HOW MUCH SHOULD WE RELY ON PERTURBATION THEORIES / SIMULATIONS

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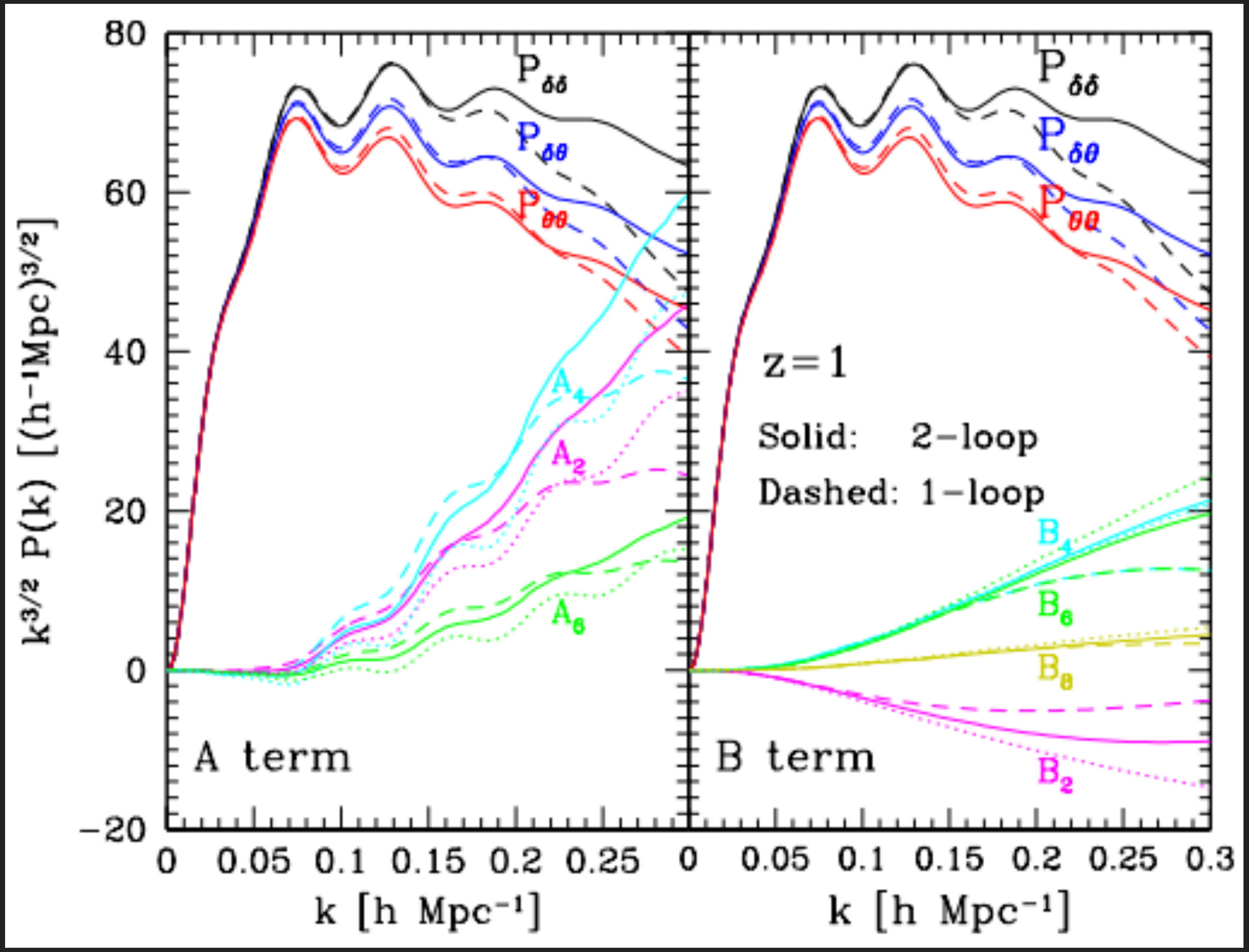
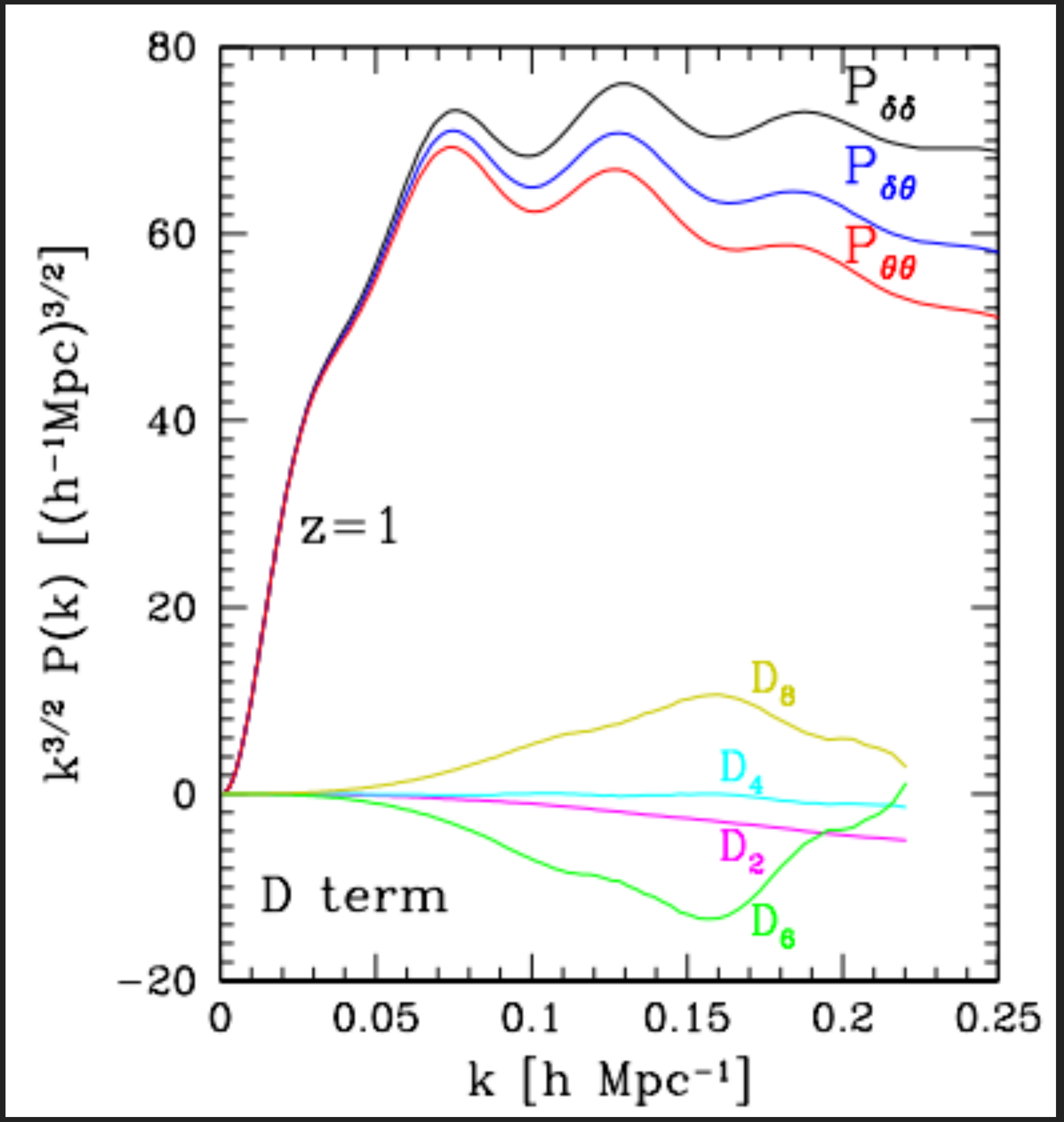
2016/09/08 KASI; Cos-KASI-ICG-NAOC-YITP Workshop 2016

ON THE RSD D(T) TERM





Taruya, Nishimichi & Bernardeau 13

KEY POINTS

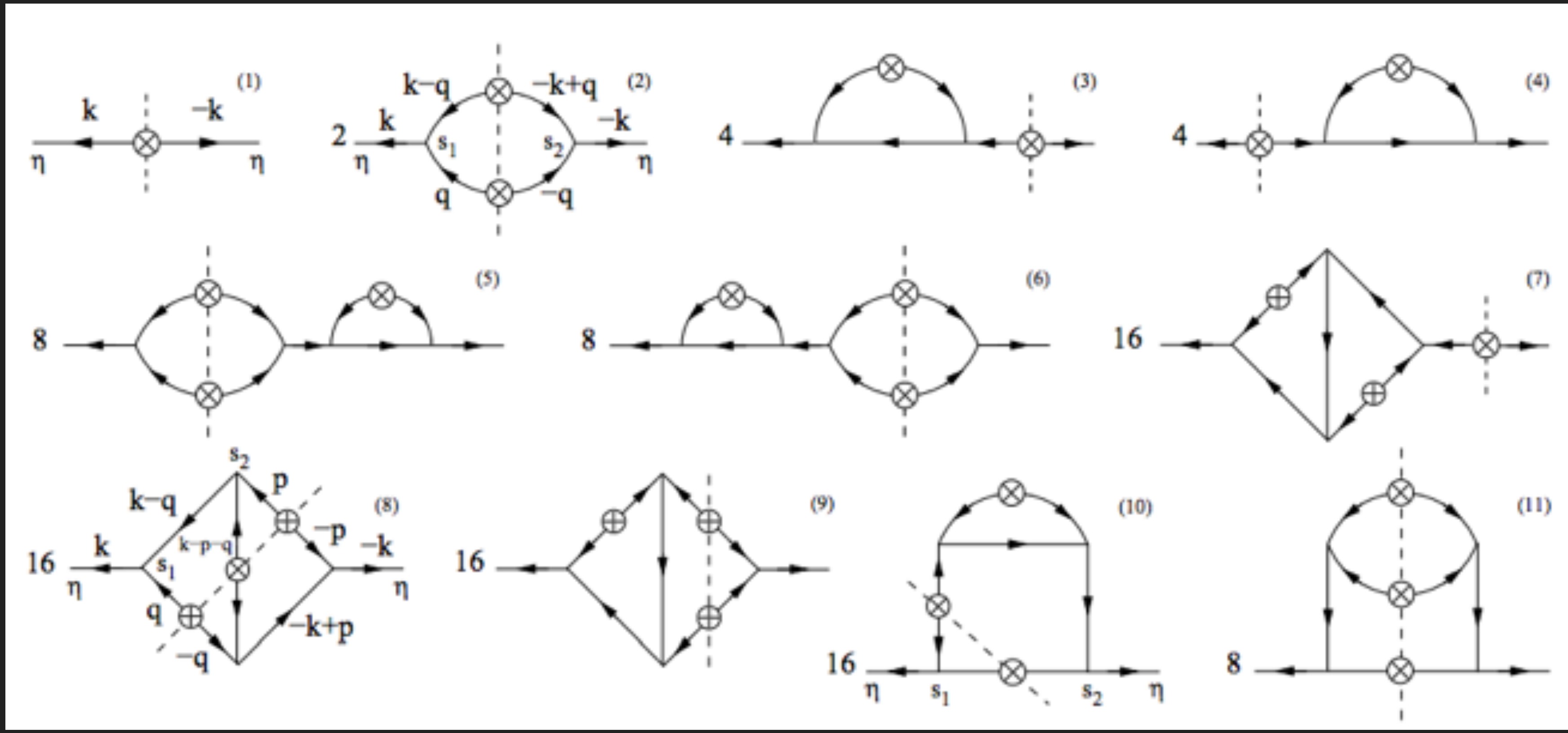
- calibration with simulations at one cosmology fine?
- cosmological analysis (MCMC) practical?
  - predictivity at every point in the cosmological parameter space
    - determine parameters from sims, and use it
    - or, treat as free parameters and determine by comparison with observation
  - and speed, hopefully

THE APPROACHES

- pure analytical, 1st principle calculations
  - SPT, LPT (singlestream)
- analytical but with some calibration w/ simulations or ansatz
  - RPT, RegPT, …
- analytical, but largely relying on simulations
  - halo model, EFT, Zheng-Song, distribution function approach, …
- fully based on simulations
  - emulator, fitting formulae

PURE ANALYTICAL, 1ST PRINCIPLE CALCULATIONS

- SPT
  - {F_n, G_n} kernels + linear power spectrum are the fundamental building blocks
  - diagram expressions (2-loops; only partly, 7 out of 29)

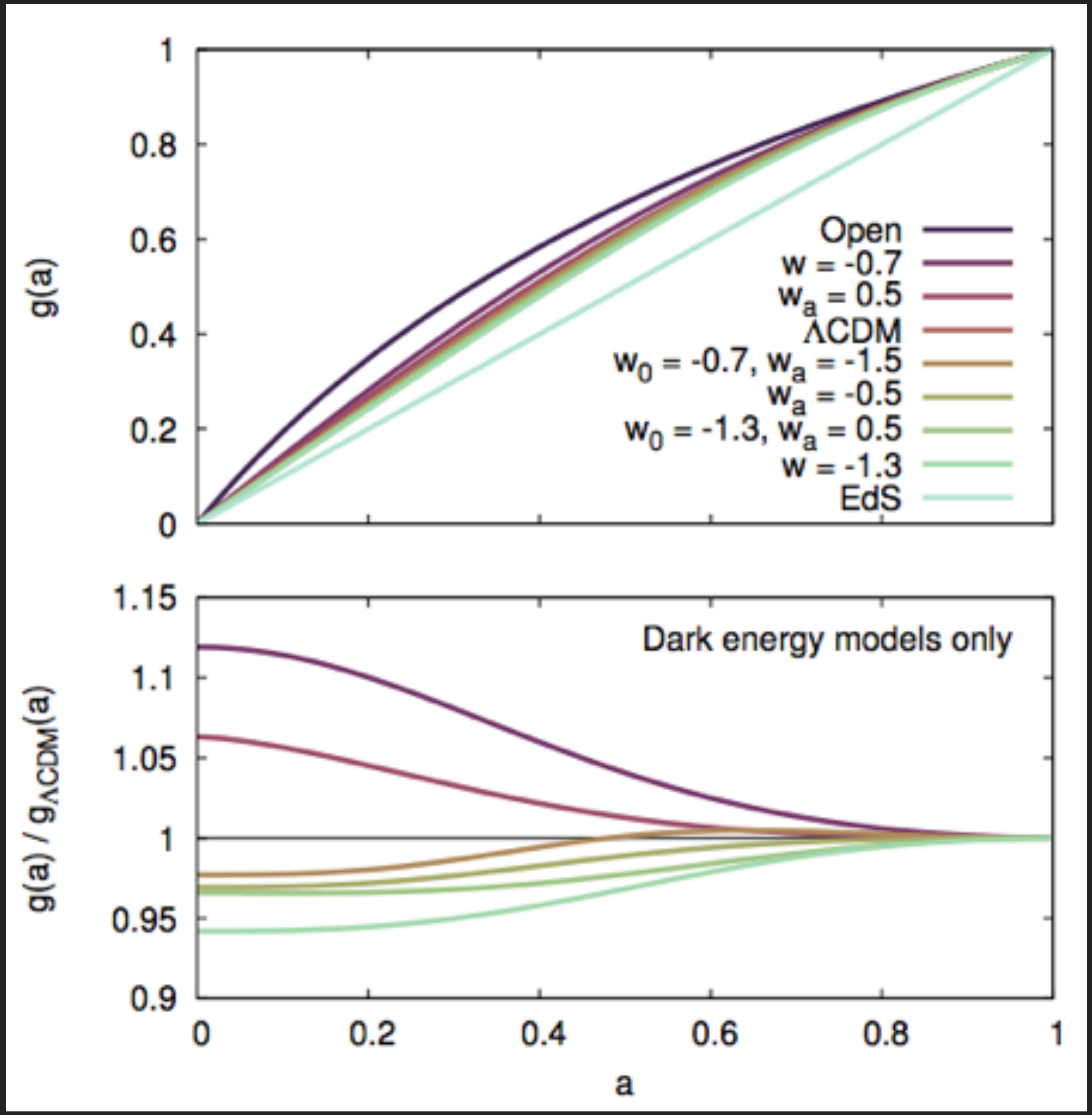


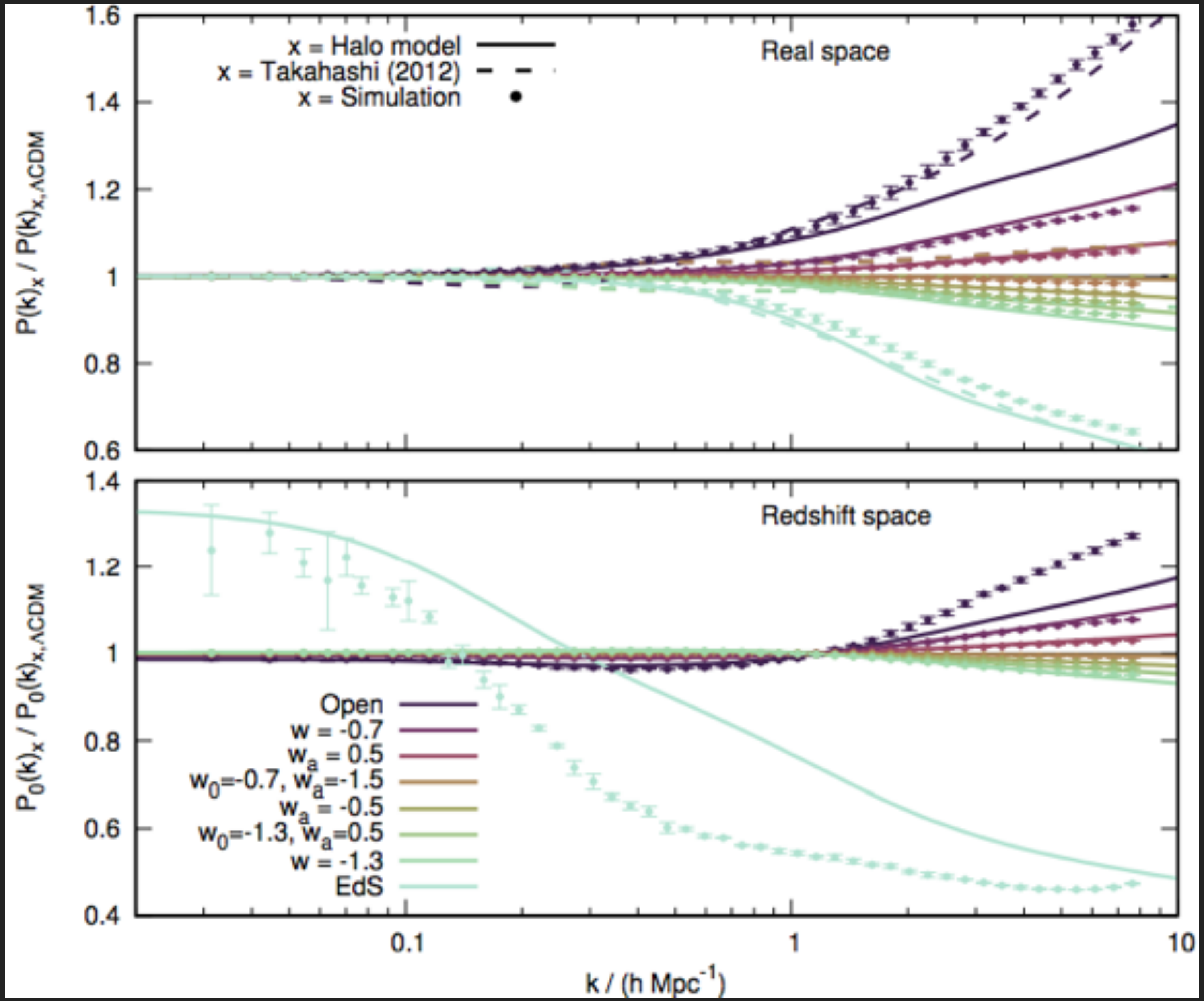
Croce & Scoccimarro 06

PURE ANALYTICAL, 1ST PRINCIPLE CALCULATIONS

- Approximations
  - single stream (Atsushi's talk yesterday)
  - {F_n, G_n} kernels borrowed from Einstein-de-Sitter solution
    - history dependence?
    - $P_{nl}(k, z) = P_{nl}[P_{lin}(q,z)](k,z)$ fine?

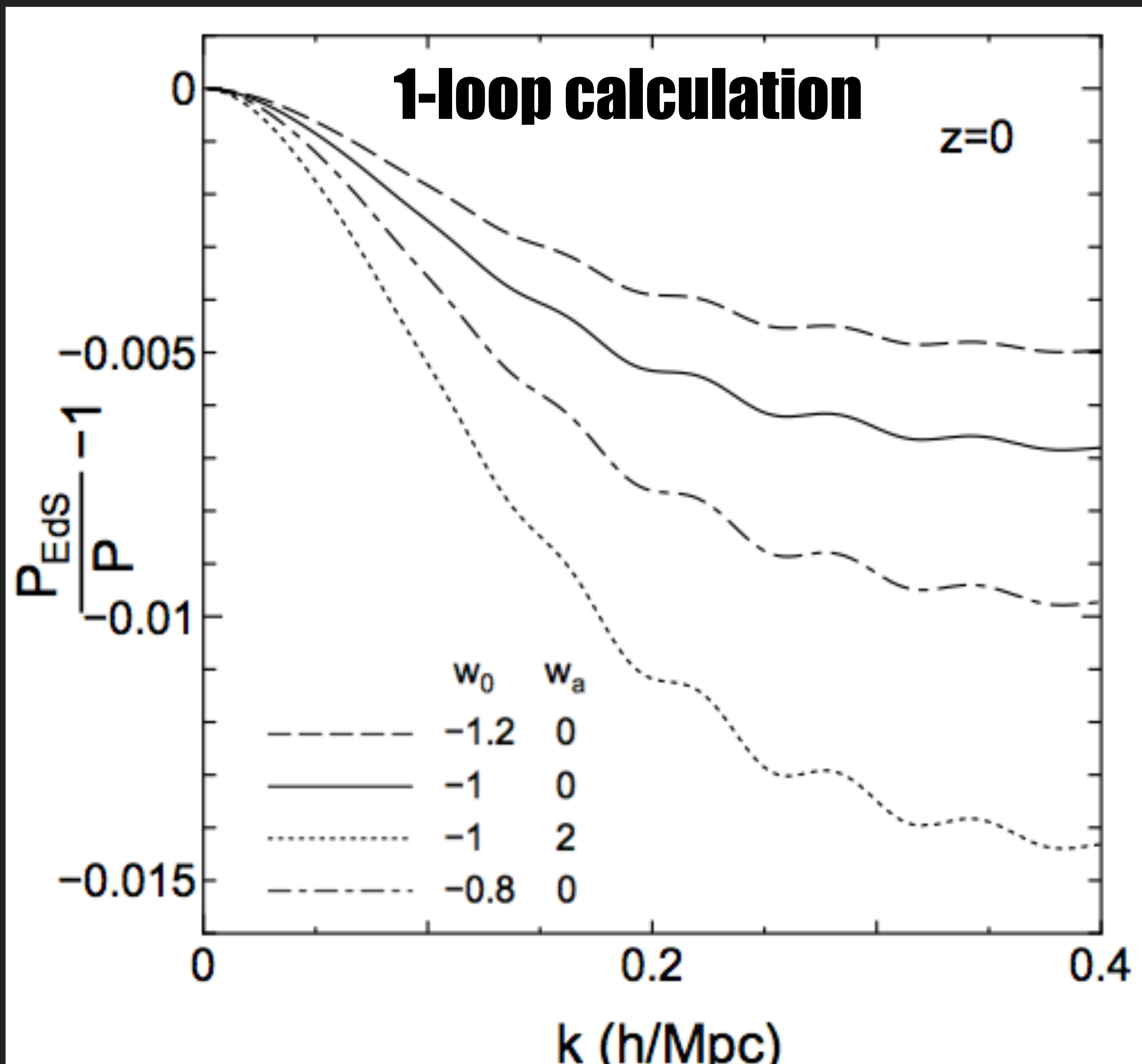
“HISTORY” DEPENDENCE



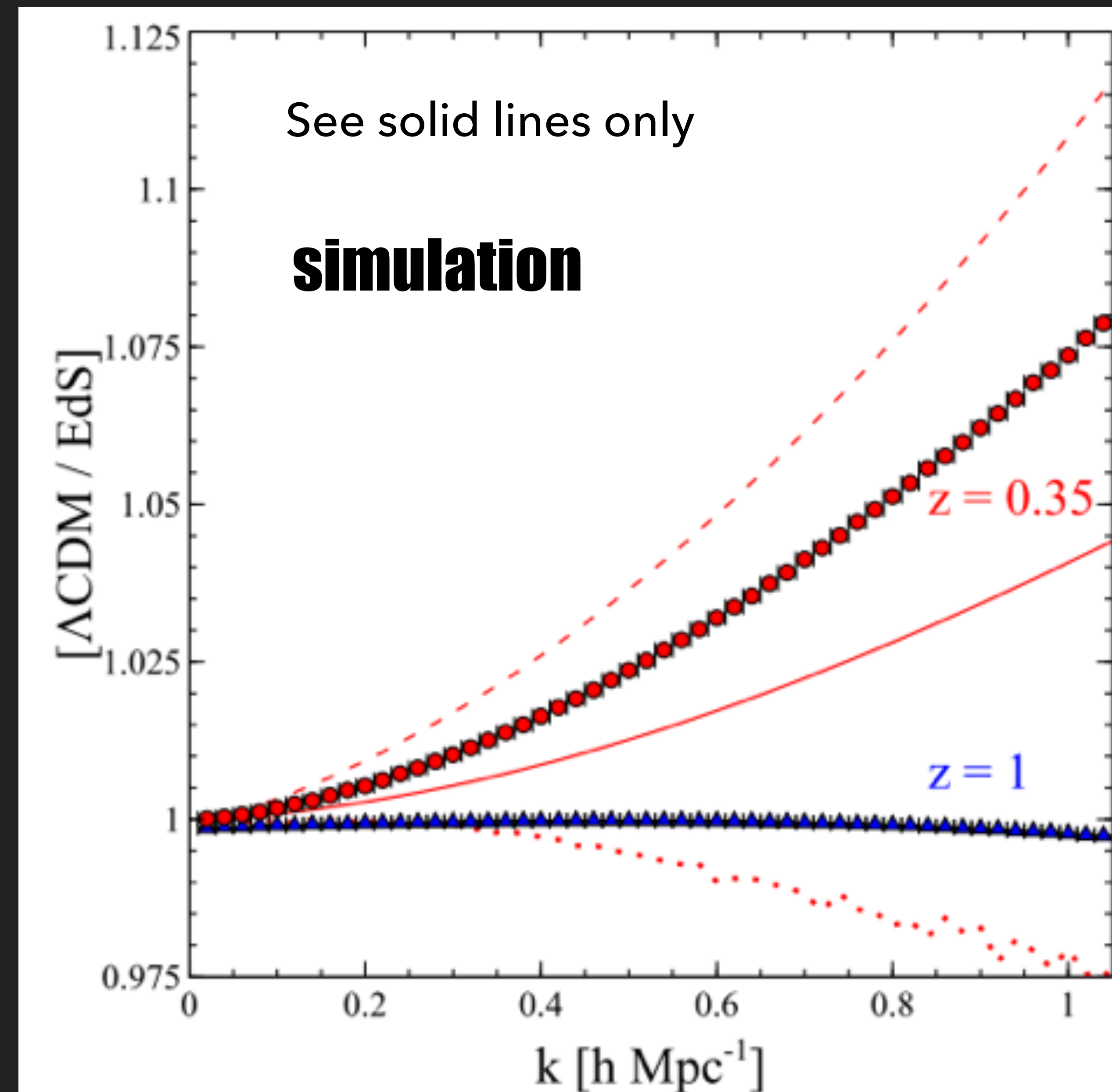


Mead 16

"HISTORY" DEPENDENCE (CONTD.)



Takahashi 08



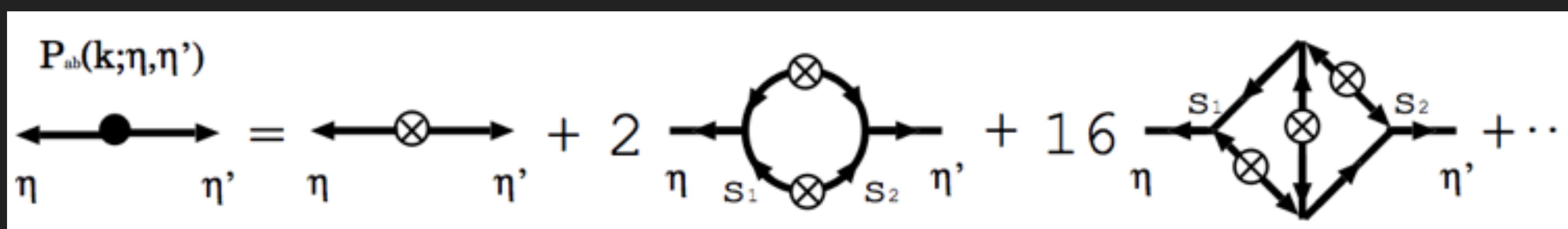
TN & Valageas 14

ANALYTICAL, BUT WITH CALIBRATION W/ SIMS.

- RPT-like approaches
  - ``Propagator" is an important building block

$$D_+(z) \longrightarrow G(k,z) = \frac{\langle \delta(\mathbf{k},z)\,\delta_0(\mathbf{k}')\rangle}{\langle \delta_0(\mathbf{k})\,\delta_0(\mathbf{k}')\rangle},$$

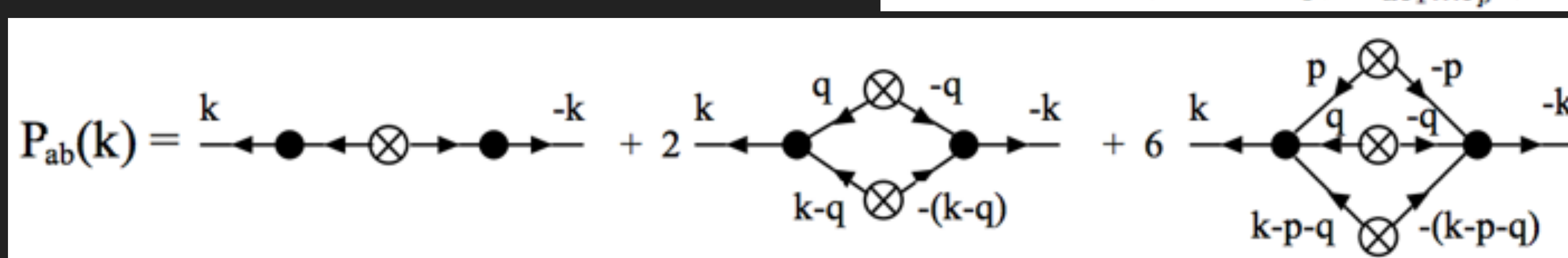
Croce & Scoccimarro 08



  - Multi-point propagators

$\{F_n, G_n\}$ kernels in PT ⟶

$$\frac{1}{p!}\left\langle \frac{\delta^p \Psi_a(\mathbf{k},s)}{\delta\phi_{b_1}(\mathbf{k}_1)\dots\delta\phi_{b_p}(\mathbf{k}_p)}\right\rangle = \delta_{\rm D}(\mathbf{k}-\mathbf{k}_{1\dots p})\,\Gamma^{(p)}_{ab_1\dots b_p}(\mathbf{k}_1,\dots,\mathbf{k}_p,s),$$

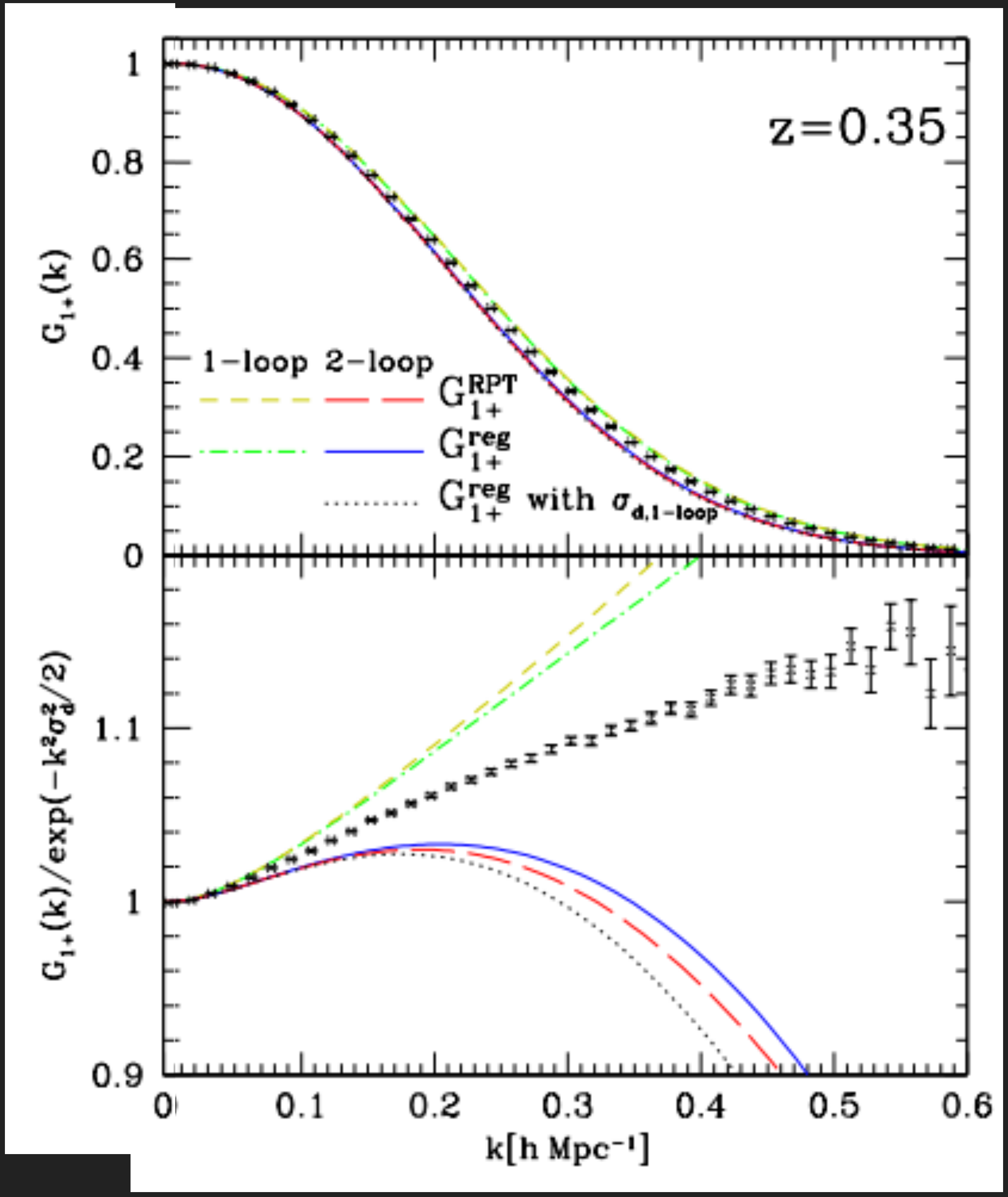


Bernardeau, Crocce & Scoccimarro 08

ANALYTICAL, BUT WITH CALIBRATION W/ SIMS.

- Propagator
  - Loss of information in the initial condition due to the motion of mass elements
  - exactly Gaussian in case of Zel'dovich dynamics
  - width of Gaussian is rms displacement in 1D

$$\sigma_{\rm d}^2 = \int \frac{{\rm d}^3\mathbf{k}}{3k^2} P_0(k).$$



Bernardeau, Takuya & Nishimichi 14

ANALYTICAL, BUT WITH CALIBRATION W/ SIMS.

- RPT
  - The alpha correction

Croce & Scoccimarro 08

The way we estimated this subdominant contribution is as follows. We notice that a sub-set of these subleading diagrams leads to power spectrum resummation. In this case the nonlinear spectrum, instead of the linear, should be used to compute the nonlinear propagator in its large-k limit (i.e. $\sigma_v^2 \to \alpha^2(z)\sigma_v^2$, with $\alpha^2(z)$ given by $\int P_{\rm nl}(k,z)d^3q/q^2/\int P_{\rm lin}(k,z)d^3q/q^2$). For simplicity, we computed this factor using `halofit` to describe $P_{\rm nl}(k,z)$, and found that it grows monotically from 1 at high redshift to $\alpha \sim 1.05$ at $z=0$. Thus we multiply by $\alpha^2(z)$ the exponents in each component of G_{ab} in Eq. (41) of [19].

ANALYTICAL, BUT WITH CALIBRATION W/ SIMS.

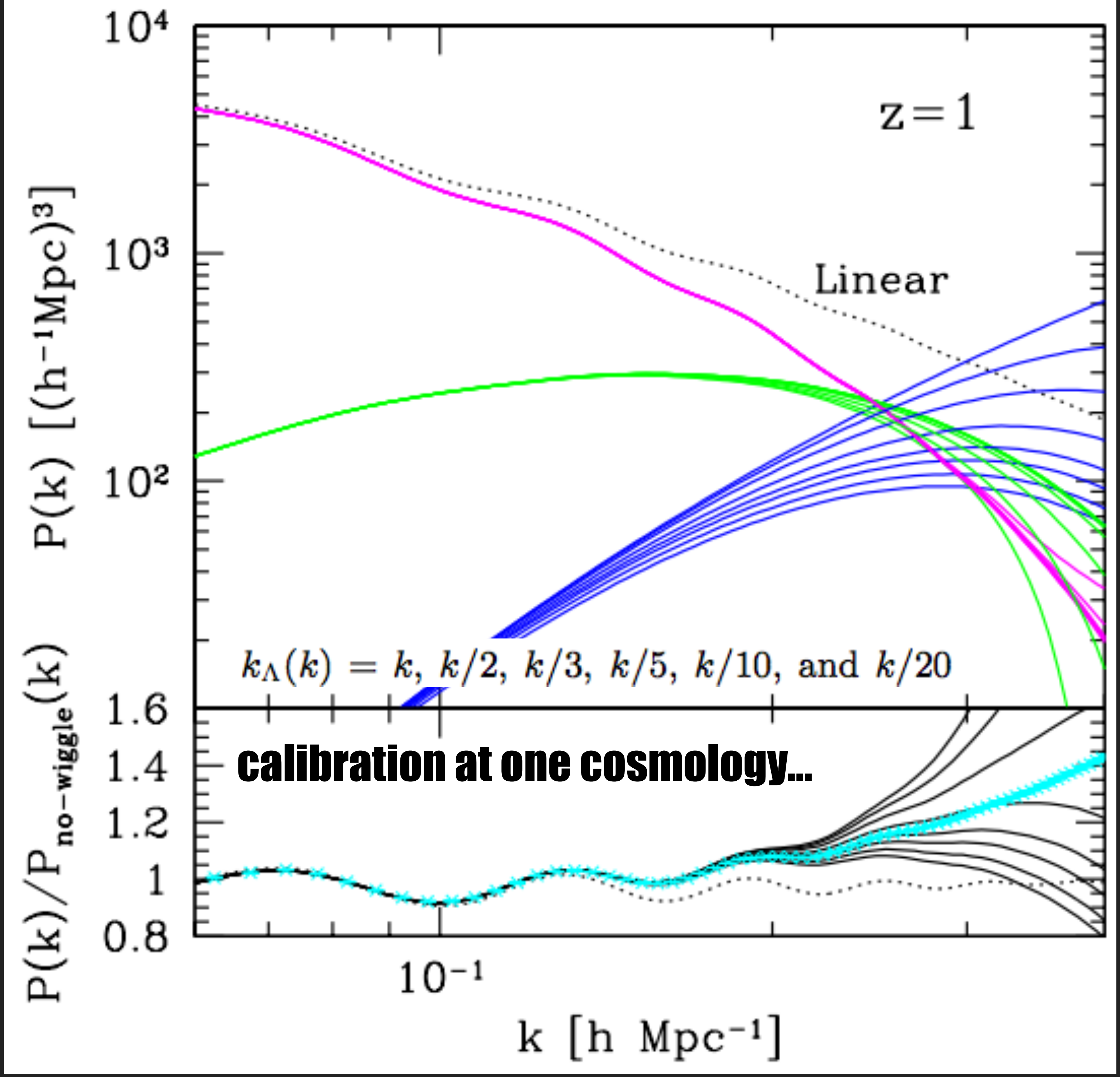
Taruya, Bernardeau, TN & Codis 12

- RegPT
  - running

$$\sigma_{\rm d}^2(k) = \int_0^{k_\Lambda(k)} \frac{dq}{6\pi^2} P_0(q).$$

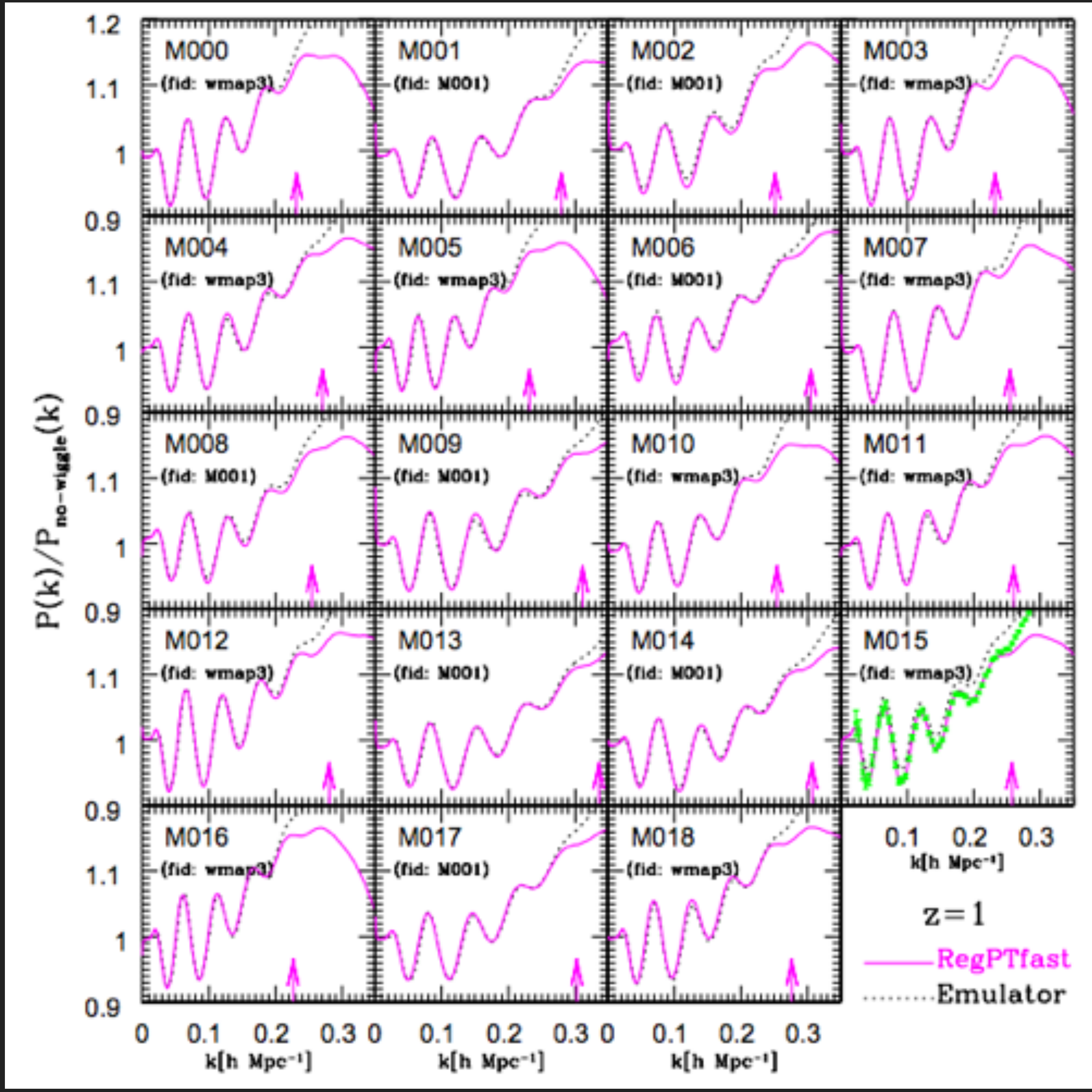
Let us take

$$k_\Lambda = k/2.$$



ANALYTICAL, BUT WITH CALIBRATION W/ SIMS.

- RegPT
  - calibration at 1 cosmology alone, but other cosmologies look fine



ANALYTICAL, BUT COEFFICIENTS FROM SIMULATIONS

- EFTofLSS

e.g., Angulo + 14 , Forman + 15

  - the key is take into account the stress tensor

$$(\partial\tau)_{\rho_l}{}^{i} \equiv \rho^{-1}\partial_j\tau^{ij}$$

$$(\partial\tau)_{\rho_l}{}^{i} \supset c_s^2\partial^i\partial^2\phi \sim c_s^2\partial^i\delta$$

$$(\partial\tau)_{\rho_l}{}^{i} \supset \partial^i(\partial_j v^j)$$

$$(\partial\tau)_{\rho_l}{}^{i} \supset \partial^i\left[\partial^2\phi\right]^2 + \partial^i\left[\partial^j\partial^k\phi\,\partial_j\partial_k\phi\right] + \partial^i\partial^j\phi\,\partial_j\partial^2\phi$$
$$\sim \partial^i\delta^2 + \partial^i\left[\frac{\partial^j\partial^k}{\partial^2}\delta\cdot\frac{\partial_j\partial_k}{\partial^2}\delta\right] + \frac{\partial^i\partial^j}{\partial^2}\delta\cdot\partial_j\delta\ ,$$

$$(\partial\tau)_{\rho_l}{}^{i} \quad\supset\quad (1-\delta)\times\left\{\partial^i\left(\frac{\partial_j v^j}{-\mathcal{H}(a)f}-\delta\right),\ \partial^i\left[\partial^2\phi\right]^2,\ \partial^i\left[\partial^j\partial^k\phi\,\partial_j\partial_k\phi\right],\ \partial^i\partial^j\phi\,\partial_j\partial^2\phi\right\}.$$

$$(\partial\tau)_{\rho_l}{}^{i} \quad\supset\quad \partial^2\partial^i\delta\ ;$$

$$(\partial\tau)_{\rho_l}{}^{i} \quad\supset\quad \partial^i\Delta\tau\ .$$

$$(\partial\tau)_{\rho_l}{}^{i} \quad\supset\quad \partial^i\delta^3,\ \ldots\ .$$

Since there are many of those, we just wrote a representative one.

ANALYTICAL, BUT COEFFICIENTS FROM SIMULATIONS

e.g., Carrasco + 14

- EFTofLSS
  - corresponding power spectrum corrections (standard)

$$P_{\text{EFT-1-loop}}(k,z) = [D_1(z)]^2 P_{11}(k) + [D_1(z)]^4 P_{\text{1-loop}}(k) + P^{(c_s)}_{\text{tree}}(k,z) \,,$$

$$P^{(c_s)}_{\text{tree}}(k,z) = -2(2\pi) c^2_{s(1)}(z) [D_1(z)]^2 \frac{k^2}{{k_{\text{NL}}}^2} P_{11}(k) \,,$$

$$P_{\text{EFT-2-loop}}(k,z) = P_{\text{EFT-1-loop}}(k,z) + [D_1(z)]^6 P_{\text{2-loop}}(k) - 2(2\pi) c^2_{s(2)}(z) \frac{k^2}{{k_{\text{NL}}}^2} P_{11}(k)$$
$$+ (2\pi) c^2_{s(1)}(z) [D_1(z)]^4 P^{(c_s)}_{\text{1-loop}}(k) + (2\pi)^2 \left(1 + \frac{\zeta + \frac{5}{2}}{2(\zeta + \frac{5}{4})}\right) [c^2_{s(1)}(z)]^2 [D_1(z)]^2 \frac{k^4}{{k_{\text{NL}}}^4} P_{11}(k)$$

  - gives ``physically well motivated'' parameterization

ANALYTICAL, BUT COEFFICIENTS FROM SIMULATIONS

- EFTofLSS

Forman, Perrier & Senatore + 15

  - corresponding power spectrum corrections (more complete)

- quadratic counterterms:

$$P_{\text{quad. counterterms}}(k,z) = \frac{(2\pi)}{k_{\rm NL}{}^2} D_1(z)^4 \left(c_0(z)\, P_{\text{1-loop}}^{(\text{quad},0)}(k) + c_1(z)\, P_{\text{1-loop}}^{(\text{quad},1)}(k) + c_2(z)\, P_{\text{1-loop}}^{(\text{quad},2)}(k) + c_3(z)\, P_{\text{1-loop}}^{(\text{quad},3)}(k) \right) , \tag{12}$$

- higher-derivative counterterm:

$$P_{\text{4-deriv. counterterm}}(k,z) = 2(2\pi)^2 D_1(z)^2 c_4(z) \left(\frac{k}{k_{\rm NL}}\right)^4 P_{11}(k) \, . \tag{13}$$

- stochastic counterterm:

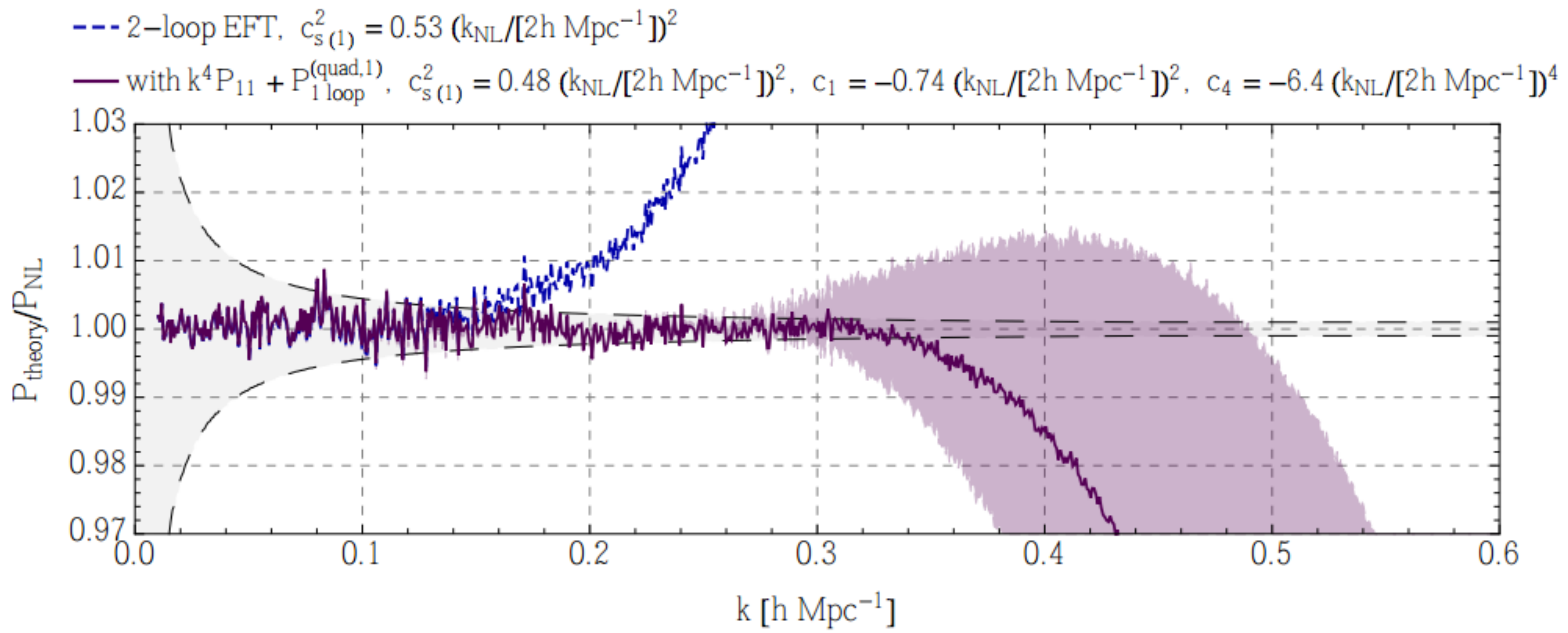
$$P_{\rm stoch}(k,z) = (2\pi)^2 D_1(z)^2 c_{\rm stoch}(z) \left(\frac{k}{k_{\rm NL}}\right)^4 \frac{1}{k_{\rm NL}{}^3} \, . \tag{14}$$

- cubic counterterms: they are degenerate with $k^2 P_{11}(k)$, and therefore do not need to be included in the calculation.

ANALYTICAL, BUT COEFFICIENTS FROM SIMULATIONS

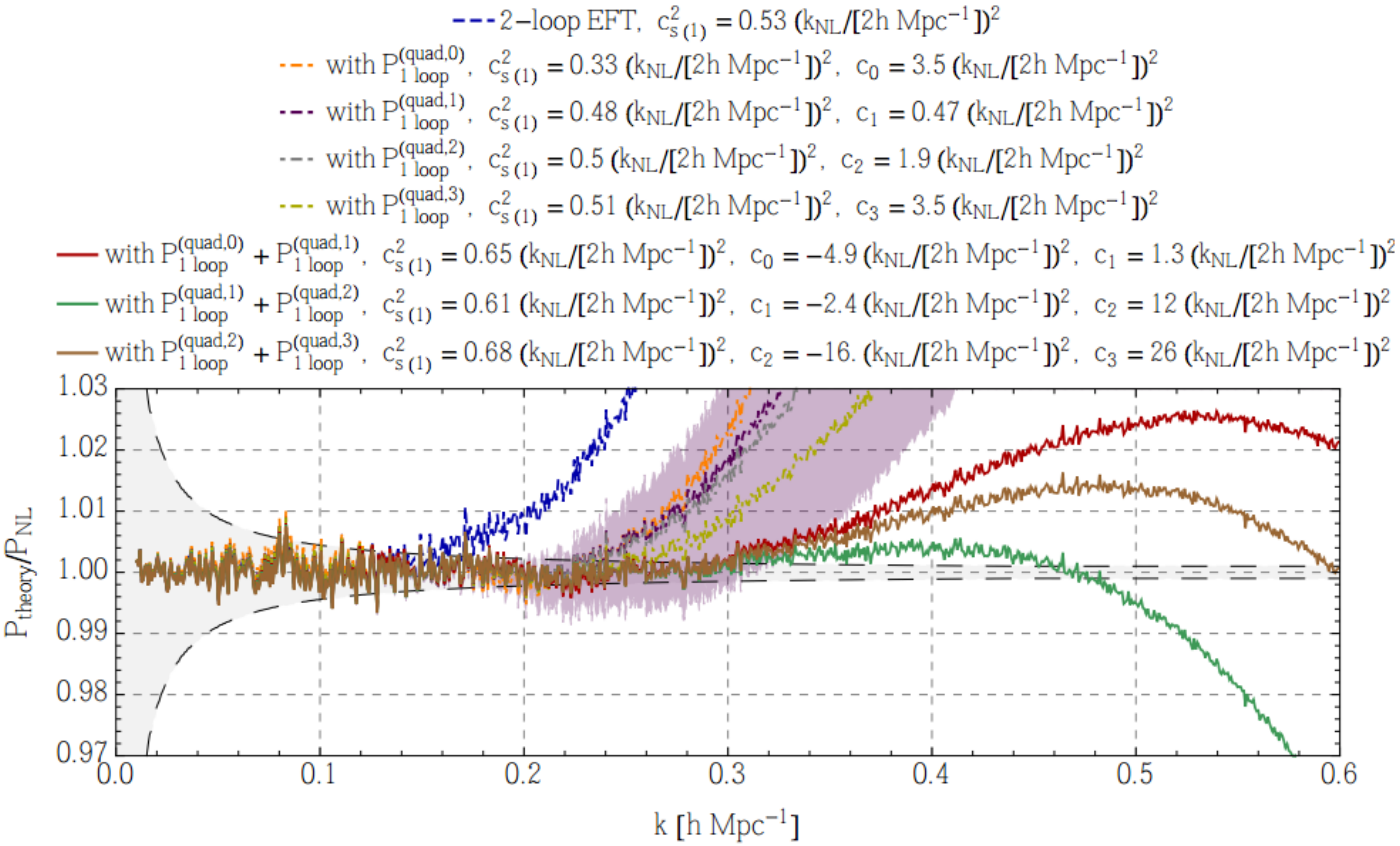
Forman, Perrier & Senatore + 15

- EFTofLSS
  - determine the parameter to fit dark sky simulation



ANALYTICAL, BUT COEFFICIENTS FROM SIMULATIONS

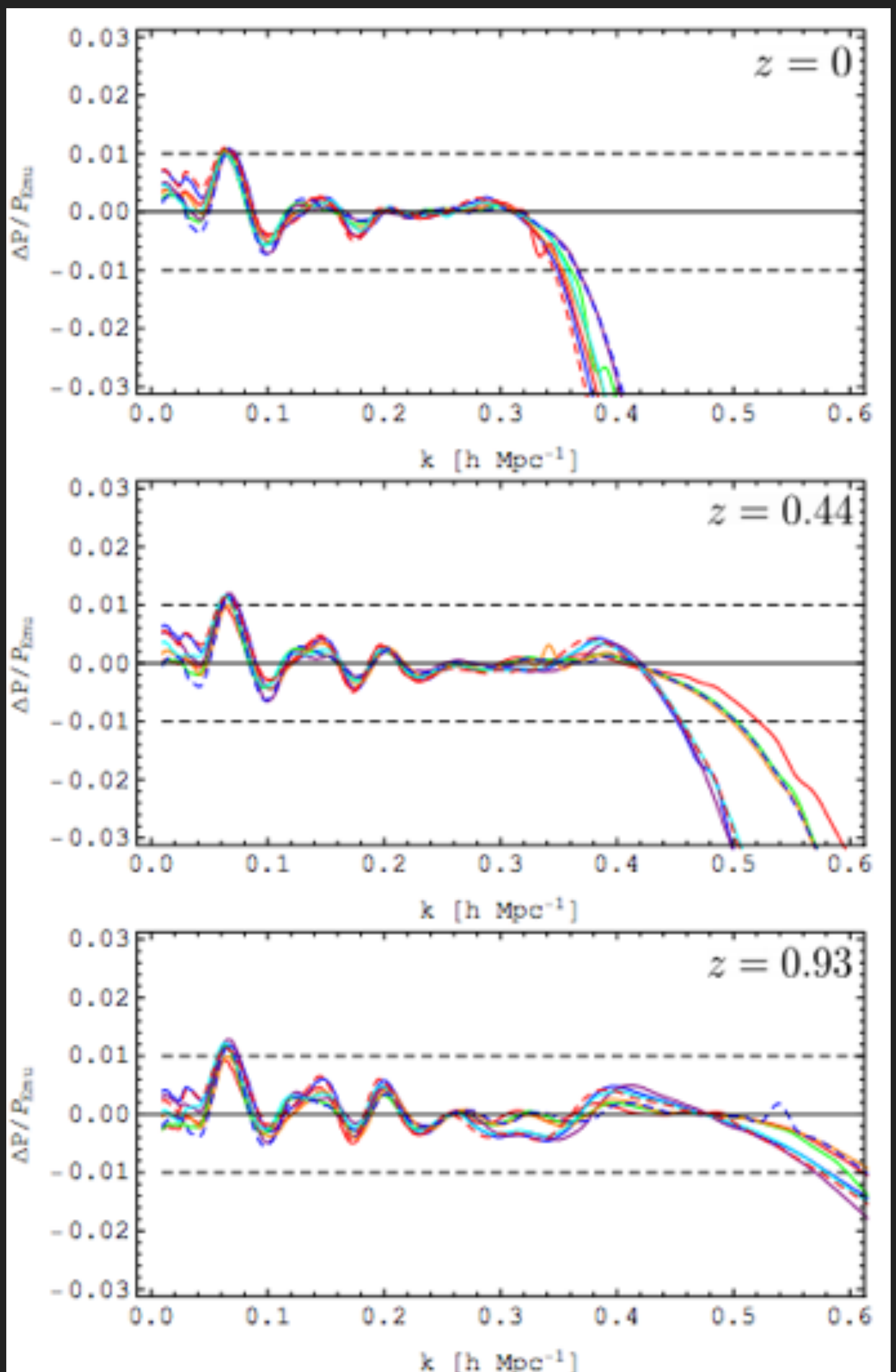
Forman, Perrier & Senatore + 15



ANALYTICAL, BUT COEFFICIENTS FROM SIMULATIONS

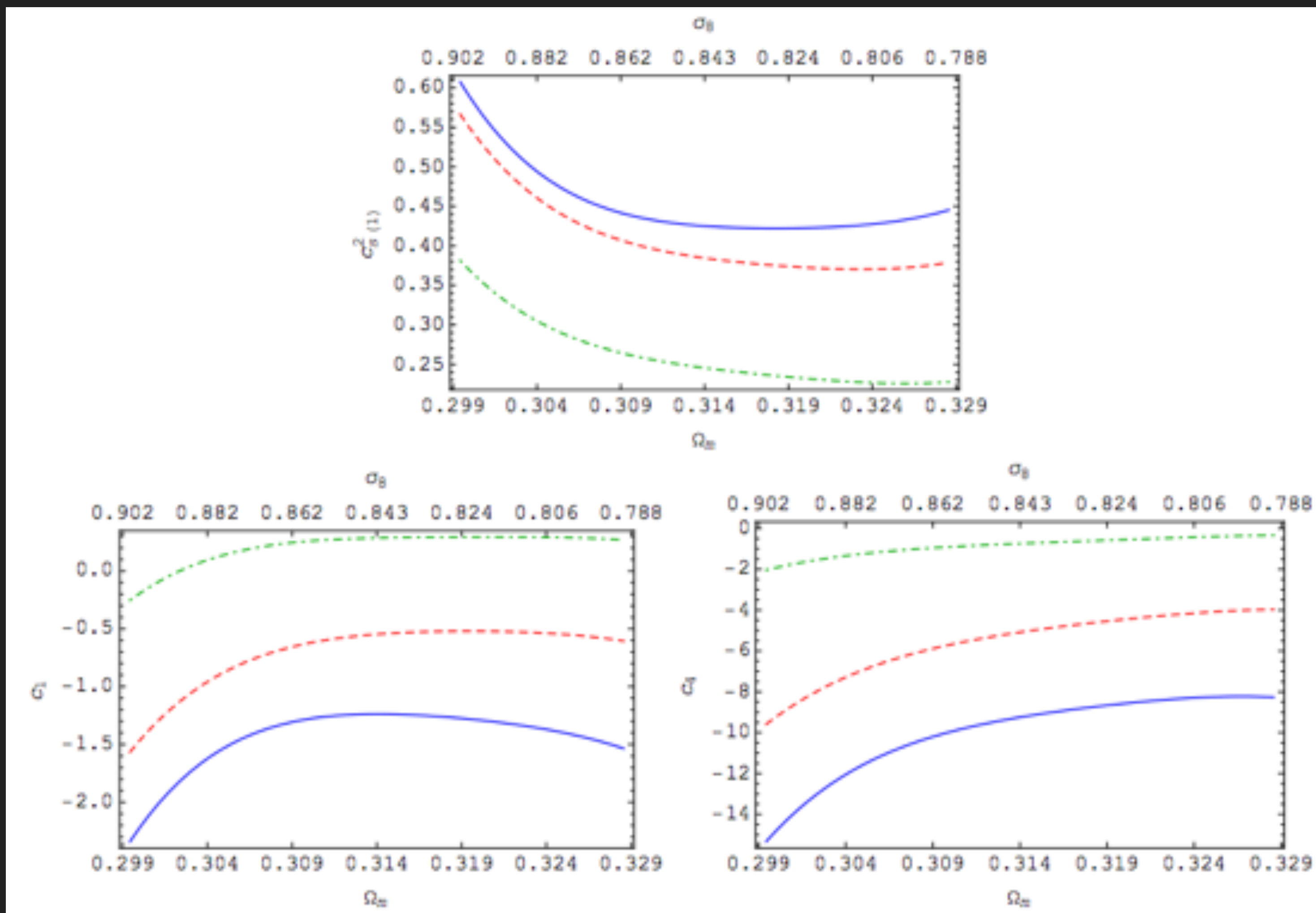
- EFTofLSS
  - fit the model to the emulator at various cosmologies, and Taylor expand the cosmology dependence of the EFT parameters
  - quick evaluation of the loop integral by degrading the accuracy parameters and apply further Taylor expansion

Cataneo, Forman & Senatore + 16



$$
\begin{aligned}
P_{\text{EFT-2-loop}}(k,z) = {} & P_{11}(k,z)_{\|2} + P_{\text{1-loop}}(k,z)_{\|1} - 2(2\pi)c^2_{s(1)}\left(\frac{k^2}{k^2_{\text{NL}}}P_{11}(k,z)\right)_{\|1} \\
& + P_{\text{2-loop}}(k,z)_{\|0} - 2(2\pi)c^2_{s(2)}\left(\frac{k^2}{k^2_{\text{NL}}}P_{11}(k,z)\right)_{\|0} \\
& + (2\pi)c^2_{s(1)}P^{(c_s)}_{\text{1-loop}}(k,z)_{\|0} + (2\pi)^2\left(c^2_{s(1)}\right)^2\left(1+\frac{\zeta+\frac{5}{2}}{2\left(\zeta+\frac{5}{4}\right)}\right)\left(\frac{k^4}{k^4_{\text{NL}}}P_{11}(k,z)\right)_{\|0} \\
& + (2\pi)c_1 P^{(\text{quad},1)}_{\text{1-loop}}(k,z)_{\|0} + 2(2\pi)^2 c_4\left(\frac{k^4}{k^4_{\text{NL}}}P_{11}(k,z)\right)_{\|0}. \qquad (4.1)
\end{aligned}
$$

ANALYTICAL, BUT COEFFICIENTS FROM SIMULATIONS



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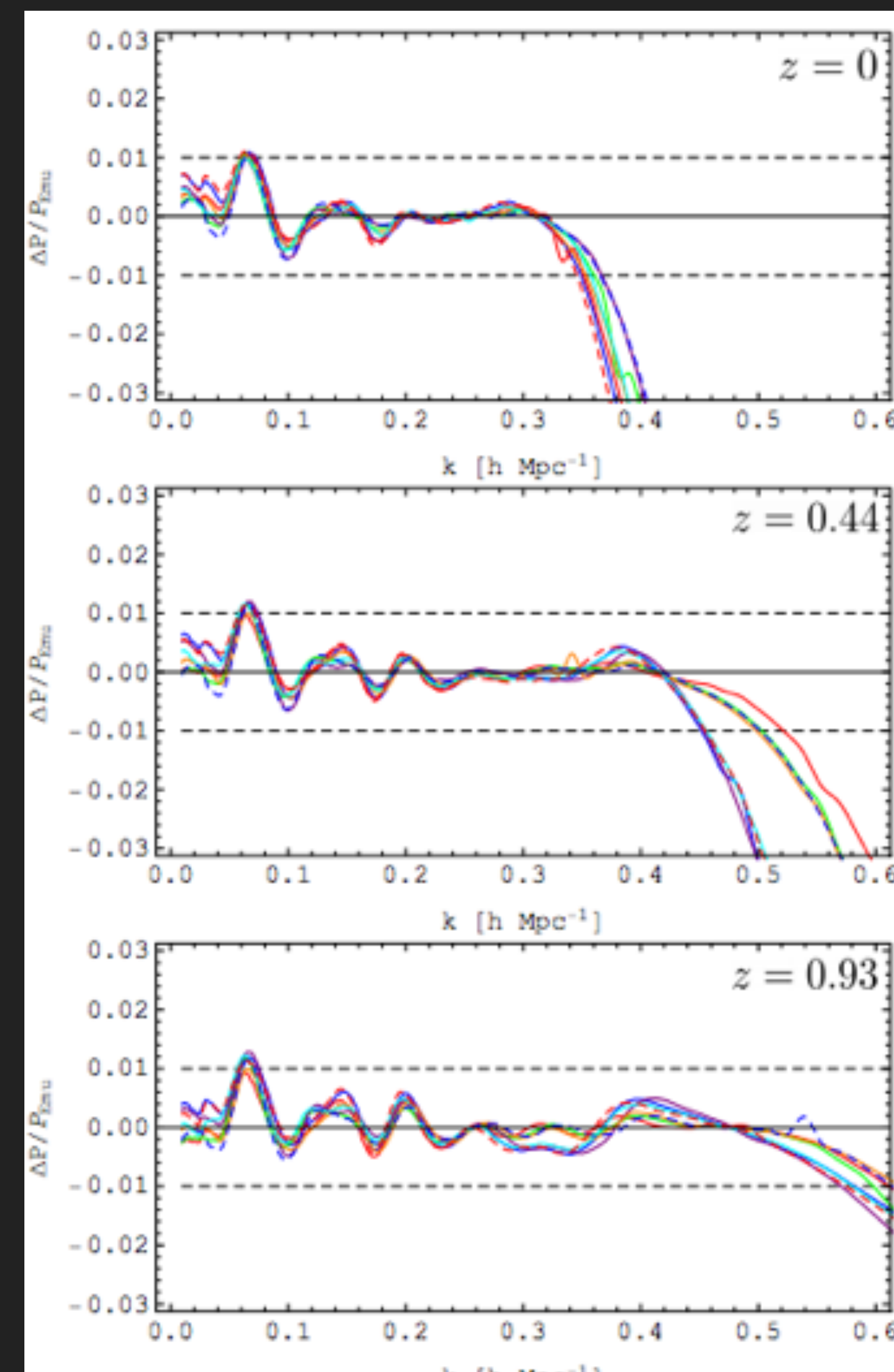

$$
\begin{aligned}
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\end{aligned}
$$

RESPONSE FUNCTION

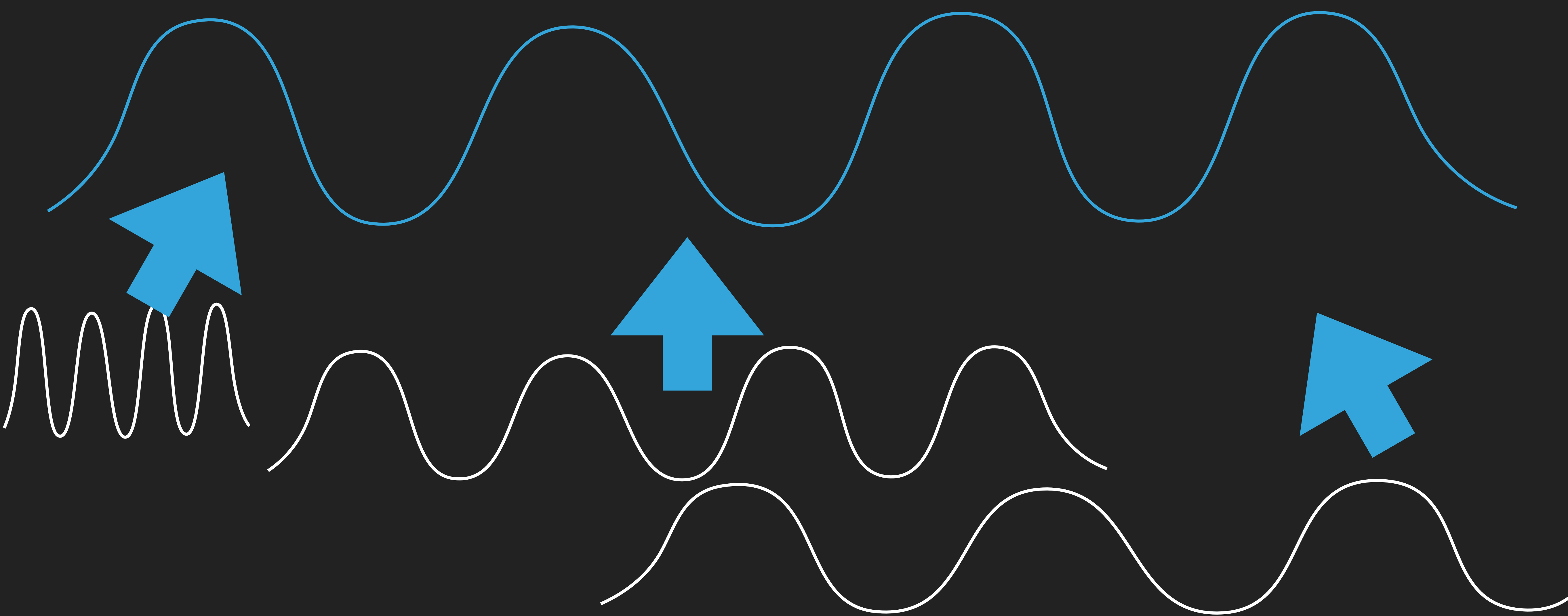
$X_{nl}\,(k;\, t,\, \Omega_m, \sigma_8, \ldots) = X_{nl}\,[P_{lin}(t,\, \Omega_m, \sigma_8, \ldots)]\,(k)$

any quantity @ final state

$$K_{\rm X}(k,q) = q\frac{\delta X(k)}{\delta P_0(q)}$$

initial spectrum

I want to study this mode at some late time t

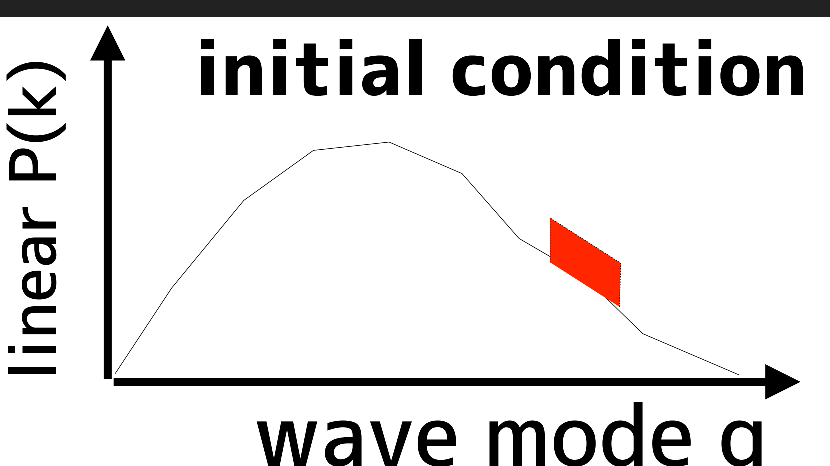


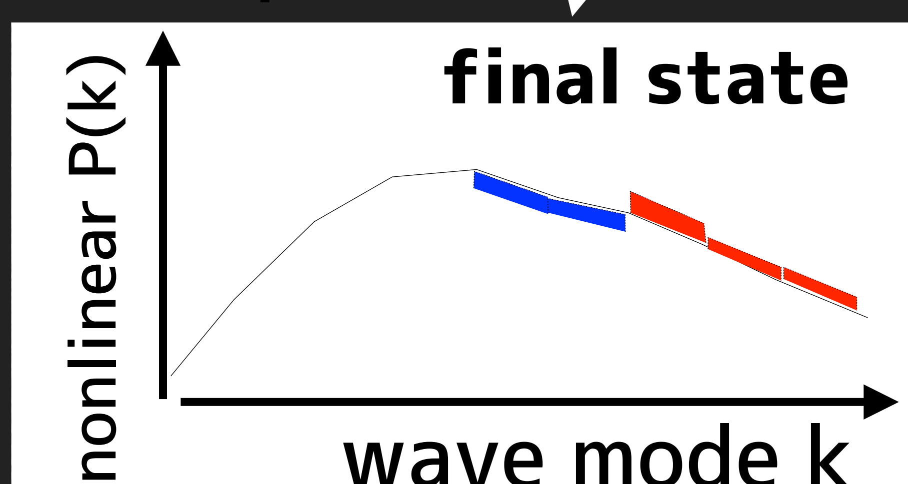
what is the impact from wave mode q at the initial time t_0?

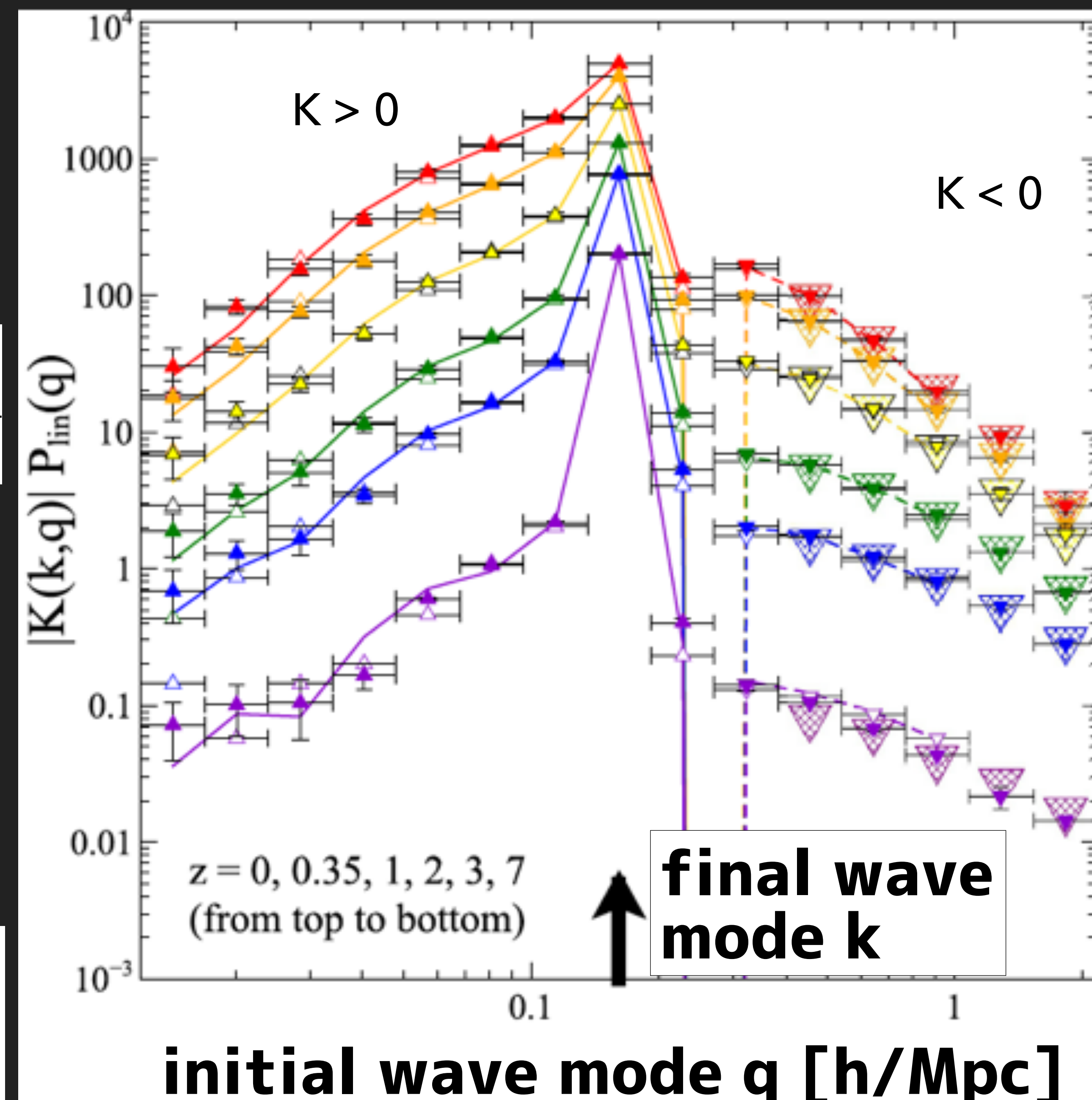
DIRECT MEASUREMENT OF MODE COUPLING

- from order-by-order to the full order discussion
- estimate the derivative from simulations

$$\hat{K}_{i,j} P_j^{\rm lin} \equiv \frac{P_i^{\rm nl}[P_{+,j}^{\rm lin}] - P_i^{\rm nl}[P_{-,j}^{\rm lin}]}{\Delta \ln P^{\rm lin} \Delta \ln q}$$



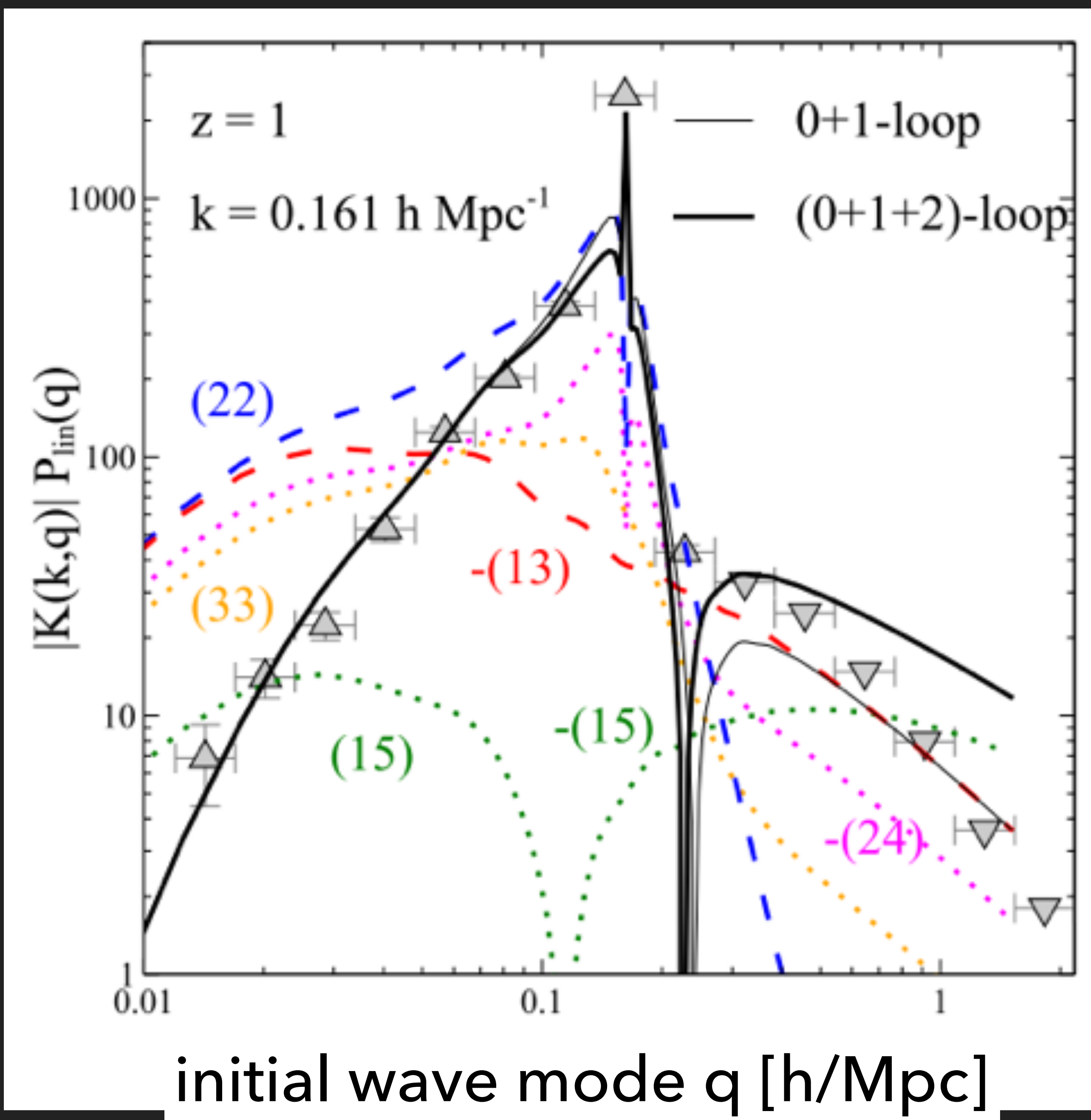




TN, Bernardeau, Taruya '14

N-BODY VS STANDARD PT

$$K(k,q;z) = q\frac{\delta P^{\rm nl}(k;z)}{\delta P^{\rm lin}(q;z)}$$

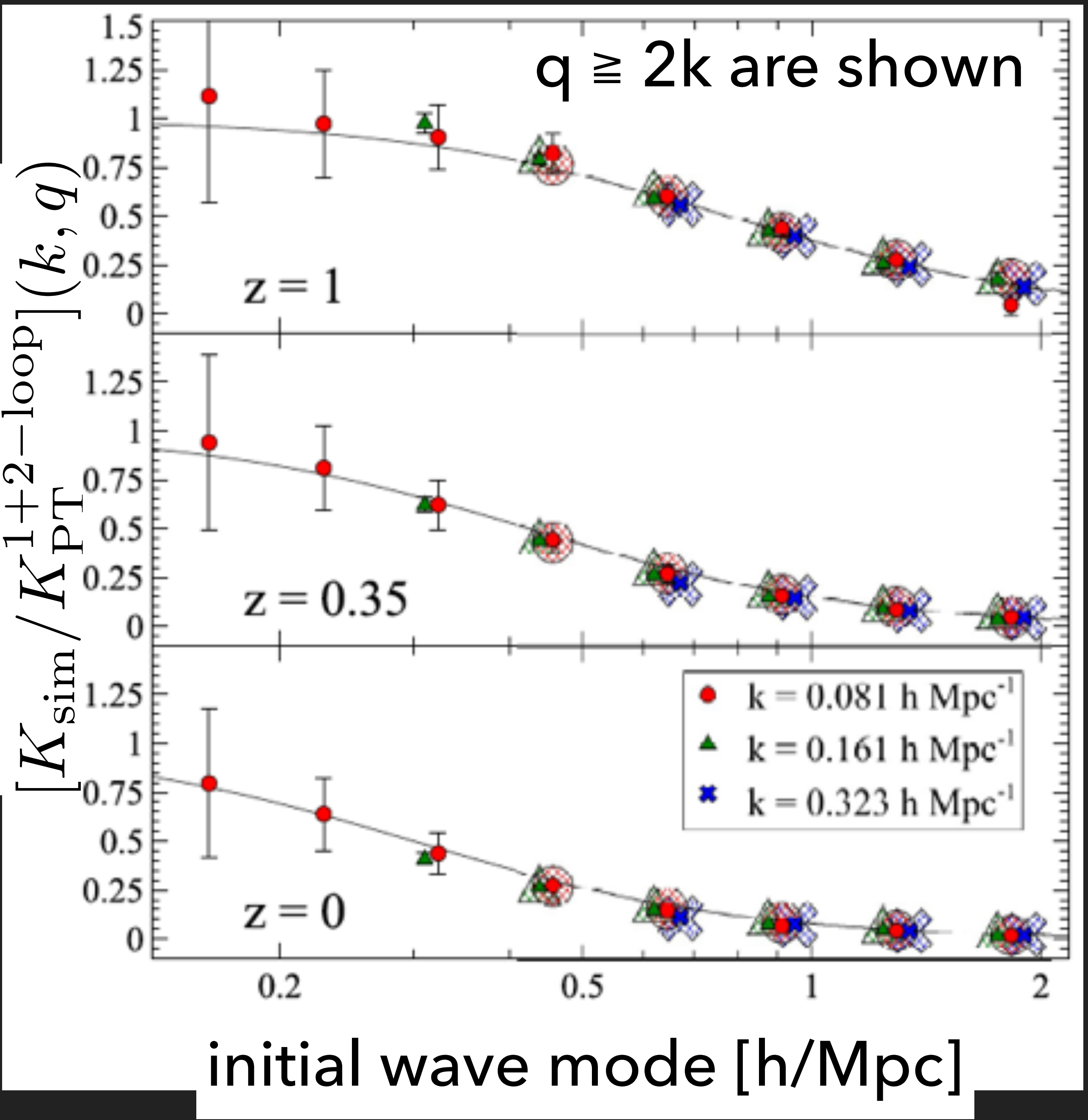


TN, Bernardeau, Taruya '14

- Overall shape fine
- Cancellation of terms at UV
  - (P_13 + P_22)
  - (P_15 + P_24 + P_33)
- UV looks more problematic
  - 2-loop > 1-loop > N-body
- Dominant terms at UV are:
  - P_13 (@ 1-loop), P_15 (@ 2-loop)
  - i.e., terms containing high-order 2-pt propagator

SUPPRESSION OF SMALL TO LARGE SCALE TRANSFER

$$K(k,q;z) = q\frac{\delta P^{\rm nl}(k;z)}{\delta P^{\rm lin}(q;z)}$$



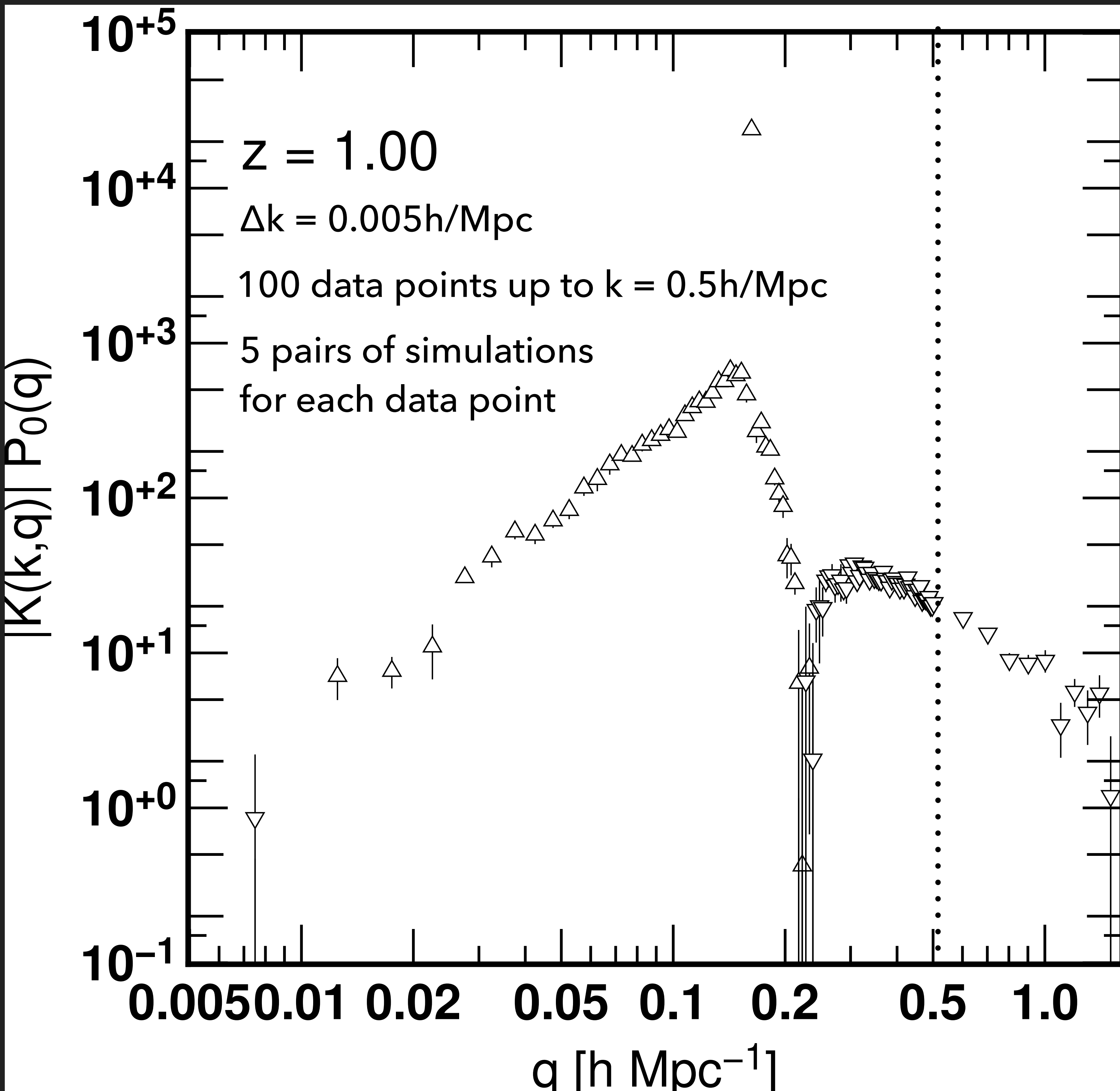
TN, Bernardeau, Taruya '14

- SPT >> simulation high q
- This is exactly the place where PT breaks down
- Simple Lorentzian form can nicely explain the suppression

$$\frac{1}{1+(q/q_0)^2}; \qquad q_0 \text{ independent of k}$$

- Large scale modes somehow protected from small scale uncertainty?
  - shell crossing? → Effective Field Theory?
  - The formula gives a quantitative guide to construct UV-safe models

FINE STRUCTURE IN THE RESPONSE FUNCTION



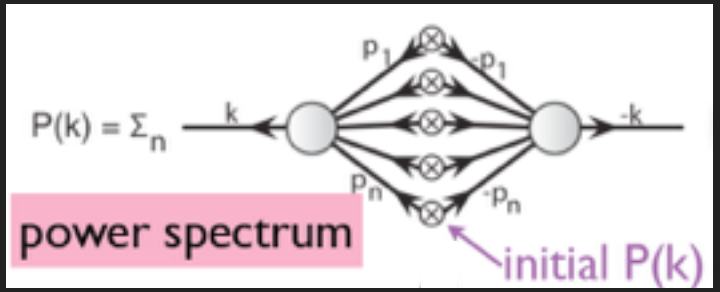
$$K(k,q;z) = q\frac{\delta P^{\rm nl}(k;z)}{\delta P^{\rm lin}(q;z)}$$

1400 simulations

10 sparsely sampled data points at 0.5 < k < 1.5

20 pairs of simulations for each data point

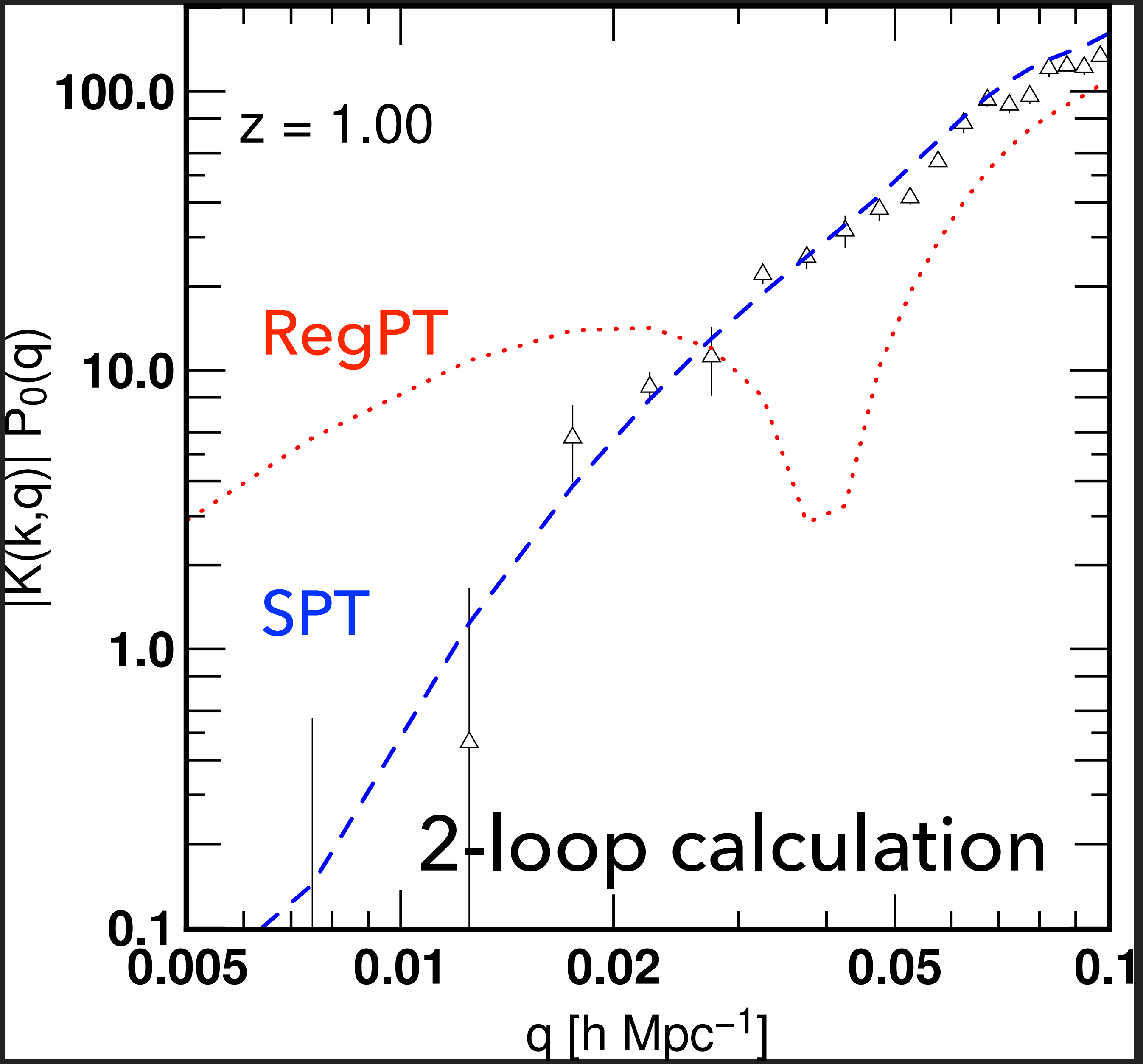
RENORMALIZATION AND RESPONSE



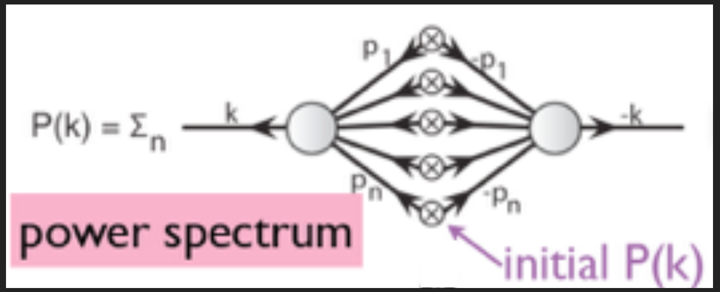
Γ expansion

- "Success" of 2-loop renormalized PT, while SPT 3-loop clearly breaks down
  - SPT is explicitly used to construct low k propagator in the calculations
  - Not a solution to the high-q crisis
- Γ expansion efficiently computes the mode coupling around q ~ k
  - very successful at high-z as this is the dominant place where modes couple
  - Galilean invariance is violated at low q
- Cover a wide dynamic range seamlessly by combining 3 models

data: 1400 sims.



RENORMALIZATION AND RESPONSE



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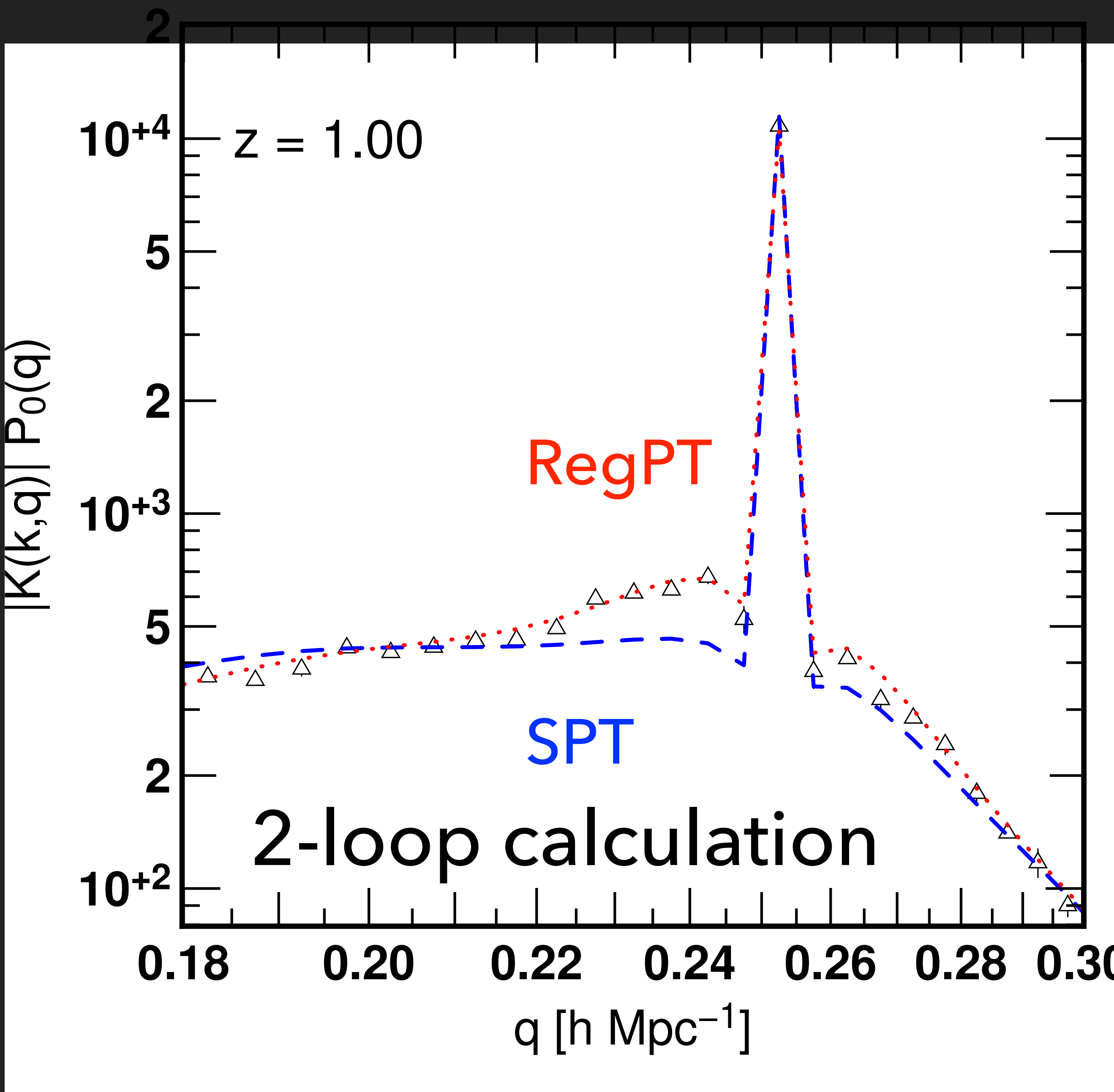

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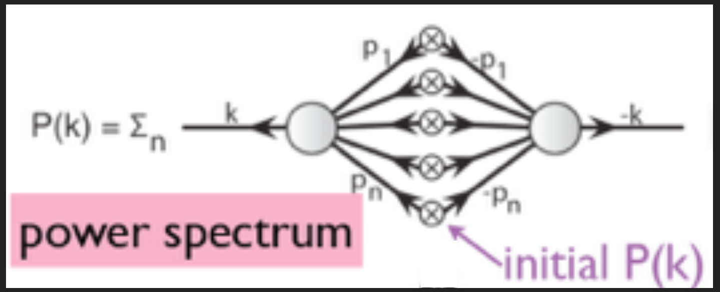

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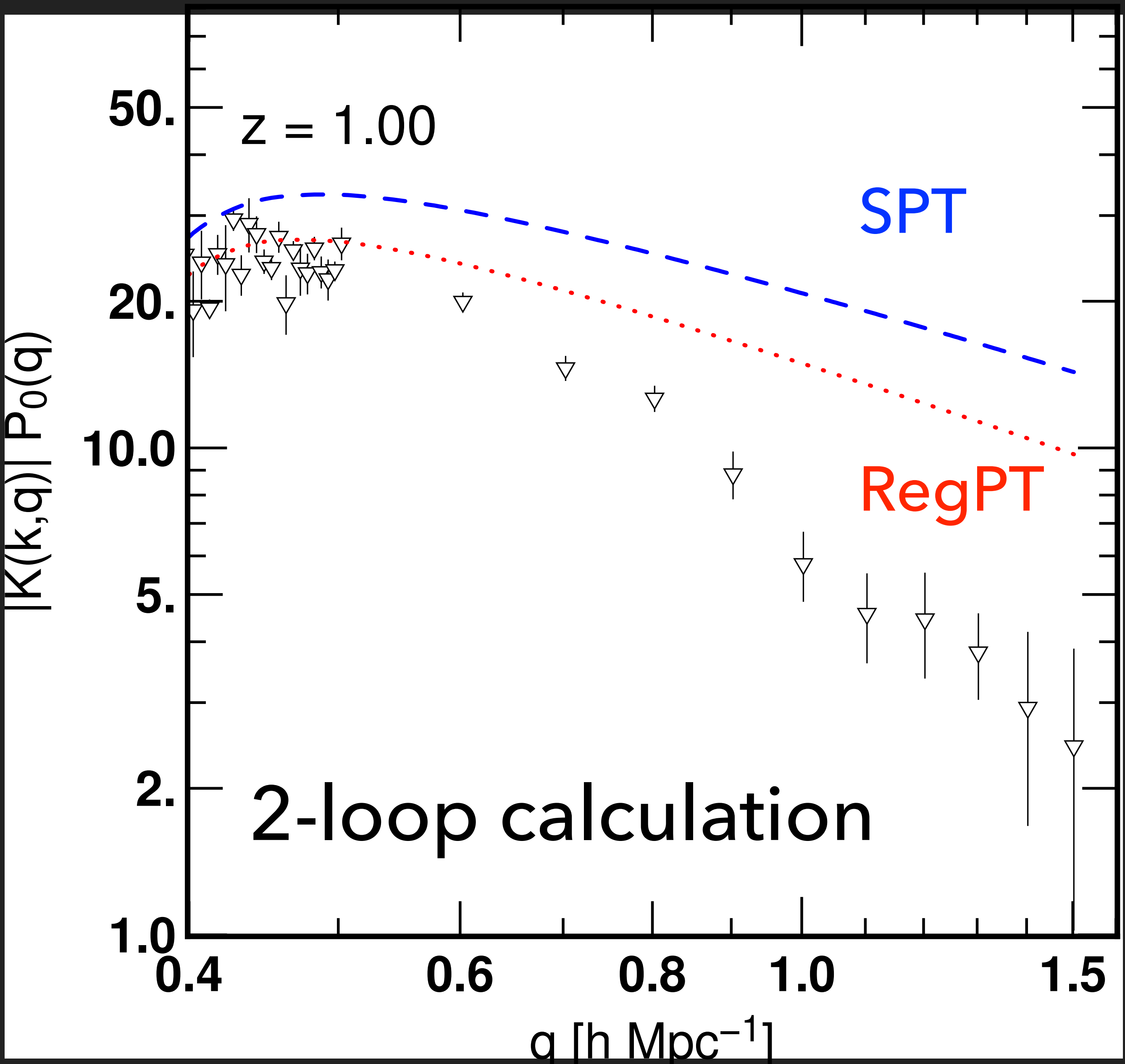
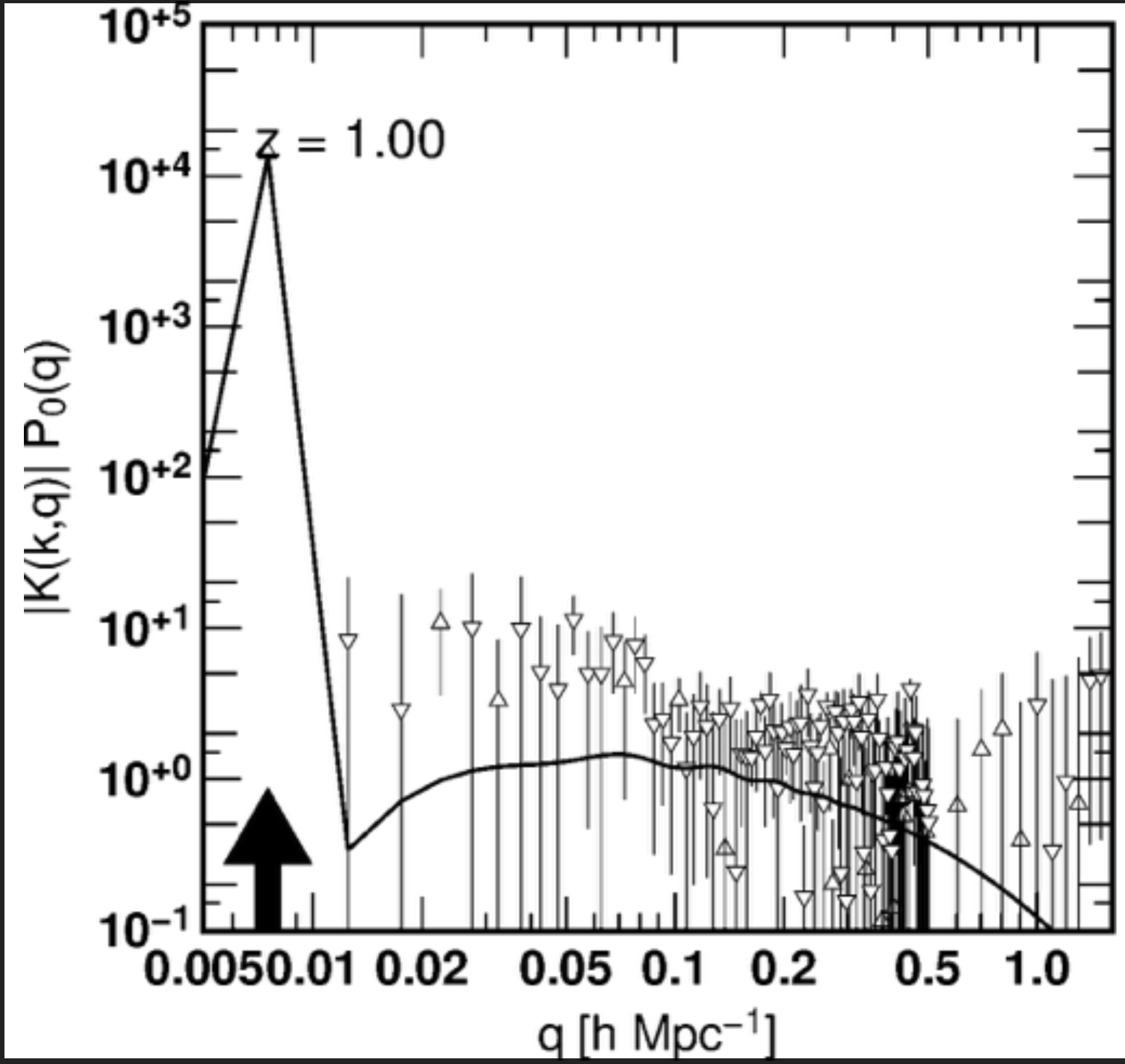


EFFECTIVE MODEL

data: 1400 sims.

- Calculation based on SPT
  - low-q is fine, automatically.
- 2-phenomenological regularizations
  - one, $\exp(-k^2\sigma_d^2)$ like RegPT
  - the other, $\exp(-q^2\sigma_d^2)$
- Cover a wide dynamic range seamlessly

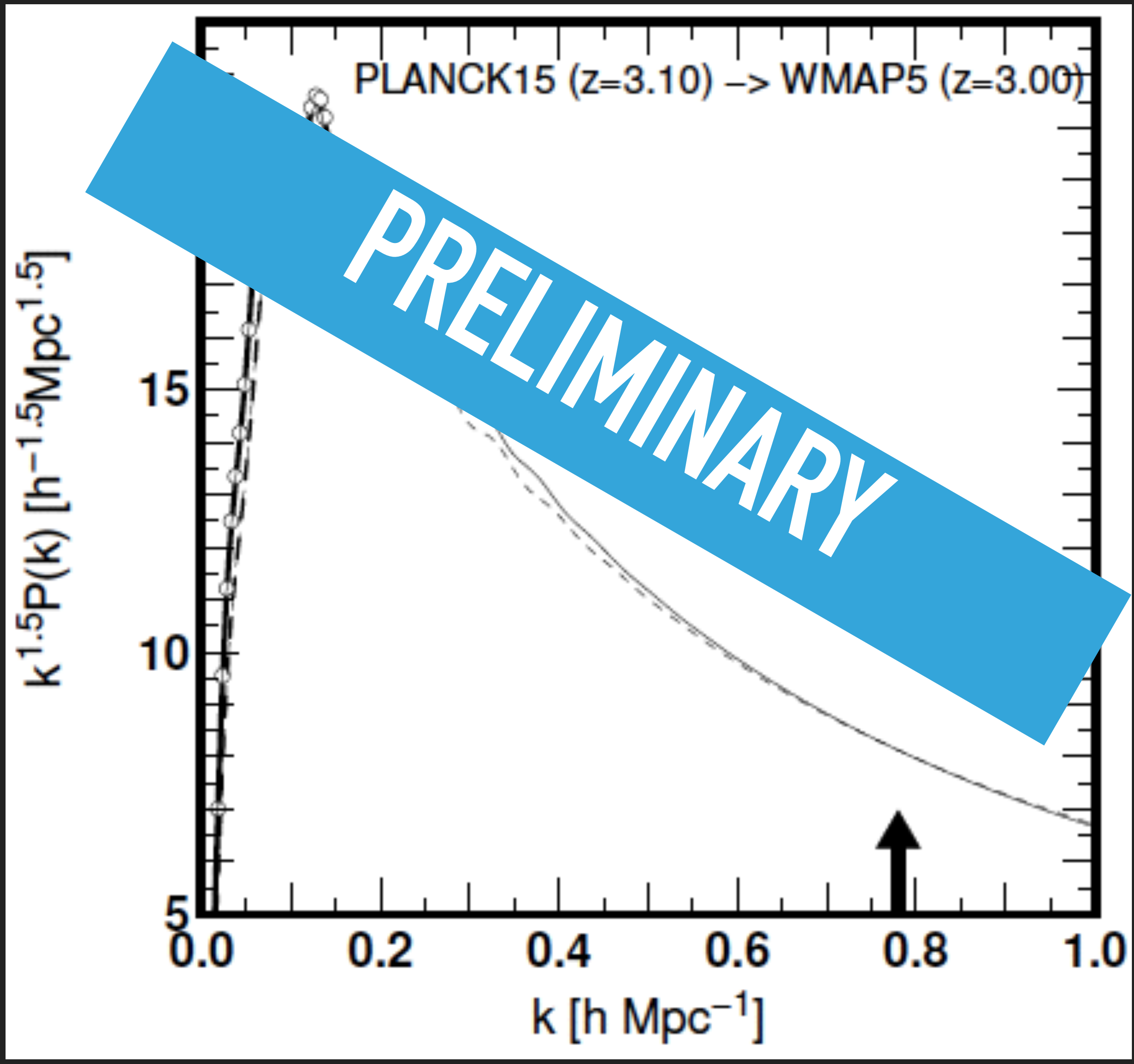


ANALYTICAL MODEL WITH HYBRID RESPONSE FUNCTION

from the definition of functional derivative

$$\Delta P_{\rm nl}(k) = \int {\rm d}\ln q\, K(k,q)\Delta P_{\rm lin}(q)$$

TN et al. in prep



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- ✓ suppressed variance by “fixed-and-paired” method (Angulo, Pontzen’16)
- ✓ -0.4 < z < 5, 20 outputs
- ✓ alias correction by “interlacing” method (Sefusatti+’16)

→ Prediction for WMAP5 cosmology

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ANALYTICAL MODEL WITH HYBRID RESPONSE FUNCTION

from the definition of functional derivative

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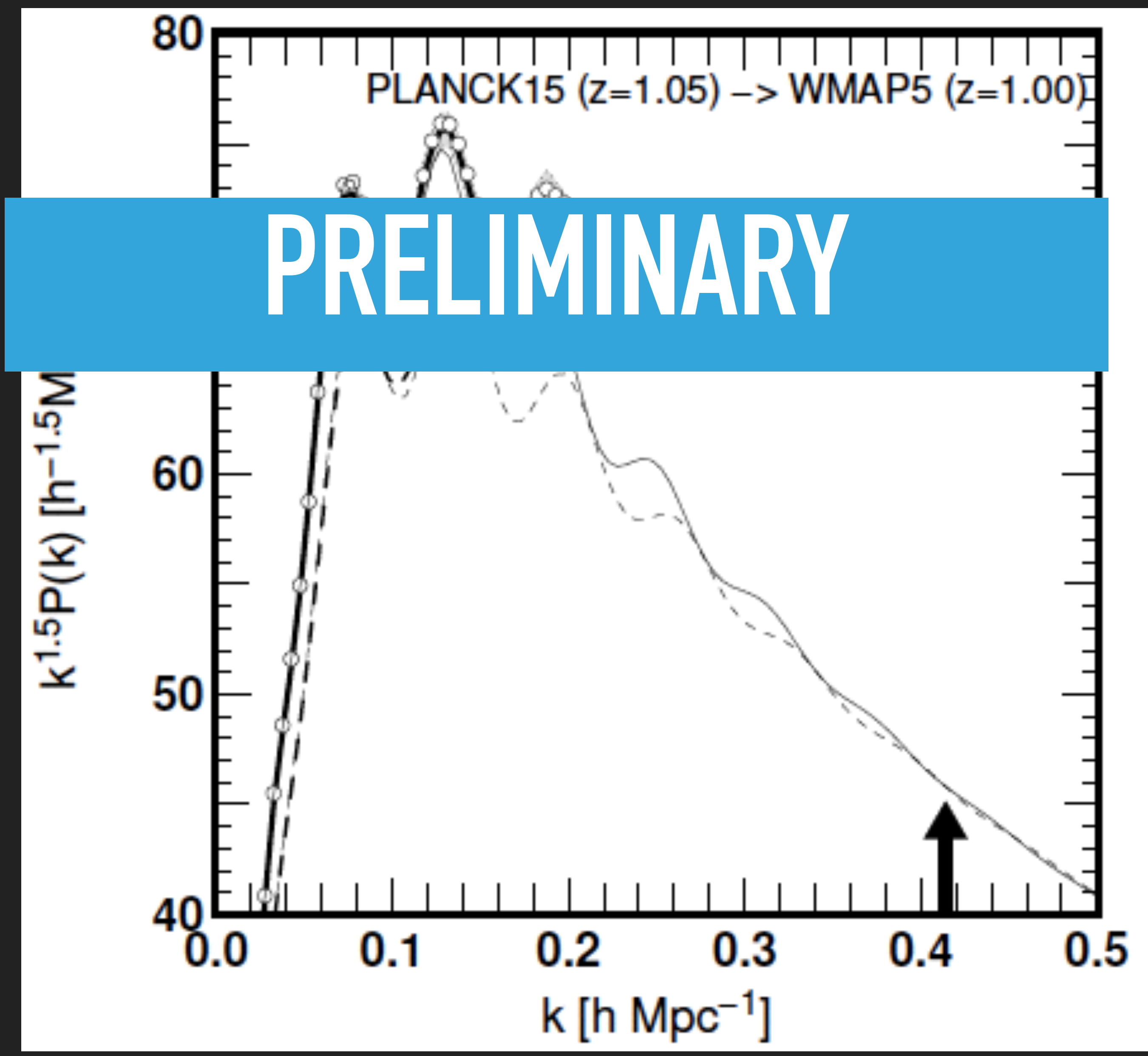

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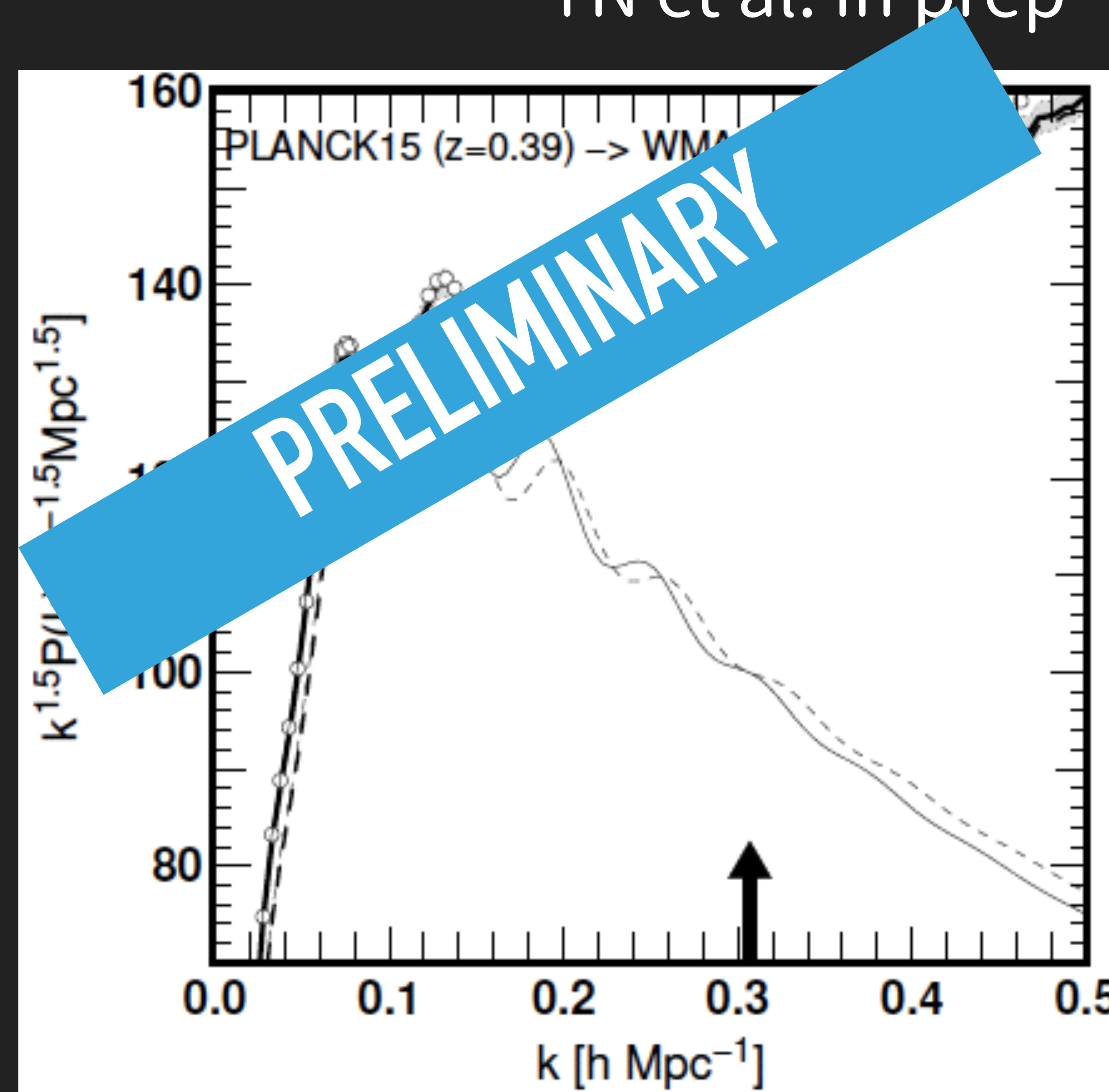

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