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CoSKASI-ICG-NAOC-YITP workshop

Toward

Perturbation Theory of Large-scale Structure beyond Shell-crossing

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Crisis in perturbation theory ?!

Perturbation theory (PT) of large-scale structure as a tool for precision cosmology is now getting in trouble

***Single-stream* approximation of Vlasov-Poisson system**

Phase-space distribution function

$$f(\boldsymbol{x},\,\boldsymbol{v};\,t)$$

$$\rightarrow\ \overline{\rho}(t)\ \{1+\delta(\boldsymbol{x};\,t)\}\ \delta_{\rm D}\,(\boldsymbol{v}-\boldsymbol{v}(\boldsymbol{x};\,t))$$

$$\frac{\partial\delta}{\partial t}+\frac{1}{a}\vec{\nabla}\cdot\left[(1+\delta)\vec{v}\right]=0$$

$$\frac{\partial\vec{v}}{\partial t}+\frac{\dot{a}}{a}\vec{v}+\frac{1}{a}(\vec{v}\cdot\vec{\nabla})\vec{v}=-\frac{1}{a}\vec{\nabla}\Phi$$

$$\frac{1}{a^2}\nabla^2\Phi=4\pi G\,\overline{\rho}_{\rm m}\,\delta$$

$$\Rightarrow\quad \delta=\delta^{(1)}+\delta^{(2)}+\cdots$$

- Higher-order mode-coupling gets a larger UV contribution

Blas, Garny & Konstandin ('14), Bernardeau, AT & Nishimichi ('14)

However !

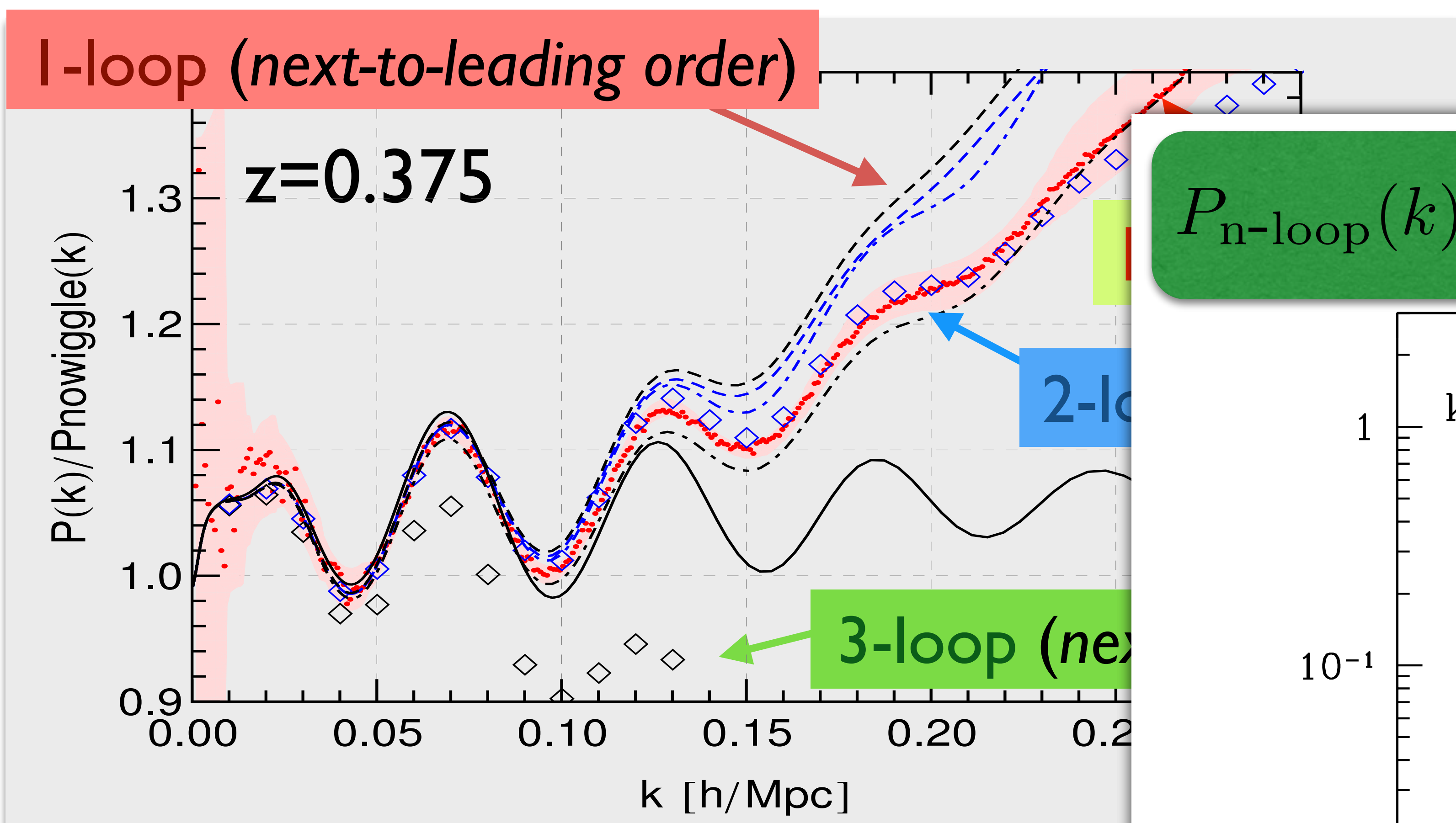
- In simulation, actual UV contribution is suppressed

Nishimichi, Bernardeau & AT ('14, '16 in prep.)

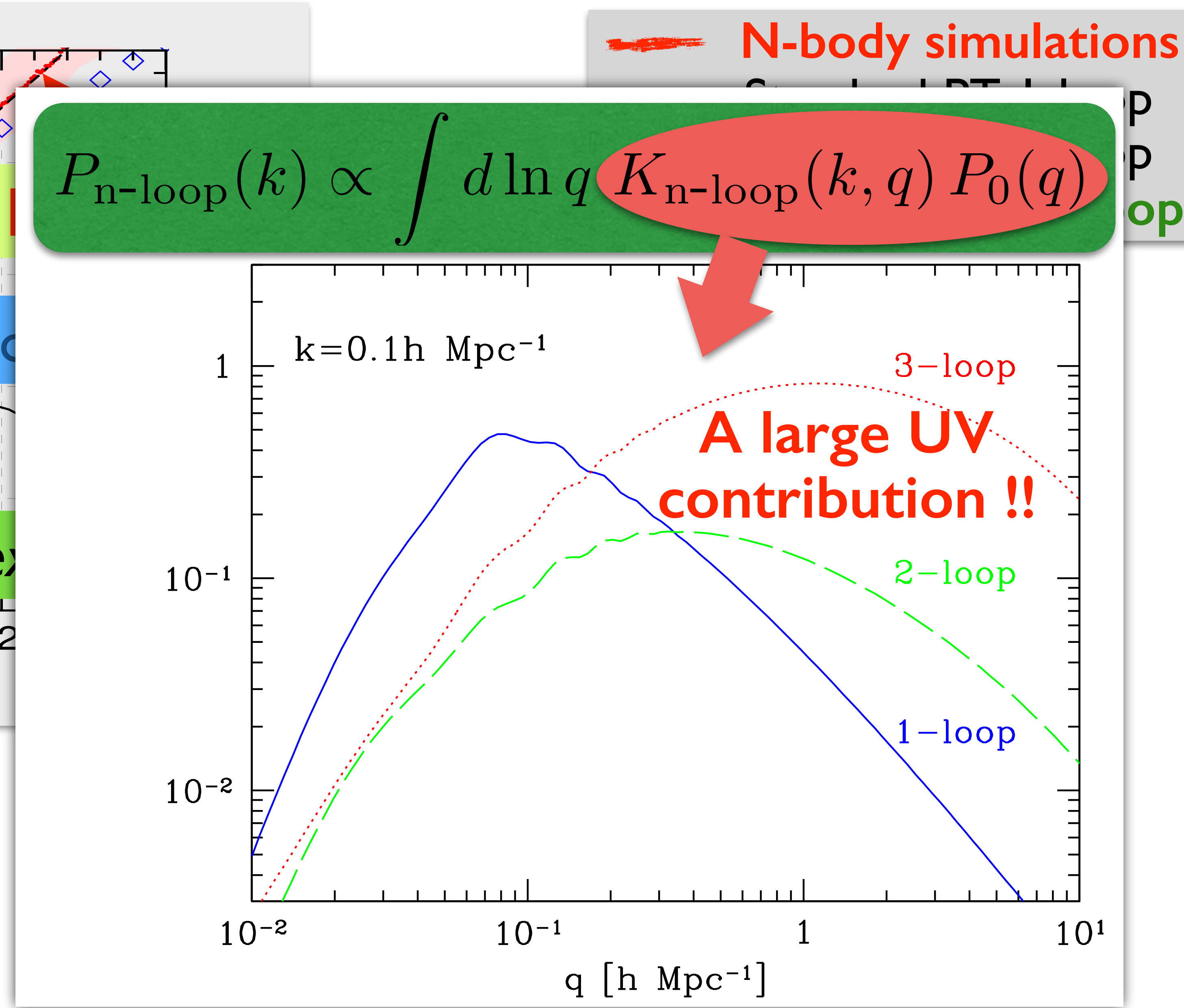
→ Taka's Talk

UV problem

When we go to ***3-loop*** (*next-to-next-to-next-to-leading order*), we eventually get a very big correction !!



Blas et al. ('14)



Bernardeau, AT & Nishimichi ('14)

What's wrong ?

"*UV-sensitive*" behavior in PT indicates that single-stream PT cannot properly deal with <u>small-scale physics</u>

(e.g., formation/merging of halos)

Break down of single-stream PT

What should we do ?

- *<u>EFT approach</u>* to remove the UV sensitivity in single-stream PT

Baumann et al. ('12), Carrasco, Herzberg & Senatore ('12), Carrasco et al. ('13ab), …

- *<u>Full-numerical approach</u>* with N-body simulations

Lawrence et al. ('10), Heitmann et al. ('14), …

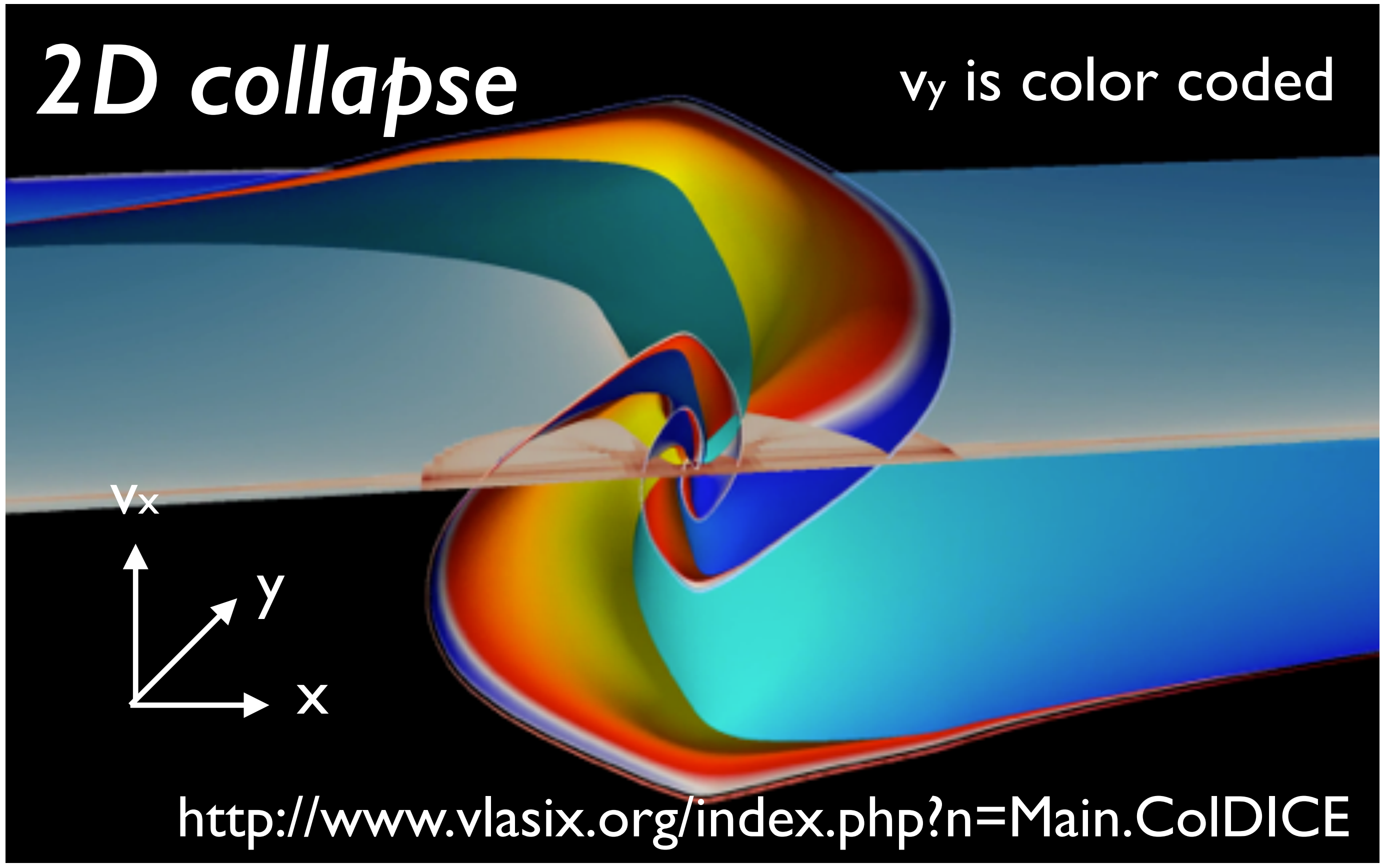
But, what more ?

Beyond single-stream PT

Is there a way to go beyond single-stream approximation ?

Key ingredient

Multi-stream dynamics



Aim of this work

- A perturbative description of multi-steam flow
- A simple recipe to '*regularize*' impact of multi-stream dynamics

→ Learn something in simple *1D cosmology*

1D Zel'dovich solution

(Zel'dovich '70)

Exact solution

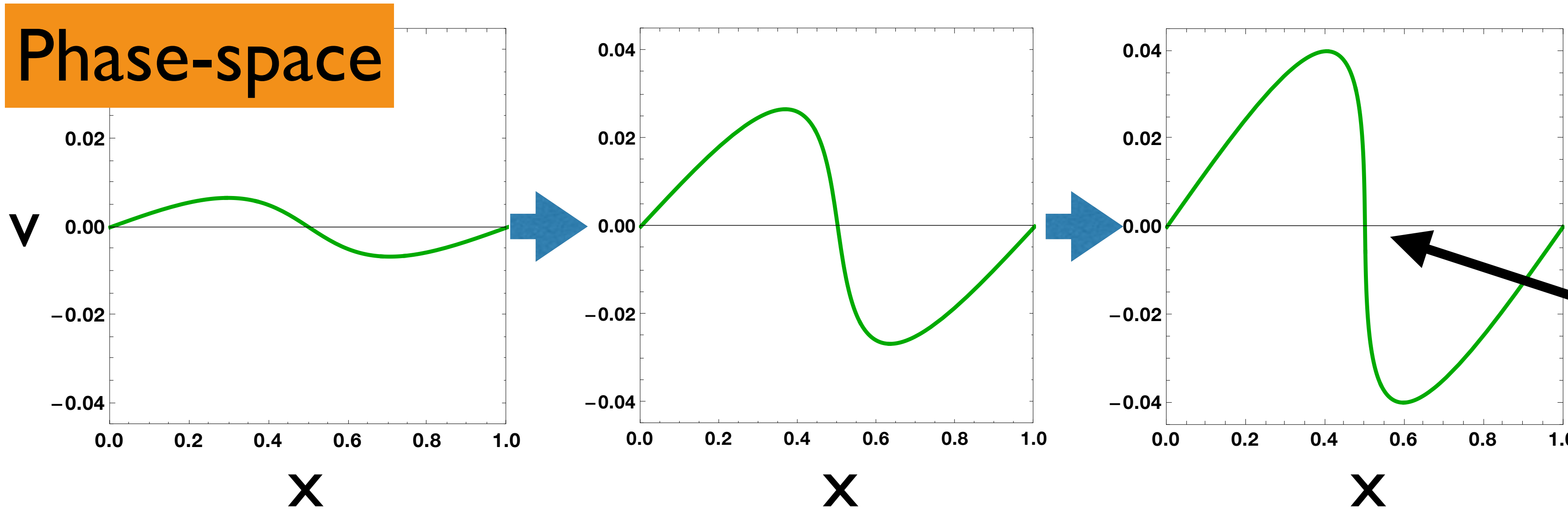
$$x(q;\tau) = q + \psi(q)\, D_+(\tau)$$

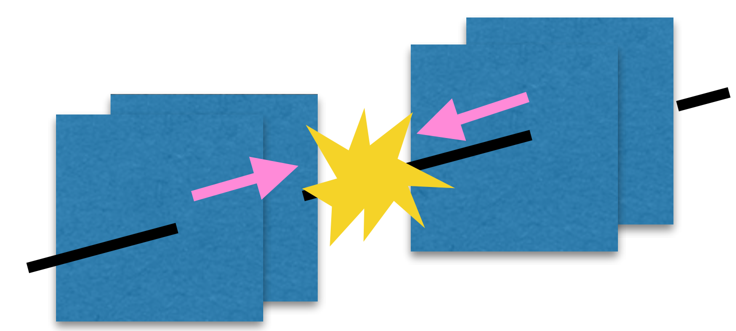
$$\mathrm{v}(q;\tau) = \psi(q)\, \frac{dD_+(\tau)}{d\tau}$$

$D_+(\tau)$: linear growth factor

$\psi(q)$: displacement field

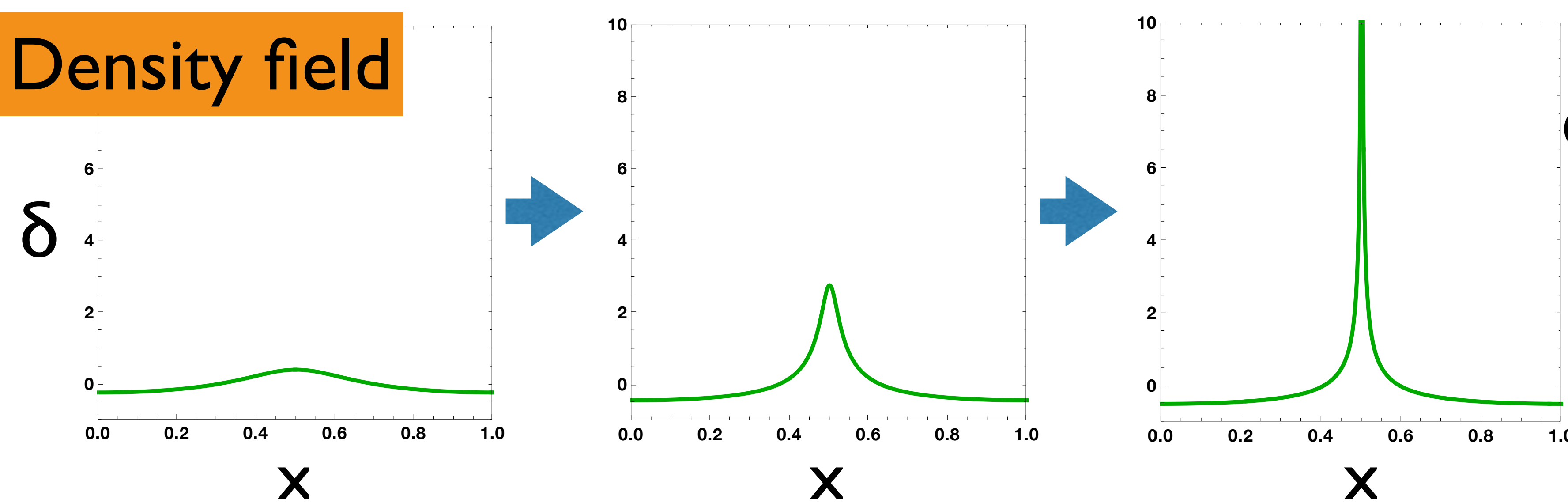
Phase-space





Shell crossing

Density field



Solution is no longer exact after shell crossing

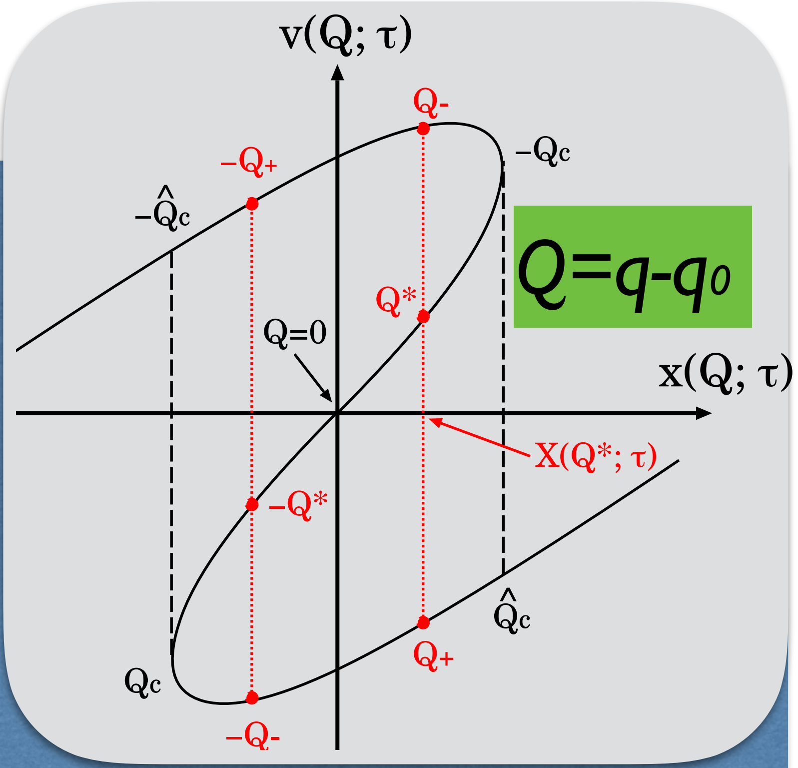
multi-stream flow

Post-collapse PT: *beyond shell crossing*

Colombi ('15), AT & Colombi ('16, to appear soon)

Basic treatment

Starting with Zel'dovich flow,



1. Find shell-crossing (collapse) point, q_0
2. Expand the displacement field around q_0:

$$x(q;\tau) \simeq A(q_0;\tau) - B(q_0;\tau)(q-q_0) + C(q_0;\tau)(q-q_0)^3$$

3. Compute force $F(x(q;\tau)) = -\nabla_x \Phi(x(q;\tau))$ at multi-stream region

Back-reaction to the Zel'dovich flow :

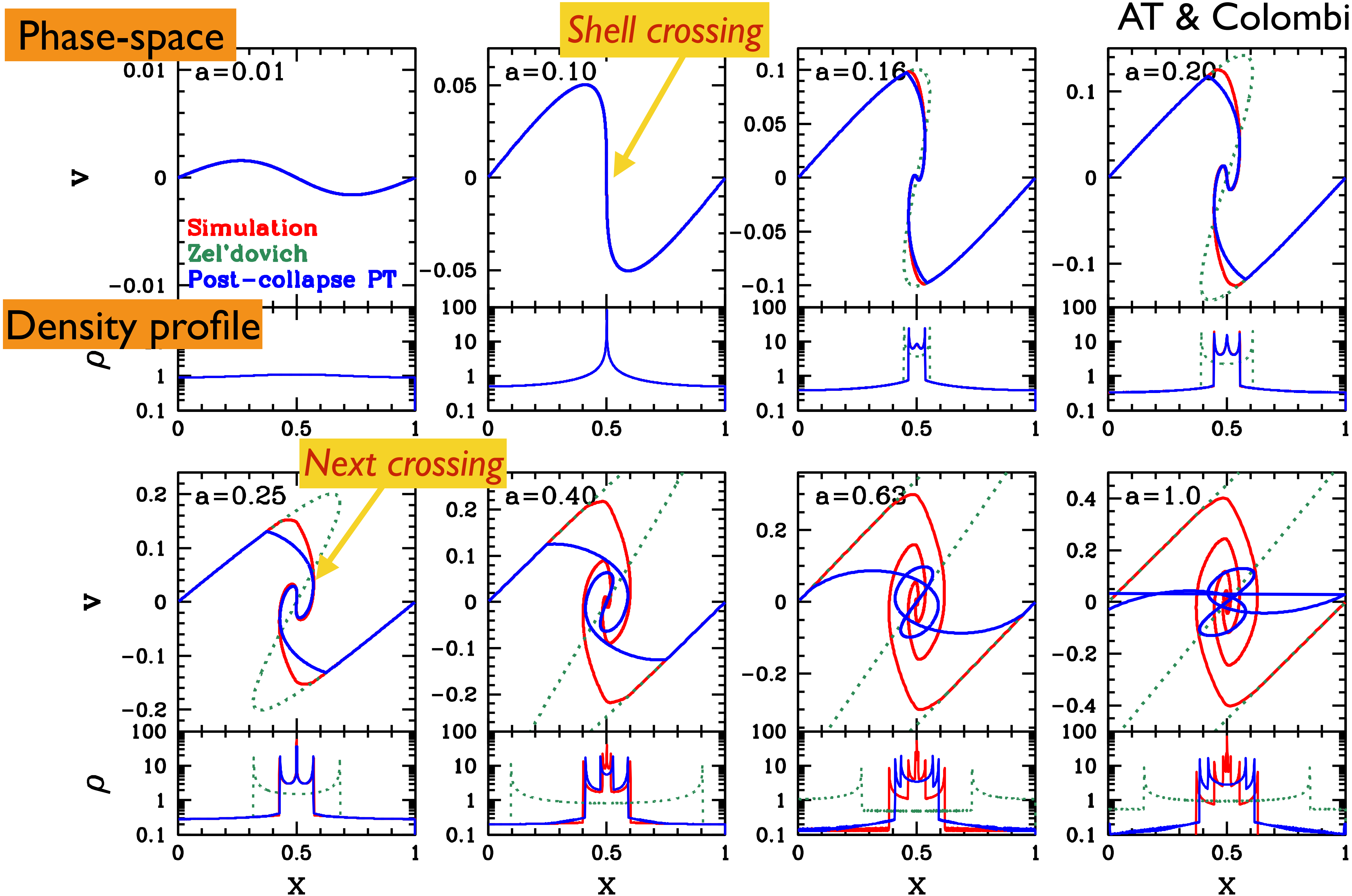
$$\Delta \mathrm{v}(Q;\tau,\,\tau_{\mathrm{q}}) = \int_{\tau_{\mathrm{q}}}^{\tau} d\tau'\,F(x(Q,\tau')), \quad \Delta x(Q;\tau,\,\tau_{\mathrm{q}}) = \int_{\tau_{\mathrm{q}}}^{\tau} d\tau'\,\Delta \mathrm{v}(Q;\tau',\tau_{\mathrm{q}})$$

…… polynomial function of $Q=q-q_0$ *up to 7th order*

(expressed in terms of the density peaks)

Single-cluster formation

AT & Colombi ('16)

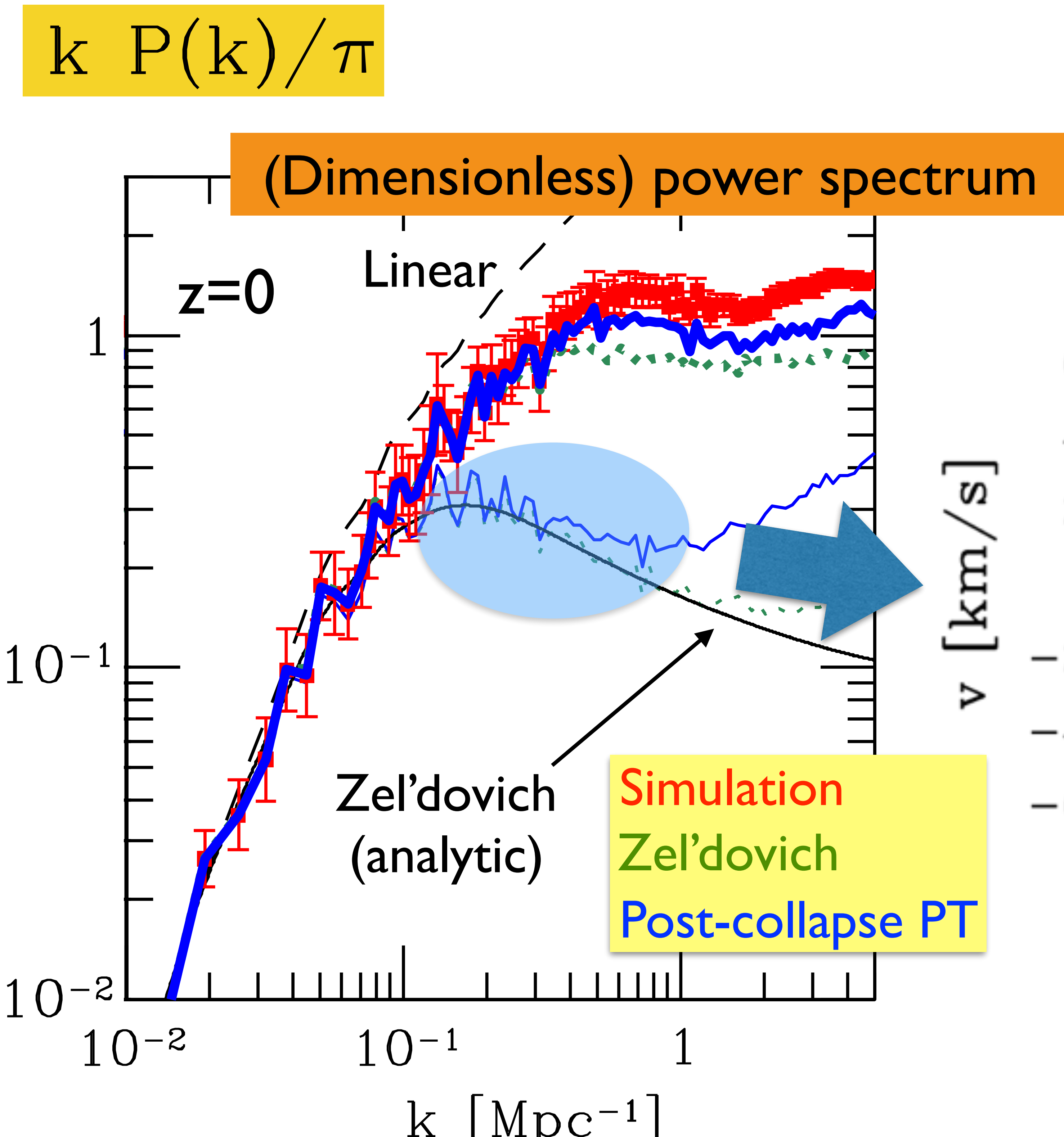


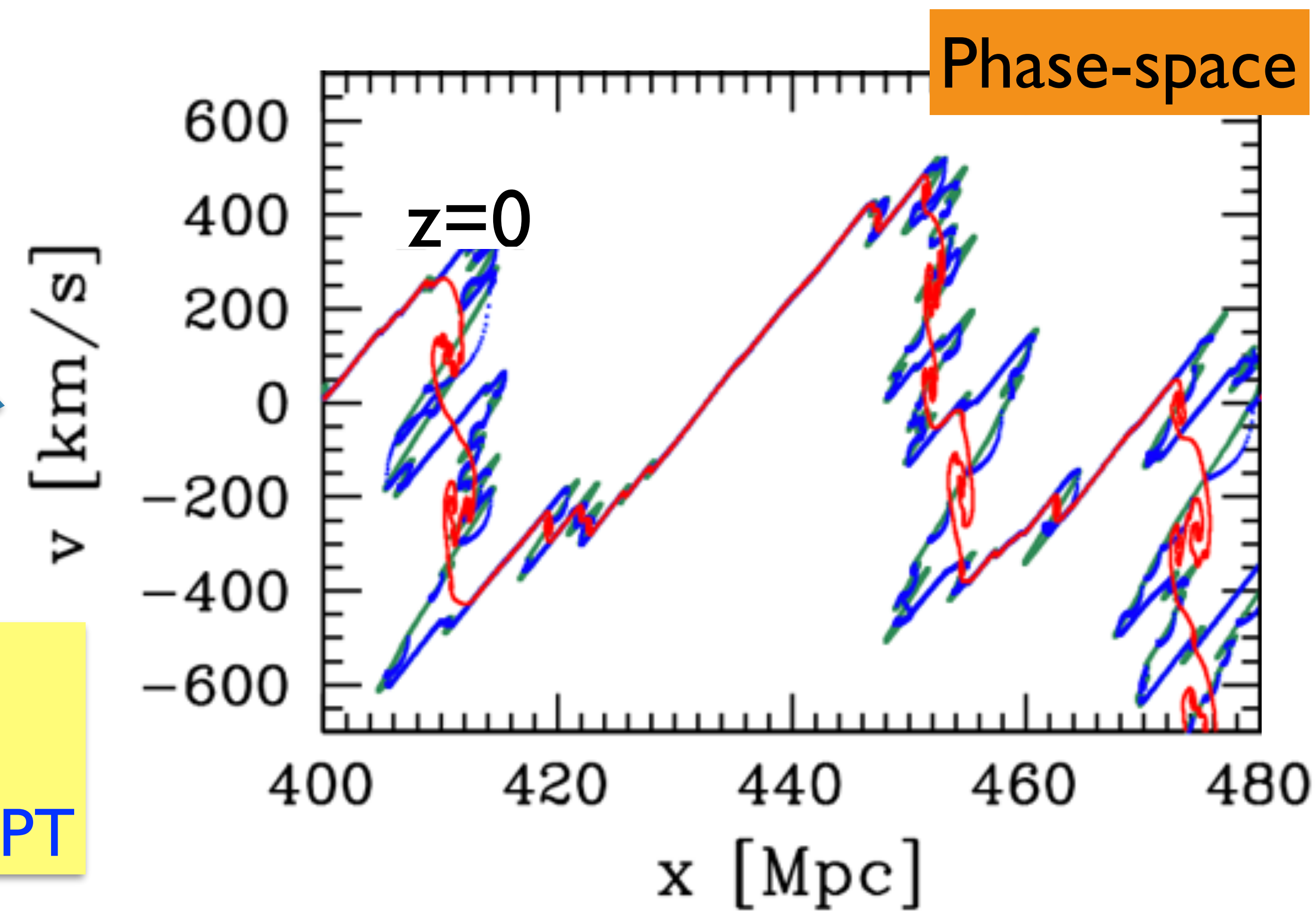
Results: CDM-like initial condition

AT & Colombi ('16)

$$P_{\rm 1D}(k) = \frac{k^2}{2\pi} P_{\rm 3D}(k)$$

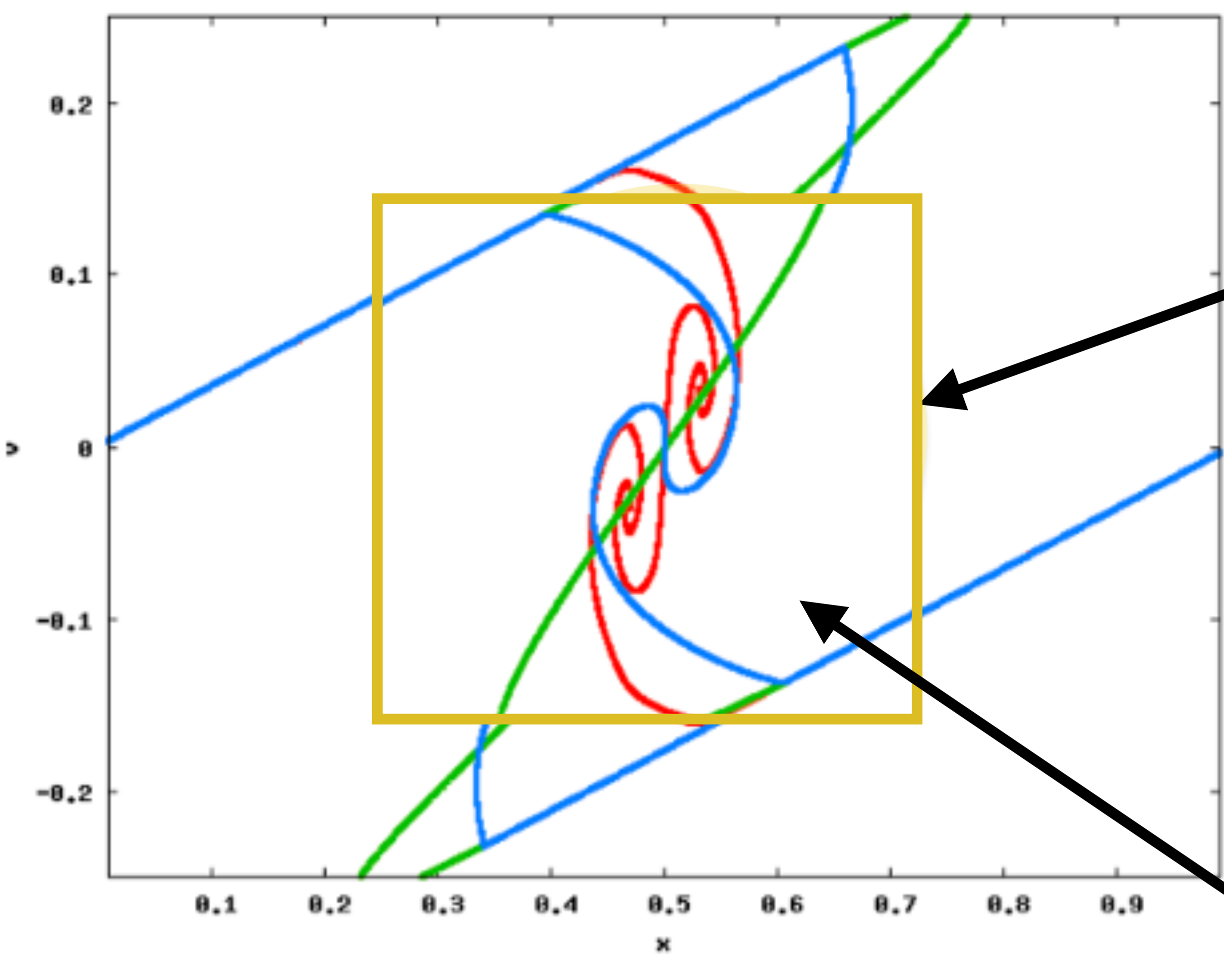
Planck ΛCDM





‘Regularizing’ small-scale structure

Adaptive smoothing
(‘object-by-object’ smoothing)



similar to peak-patch treatment
by Bond & Myers (’96)

1. Construct smoothed initial density fields at various scales:

sharp-k filter: $W(k)=\Theta(k_{*}-k)$

2. Choose the filter scale with which the shell-crossing point *first exceeds* next-crossing time

$$\tau_{\rm cross}(q_0)\equiv\frac{\delta_{\rm L}(q_0)\,\frac{dD_+(\tau_0)}{d\tau_0}}{\frac{3}{2}H_0^2\Omega_{\rm m,0}\,a(\tau_0)}$$

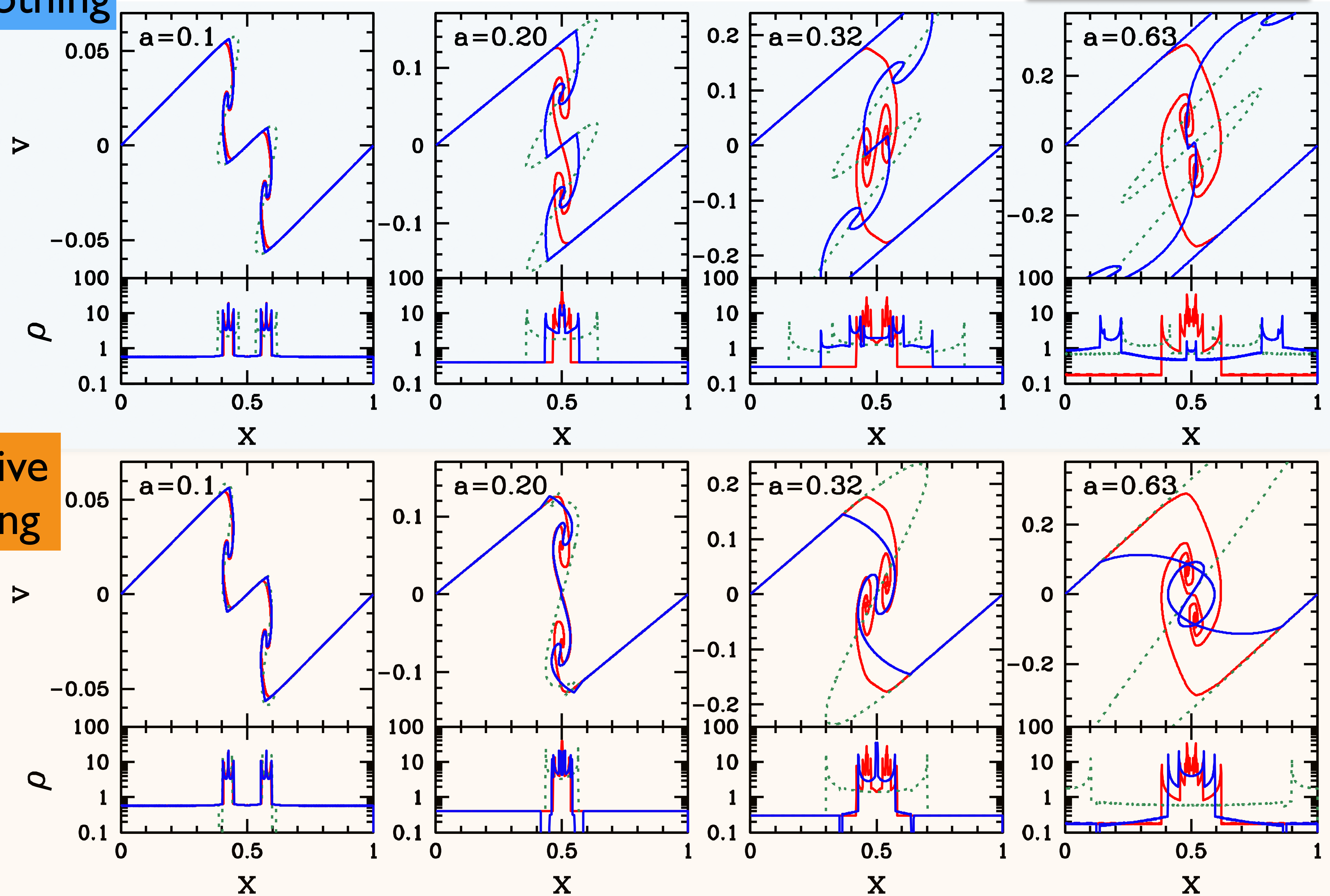
3. Apply post-collapse PT to the smoothed initial density field

Merging clusters

Simulation
Zel'dovich
Post-collapse PT

No smoothing

w/ adaptive smoothing

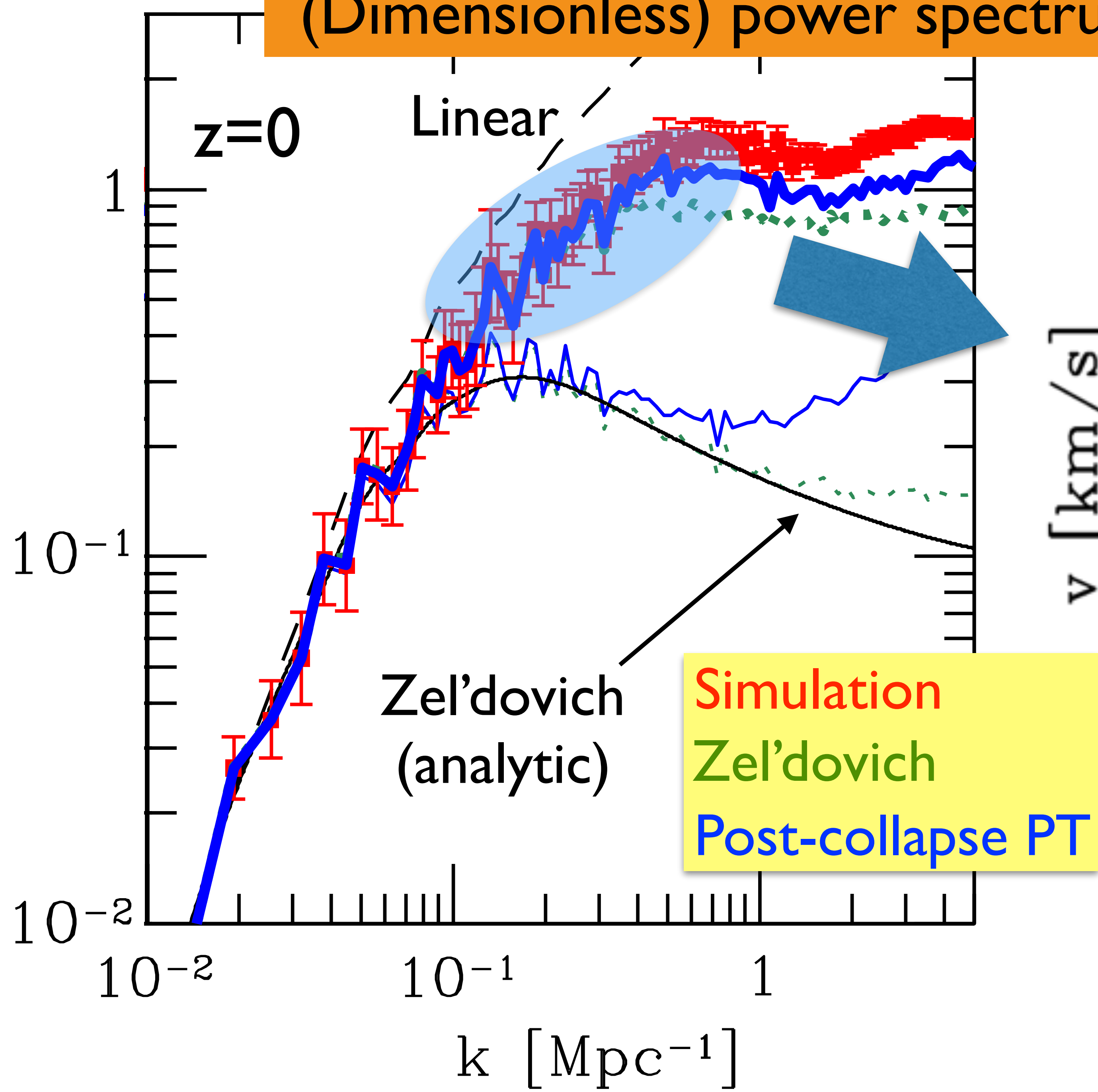


Results: CDM-like initial condition

AT & Colombi ('16)

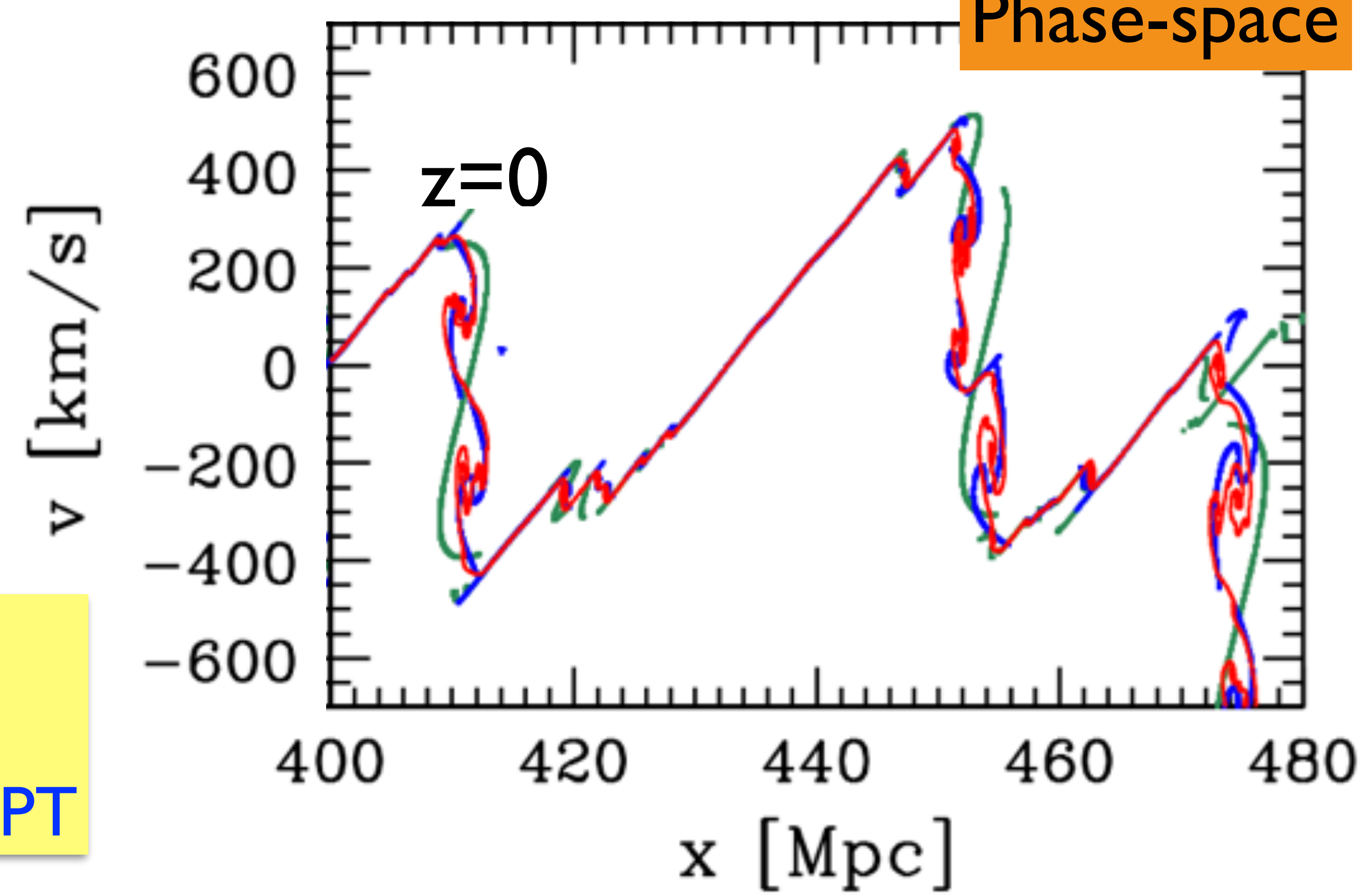
$k\ P(k)/\pi$

(Dimensionless) power spectrum



with adaptive smoothing

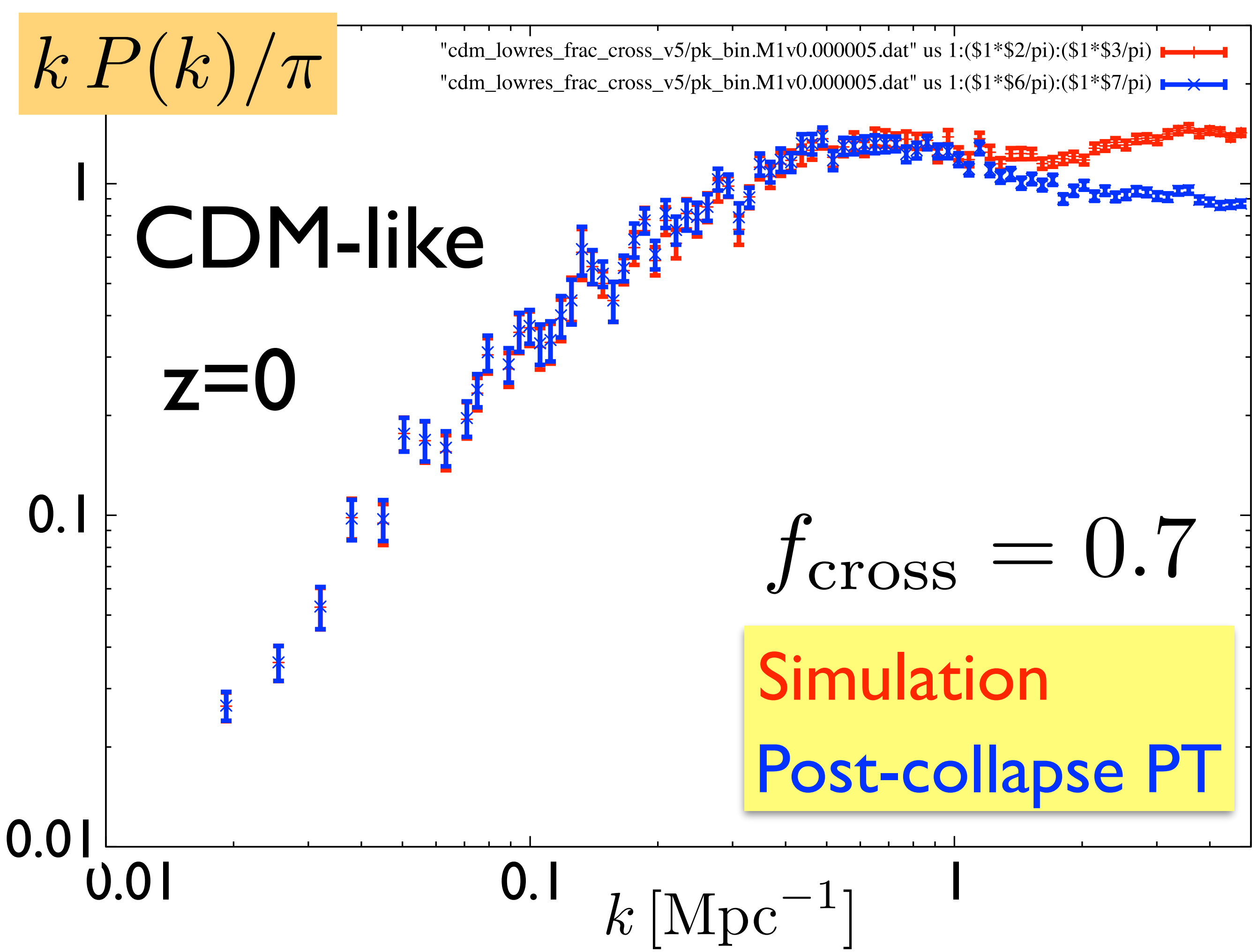
Phase-space

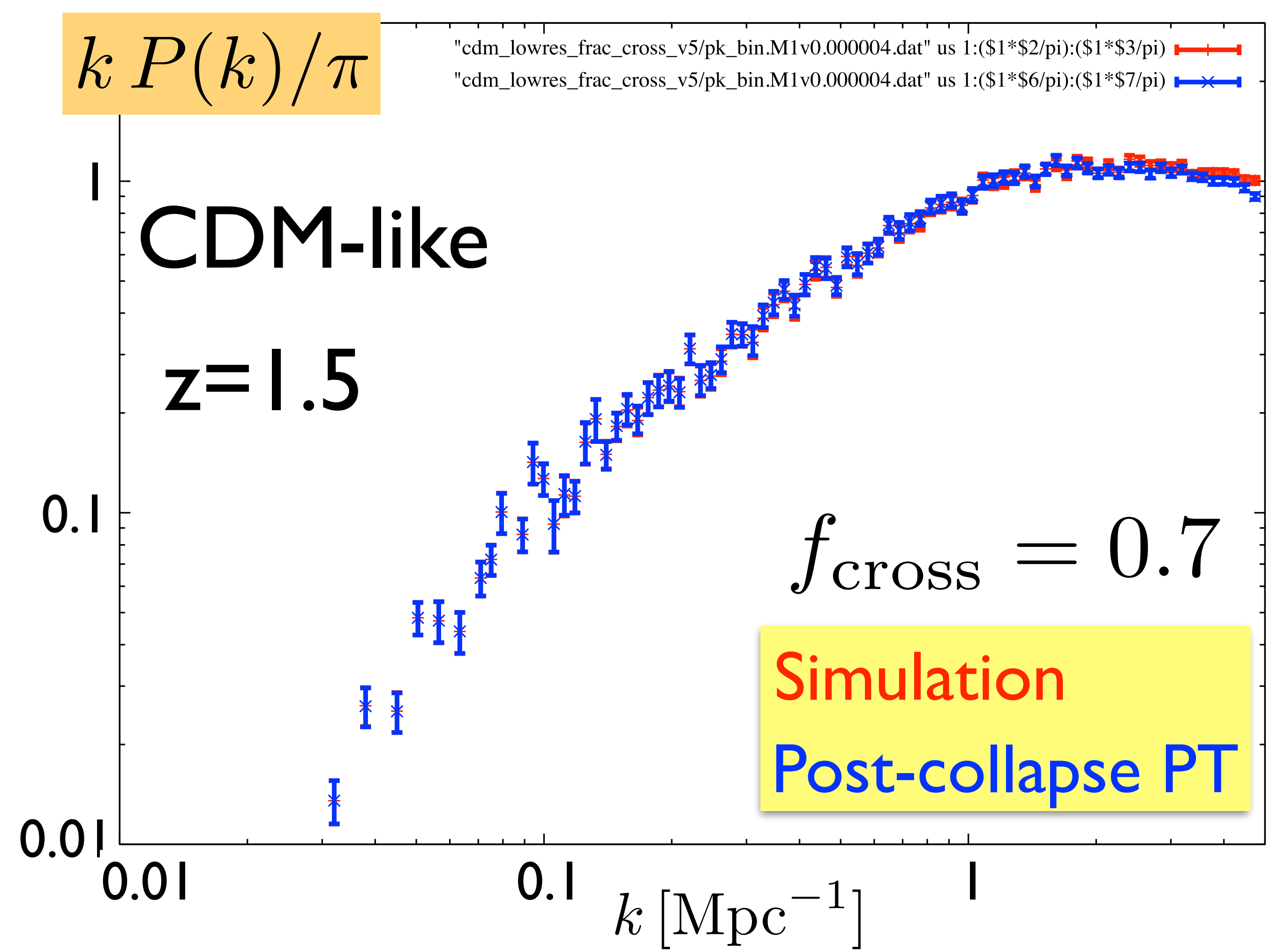


Tuning filter scales

Tuning the parameter to choose the filter scale can further improve the predictions :

2. Choose the filter scale with which the shell-crossing point *first* *exceeds* next-crossing time $\tau_{\rm cross}$ ⟶ $f_{\rm cross}\,\tau_{\rm cross}$





Summary

1D cosmology offers a testing ground for new PT treatment beyond single-stream approx.

- ✦ "Post-collapse perturbation theory" that can deal with multi-stream dynamics
- ✦ "Adaptive smoothing" that can handle small-scale dynamics

Improved predictions beyond single-stream regime

(good agreement with simulations at k<0.6 Mpc^-1)

Hints & clues for extension to 3D cosmology