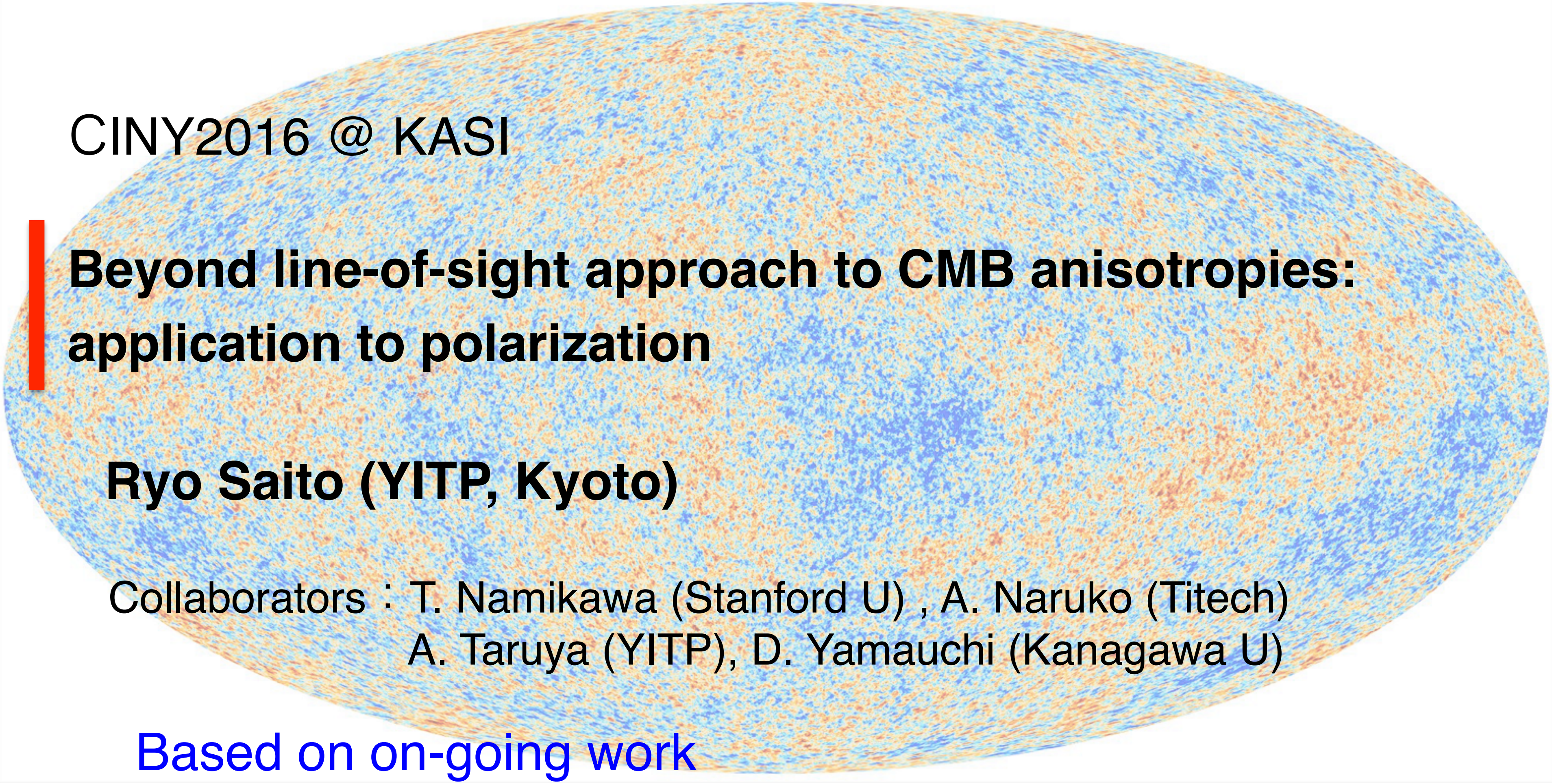

CINY2016 @ KASI

Beyond line-of-sight approach to CMB anisotropies: application to polarization

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Collaborators : T. Namikawa (Stanford U) , A. Naruko (Titech)
A. Taruya (YITP), D. Yamauchi (Kanagawa U)

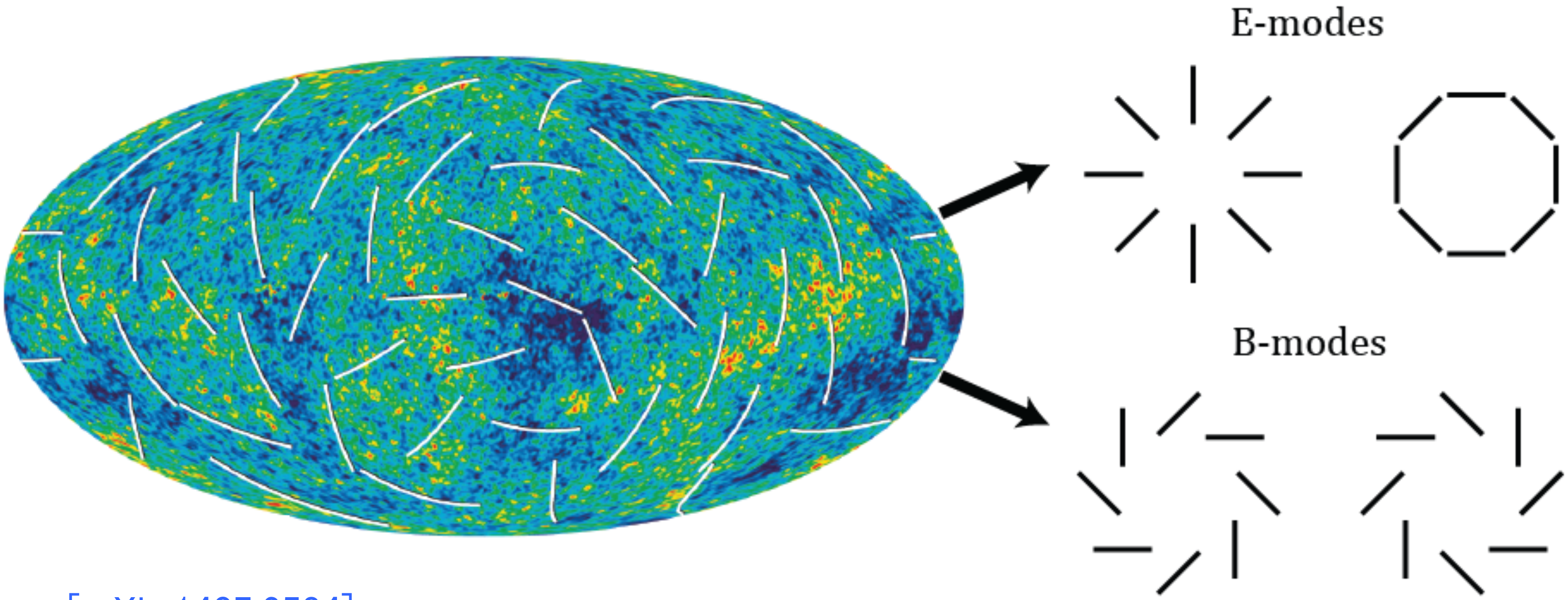
Based on on-going work

CMB polarizations

Parity decomposition of CMB polarization map:

$$E_{lm} \pm iB_{lm} \sim \int \mathrm{d}^2 n \left[Y_{lm}^{\mp 2}(\mathbf{n}) \right] \left[Q(\mathbf{n}) \pm iU(\mathbf{n}) \right]$$

Stokes parameters



From [arXiv:1407.2584]

Two important sources of B-modes: primordial GW and **lensing**

lensing induced B-modes

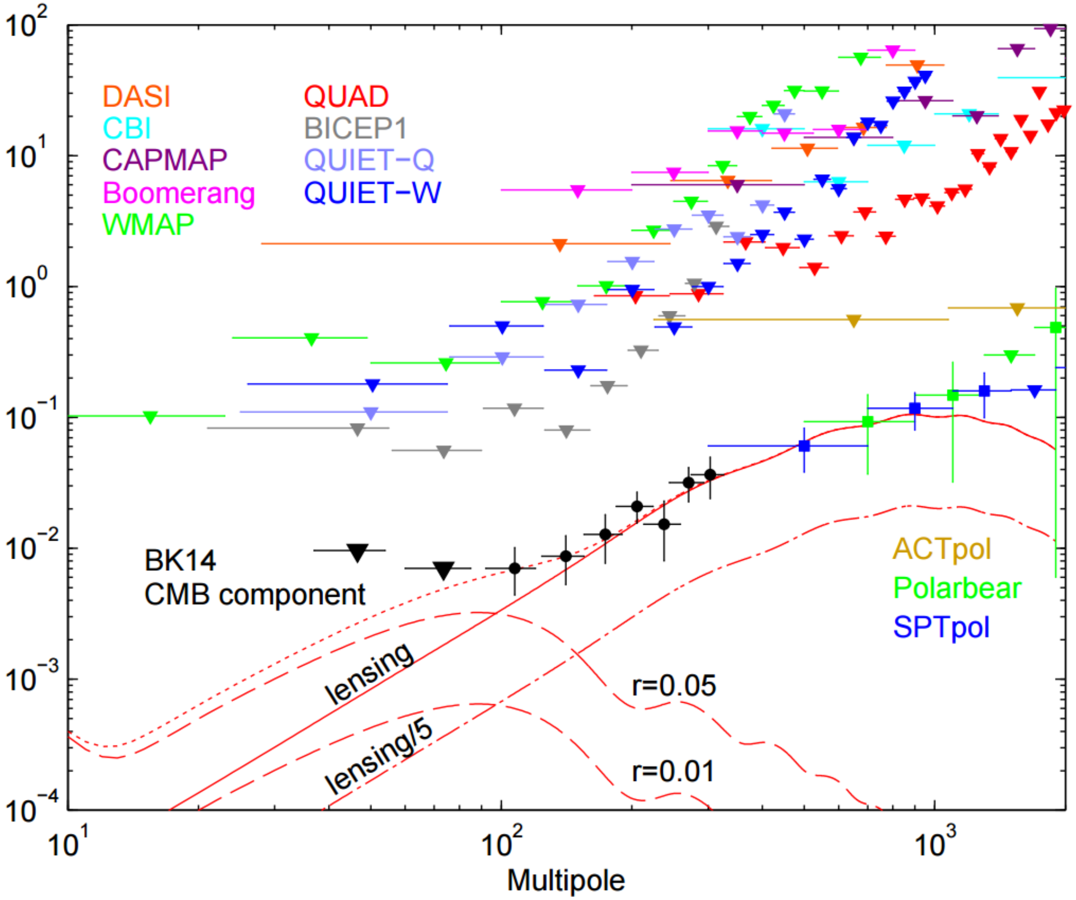
- Leakages from E-modes due to E-B mixing by lensing.

- Main contrib. on small scales.

- “Detected” via the correlations w/ CIB, LSS, E-modes.

- Informative of
  the Universe after LSS
  (DE, neutrinos,…)

- Contaminant for
  the inflationary signal
  (B-modes from primordial GW)



[http://polar-array.stanford.edu/science.html]

- Can be subtracted (**delensing**), *if we know well how CMB is lensed.*

[Knox+ 02; Kesden+ 02; Seljak & Hirata 04]

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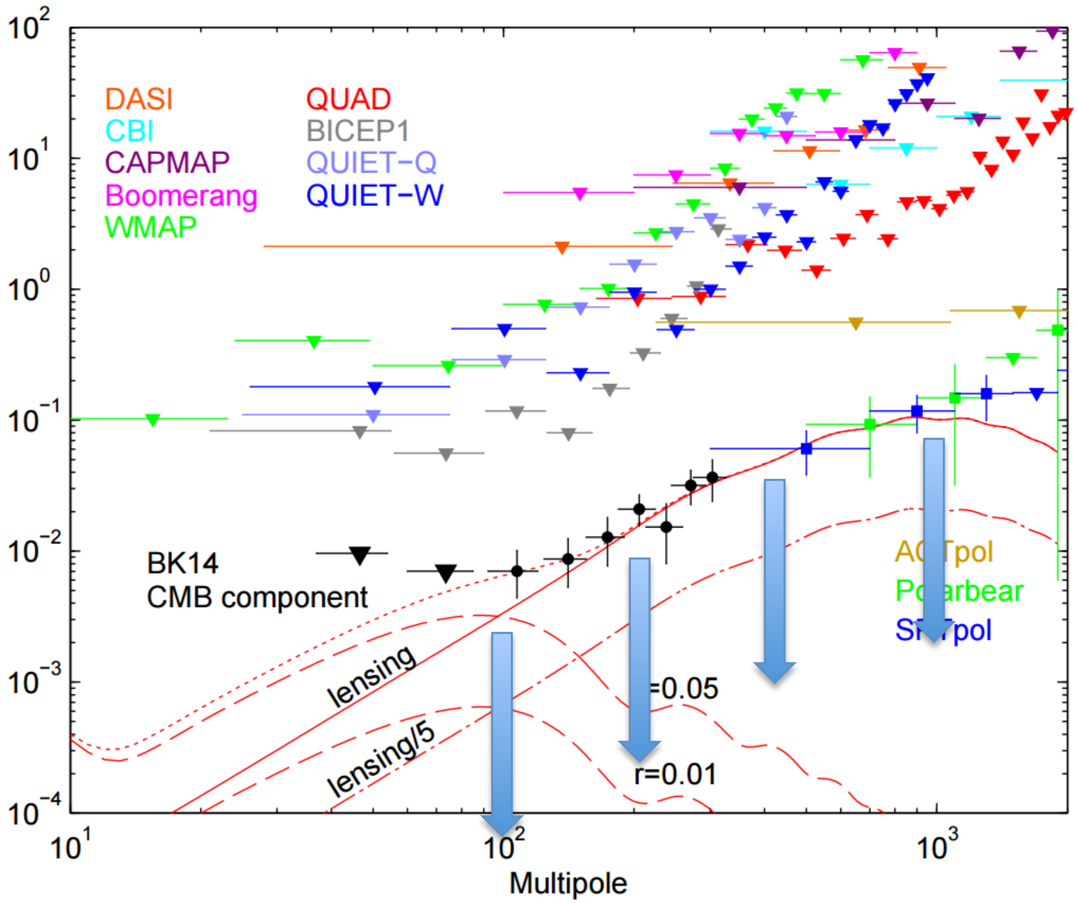

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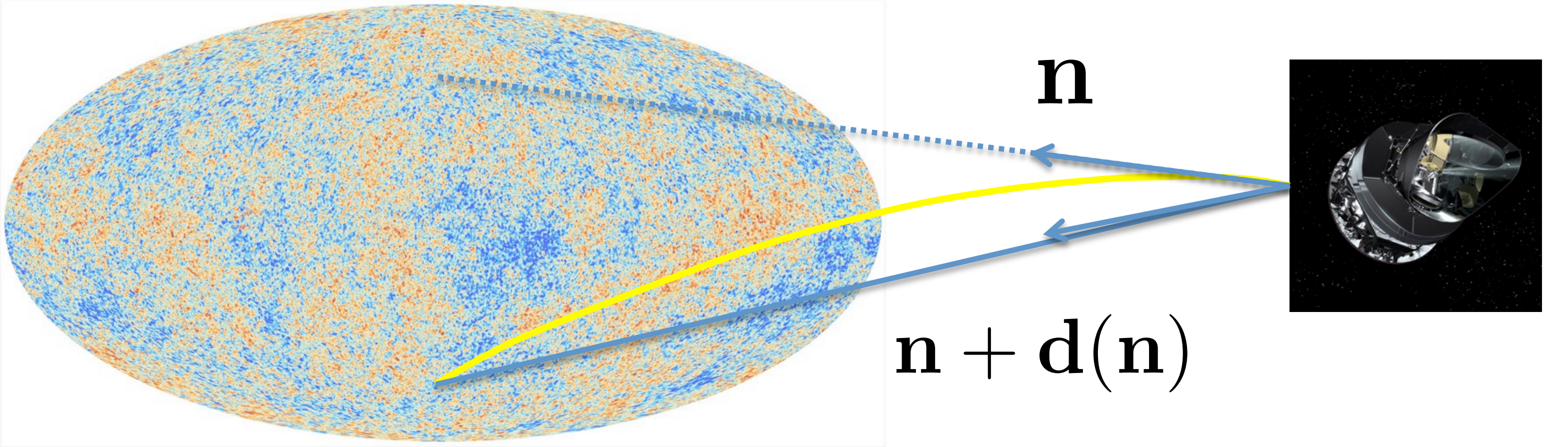
- Can be subtracted (**delensing**), *if we know well how CMB is lensed.*

[Knox+ 02; Kesden+ 02; Seljak & Hirata 04]

Standard approach: Remapping

The lensing effects on CMB can be modeled well by **remapping**:

$$\begin{aligned} \Delta T(\mathbf{n})_{\rm lensed} &= \Delta T(\mathbf{n}+\mathbf{d}(\mathbf{n}))_{\rm unlensed} \\ (Q \pm iU)(\mathbf{n})_{\rm lensed} &= (Q \pm iU)(\mathbf{n}+\mathbf{d}(\mathbf{n}))_{\rm unlensed} \end{aligned}$$



$$\mathbf{d}(\mathbf{n}) = \nabla\phi \quad ; \quad \phi \equiv -\int_{\eta'}^{\eta_0} {\rm d}\eta_1 \frac{\eta_1-\eta'}{(\eta_0-\eta')(\eta_0-\eta_1)} \underbrace{(\Psi-\Phi)}_{\text{Gravitational potentials after LSS}}$$

Accuracy of remapping approach

Inaccuracies of the remapping approach introduce systematics in delensing.

Contributions beyond remapping:

- Multiple lensing/ post-Born : < 0.2 % (at l < 3000)

[Hagstotz+ 15; Calabrese+ 15; Pratten & Lewis 16]

- Gravitational time delay : 0.01 %

[Hu & Cooray 01]

- Gravitational rotation of polarization : no contrib. from scalar

[Dai 14]

- 2^{nd} order collision + redshift : 10^{-4} %

[Fidler+ 14]

Other contributions? We need the fist-principle method.

“Curve”-of-sight approach :

Non-linear extension of line-of-sight approach

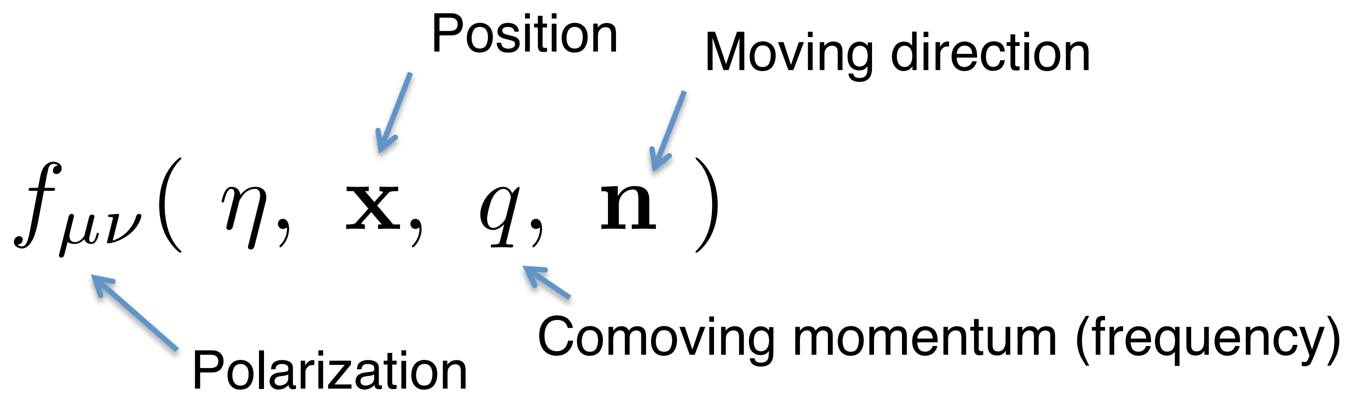
Temperature: RS, Naruko, Hiramatsu, & Sasaki JCAP1410 (2014)
Polarization: On-going work with Namikawa, Naruko, Taruya, & Namikawa

Other approaches: [Su & Lim 14] [Fidler, Koyama, & Pettinari 14]

Polarization of CMB photons

All information of CMB (incl. polarization) is encoded in

tensor-valued distribution function



Position

Moving direction

$$f_{\mu\nu}(\ \eta,\ \mathbf{x},\ q,\ \mathbf{n}\ )$$

Polarization

Comoving momentum (frequency)

$\epsilon_a{}^{\mu}[\epsilon_a{}^{\nu}]^* f_{\mu\nu}$ gives the phase-space number density for a polarization $\epsilon_a{}^{\mu}$ $(a=\pm)$

The Stokes parameters Q, U (and then E, B) can be calculated by $f_{\mu\nu}$

Difficulty: Boltzmann hierarchy

The evolution of the distribution function is determined by Boltzmann eq.

Collision term

$$\frac{\mathcal{D}}{\mathcal{D}\eta} f_{\mu\nu} = \dot{\tau} C_{\mu\nu}$$

Covariant derivative along a photon trajectory

It gives coupled differential equations for the multipole moments:

$$\frac{\partial f_{**,l}}{\partial \eta} = ikC^{+,l} f_{**,l+1} - ikC^{-,l} f_{**,l-1} + \cdots$$

Very heavy calculation to determine the multipole moments up to $l_{max} \sim 10^3$.

Line-of-sight approach

At linear order, the problem is solved by the line-of-sight integration method.

[Seljak & Zalddariaga 96]

Rewrite the Boltzmann eq. *in an integral form along the line of sight*:

$$f_{\mu\nu}(\eta_0, \mathbf{x}_0, \mathbf{q}_0) = \int_0^{\eta_0} \mathrm{d}\eta \, \underline{\left[e^{-\bar{\tau}} \overset{\text{ISW term}}{D_{\mu\nu}} - \dot{\bar{\tau}} e^{-\bar{\tau}} (f_{\mu\nu} - C_{\mu\nu}) \right]} \Big|_{\mathrm{at} \ (\eta, \bar{\mathbf{x}}(\eta), \bar{\mathbf{q}}(\eta))}$$

$$\equiv S_{\mu\nu}$$ Source term

where

$$\bar{x}^i(\eta') = n_0^i(\eta' - \eta_0) + x_0^i$$

$$\bar{q}^i(\eta') = q_0^i = q_0 n_0^i$$

This equation is *not* solved for $f_{\mu\nu}$, but it enable us to calculate $f_{\mu\nu}$ efficiently.

Moving to Fourier space:

$$f_{\mu\nu}(\eta_0, \mathbf{x}_0, \mathbf{q}_0) =$$

$$\int \mathrm{d}^3k \int_0^{\eta_0} \mathrm{d}\underline{\eta e^{i\mathbf{k}\cdot\mathbf{x}_0} S_{\mu\nu}}\, \underline{e^{i\mathbf{k}\cdot\mathbf{n}_0(\eta-\eta_0)}}$$

↓ Multipole moments ↓

$j_l[k(\eta - \eta_0)]$

Known function

Written in terms of six functions:

Φ, Ψ, $f_{I,00}$, $f_{I,20}$, $f_{E,20}$, v_e

All higher multipole moments are determined by the six functions.

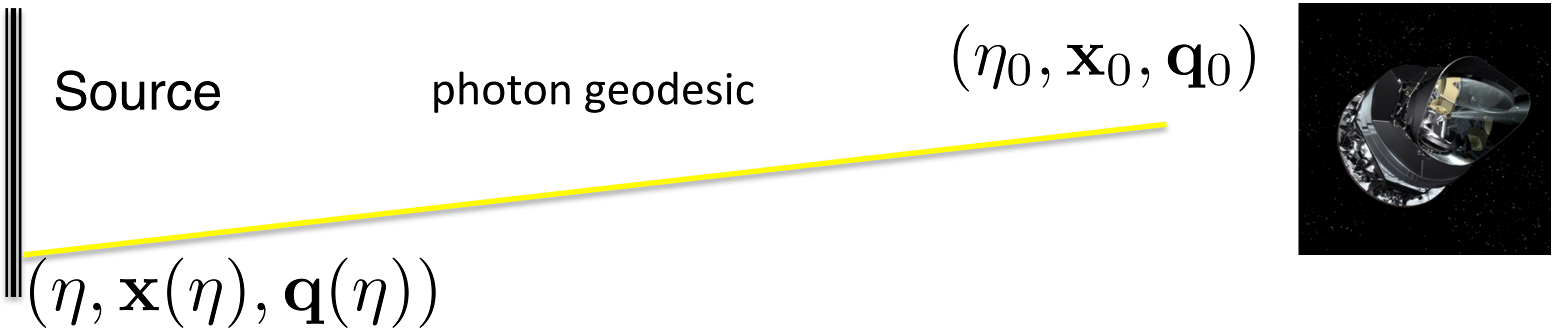
Is it possible to extend the LOS approach incl. lensing?

Why the LOS approach works

Liouville's theorem

Parallel transportation op.

$$f_{\mu\nu}(\eta_0, \mathbf{x}_0, \mathbf{q}_0) = \mathcal{P}_{\mu}{}^{\mu'}\mathcal{P}_{\nu}{}^{\nu'} f_{\mu'\nu'}(\eta, \mathbf{x}(\eta), \mathbf{q}(\eta))$$



Source

photon geodesic

$(\eta_0, \mathbf{x}_0, \mathbf{q}_0)$

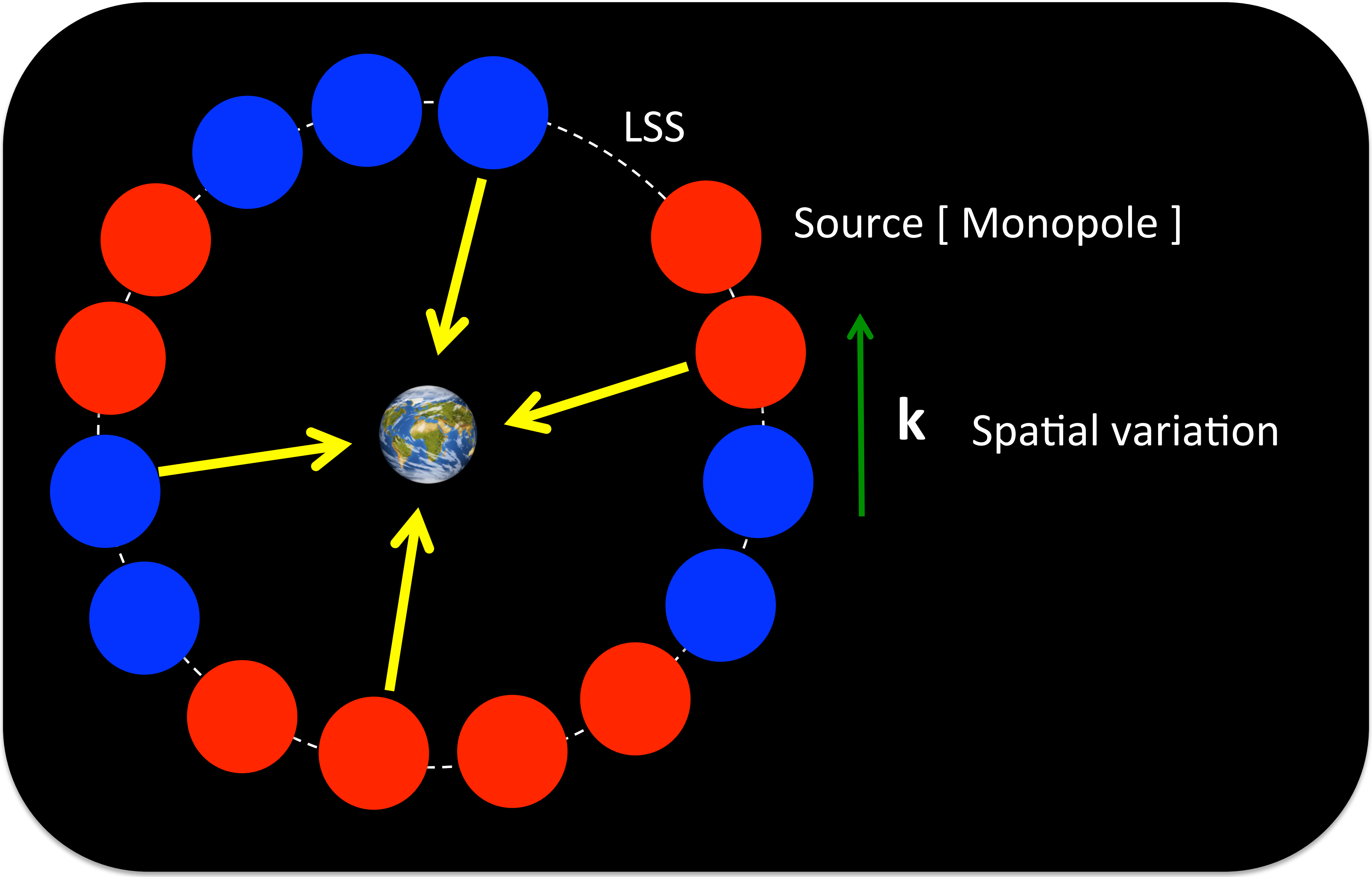
$(\eta, \mathbf{x}(\eta), \mathbf{q}(\eta))$

The distribution function at present is determined only by

information of sources (source function)

information of photon geodesics (straight line at linear order)

Six functions: Φ, Ψ, $f_{I,00}$, $f_{I,20}$, $f_{E,20}$, v_e


LSS
Source [Monopole]
k
Spatial variation

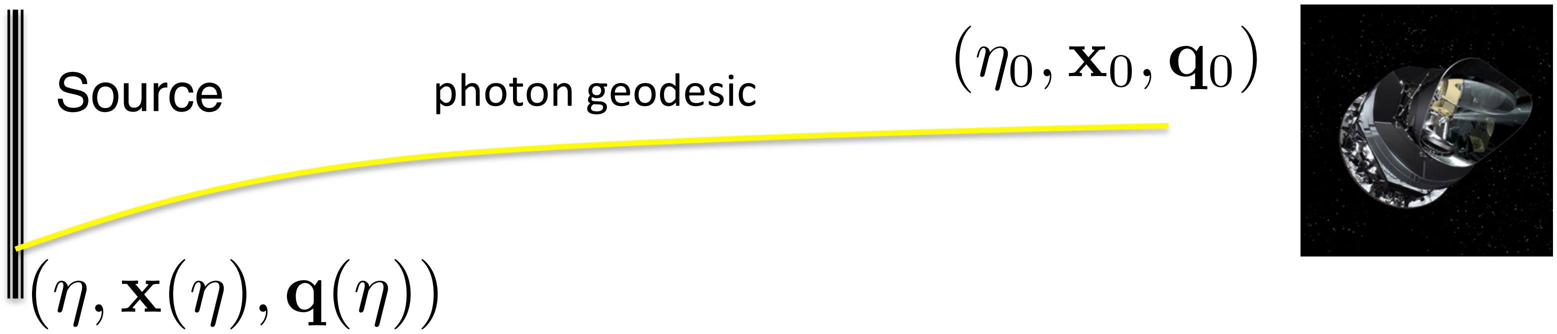
At higher orders (w/ Lensing effect)

No change in the conclusion

Liouville's theorem

Parallel transportation op.

$$f_{\mu\nu}(\eta_0,\mathbf{x}_0,\mathbf{q}_0) = \mathcal{P}_\mu{}^{\mu'}\mathcal{P}_\nu{}^{\nu'} f_{\mu'\nu'}(\eta,\mathbf{x}(\eta),\mathbf{q}(\eta))$$



Source

photon geodesic

$(\eta_0,\mathbf{x}_0,\mathbf{q}_0)$

$(\eta,\mathbf{x}(\eta),\mathbf{q}(\eta))$

The distribution function at present is determined only by

information of sources (source function)

information of photon geodesics (**gravitational potentials**)

Six functions: Φ, Ψ, $f_{I,00}$, $f_{I,20}$, $f_{E,20}$, v_e

“Curve”-of-sight approach

[On-going work with Namikawa, Naruko, Taruya, & Yamauchi; RS+ 14 for temperature]

Rewrite the Boltzmann eq. *in an integral form along the full geodesic curve*:

$$\equiv -\dot{\bar{\tau}} e^{-\bar{\tau}}$$ Visibility function

$$f_{\mu\nu}(\eta_0, \mathbf{x}_0, \mathbf{q}_0) = \int_0^{\eta_0} \mathrm{d}\eta \, \mathcal{P}_\mu{}^{\mu'} \mathcal{P}_\nu{}^{\nu'} g_v(\eta) \left(f_{\mu'\nu'} - C_{\mu'\nu'} \right) \Big|_{\text{at } \underline{(\eta, \mathbf{x}(\eta), \mathbf{q}(\eta))}}$$

A point at the full geodesic curve

- The source term is localized at photon sources $S_{\mu\nu} \equiv g_v(f_{\mu\nu} - C_{\mu\nu})$
- $(\mathbf{x}(\eta), \mathbf{q}(\eta))$ is obtained by solving the geodesic eqs. with $(\mathbf{x}(\eta_0), \mathbf{q}(\eta_0)) = (\mathbf{x}_0, \mathbf{q}_0)$

The distribution function at present is determined by

mapping the source term through the geodesic curve

Its structure is similar to the formula of remapping.

Projection [technical note]

The (phase space) number density for a given polarization,

$$\epsilon_a{}^{\mu}(\eta_0)[\epsilon_b{}^{\nu}(\eta_0)]^* f_{\mu\nu} =$$

$$\int_0^{\eta_0} \mathrm{d}\eta \, U_a{}^{a'}[U_b{}^{b'}]^* g_v(\eta) \, \underline{\epsilon_{a'}{}^{\mu'}(\eta)[\epsilon_{b'}{}^{\nu'}(\eta)]^* S_{\mu'\nu'}}\Big|_{\mathrm{at}\ (\eta, \mathbf{x}(\eta), \mathbf{q}(\eta))}$$

The source function is also projected.

We used

$$\epsilon_a{}^{\mu}(\eta_0)\mathcal{P}^{\mu}{}_{\mu'} = U_a{}^{a'}\epsilon_{a'}{}^{\mu'}(\eta) \quad (\mathrm{mod}\ q^{\mu}(\eta)\ [\mathrm{gauge\ tr.}])$$

$q^{\mu}S_{\mu\nu} = 0$ (Ward identity)

$$\left(\begin{array}{cc} f_{++} & f_{+-} \\ f_{-+} & f_{--} \end{array} \right) = \left(\begin{array}{cc} f_I - f_V & f_Q - i f_U \\ f_Q + i f_U & f_I + f_V \end{array} \right) .$$

“Curve”-of-sight approach

Its structure is similar to the formula of remapping:

$$f_{ab}(\eta_0, \mathbf{x}_0, \mathbf{q}_0) = \int_0^{\eta_0} \mathrm{d}\eta U_a{}^{a'} [U_b{}^{b'}]^* g_v(\eta) S_{a'b'} \underline{[\eta, \bar{\mathbf{x}} + \delta\mathbf{x}, \bar{\mathbf{q}} + \delta\mathbf{q}]}$$

$$U_a{}^b = \begin{pmatrix} U_+{}^+ & U_+{}^- \\ U_-{}^+ & U_-{}^- \end{pmatrix} = \begin{pmatrix} e^{-i\psi} & 0 \\ 0 & e^{i\psi} \end{pmatrix} .$$

decomposing the coordinates into the background and perturbed parts.

No approximation was used in the derivation.

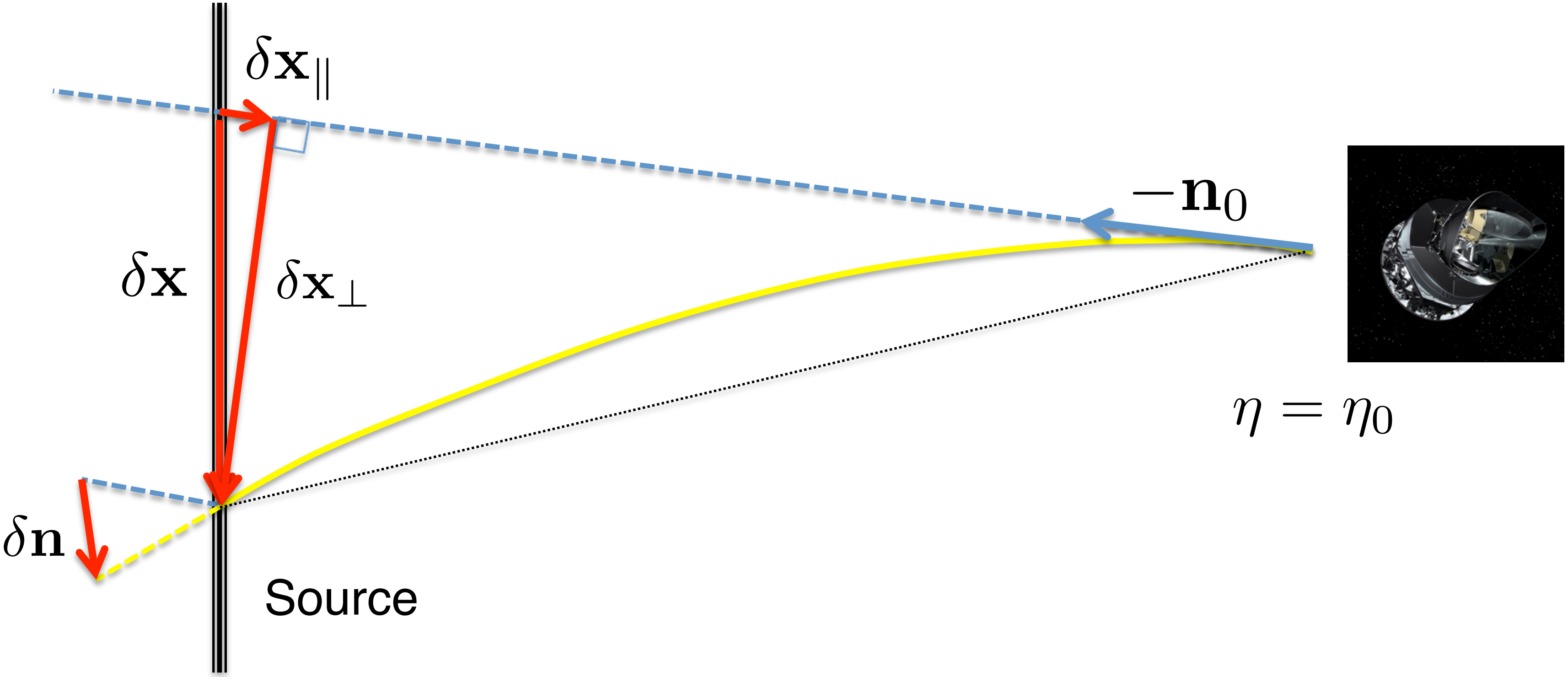
It is just an integral form of the Boltzmann equation.

Gravitational effects + α

Rotation **Redshift**

$$\int_0^{\eta_0} \mathrm{d}\eta\, U_a{}^{a'} [U_b{}^{b'}]^* g_v(\eta) S_{a'b'} \left[\eta, \bar{\mathbf{x}}+\delta\mathbf{x}, q+\delta q, \bar{\mathbf{n}}+\delta\mathbf{n}\right]$$

Time delay + Lensing



Gravitational effects + α

Rotation

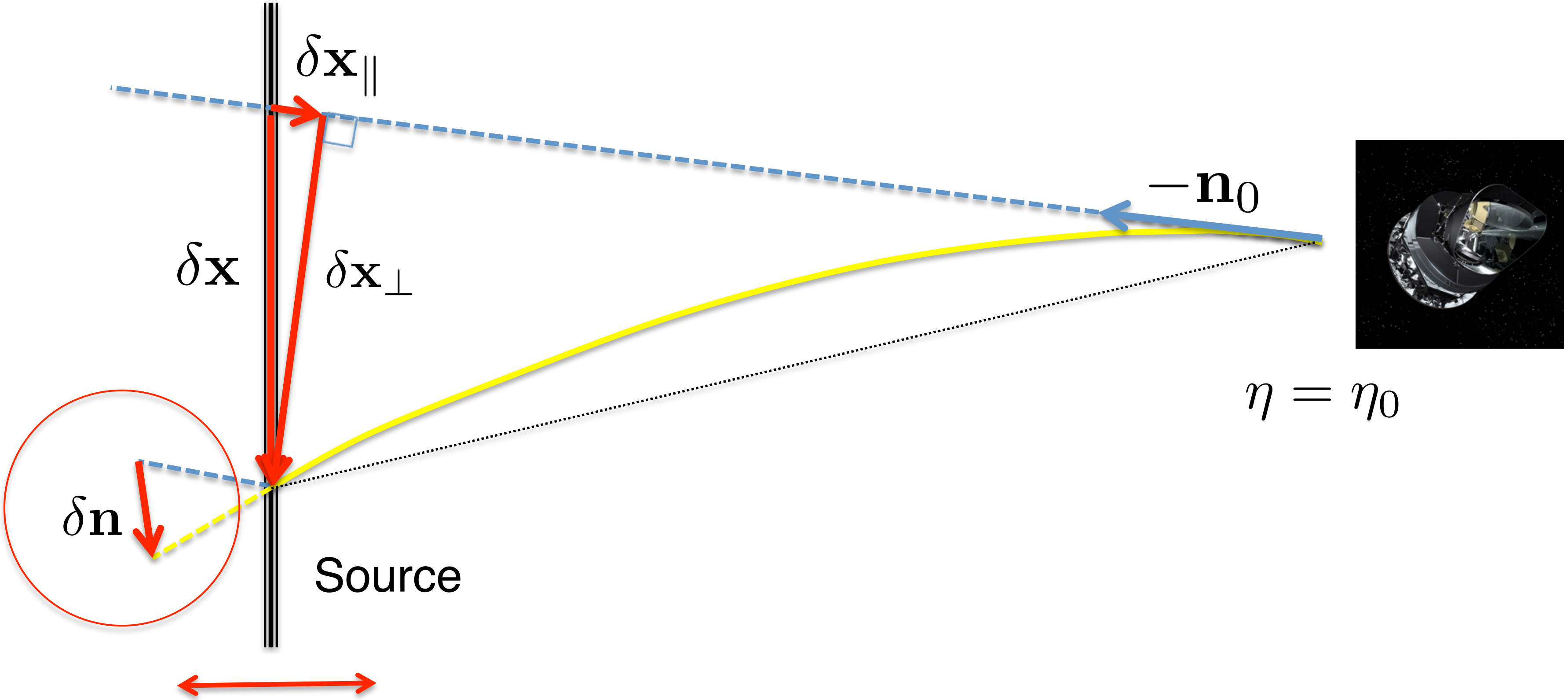
Redshift

Deflection

$$\boxed{\int_0^{\eta_0}} d\eta U_a{}^{a'} [U_b{}^{b'}]^* g_v(\eta) S_{a'b'} [\eta, \bar{\mathbf{x}} + \delta\mathbf{x}, q + \delta q \boxed{, \bar{\mathbf{n}} + \delta\mathbf{n}]}$$

Time delay + Lensing

Finite width of source



Relation to remapping

$\bar{\mathbf{n}} - \mathbf{d}$

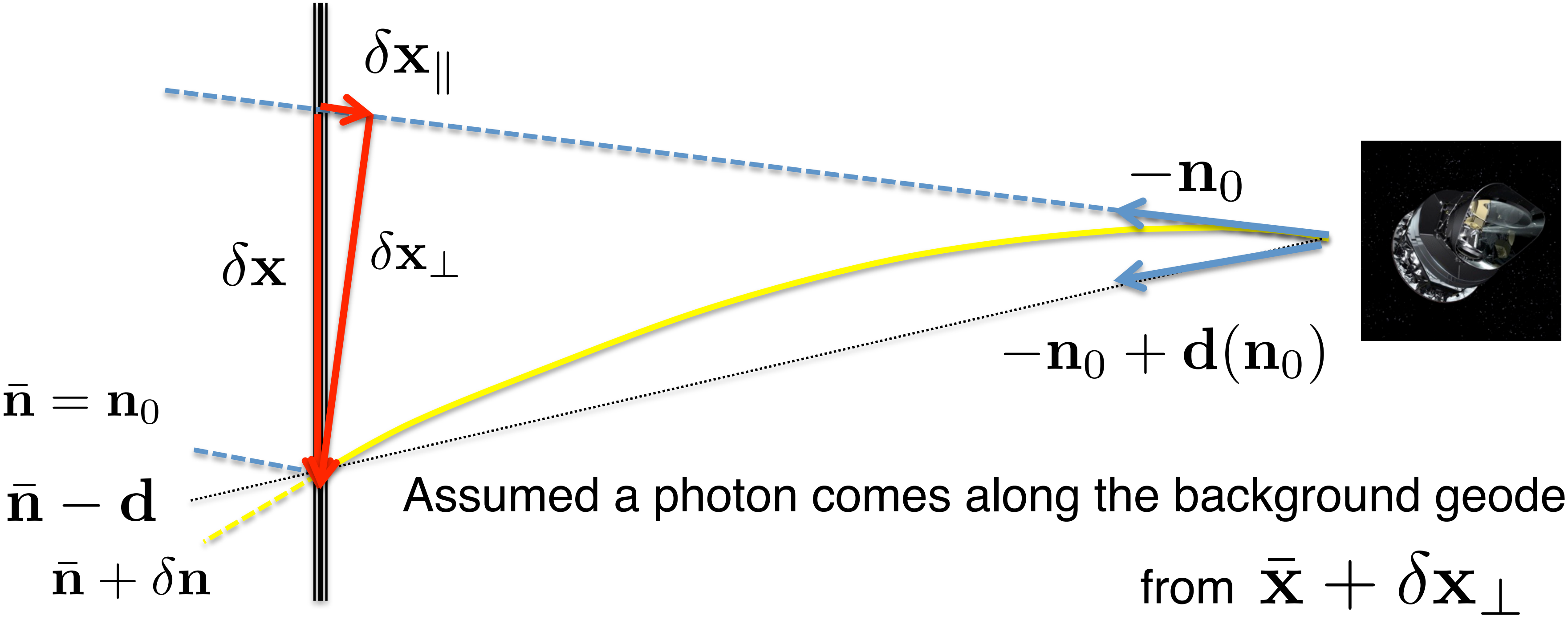
~~Rotation~~

~~Redshift~~

$$\boxed{\int_0^{\eta_0} \mathrm{d}\eta} U_a{}^{a'} [U_b{}^{b'}]^* g_v(\eta) S_{a'b'} \left[\eta, \bar{\mathbf{x}} + \delta\mathbf{x}, q + \delta q, \boxed{\bar{\mathbf{n}} + \delta\mathbf{n}}\right]$$

~~Time delay~~ + Lensing

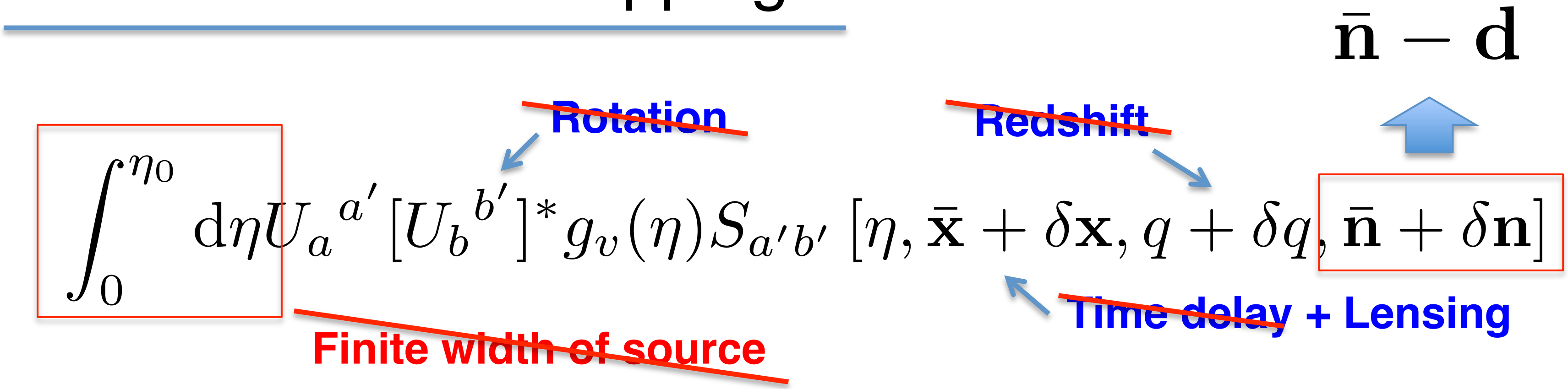
~~Finite width of source~~



Assumed a photon comes along the background geodesic

from $\bar{\mathbf{x}} + \delta\mathbf{x}_{\perp}$

Corrections to remapping



$$\bar{\mathbf{n}} - \mathbf{d}$$

~~Rotation~~

~~Redshift~~

$$\int_0^{\eta_0} \mathrm{d}\eta U_a{}^{a'} [U_b{}^{b'}]^* g_v(\eta) S_{a'b'} \left[\eta, \bar{\mathbf{x}} + \delta\mathbf{x}, q + \delta q, \bar{\mathbf{n}} + \delta\mathbf{n}\right]$$

~~Time delay~~ + Lensing

~~Finite width of source~~

✔ Time delay [Hu & Cooray 01]

✔ Rotation [Dai 14]

✔ Redshift [Fidler+ 14]

[New] **Deflection** $\delta\tilde{\mathbf{n}} \equiv \delta\mathbf{n} + \mathbf{d}$

[New] **Finite width of source**

Corrections to remapping

$$\bar{\mathbf{n}} - \mathbf{d}$$

~~Rotation~~

~~Redshift~~

$$\boxed{\int_0^{\eta_0} \mathrm{d}\eta} U_a{}^{a'} [U_b{}^{b'}]^* g_v(\eta) S_{a'b'} \left[\eta, \bar{\mathbf{x}} + \delta\mathbf{x}, q + \delta q, \boxed{\bar{\mathbf{n}} + \delta\mathbf{n}]}\right.$$

~~Time delay~~ + Lensing

~~Finite width of source~~

✔ Time delay [Hu & Cooray 01]

✔ Rotation [Dai 14]

✔ Redshift [Fidler+ 14]

[New] **Deflection** $\delta\tilde{\mathbf{n}} \equiv \delta\mathbf{n} + \mathbf{d}$

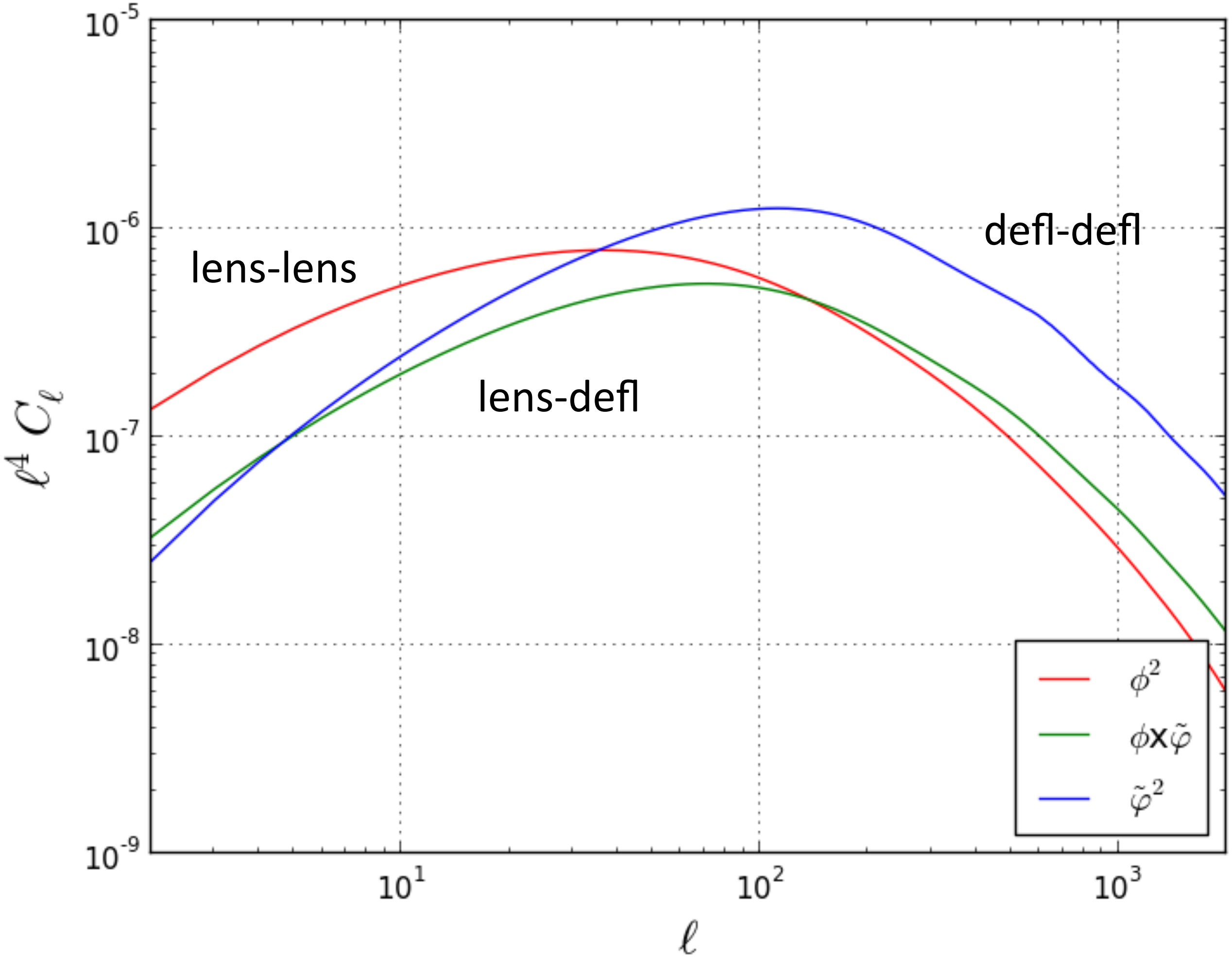
[New] **Finite width of source**

Intrinsic B-mode polarization

from deflection

Lensing & deflection potentials

$$\delta\tilde{\mathbf{n}} = \nabla\tilde{\phi} \quad ; \quad \tilde{\phi} \equiv -\frac{1}{(\eta_0 - \eta_{\rm LSS})}\int_{\eta_{\rm LSS}}^{\eta_0} d\eta_1 (\Psi - \Phi)$$



Remapping with deflection $\delta\tilde{\mathbf{n}}$

The expansion of the COS formula can be reorganized as,

$$f_{\mp\pm}(\eta_0, \mathbf{n}_0) =$$

$$\underbrace{f_{\mp\pm}^{\mathrm{unlens}} + d^a \frac{\partial f_{\mp\pm}^{\mathrm{unlens}}}{\partial n_0^a} + \frac{1}{2} d^a d^b \frac{\partial^2 f_{\mp\pm}^{\mathrm{unlens}}}{\partial n_0^a \partial n_0^b}}_{\text{standard lensing}}$$

Derivative for line of sight

$$\frac{\partial}{\partial n_0^a} = \underline{(\eta_0 - \eta_{\mathrm{LSS}})} \frac{\partial}{\partial x^a} + \frac{\partial}{\partial n^a}$$

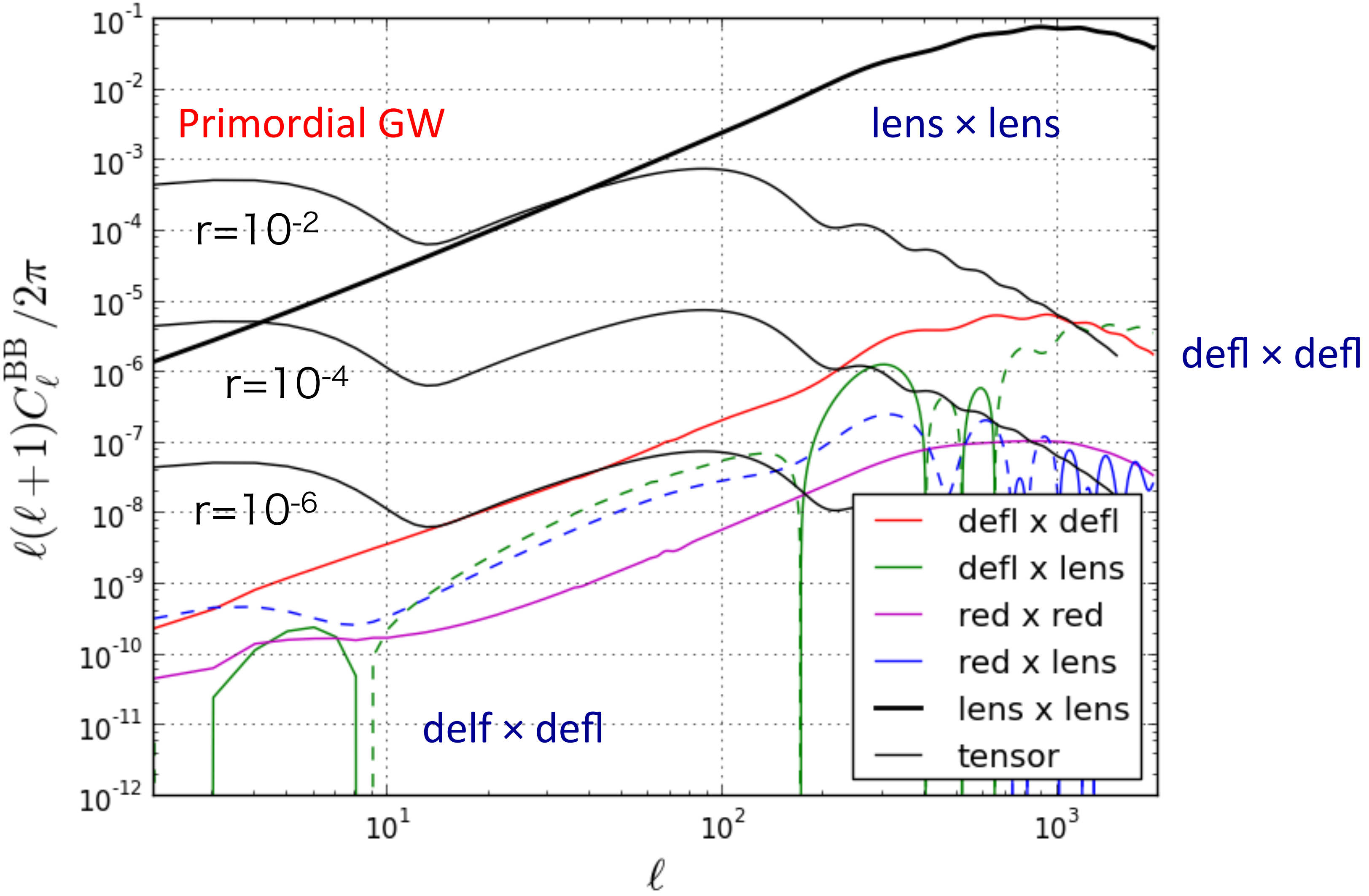
$$\underline{+ \left(\delta\tilde{n}^a \frac{\partial}{\partial n^a} + \frac{1}{2} \delta\tilde{n}^a \delta\tilde{n}^b \frac{\partial^2}{\partial n^a \partial n^b} + d^a \delta\tilde{n}^b \frac{\partial^2}{\partial n_0^a \partial n^b} \right) f_{\mp\pm} \Big|_{\mathbf{n}=\mathbf{n}_0}}$$

New contributions from deflection

The effects of lensing are enhanced by,

$$\partial_{n_0} / \partial_{\tilde{n}} \sim k(\eta_0 - \eta_{\mathrm{LSS}}) \sim l_* \sim 100 - 500$$

The intrinsic B-modes



Auto & cross correlations: 0.01% - 0.001% corrections $\sim l_*^{-2}$

Why small cross correlation?

Where does the additional suppression factor, l_*^{-1} , come from?

→ **Phase cancellation**

$$f_B^{\rm lens} \sim \underline{T^{\rm lens}} f_E * \phi$$

$$\sim \frac{\cos\left[k(\eta_0 - \eta) + (l+1)\pi/2\right]}{l} + \mathcal{O}(l^{-2})$$

$$f_B^{\rm defl} \sim \underline{T^{\rm defl}} f_E * \tilde{\phi}$$

$$\sim \frac{\sin\left[k(\eta_0 - \eta) + (l+1)\pi/2\right]}{l} + \mathcal{O}(l^{-2})$$

Geometric reason. Applicable for any theory (e.g., Modified gravity)

Summary

- Framework to discuss the general corrections to the remapping:

“Curve”-of-sight approach

- New corrections: **deflection**, finite width of source

- **Deflection**: 0.01% - 0.001% corrections, very generally

Only important for $r = 10^{-(5\text{-}6)}$

- Effects of a finite width of source?