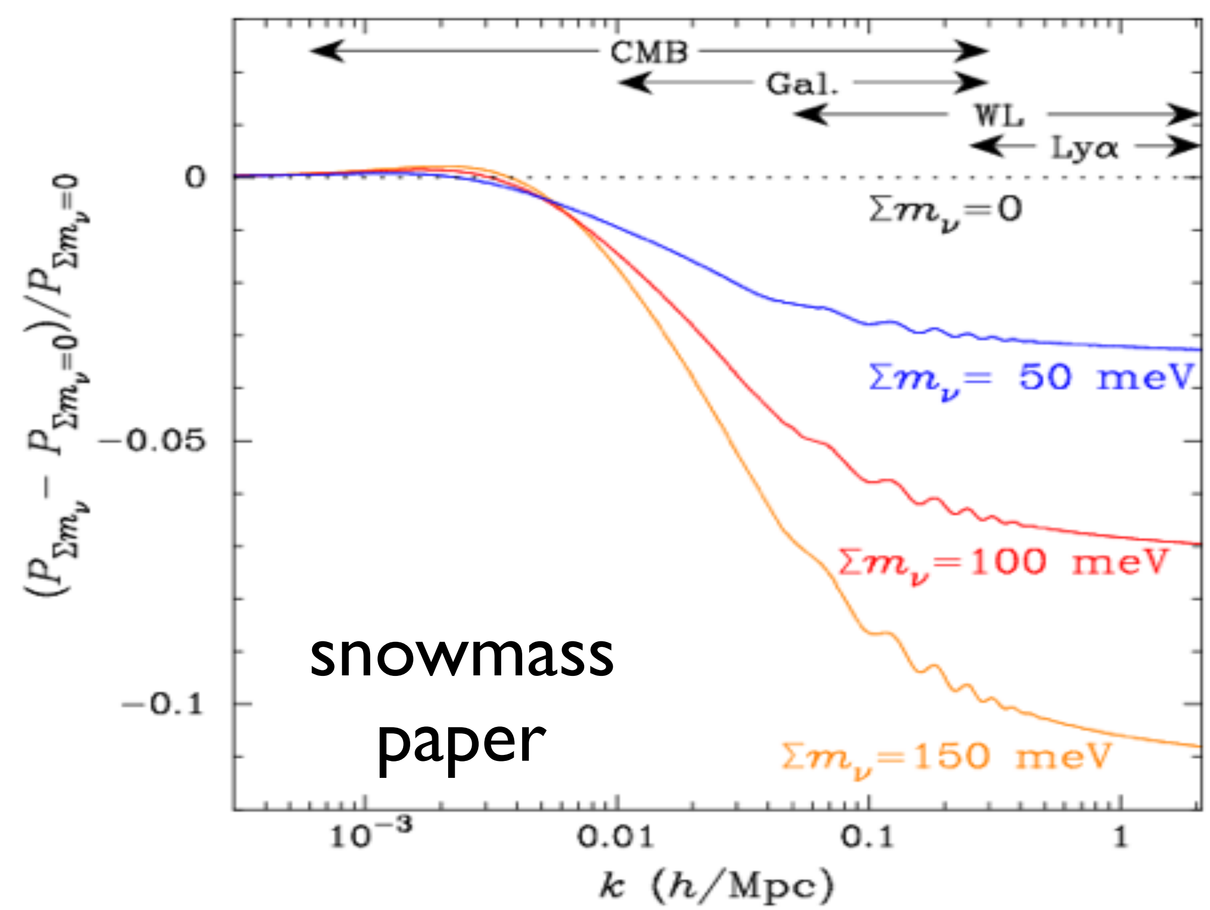
With Naoya Ohishi & Takashi Hiramatsu

Massive neutrinos & LSS

- Weighing mass of neutrinos is fundamental issue in particle physics (beyond standard model)
- Signature of massive neutrinos is imprinted on large-scale structure via *free-streaming suppression*



Planck 2015



Precision LSS observation would be the key

Describing massive ν

Massive ν is fundamentally described by collisionless Boltzmann eq.

… but this is difficult

How well we can approximately but accurately describe structure formation with massive ν ?

Remarks

- Co-existence of *very* hot & *very* cold (CDM) components
- Tiny amount of neutrinos : $\delta = f_\nu \delta_\nu + (1 - f_\nu)\delta_{cb}$

$$f_\nu \equiv \frac{\Omega_\nu h^2}{\Omega_m h^2} = \frac{1}{\Omega_m h^2} \frac{\sum_i m_{\nu,i}}{94.1\,\mathrm{eV}} \lesssim 0.02 \quad \text{for} \quad \sum_i m_{\nu,i} < 0.3\ \mathrm{eV}$$

------▸ different dynamic range in phase-space

Numerical and analytic approaches

Simulation

- N-body particles — e.g., Brandbyge et al. ('08), Viel et al. ('10), …
- Linear Boltzmann on grids — e.g., Brandbyge & Hannestad ('09)
- Fluid with pressure by SPH — Hannestad, Haugbølle, Schultz ('12)
- Hybrid (particles and fluid) — Banerjee & Dalal ('16)

Perturbation theory

- Linear Boltzmann — e.g., Saito, Takada & Taruya ('08), Wong ('08)
- Single-fluid with pressure — e.g., Shoji & Komatsu ('09), Blas et al. ('14)
- Collection of single-stream flow — Dupuy & Bernardeau ('14)

Validity of fluid treatment

To what extent massive ν is described by fluid ?

In linear theory,

fluid treatment is shown to be a good approx. (at sub-percent) in non-relativistic regime for $\sum m_\nu \gtrsim 0.05\ \mathrm{eV}$ at $k \lesssim 1\, h\, \mathrm{Mpc}^{-1}$

Shoji & Komatsu ('10)

Beyond linear regime,

- Gravitational clustering is followed by formation of CDM halos
- Massive ν would be clustered around CDM halos

Can we accurately predict amount of clustered massive ν ?

Setup

- CDM halo described by NFW profile

$$\rho_{\rm halo}(r) \propto \frac{1}{(r/r_s)(1+r/r_s)^2}$$

- Initially homogeneous Fermi-Dirac dist.

$$f_{\rm FD}(p) \propto \frac{1}{1+e^{pc/(k_{\rm B}T_\nu)}}$$

$p = m_\nu \, v$

- Ignoring self-gravity of massive ν,

solving time evolution of neutrino clustering in two ways :

✓ Collisionless Boltzmann eq.

✓ Fluid equations

Solving collisionless Boltzmann eq.

'N-one body approach' (Ringwald & Wong '04)

$\approx$ ray-tracing simulation with non-relativistic massive particles

Liouville theorem implies

distribution function

$$f[\vec{x}(t_{\rm final}),\vec{v}(t_{\rm final})] = f[\vec{x}(t_{\rm init}),\vec{v}(t_{\rm init})]$$

same particle's trajectory but with different time

- Backward approach is less costly calculation:

= Fermi-Dirac distribution

- Using many backward trajectories,

$$\rho_\nu(r;t_{\rm fin}) = \sum_i f_{\rm FD}\Big(\vec{v}_i(t_{\rm init})\Big)\, w_i(t_{\rm fin})$$

phase-space volume given at radius *r* & $t_{\rm final}$

Fluid treatment

Solve moment eqs. under spherical symmetry:

$$\frac{\partial \delta_\nu}{\partial t} + \frac{1}{a}\nabla\left[(1+\delta_\nu)\vec{v}\right] = 0$$

NFW halo Effective pressure

$$\frac{\partial \vec{v}}{\partial t} + H\,\vec{v} + \frac{1}{a}(\vec{v}\cdot\nabla)\vec{v} = -\frac{1}{a}\nabla\Phi_{\rm ext} - \frac{1}{a}\frac{1}{\overline{\rho}_\nu(1+\delta_\nu)}\nabla P_\nu$$

$$\nabla P_\nu \rightarrow \overline{\rho}_\nu\, c_s^2\, \nabla\delta_\nu$$

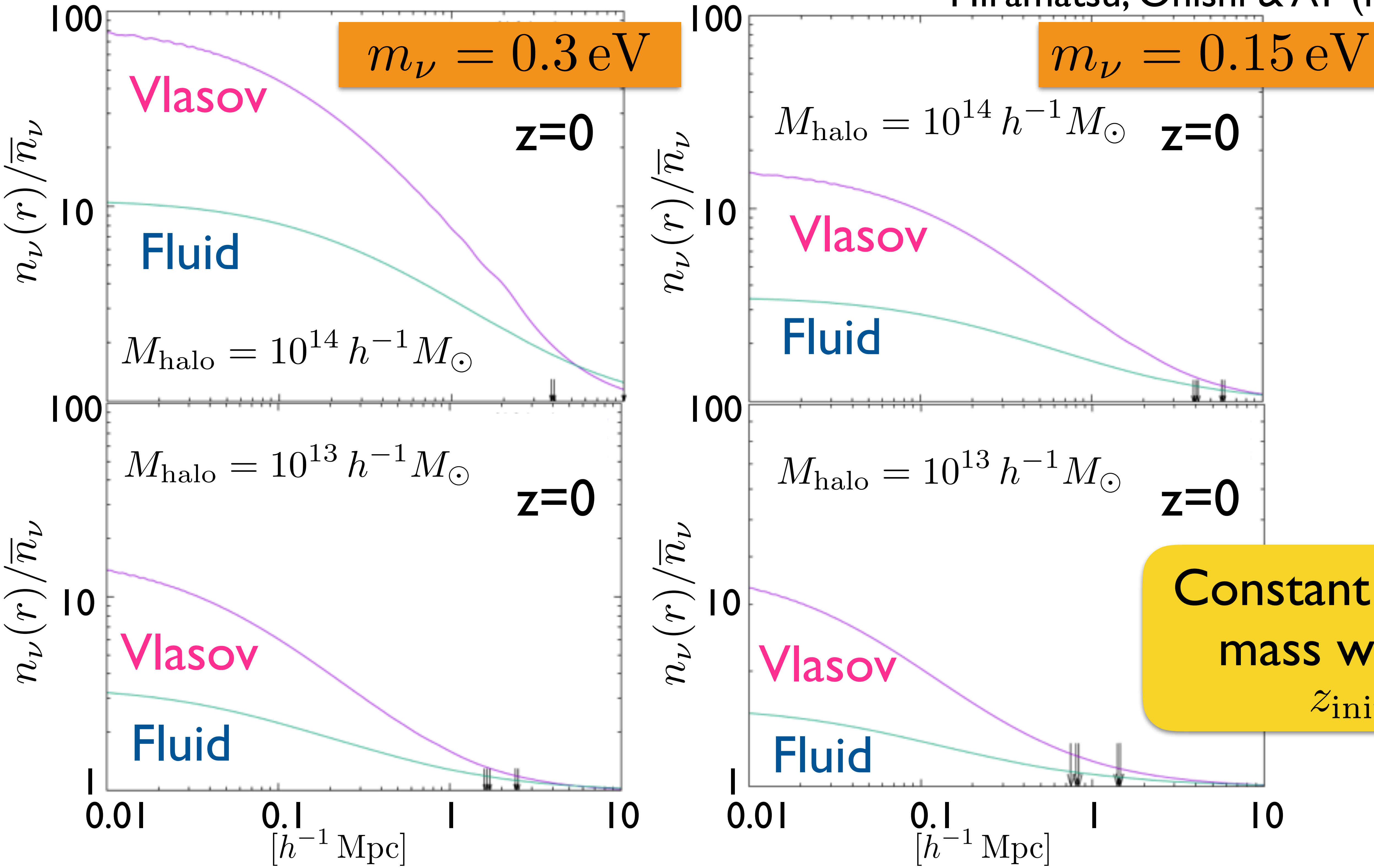
Shoji & Komatsu ('10)

with $c_s(z) \simeq \sqrt{\frac{5}{9}}\,\sigma_\nu(z) \simeq \left(\frac{15\zeta(5)}{\zeta(3)}\right)^{1/2}\frac{T_\nu(z)}{m_\nu}$ in units of c (valid for $T_\nu \ll m_\nu$)

Start simulation with homogeneous distribution

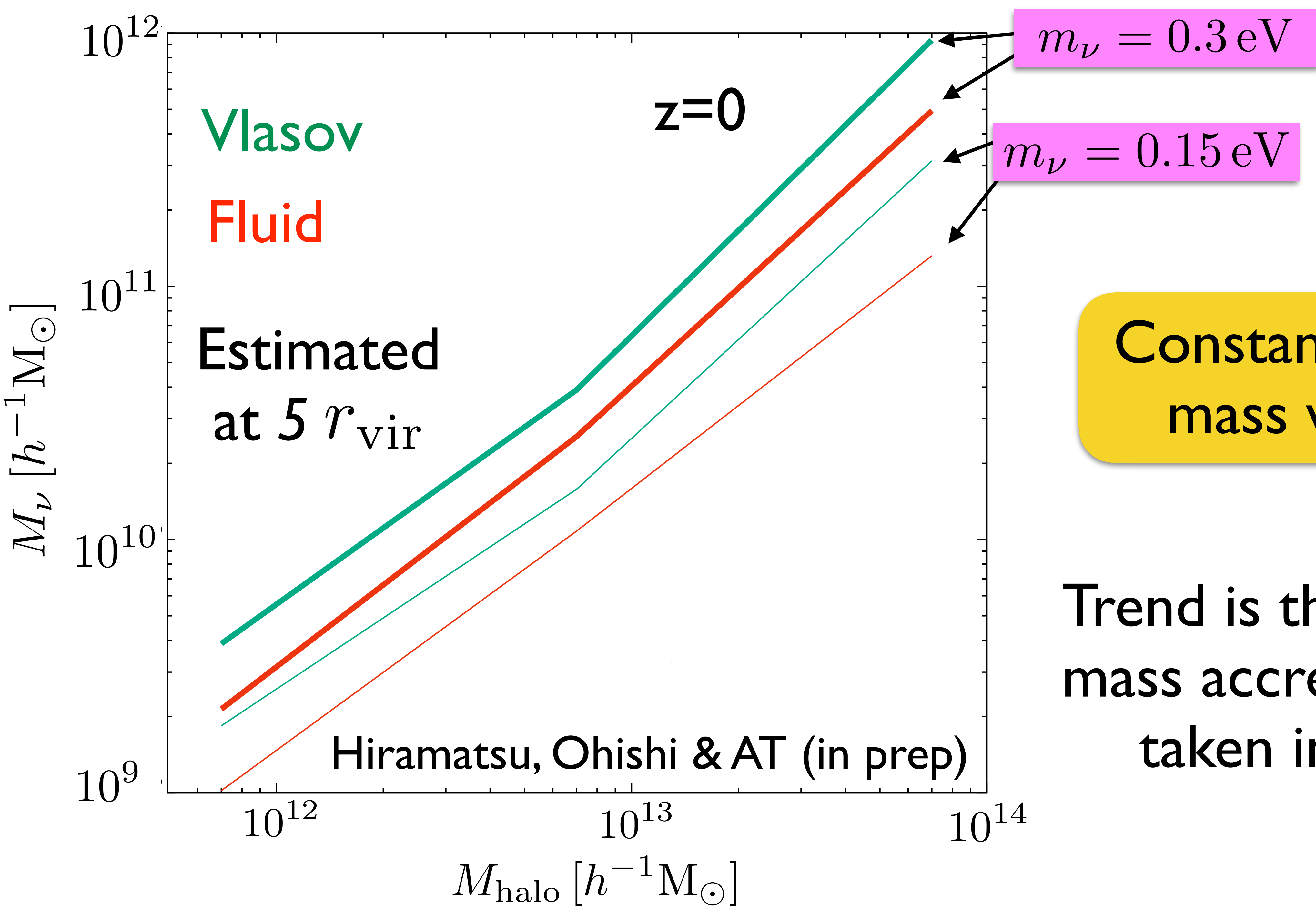
Results: density profile

Hiramatsu, Ohishi & AT (in prep)



Constant halo mass with $z_\mathrm{init} = 3$

Results: clustered mass of ν



Constant halo mass with $z_{\rm init} = 3$

Trend is the same even if mass accretion history is taken into account

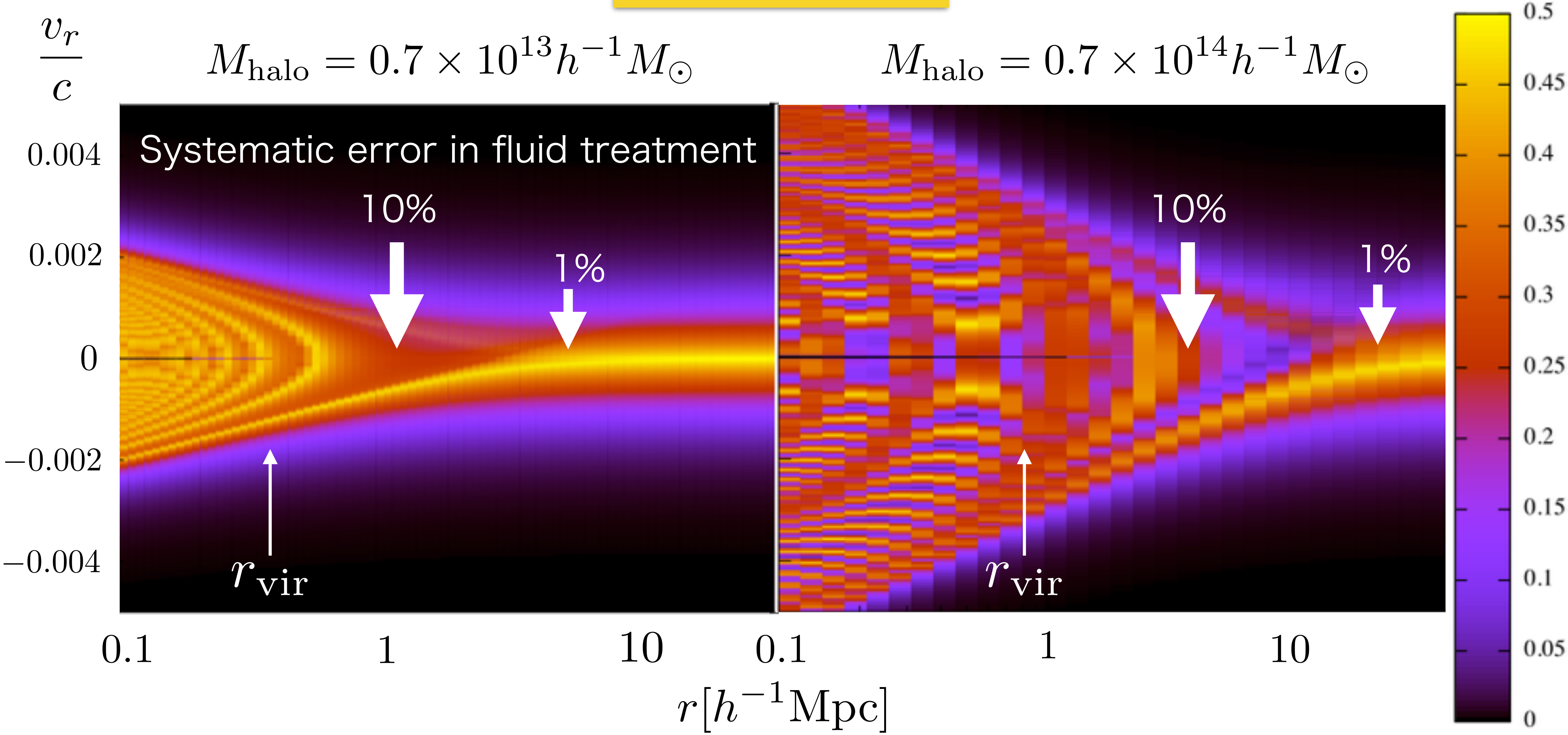
Fluid treatment generally underestimates the clustered mass by factor of 2~3

Phase-space structure

From Vlasov simulation

Hiramatsu, Ohishi & AT (in prep)

$m_\nu = 0.3eV$



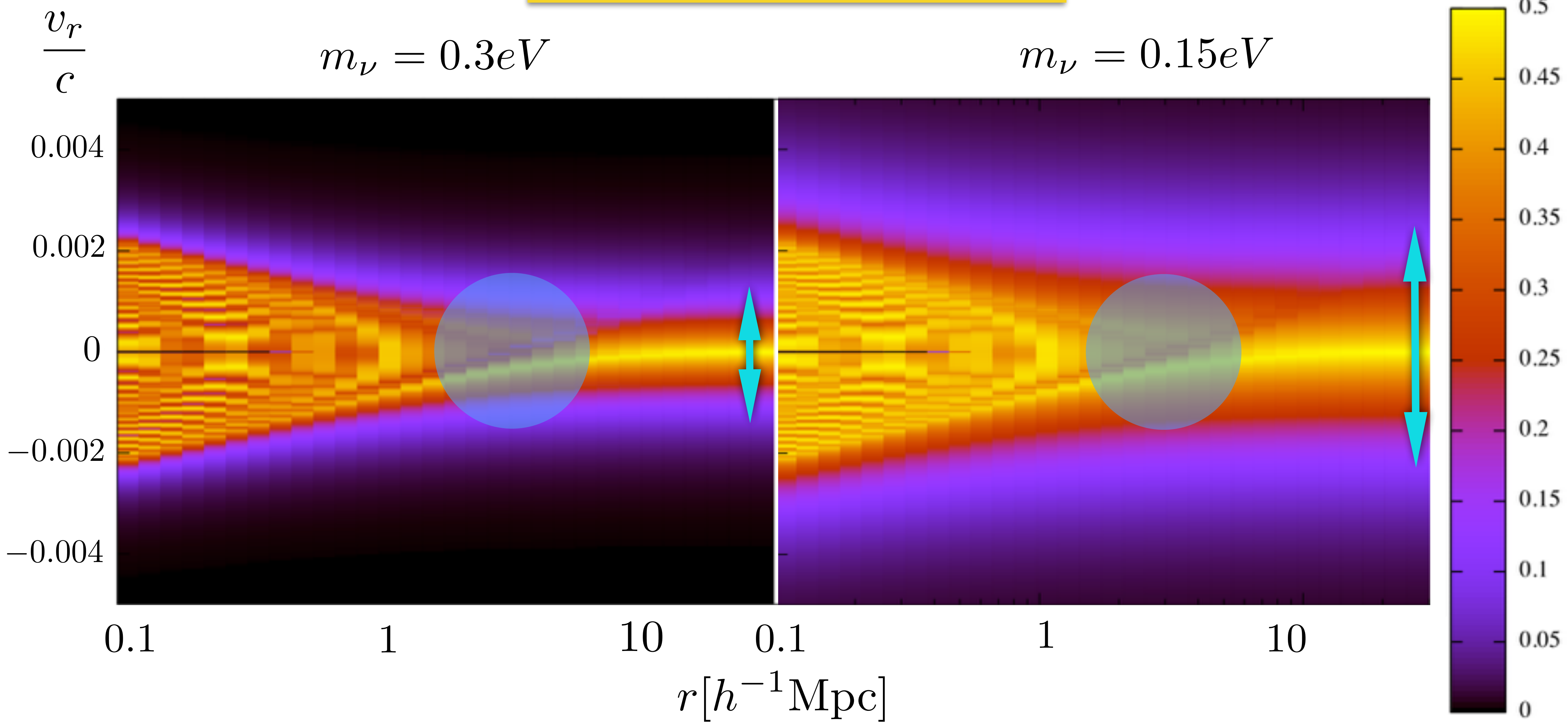
Fluid treatment starts to break down at the multi-stream region

Phase-space structure

From Vlasov simulation

Hiramatsu, Ohishi & AT (in prep)

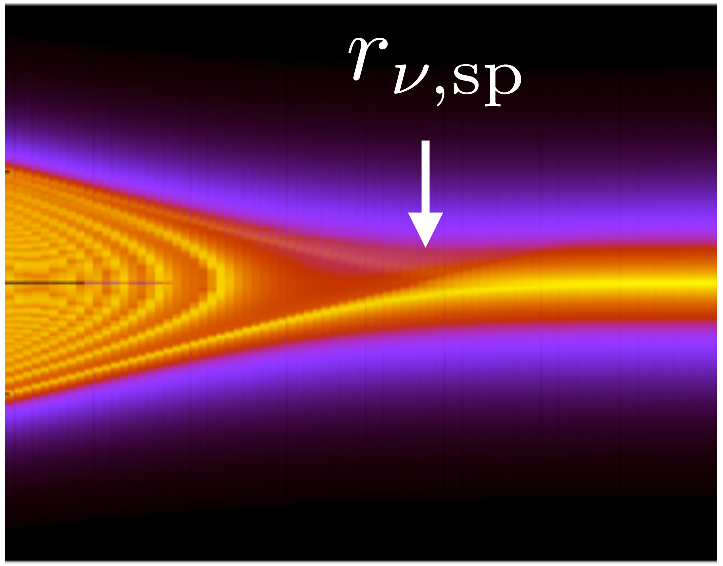
$M_{\rm halo} = 0.7 \times 10^{13} h^{-1} M_\odot$



Structure of multi-stream flow looks the same with different m_ν

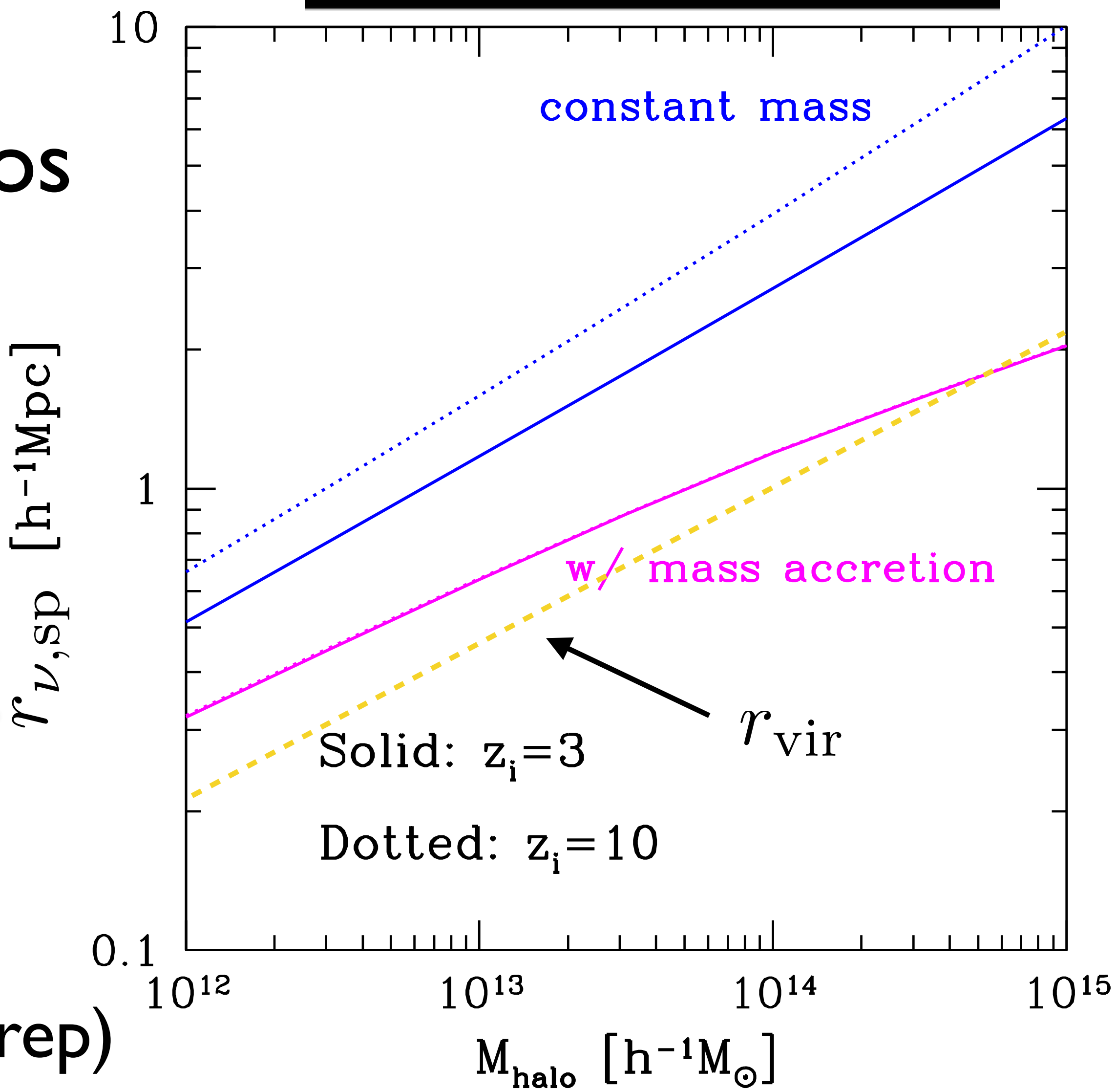
Neutrino's splashback radius

$r_{\nu,\mathrm{sp}}$: Location of outermost caustic



good indicator for the boundary where fluid treatment start to be broken down

- Roughly insensitive to mass of neutrinos
- Larger than viral radius of CDM halo
- Trend can be qualitatively explained by zero-AM orbits



Hiramatsu, Ohishi & AT (in prep)

Summary

Validity of fluid treatment of massive neutrinos in cosmology

Clustering of neutrinos around a CDM halo :

- A simple fluid treatment is invalid and is broken down when the ***multi-stream*** flow appears

  ⟶ underestimate clustering ν's mass by factor of 2-3

- Neutrino's splashback radius is the boundary

Actual impact of neutrino's multi-stream flow ?

precision large-scale structure calculations (simulation)