Cosmology in generalized Proca theories

Shinji Tsujikawa
(Tokyo University of Science)

Collaboration with
A. De Felice, L. Heisenberg, R. Kase, S. Mukohyama, Y. Zhang

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There have been many attempts for constructing dark energy models in the framework of scalar-tensor theories.

Most of them belong to the so-called Horndeski theories:

$$S = \int d^4x\sqrt{-g}\, L$$

Most general scalar-tensor theories with second-order equations

$$\begin{aligned} L \;=\; & G_2(\phi, X) + G_3(\phi, X)\Box\phi + G_4(\phi, X)R - 2G_{4,X}(\phi, X)\left[(\Box\phi)^2 - \phi^{;\mu\nu}\phi_{;\mu\nu}\right] \\ & + G_5(\phi, X)G_{\mu\nu}\phi^{;\mu\nu} + \frac{1}{3}G_{5,X}(\phi, X)[(\Box\phi)^3 - 3(\Box\phi)\phi_{;\mu\nu}\phi^{;\mu\nu} + 2\phi_{;\mu\nu}\phi^{;\mu\sigma}\phi^{;\nu}{}_{;\sigma}] \end{aligned}$$

Single scalar field ϕ with $X = g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$

Horndeski (1973)

R and $G_{\mu\nu}$ are the 4-dimensional Ricci scalar and the Einstein tensors, respectively.

- General Relativity corresponds to $G_4 = M_{\rm pl}^2/2$.
- Horndeski theories accommodate a wide variety of gravitational theories like Brans-Dicke theory, $f(R)$ gravity, and covariant Galileons.

What happens for a vector field instead of a scalar field ?

(i) Maxwell field (massless)

Lagrangian: $\mathcal{L}_F = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$

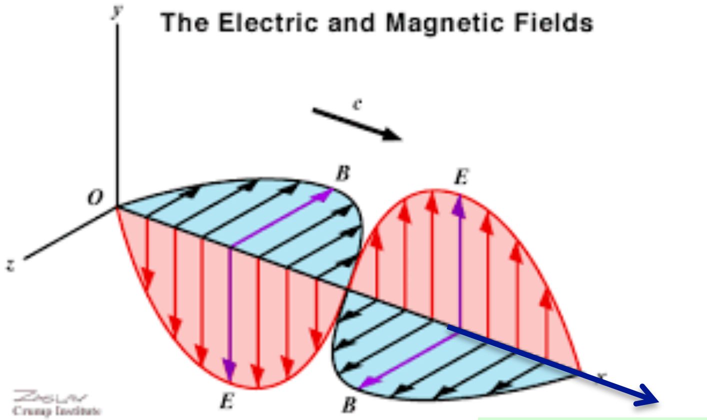
There are two transverse polarizations (electric and magnetic fields).

(ii) Proca field (massive)

Lagrangian: $\mathcal{L}_F = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} m^2 A_\mu A^\mu$

Introduction of the mass m of the vector field A_μ allows the propagation in the longitudinal direction due to the breaking of $U(1)$ gauge invariance.

2 transverse and 1 longitudinal
= 3 DOFs



Longitudinal propagation

Generalized Proca theories

In a curved background, it is possible to extend the massive Proca theories to those containing three DOFs (besides two tensor polarizations).

Heisenberg Lagrangian (2014)

See also Tasinato (2014)

$$
\begin{aligned}
\mathcal{L}_F &= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}\,,\\
\mathcal{L}_2 &= G_2(X)\,,\\
\mathcal{L}_3 &= G_3(X)\nabla_\mu A^\mu\,,\\
\mathcal{L}_4 &= G_4(X)R + G_{4,X}(X)\left[(\nabla_\mu A^\mu)^2 + c_2\nabla_\rho A_\sigma \nabla^\rho A^\sigma - (1+c_2)\nabla_\rho A_\sigma \nabla^\sigma A^\rho\right]\,,\\
\mathcal{L}_5 &= G_5(X)G_{\mu\nu}\nabla^\mu A^\nu - \frac{1}{6}G_{5,X}(X)[(\nabla_\mu A^\mu)^3 - 3d_2\nabla_\mu A^\mu \nabla_\rho A_\sigma \nabla^\rho A^\sigma - 3(1-d_2)\nabla_\mu A^\mu \nabla_\rho A_\sigma \nabla^\sigma A^\rho\\
&\quad +(2-3d_2)\nabla_\rho A_\sigma \nabla^\gamma A^\rho \nabla^\sigma A_\gamma + 3d_2\nabla_\rho A_\sigma \nabla^\gamma A^\rho \nabla_\gamma A^\sigma]\,.
\end{aligned}
$$

where $X = -A_\mu A^\mu/2$ and $F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu$.

The terms proportional to c_2 and d_2 can be expressed in terms of $F_{\mu\nu}$, so they correspond to pure vector modes.

The non-minimal derivatives couplings like $G_4(X)R$ are required to keep the equations of motion up to second order.

Cosmology in generalized Proca theories

De Felice et al, JCAP 1606, 048 (2016)

Can we realize a viable cosmology with the late-time acceleration?

Vector field: $A^\mu = (\phi(t), 0, 0, 0)$ (which does not break spatial isotropy)

Variation of the Heisenberg action with respect to $g_{\mu\nu}$ on the flat FLRW background leads to

$$G_2 - G_{2,X}\phi^2 - 3G_{3,X}H\phi^3 + 6G_4H^2 - 6(2G_{4,X} + G_{4,XX}\phi^2)H^2\phi^2 + G_{5,XX}H^3\phi^5 + 5G_{5,X}H^3\phi^3 = \rho_M\,,$$
$$G_2 - \dot{\phi}\phi^2 G_{3,X} + 2G_4\,(3H^2 + 2\dot{H}) - 2G_{4,X}\phi\,(3H^2\phi + 2H\dot{\phi} + 2\dot{H}\phi) - 4G_{4,XX}H\dot{\phi}\phi^3$$
$$+ G_{5,XX}H^2\dot{\phi}\phi^4 + G_{5,X}H\phi^2(2\dot{H}\phi + 2H^2\phi + 3H\dot{\phi}) = -P_M\,.$$

The matter density ρ_M and the pressure P_M obey the continuity equation

$$\dot{\rho}_M + 3H(\rho_M + P_M) = 0$$

Variation of the action with respect to A^μ leads to

$$\phi\left(G_{2,X} + 3G_{3,X}H\phi + 6G_{4,X}H^2 + 6G_{4,XX}H^2\phi^2 - 3G_{5,X}H^3\phi - G_{5,XX}H^3\phi^3\right) = 0\,.$$

The branch $\phi \neq 0$ gives the solution where ϕ depends on H alone, which allows the existence of de Sitter solutions with constant ϕ and H.

Vector Galileons

The Lagrangian of vector Galileons which recover the Galilean symmetry in the scalar limit ($A_\mu \to \partial_\mu \pi$) on the flat space-time is given by

$$G_2(X) = b_2 X\,, \qquad G_3(X) = b_3 X\,, \qquad G_4(X) = \frac{M_{\rm pl}^2}{2} + b_4 X^2\,, \qquad G_5(X) = b_5 X^2.$$

We substitute these functions into the vector-field equation:

$$G_{2,X} + 3G_{3,X} H\phi + 6G_{4,X} H^2 + 6G_{4,XX} H^2 \phi^2 - 3G_{5,X} H^3 \phi - G_{5,XX} H^3 \phi^3 = 0$$

Taking note that $X = \phi^2/2$, the background EOM admits the solution

$$\phi H = \text{constant}.$$

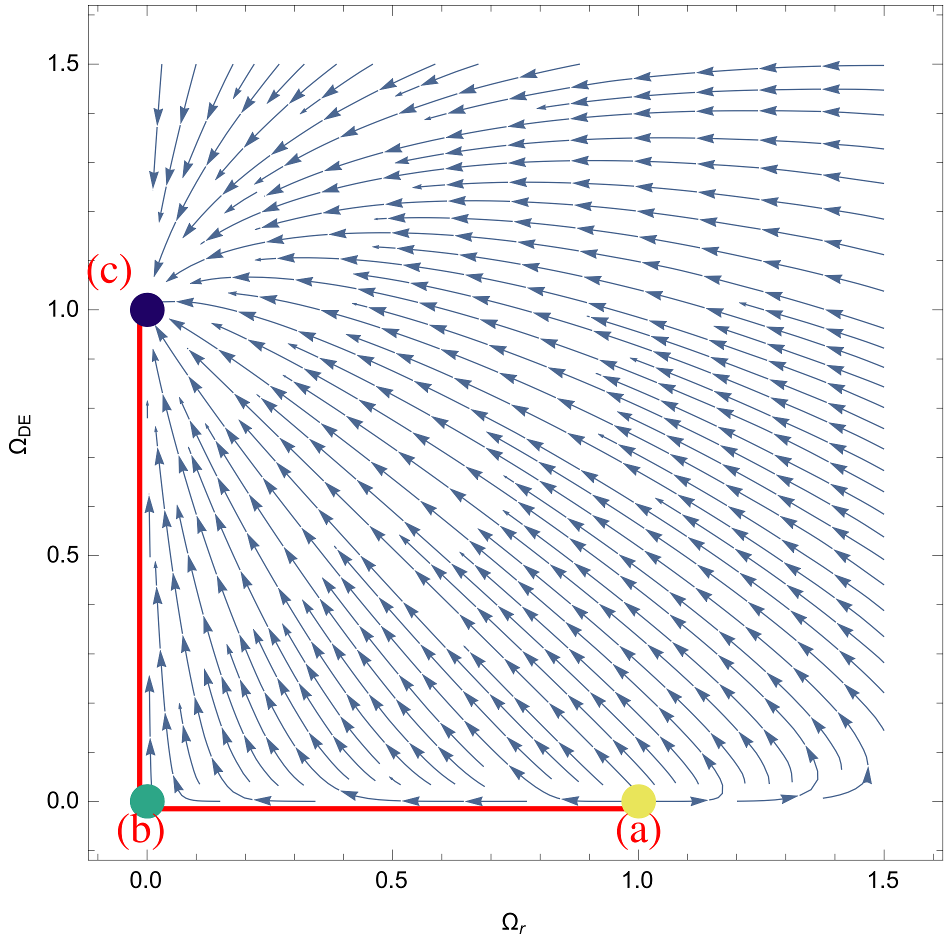
⇓

The temporal component ϕ is small in the early cosmological epoch, but it grows with the decrease of H.

The solution finally approaches the de Sitter attractor characterized by

$$\phi = \text{constant}, \qquad H = \text{constant}.$$

Phase-space trajectories for vector Galileons



(a) Radiation point
(b) Matter point
(c) De Sitter point

The de Sitter fixed point (c) is always stable against homogeneous perturbations, so it corresponds to the late-time attractor.

The dark energy equation of state $w_{\rm DE}$ is -2 during the matter era.

⇩

This behavior is the same as a tracker solution of scalar Galileons, which is in tension with the observational data (Nesseris, De Felice, ST, 2010).

Generalizations of vector Galileons

Let us consider the case in which ϕ is related with H according to

$$\phi^p \propto H^{-1} \qquad (p > 0)$$

This solution can be realized for

$$G_2(X) = b_2 X^{p_2}\,, \qquad G_3(X) = b_3 X^{p_3}\,, \qquad G_4(X) = \frac{M_{\rm pl}^2}{2} + b_4 X^{p_4}\,, \qquad G_5(X) = b_5 X^{p_5}\,,$$

where

$$p_3 = \frac{1}{2}(p + 2p_2 - 1)\,, \qquad p_4 = p + p_2\,, \qquad p_5 = \frac{1}{2}(3p + 2p_2 - 1)\,.$$

→ The vector Galileon corresponds to $p_2 = p = 1$.

The dark energy and radiation density parameters obey

$$\Omega'_{\rm DE} = \frac{(1+s)\Omega_{\rm DE}(3 + \Omega_r - 3\Omega_{\rm DE})}{1 + s\,\Omega_{\rm DE}}\,,$$
$$\Omega'_r = -\frac{\Omega_r[1 - \Omega_r + (3+4s)\Omega_{\rm DE}]}{1 + s\,\Omega_{\rm DE}}\,,$$

where $s \equiv \dfrac{p_2}{p}\,.$

→ There are 3 fixed points:

(a) $(\Omega_{\rm DE}, \Omega_r) = (0, 1)$

(b) $(\Omega_{\rm DE}, \Omega_r) = (0, 0)$

(c) $(\Omega_{\rm DE}, \Omega_r) = (1, 0)$

The dark energy equation of state

$$w_{\rm DE} = -\frac{3(1+s)+s\,\Omega_r}{3(1+s\,\Omega_{\rm DE})}\,.$$

(a) $w_{\rm DE} = -1 - 4s/3$ in the radiation era,
(b) $w_{\rm DE} = -1 - s$ in the matter era,
(c) $w_{\rm DE} = -1$ in the de Sitter era

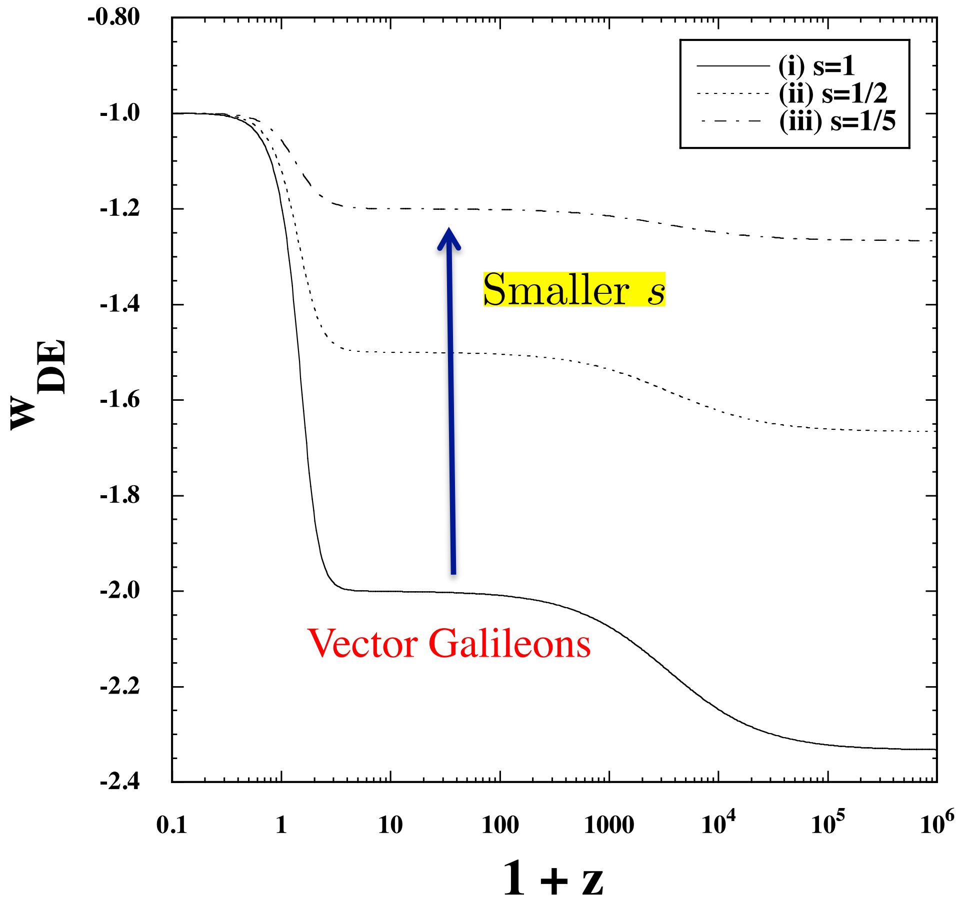
For smaller $s = p_2/p$ close to 0, $w_{\rm DE} = -1 - s$ approaches -1.

The joint data analysis of SNIa, CMB, and BAO give the bound

$$0 \le s \le 0.36 \quad (95\,\%\text{CL})$$

(De Felice and ST, 2012)

For larger p the field ϕ evolves more slowly as $\phi \propto H^{-1/p}$, so $w_{\rm DE}$ approaches -1.



Cosmological perturbations in generalized Proca theories

We need to study perturbations on the flat FLRW background to study

(i) Conditions for avoiding ghosts and instabilities,
(ii) Observational signatures for the matter distribution in the Universe.

In doing so, let us consider the perturbed metric in flat gauge:

$$ds^2 = -(1+2\alpha)\,dt^2 + 2\,(\partial_i \chi + V_i)\,dt\,dx^i + a^2(t)\,(\delta_{ij} + h_{ij})\,dx^i dx^j\,,$$

where α, χ are scalar perturbations, V_i and h_{ij} are the vector and tensor perturbations, respectively, obeying

$$\partial^i V_i = 0\,,$$
$$\partial^i h_{ij} = 0\,, \qquad {h_i}^i = 0\,.$$

We also consider the perturbations of the vector field, as

$$A^0 = \phi(t) + \delta\phi\,,$$
$$A^i = \frac{1}{a^2}\delta^{ij}\left(\partial_j \chi_V + E_j\right)$$

where $\delta\phi$ and χ_V are scalar perturbations, while E_j is the vector perturbation obeying $\partial^j E_j = 0$.

Tensor perturbations : 2 Dofs

There are two polarization modes h_+ and $h_\times$ for the tensor perturbation:

$$h_{ij} = h_+ e_{ij}^+ + h_\times e_{ij}^\times$$

Expanding the Heisenberg action up to second order in tensor perturbations, the resulting second-order action is given by

$$S_T^{(2)} = \sum_{\lambda=+,\times} \int dt\, d^3x\, a^3 \frac{q_T}{8} \left[\dot{h}_\lambda^2 - \frac{c_T^2}{a^2} (\partial h_\lambda)^2 \right] ,$$

where

$$q_T = 2G_4 - 2\phi^2 G_{4,X} + H\phi^3 G_{5,X} ,$$

$$c_T^2 = \frac{2G_4 + \phi^2 \dot{\phi}\, G_{5,X}}{q_T} .$$

The tensor perturbation obeys (in Fourier space)

$$\ddot{h}_\lambda + \left(3H + \frac{\dot{q}_T}{q_T} \right) \dot{h}_\lambda + c_T^2 \frac{k^2}{a^2} h_\lambda = 0$$

The ghost and instability can be avoided for

$$q_T > 0 , \qquad c_T^2 > 0$$

Vector perturbations : 2 Dofs

Besides the vector field, we take into account a single perfect fluid described by the Schutz-Sorkin action:

$$S_M = -\int d^4x \left[\sqrt{-g}\,\rho_M(n) + J^\mu(\partial_\mu \ell + \mathcal{A}_1\partial_\mu\mathcal{B}_1 + \mathcal{A}_2\partial_\mu\mathcal{B}_2)\right]$$

Related with the number density, as

$$n = \sqrt{J^\alpha J^\beta g_{\alpha\beta}/g}$$

Scalar part

Vector part

After integrating out the matter action, introducing the combination $Z_i = E_i + \phi(t)V_i$, and finally taking the small-scale limit, the resulting vector action (for two dofs Z_1, Z_2) reads

$$S_V^{(2)} \simeq \sum_{i=1}^{2}\int dt\, d^3x\, \frac{a q_V}{2}\left(\dot{Z}_i^2 + \frac{k^2}{a^2}c_V^2 Z_i^2\right),$$

where

$$q_V = 1 - 2c_2 G_{4,X} - 2d_2 H\phi\, G_{5,X}\,,$$

$$c_V^2 = 1 + \frac{\phi^2(2G_{4,X} - G_{5,X}H\phi)^2}{2q_T q_V} + \frac{d_2 G_{5,X}(H\phi - \dot{\phi})}{q_V}\,.$$

Scalar perturbations : 2 Dofs (1 scalar +1 matter)

The second-order Lagrangian for scalar perturbations is given by

$$L_S^{(2)} = a^3 \left(\dot{\vec{\mathcal{X}}}^t \boldsymbol{K} \dot{\vec{\mathcal{X}}} + \frac{k^2}{a^2} \vec{\mathcal{X}}^t \boldsymbol{G} \vec{\mathcal{X}} - \vec{\mathcal{X}}^t \boldsymbol{M} \vec{\mathcal{X}} - \vec{\mathcal{X}}^t \boldsymbol{B} \dot{\vec{\mathcal{X}}} \right) ,$$

$$\vec{\mathcal{X}}^t = (\psi, \delta\rho_M) \, .$$

where $\psi = \chi_V + \phi(t)\chi$ and $\delta\rho_M$ is the matter perturbation.

If the two eigenvalues of the 2×2 matrix $\mathbf{K}$ are positive, the ghosts are absent. One of them is $\rho_M + P_M > 0$, and another is

$$Q_S = \frac{a^3 H^2 q_T (3w_1^2 + 4q_T w_4)}{\phi^2 (w_1 - 2w_2)^2}$$

In the small-scale limit, the dispersion relation is given by

$$\det \left(\omega^2 \boldsymbol{K} - \frac{k^2}{a^2} \boldsymbol{G} \right) = 0$$

One of the solutions is the matter propagation speed squared, while another one is

$$\begin{aligned} c_S^2 \;=\; & \frac{1}{\Delta} \Big\{ 2\, w_2^2 w_3 (\rho_M + P_M) - w_3 (w_1 - 2w_2) \left[w_1 w_2 + \phi (w_1 - 2w_2) w_6 \right] \left(\dot{\phi}/\phi - H \right) - w_3 \left(2 w_2^2 \dot{w}_1 - w_1^2 \dot{w}_2 \right) \\ & + \phi\, (w_1 - 2w_2)^2 \left[w_3 \dot{w}_6 + \phi (2 w_3 w_7 + w_6^2) \right] + w_1 w_2 \left[w_1 w_2 + (w_1 - 2w_2)(2\phi\, w_6 - w_3 \dot{\phi}/\phi) \right] \Big\} , \end{aligned}$$

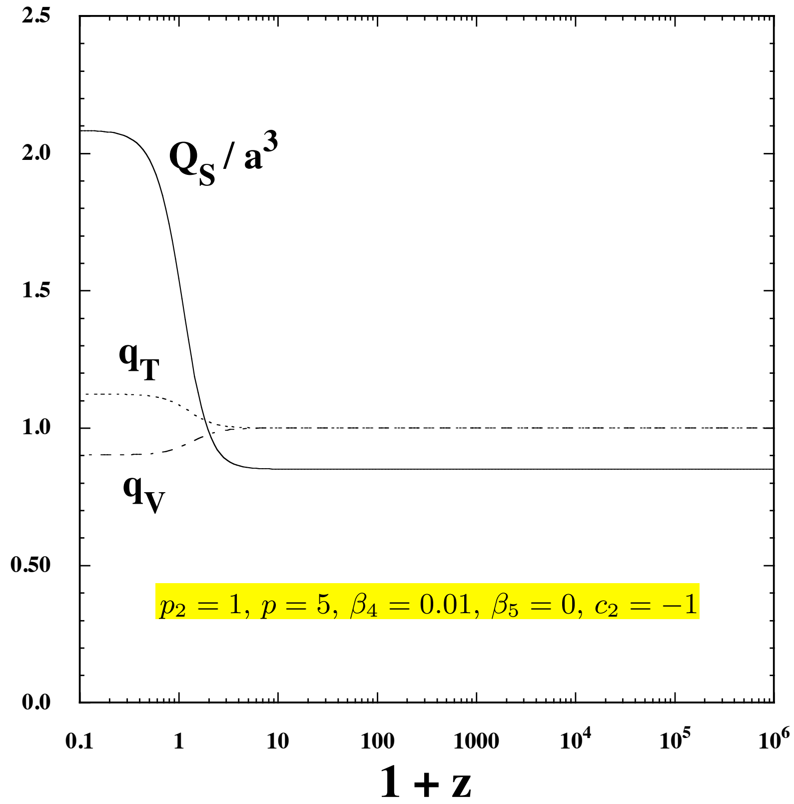
where $\Delta = 8H^2\phi^2 q_T q_V q_S$, and w_1 etc are the known from the background.

A model consistent with no-ghost and stability conditions

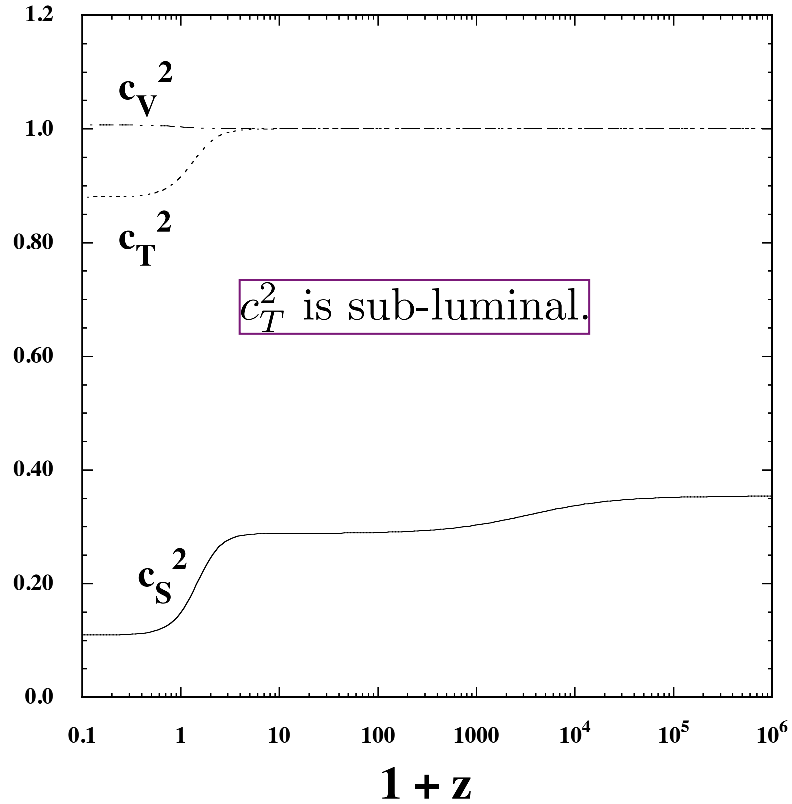
$$G_2(X) = b_2 X\,, \qquad G_3(X) = b_3 X^{p_3}\,, \qquad G_4(X) = \frac{M_{\rm pl}^2}{2} + b_4 X^{p_4}\,, \qquad G_5(X) = 0.$$

Provided that $0 < \beta_4 < 1/[6(2p+1)]$, there exists the parameter space in which all the theoretically consistent conditions are satisfied.

$q_T > 0, q_V > 0, Q_S > 0$



$c_T^2 > 0, c_V^2 > 0, c_S^2 > 0$



Effective gravitational couplings for the cosmic growth

Under the quasi-static approximation on sub-horizon scales, the matter perturbation obeys

De Felice et al, PRD (2016)

$$\ddot{\delta}_M + 2H\dot{\delta}_M - 4\pi G_{\rm eff}\rho_M \delta_M \simeq 0$$

where the effective gravitational coupling is

$$G_{\rm eff} = \frac{\xi_2 + \xi_3}{\xi_1}$$

$$\xi_1 = 4\pi\phi^2 \left(w_2 + 2Hq_T\right)^2 \,,$$

$$\xi_2 = \left[H\left(w_2 + 2Hq_T\right) - \dot{w}_1 + 2\dot{w}_2 + \rho_M\right]\phi^2 - \frac{w_2^2}{q_V}\,,$$

$$\xi_3 = \frac{1}{8H^2\phi^2 q_S^3 q_T c_S^2}\Big[2\phi^2\left\{q_S[w_2\dot{w}_1 - (w_2 - 2Hq_T)\dot{w}_2] + \rho_M w_2[3w_2(w_2 + 2Hq_T) - q_S]\right\} - \frac{q_S}{q_V}w_2\left\{w_6\phi(w_2 + 2Hq_T) - w_2(w_2 - 2Hq_T)\right\}\Big]^2 .$$

ξ_3 is positive under the no-ghost and stability conditions (which enhances the gravitational attraction).

For smaller q_V close to 0, there is a tendency that $G_{\rm eff}$ decreases.

Additional contribution to the Heisenberg Lagrangian

$$\mathcal{L}_6 = G_6(X) L^{\mu\nu\alpha\beta} \nabla_\mu A_\nu \nabla_\alpha A_\beta + \frac{1}{2} G_{6,X}(X) \tilde{F}^{\alpha\beta} \tilde{F}^{\mu\nu} \nabla_\alpha A_\mu \nabla_\beta A_\nu$$ (Horndeski, 1976)

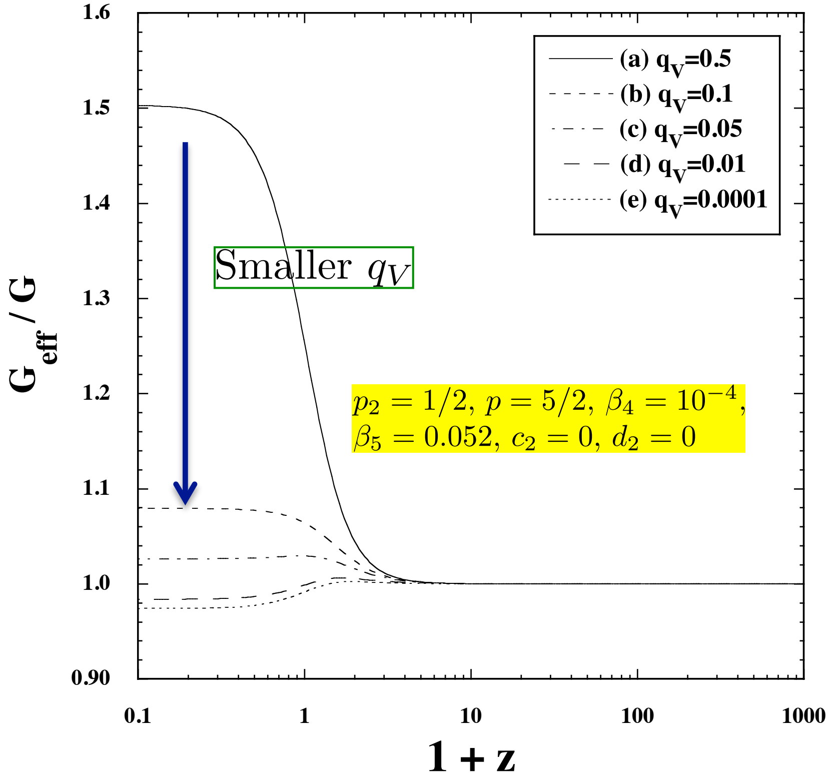
$G_{\rm eff}$ is modified only through the change of q_V :

$$q_V = 1 - 2c_2 G_{4,X} - 2d_2 H\phi G_{5,X} + 2G_6 H^2 + 2G_{6,X} H^2 \phi^2$$

The term G_6 can decrease q_V.

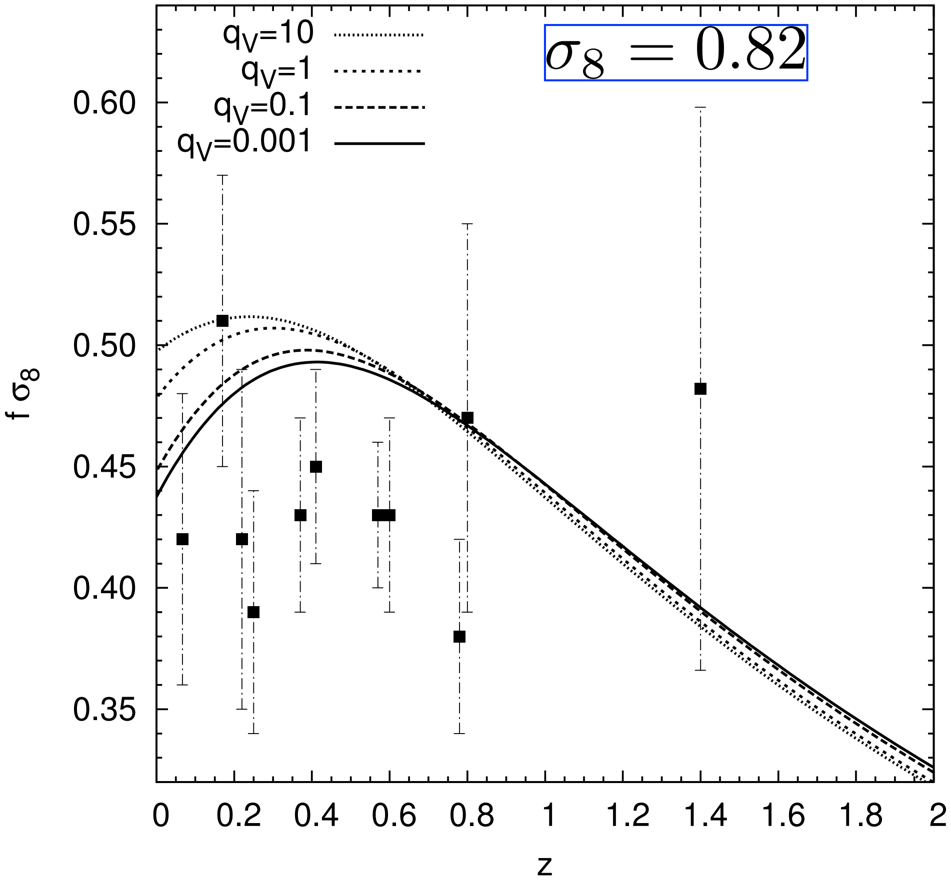
For smaller q_V approaching 0, the effect of the vector field tends to reduce the gravitational attraction.

It is possible to distinguish the models with $G_6 = 0$ and $G_6 \neq 0$ observationally.



Observational signatures in red-shift space distortions (RSD)

From the RSD measurement we can constrain the growth rate of matter perturbations: $f = \dot{\delta}_m/(H\delta_m)$.



For smaller q_V, the values of $f\sigma_8$ tend to be smaller.

The present $f\sigma_8$ data alone are not sufficient to distinguish between the models with different q_V.

This situation will be improved in the future.

Beyond generalized Proca theories

Heisenberg, Kase, ST, PLB (2016)

The Heisenberg Lagrangian contains the Galileon-like contributions:

$$\mathcal{L}^{\rm Ga}_{i+2} = g_{i+2}\,\hat{\delta}^{\beta_1\cdots\beta_i\gamma_{i+1}\cdots\gamma_4}_{\alpha_1\cdots\alpha_i\gamma_{i+1}\cdots\gamma_4}\nabla_{\beta_1}A^{\alpha_1}\cdots\nabla_{\beta_i}A^{\alpha_i}$$

We can consider the generalized Lagrangians like

$$\mathcal{L}^{\rm N}_4 = f_4\hat{\delta}^{\beta_1\beta_2\beta_3\gamma_4}_{\alpha_1\alpha_2\alpha_3\gamma_4}A^{\alpha_1}A_{\beta_1}\nabla^{\alpha_2}A_{\beta_2}\nabla^{\alpha_3}A_{\beta_3}\,,$$

$$\mathcal{L}^{\rm N}_5 = f_5\hat{\delta}^{\beta_1\beta_2\beta_3\beta_4}_{\alpha_1\alpha_2\alpha_3\alpha_4}A^{\alpha_1}A_{\beta_1}\nabla^{\alpha_2}A_{\beta_2}\nabla^{\alpha_3}A_{\beta_3}\nabla^{\alpha_4}A_{\beta_4}$$

→ In the scalar limit $A^\mu \to \nabla^\mu\pi$, these recover the Lagrangains of Gleyzes-Langlois-Piazza-Verinizz theories.

The analysis of linear perturbations on the flat FLRW background and on the anisotropic cosmological background showed that there are no additional ghostly DOF even with these new Lagrangians.

→ The theories may be healthy on general backgrounds.

Anisotropic cosmology in beyond-generalized Proca theories

Heisenberg, Kase, ST, arXiv/1607.03175

Second-order generalized Proca theories with four new Lagrangians :

$$\mathcal{L}_4^{\rm N} = f_4(X)\hat{\delta}^{\beta_1\beta_2\beta_3\gamma_4}_{\alpha_1\alpha_2\alpha_3\gamma_4} A^{\alpha_1} A_{\beta_1} \nabla^{\alpha_2} A_{\beta_2} \nabla^{\alpha_3} A_{\beta_3}\,,$$
$$\mathcal{L}_5^{\rm N} = f_5(X)\hat{\delta}^{\beta_1\beta_2\beta_3\beta_4}_{\alpha_1\alpha_2\alpha_3\alpha_4} A^{\alpha_1} A_{\beta_1} \nabla^{\alpha_2} A_{\beta_2} \nabla^{\alpha_3} A_{\beta_3} \nabla^{\alpha_4} A_{\beta_4}$$
$$\tilde{\mathcal{L}}_5^{\rm N} = \tilde{f}_5(X)\hat{\delta}^{\beta_1\beta_2\beta_3\beta_4}_{\alpha_1\alpha_2\alpha_3\alpha_4} A^{\alpha_1} A_{\beta_1} \nabla^{\alpha_2} A^{\alpha_3} \nabla_{\beta_2} A_{\beta_3} \nabla^{\alpha_4} A_{\beta_4}$$
$$\mathcal{L}_6^{\rm N} = f_6(X)\hat{\delta}^{\beta_1\beta_2\beta_3\beta_4}_{\alpha_1\alpha_2\alpha_3\alpha_4} \nabla_{\beta_1} A_{\beta_2} \nabla^{\alpha_1} A^{\alpha_2} \nabla_{\beta_3} A^{\alpha_3} \nabla_{\beta_4} A^{\alpha_4}$$

See arXiv:1608.07066 (Kimura et al) for the analysis of general backgrounds up to quartic Lagrangian.

Anisotropic background:

$$ds^2 = -N^2(t)dt^2 + e^{2\alpha(t)}\left[e^{-4\sigma(t)}dx^2 + e^{2\sigma(t)}\left(dy^2+dz^2\right)\right]$$

with the vector field $A^\mu = \left(\frac{\phi(t)}{N(t)}, e^{-\alpha(t)+2\sigma(t)}\,v(t), 0, 0\right)$

The Hamiltonian constraint is

$$\frac{\partial L}{\partial N} = -\frac{\mathcal{H}}{N} = 0 \quad \Rightarrow \quad \mathcal{H} = 0$$

No ghost-like Ostrogradski instability

Conclusions and outlook

1. Generalized Proca theories give rise to interesting cosmological solutions with a late-time de Sitter attractor.
2. We derived 6 no-ghost and stability conditions associated with tensor, vector, and scalar perturbations for theconsistency of the theory.
3. We constructed a class of models in which all the theoretically consistent conditions are satisfied during the cosmic expansion history.
4. We also derived the effective gravitational coupling that can be used to put observational constraints on the models.
5. It is also possible to extend second-order generalized Proca theories in a way that additional ghostly degrees of freedom do not arise.

It will be of interest to put observational constraints on the viable parameter spaces for our proposed models.