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Study on the mapping of dark matter clustering from real space to redshift space

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Yi Zheng, Yong-Seon Song, arXiv:1603.00101,
JCAP08(2016)050

Redshift space distortion:
small scale modelling and systematic errors

5-10%

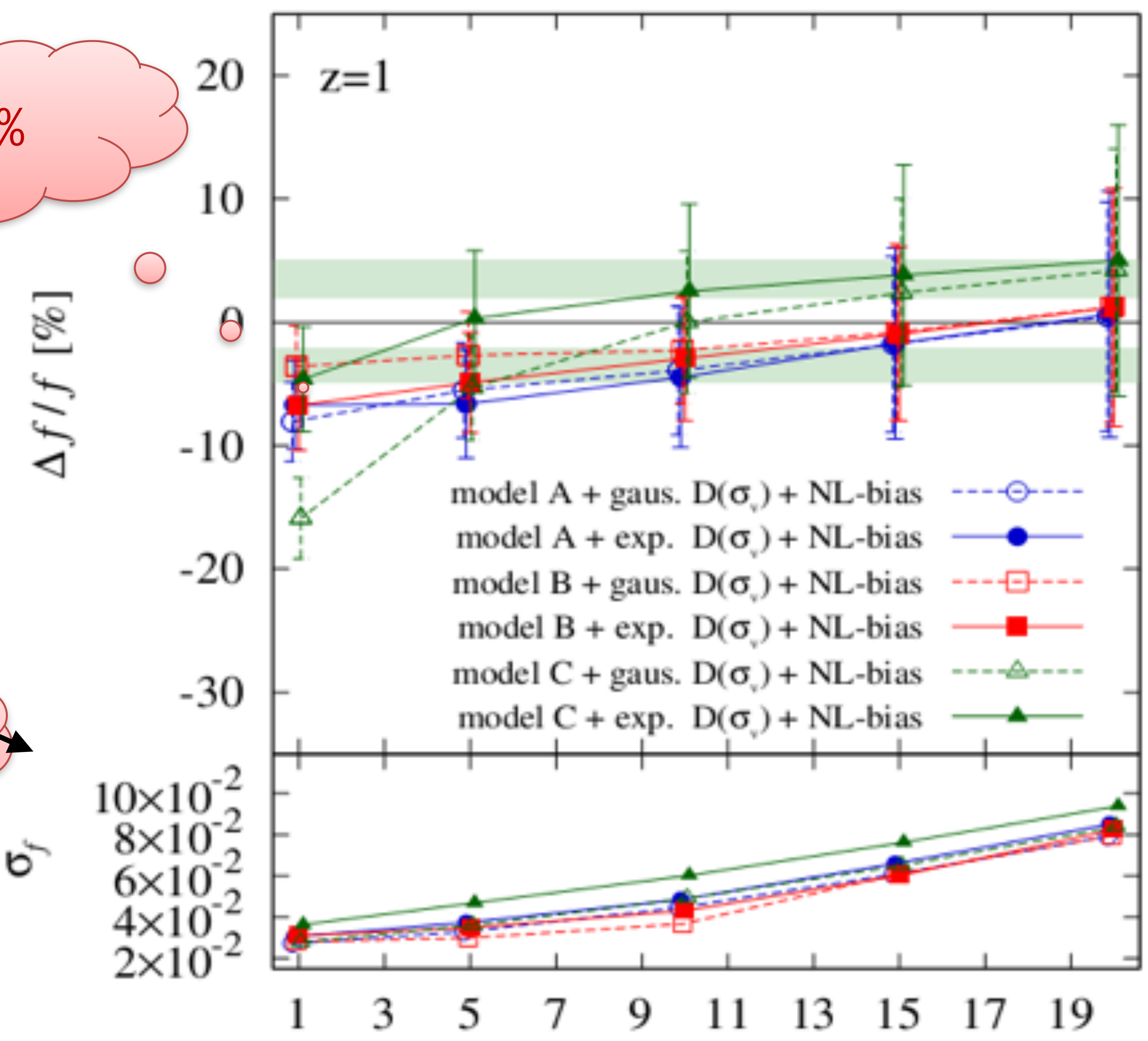
$$N_k \propto k^3$$

Theoretically:
1. Nonlinear mapping
2. Nonlinear evolution
3. Bias modelling

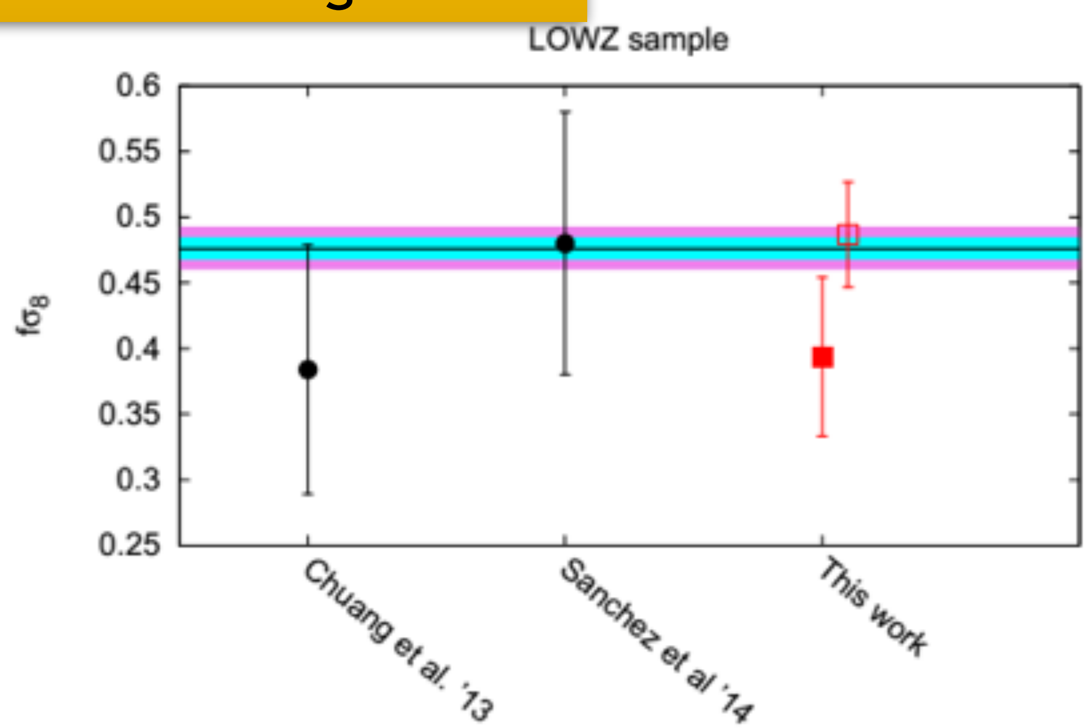
Observationally:

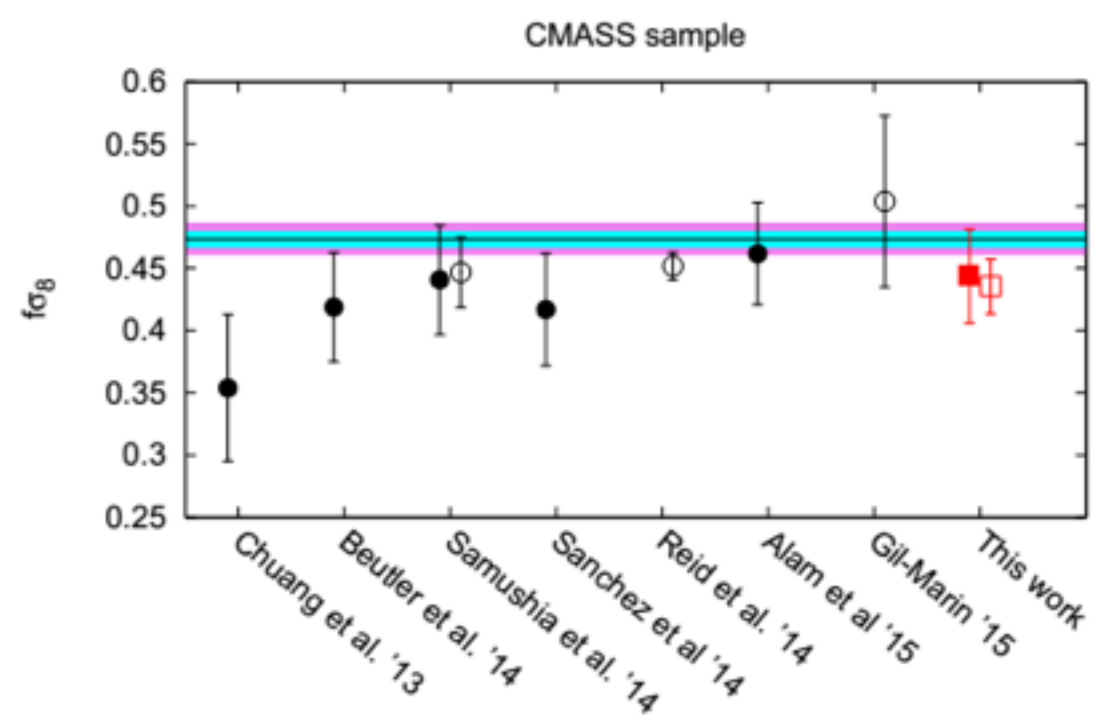
O(1%)

DESI, Euclid et all:



Torre & Guzzo, 1202.5559





Gil-Marin et al. 1509.06386

Redshift space distortion:
starting point for PS

$$P^{(\mathrm{S})}(k,\mu) = \int d^3\boldsymbol{x}\, e^{i\,\boldsymbol{k}\cdot\boldsymbol{x}} \langle e^{j_1 A_1} A_2 A_3 \rangle$$

$$\delta_D(\boldsymbol{k}) + P_s(\boldsymbol{k}) = \int \frac{d^3 r}{(2\pi)^3} \mathrm{e}^{-i\boldsymbol{k}\cdot\boldsymbol{r}} \langle \mathrm{e}^{ifk_z\Delta u_z}[1+\delta(\boldsymbol{x})] \times [1+\delta(\boldsymbol{x}')] \rangle,$$

e.g. Scoccimarro astro-ph/0407214
Zhang 1207.2722

$$P^{(\mathrm{S})}(k,\mu) = \int \frac{d^3 r}{(2\pi)^3} e^{-i\,\boldsymbol{k}\cdot\boldsymbol{r}} \langle e^{ifk_z u_z}[1+\delta(\boldsymbol{x})] e^{-ifk_z u_z'}[1+\delta(\boldsymbol{x}')] \rangle$$

Taylor expansion!

$$P^{ss}(\mathbf{k}) = \sum_{L=0}^{\infty} \frac{1}{L!^2} \left(\frac{k\mu}{H}\right)^{2L} P_{LL}(\mathbf{k}) + 2\sum_{L=0}^{\infty}\sum_{L'>L} \frac{(-1)^L}{L!\,L'!} \left(\frac{ik\mu}{H}\right)^{L+L'} P_{LL'}(\mathbf{k})$$

Seljak&McDonald 1109.1888

$$P^{(\mathrm{S})}(\boldsymbol{k}) = \int d^3\boldsymbol{x} e^{i\boldsymbol{k}\cdot\boldsymbol{x}} \langle e^{-ik\mu f\Delta u_z} \{\delta(\boldsymbol{r}) + f\nabla_z u_z(\boldsymbol{r})\} \times \{\delta(\boldsymbol{r}') + f\nabla_z u_z(\boldsymbol{r}')\} \rangle,$$

e.g. TNS 1006.0699

$$\langle e^{j_1 A_1} A_2 A_3 \rangle = \exp\{\langle e^{j_1 A_1} \rangle_c\} [\langle e^{j_1 A_1} A_2 A_3 \rangle_c + \langle e^{j_1 A_1} A_2 \rangle_c \langle e^{j_1 A_1} A_3 \rangle_c].$$

Cumulant expansion

$$P^{(\mathrm{S})}(k,\mu) = \int d^3\boldsymbol{x} e^{i\boldsymbol{k}\cdot\boldsymbol{x}} \exp\{\langle e^{j_1 A_1} \rangle_c\} [\langle e^{j_1 A_1} A_2 A_3 \rangle_c + \langle e^{j_1 A_1} A_2 \rangle_c \langle e^{j_1 A_1} A_3 \rangle_c].$$

FoG

in terms of j_1

$$\langle e^{j_1 A_1} A_2 A_3 \rangle_c + \langle e^{j_1 A_1} A_2 \rangle_c \langle e^{j_1 A_1} A_3 \rangle_c$$
$$\simeq \langle A_2 A_3 \rangle + j_1 \langle A_1 A_2 A_3 \rangle_c + j_1^2 \{\tfrac{1}{2} \langle A_1^2 A_2 A_3 \rangle_c + \langle A_1 A_2 \rangle_c \langle A_1 A_3 \rangle_c\} + \mathcal{O}(j_1^3).$$

TNS 1006.0699
Zheng 1603.00101

RSD modelling

$$P^{(\mathrm{S})}(\boldsymbol{k}) = \int d^3\boldsymbol{x} e^{i\boldsymbol{k}\cdot\boldsymbol{x}} \langle e^{-ik\mu f \Delta u_z} \{\delta(\boldsymbol{r}) + f\nabla_z u_z(\boldsymbol{r})\} \times \{\delta(\boldsymbol{r}') + f\nabla_z u_z(\boldsymbol{r}')\}\rangle,$$

Definition:

$$j_1 = -ik\mu f, \qquad A_1 = u_z(\boldsymbol{r}) - u_z(\boldsymbol{r}'),$$
$$A_2 = \delta(\boldsymbol{r}) + f\nabla_z u_z(\boldsymbol{r}), \qquad A_3 = \delta(\boldsymbol{r}') + f\nabla_z u_z(\boldsymbol{r}').$$

$$P^{(\mathrm{S})}(k,\mu) = \int d^3\boldsymbol{x} e^{i\boldsymbol{k}\cdot\boldsymbol{x}} \langle e^{j_1 A_1} A_2 A_3 \rangle,$$

$$\langle e^{j_1 A_1} A_2 A_3 \rangle = \exp\{\langle e^{j_1 A_1}\rangle_c\}[\langle e^{j_1 A_1} A_2 A_3\rangle_c + \langle e^{j_1 A_1} A_2\rangle_c \langle e^{j_1 A_1} A_3\rangle_c].$$

Important for separation of FoG

$$P^{(\mathrm{S})}(k,\mu) = \int d^3\boldsymbol{x} e^{i\boldsymbol{k}\cdot\boldsymbol{x}} \exp\{\langle e^{j_1 A_1}\rangle_c\}[\langle e^{j_1 A_1} A_2 A_3\rangle_c + \langle e^{j_1 A_1} A_2\rangle_c \langle e^{j_1 A_1} A_3\rangle_c].$$

Scale dependent part+scale independent part

$$\begin{aligned} &D^{\mathrm{FoG}}_{\mathrm{corr}}(k\mu, \boldsymbol{x}) \left[\langle e^{j_1 A_1} A_2 A_3\rangle_c + \langle e^{j_1 A_1} A_2\rangle_c \langle e^{j_1 A_1} A_3\rangle_c\right] \\ &\simeq j_1^0 \langle A_2 A_3\rangle_c + j_1^1 \langle A_1 A_2 A_3\rangle_c \\ &+ j_1^2 \left\{\langle A_1 A_2\rangle_c \langle A_1 A_3\rangle_c + \frac{1}{2}\langle A_1^2 A_2 A_3\rangle_c - \langle u_z u_z'\rangle_c \langle A_2 A_3\rangle_c\right\} \\ &+ \mathcal{O}(j_1^3), \qquad (13) \end{aligned}$$

Final model

1006.0699 & 1603.00101

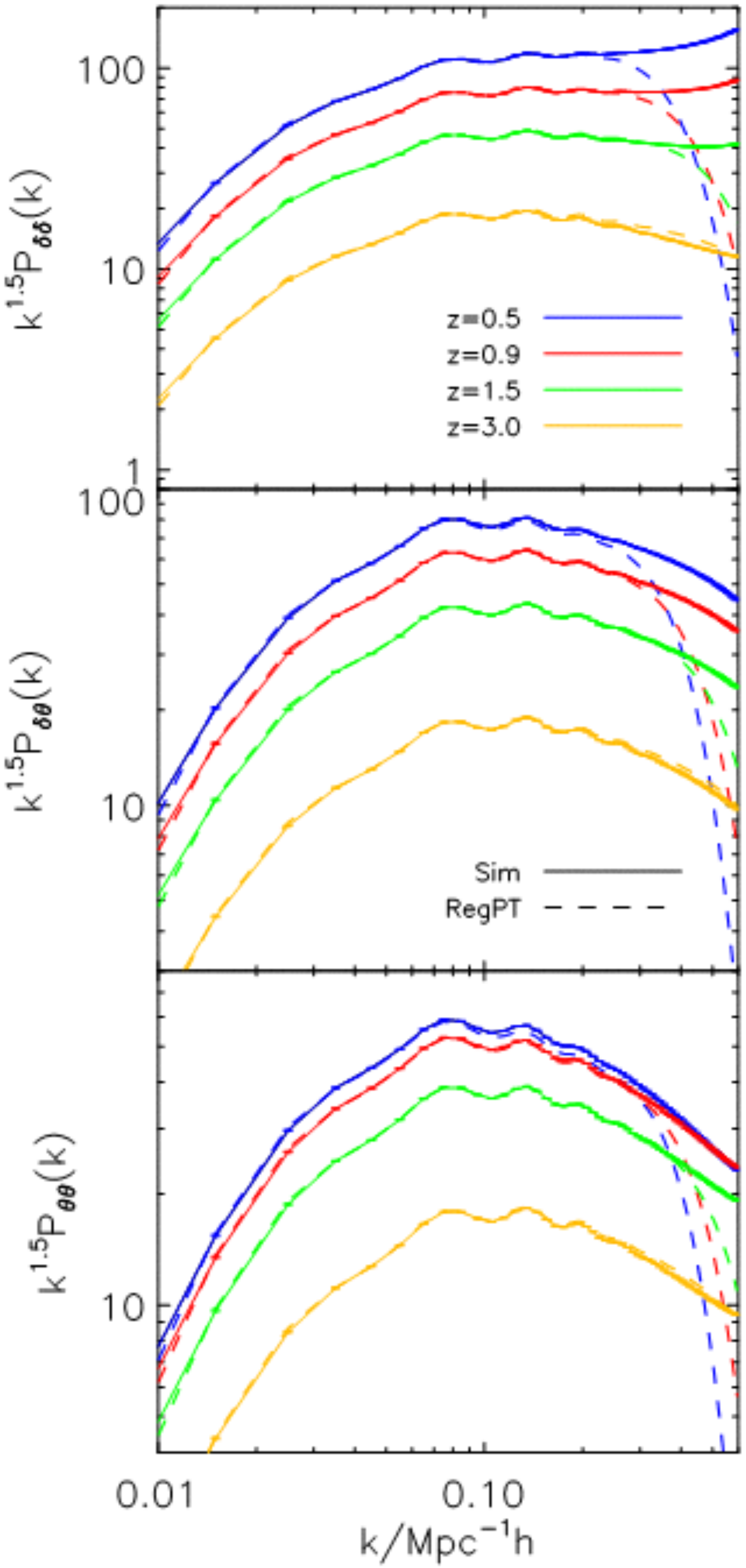
Taylor expansion in terms of $j_1 = -ik\mu$

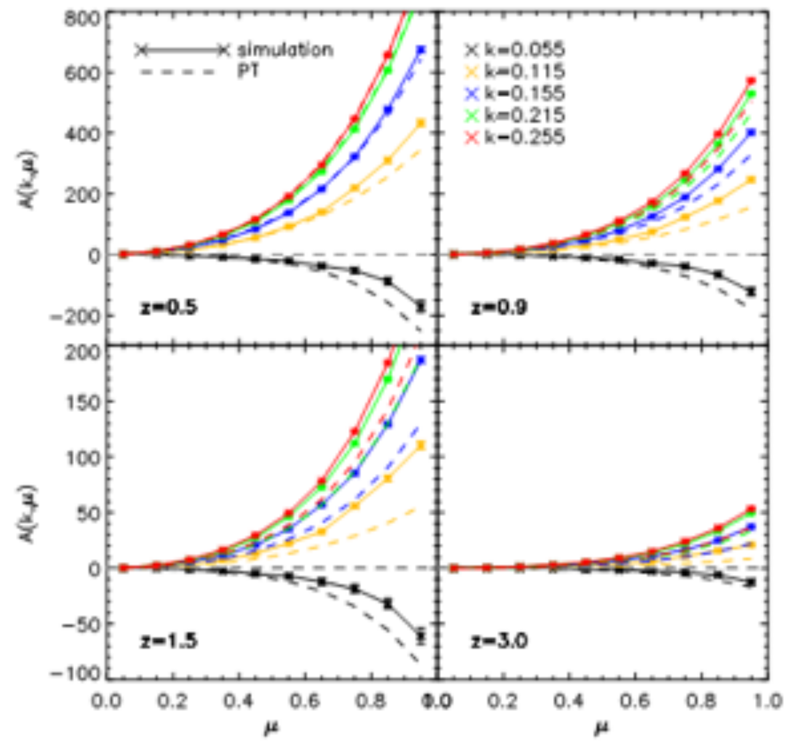
$$
\begin{aligned}
&D^{\rm FoG}_{\rm local}(k\mu,\boldsymbol{x})\left[\langle e^{j_1A_1}A_2A_3\rangle_c + \langle e^{j_1A_1}A_2\rangle_c\langle e^{j_1A_1}A_3\rangle_c\right] \\
&\simeq j_1^0\langle A_2A_3\rangle_c + j_1^1\langle A_1A_2A_3\rangle_c \\
&+j_1^2\Big\{\langle A_1A_2\rangle_c\langle A_1A_3\rangle_c + \frac{1}{2}\langle A_1^2A_2A_3\rangle_c - \langle u_zu_z'\rangle_c\langle A_2A_3\rangle_c\Big\} \\
&+\mathcal{O}(j_1^3)\,, \qquad (14)
\end{aligned}
$$

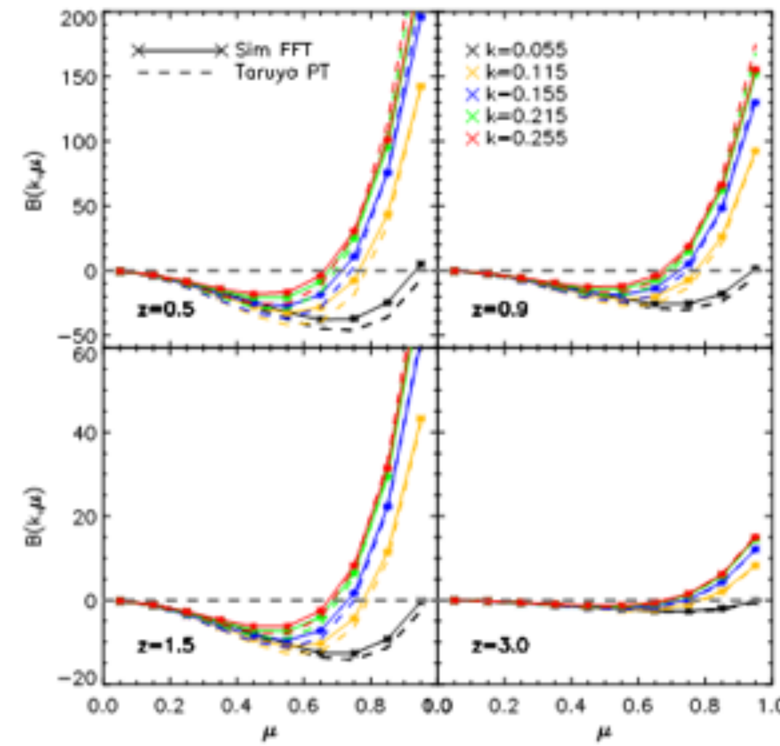
$$
\begin{aligned}
P^{(\rm S)}(k,\mu) &= D^{\rm FoG}(k\mu\sigma_z)P_{\rm perturbed}(k,\mu) \\
&= D^{\rm FoG}(k\mu\sigma_z)[P_{\delta\delta} + 2\mu^2P_{\delta\Theta} + \mu^4P_{\Theta\Theta} \quad (15) \\
&\quad +A(k,\mu) + B(k,\mu) + T(k,\mu) + F(k,\mu)].
\end{aligned}
$$

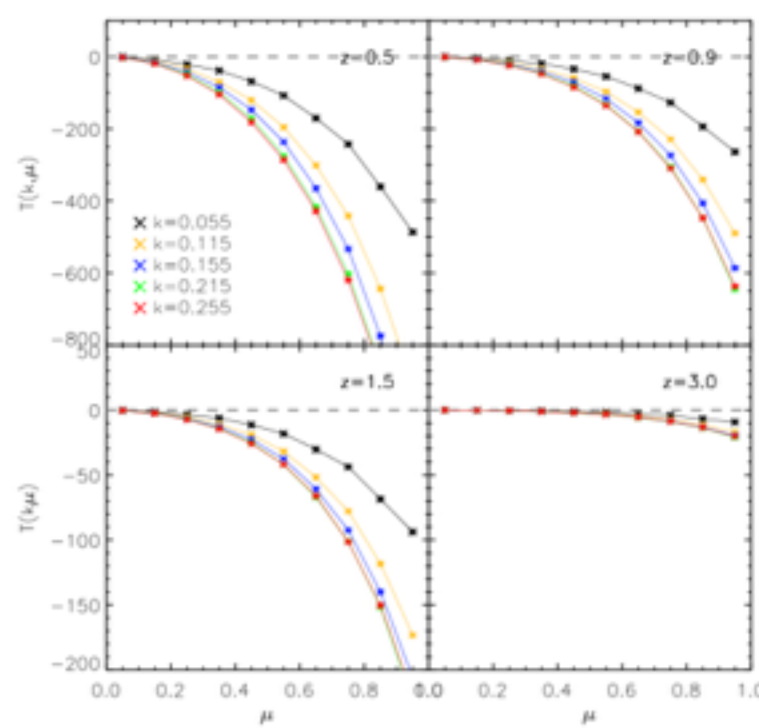
parameter	physical meaning	value
Ω_m	present fractional matter density	0.3132
Ω_Λ	$1-\Omega_m$	0.6868
Ω_b	present fractional baryon density	0.049
h	$H_0/(100\ {\rm km\ s^{-1}Mpc^{-1}})$	0.6731
n_s	primordial power spectral index	0.9655
σ_8	r.m.s. linear density fluctuation	0.829
$L_{\rm box}$	simulation box size	$1890\ h^{-1}$Mpc
$N_{\rm p}$	simulation particle number	1024^3
$m_{\rm p}$	simulation particle mass	$5.46\times10^{10}h^{-1}M_\odot$

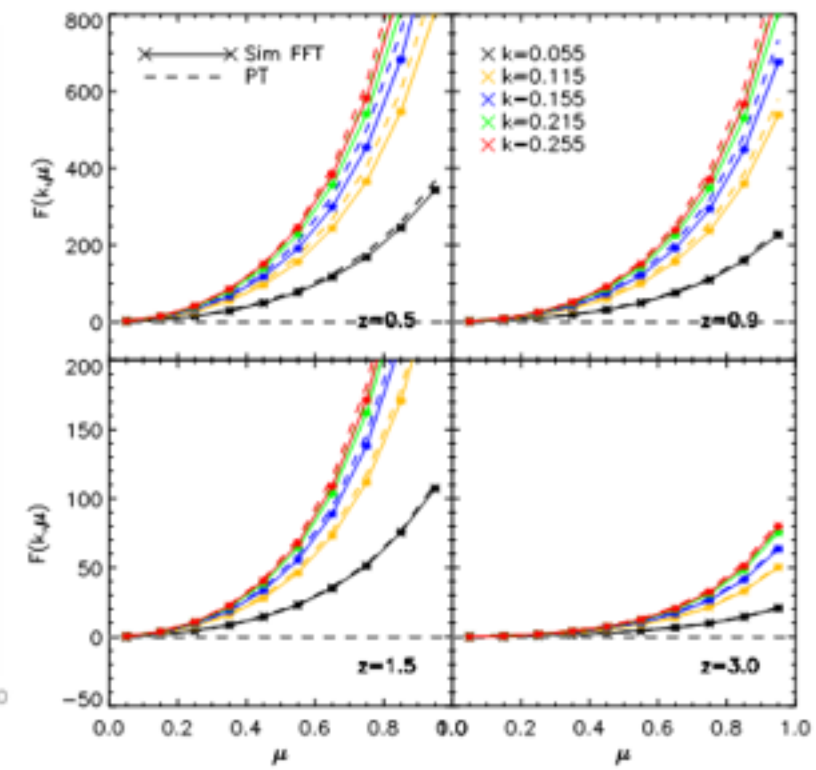
$$
\begin{aligned}
A(k,\mu) &= j_1\int d^3\boldsymbol{x}\ e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\ \langle A_1A_2A_3\rangle_c \\
&= j_1\int d^3\boldsymbol{x}\ e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\ \langle(u_z-u_z') \\
&\qquad\times(\delta+\nabla_zu_z)(\delta'+\nabla_zu_z')\rangle_c
\end{aligned}
$$







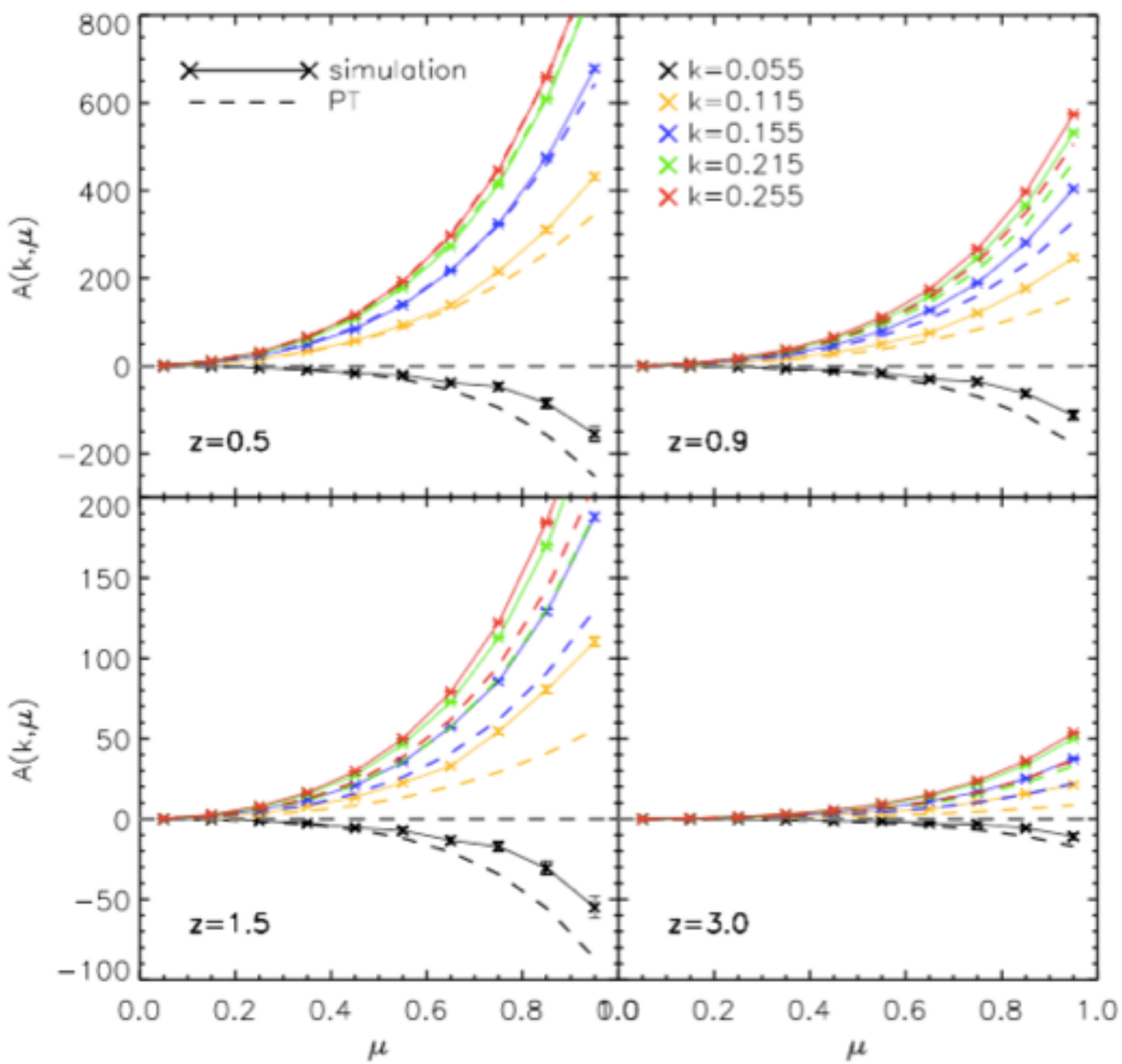




All higher order terms are important

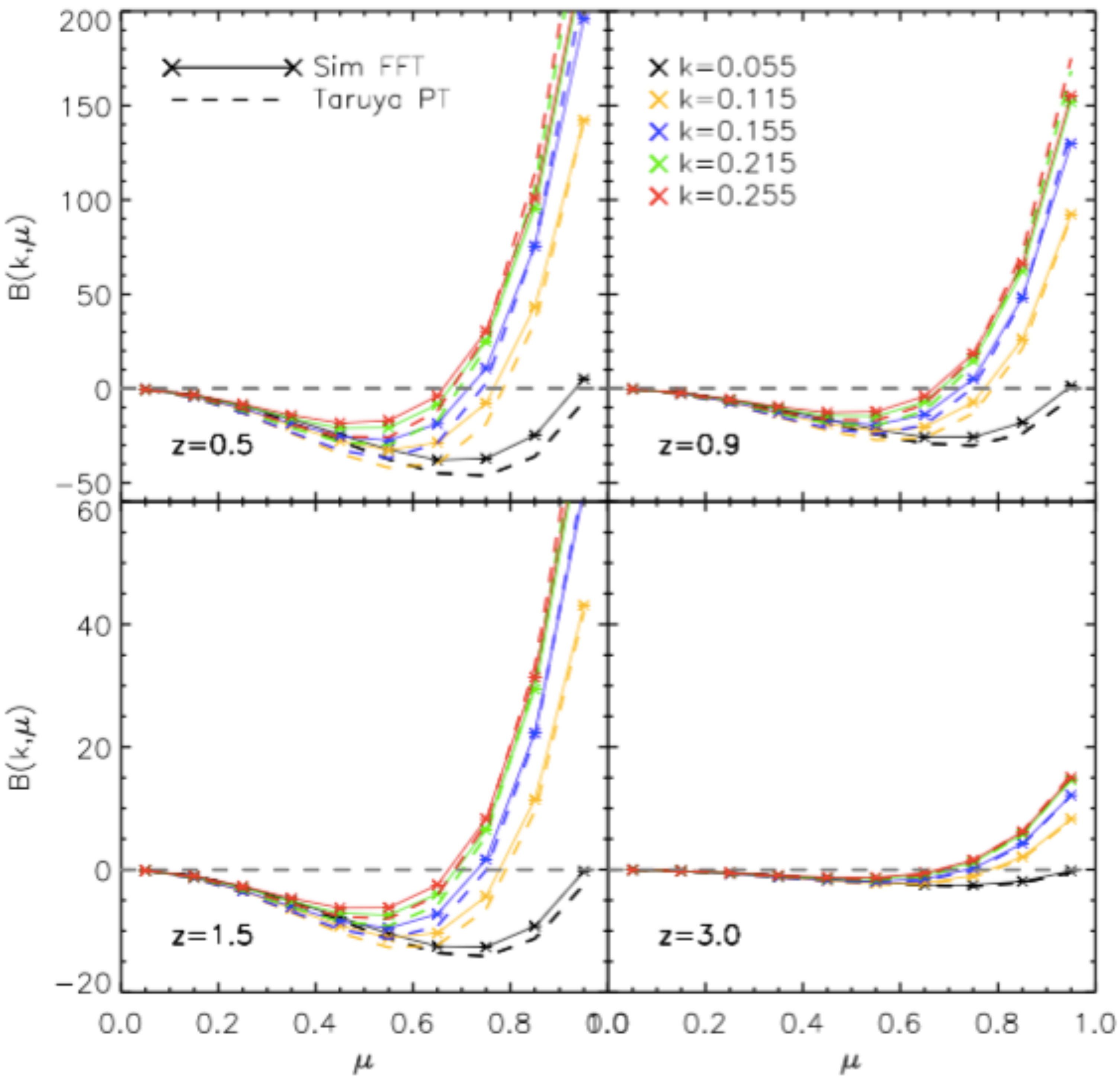
A term

$$
\begin{aligned}
A(k,\mu) &= j_1 \int d^3\boldsymbol{x}\; e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\; \langle A_1 A_2 A_3 \rangle_c \\
&= j_1 \int d^3\boldsymbol{x}\; e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\; \langle (u_z - u_z') \times (\delta + \nabla_z u_z)(\delta' + \nabla_z u_z') \rangle_c \\
&= (k\mu) \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{p_z}{p^2} \{ B_\sigma(\mathbf{p}, \boldsymbol{k}-\mathbf{p}, -\boldsymbol{k}) - \mathbf{B}_\sigma(\mathbf{p}, \boldsymbol{k}, -\boldsymbol{k}-\mathbf{p}) \},
\end{aligned}
$$



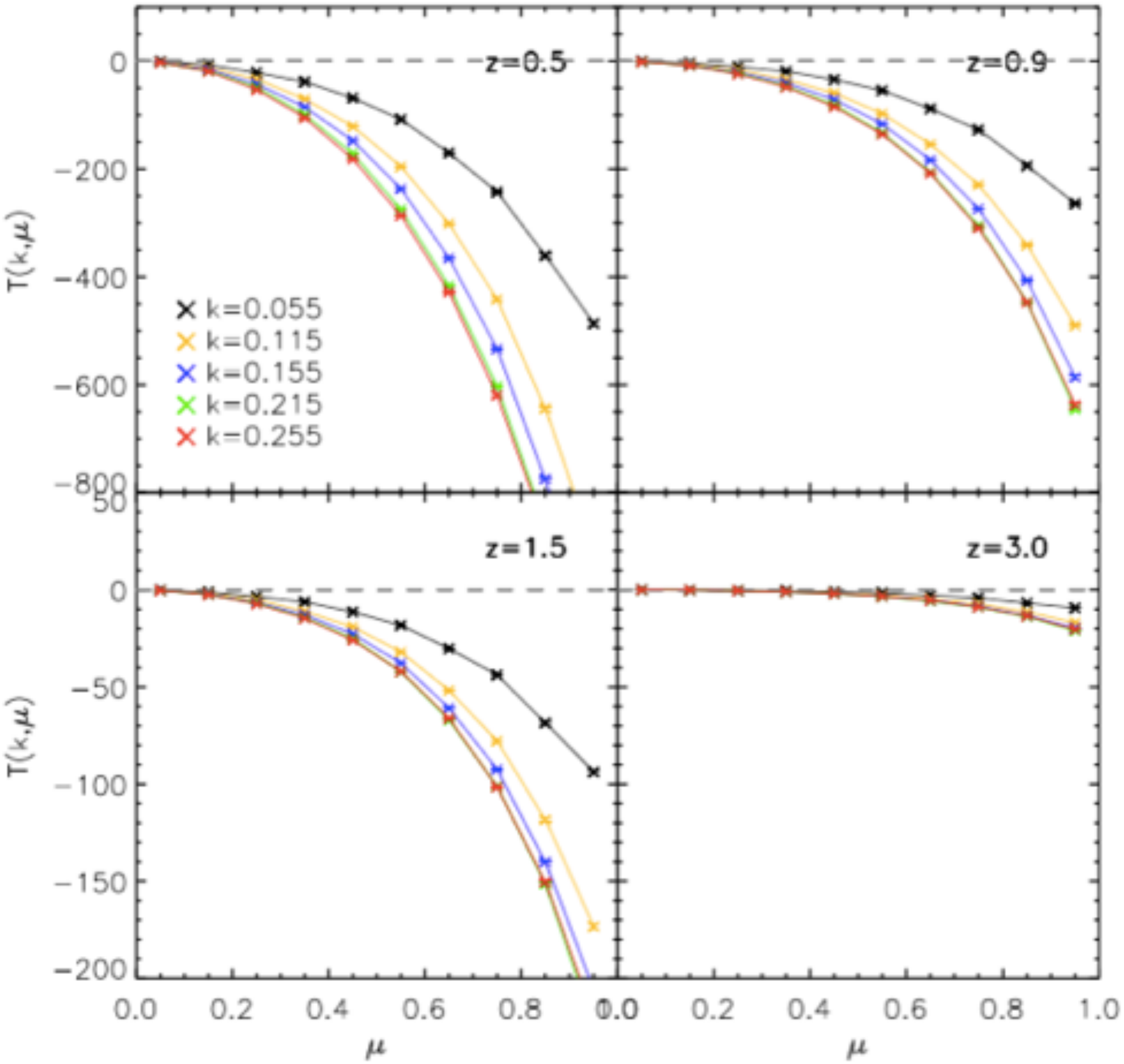
B term

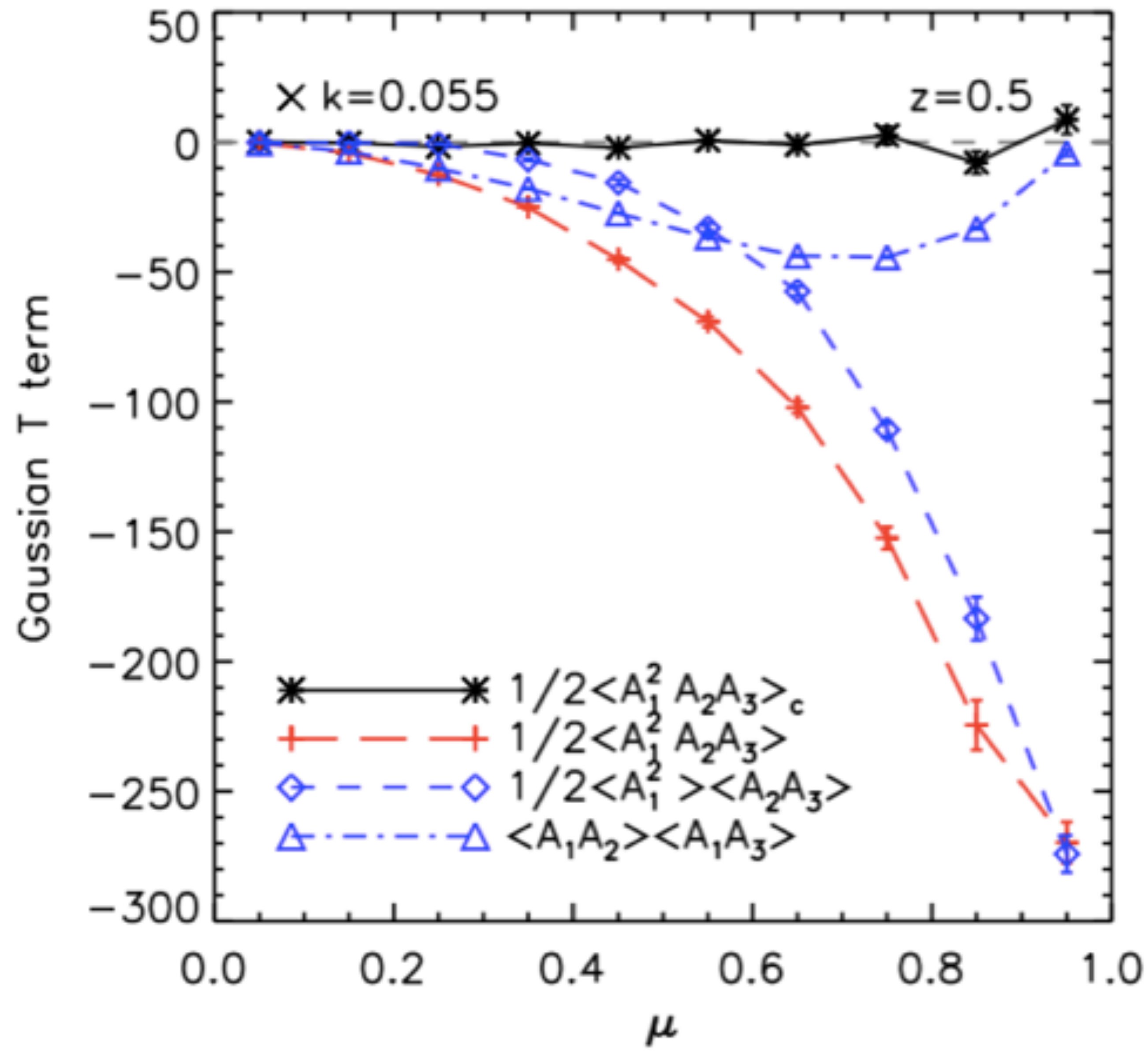
$$B(k,\mu) = j_1^2 \int d^3\boldsymbol{x}\; e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\; \langle A_1 A_2 \rangle_c \, \langle A_1 A_3 \rangle_c$$
$$= j_1^2 \int d^3\boldsymbol{x}\; e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\; \langle (u_z - u_z')(\delta + \nabla_z u_z) \rangle_c \times \langle (u_z - u_z')(\delta' + \nabla_z u_z') \rangle_c .$$



T term

$$
\begin{aligned}
T(k,\mu) &= \frac{1}{2} j_1^2 \int d^3\boldsymbol{x}\; e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\; \langle A_1^2 A_2 A_3\rangle_c \\
&= \frac{1}{2} j_1^2 \int d^3\boldsymbol{x}\; e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\; \{\langle A_1^2 A_2 A_3\rangle - \langle A_1^2\rangle\langle A_2 A_3\rangle - 2\langle A_1 A_2\rangle\langle A_1 A_3\rangle\} \\
&= \frac{1}{2} j_1^2 \int d^3\boldsymbol{x}\; e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\; \{\langle (u_z - u_z')^2 \times (\delta + \nabla_z u_z)(\delta' + \nabla_z u_z')\rangle \\
&\qquad - \langle A_1^2\rangle\langle A_2 A_3\rangle - 2\langle A_1 A_2\rangle\langle A_1 A_3\rangle\}\,.
\end{aligned}
$$

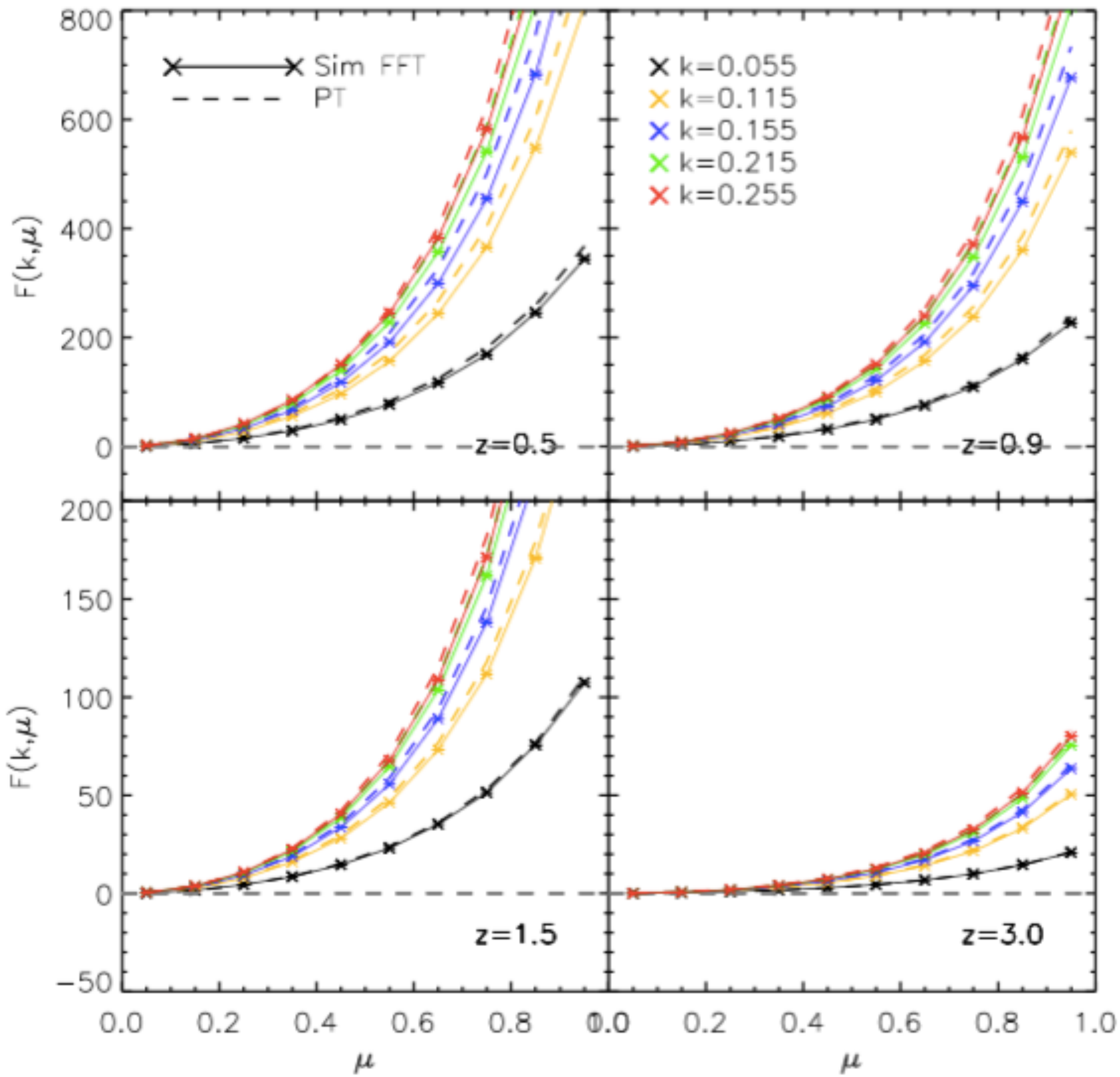




F term

$$F(k,\mu) = -j_1^2 \int d^3\boldsymbol{x}\; e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\; \langle u_z u_z' \rangle_c \langle A_2 A_3 \rangle_c$$

$$= -j_1^2 \int d^3\boldsymbol{x}\; e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\; \langle u_z u_z' \rangle_c \times \langle (\delta + \nabla_z u_z)(\delta' + \nabla_z u_z') \rangle_c.$$



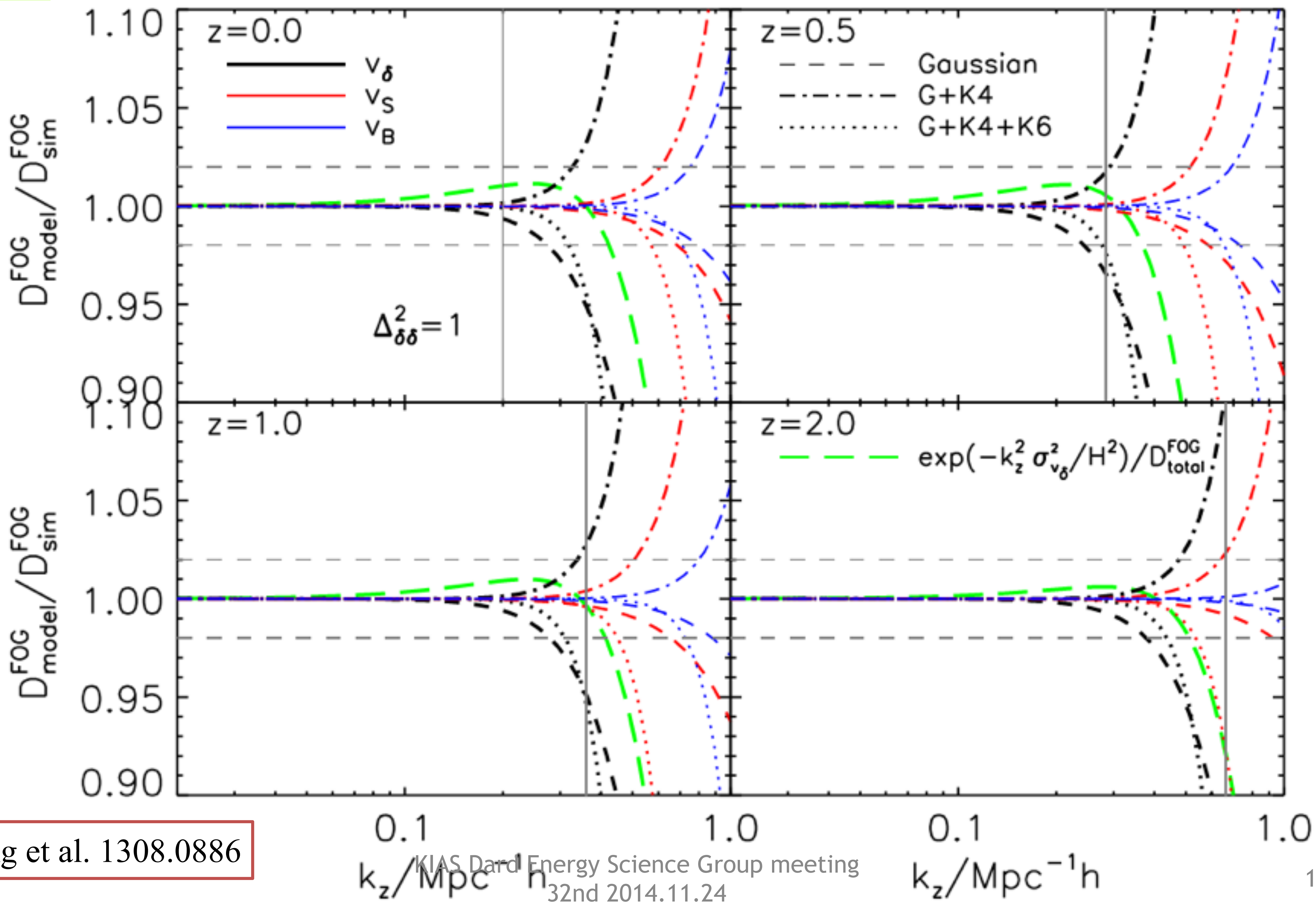
FoG effect: Gaussian approx.

$$D_{\rm 1pt}^{\rm FoG}(k\mu) = \exp\left\{ j_1^2\sigma_z^2 + 2\sum_{n=2}^{\infty} j_1^{2n}\sigma_z^{2n}\frac{K_{2n}}{(2n)!}\right\},$$

$$\sqrt{D_\alpha^{\rm FOG}(k_z)} = \int_{-\infty}^{\infty} \cos\left(\frac{k_z v_{z,\alpha}}{H}\right) P_\alpha(v_{z,\alpha}) dv_{z,\alpha}$$

Gaussian approximation works!

$$x \equiv (k_z \sigma_{v_\alpha}/H)^2$$

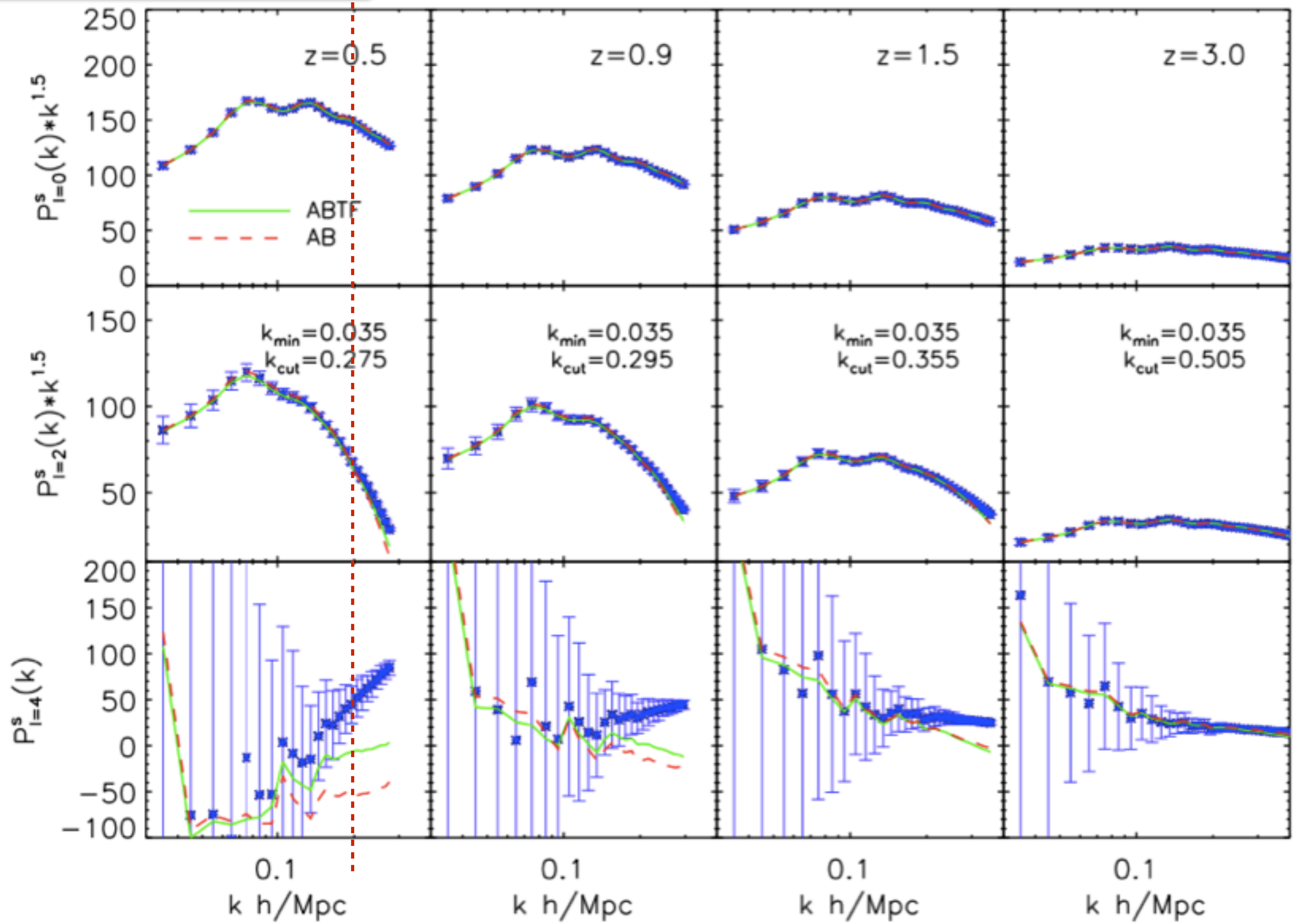


Zheng et al. 1308.0886

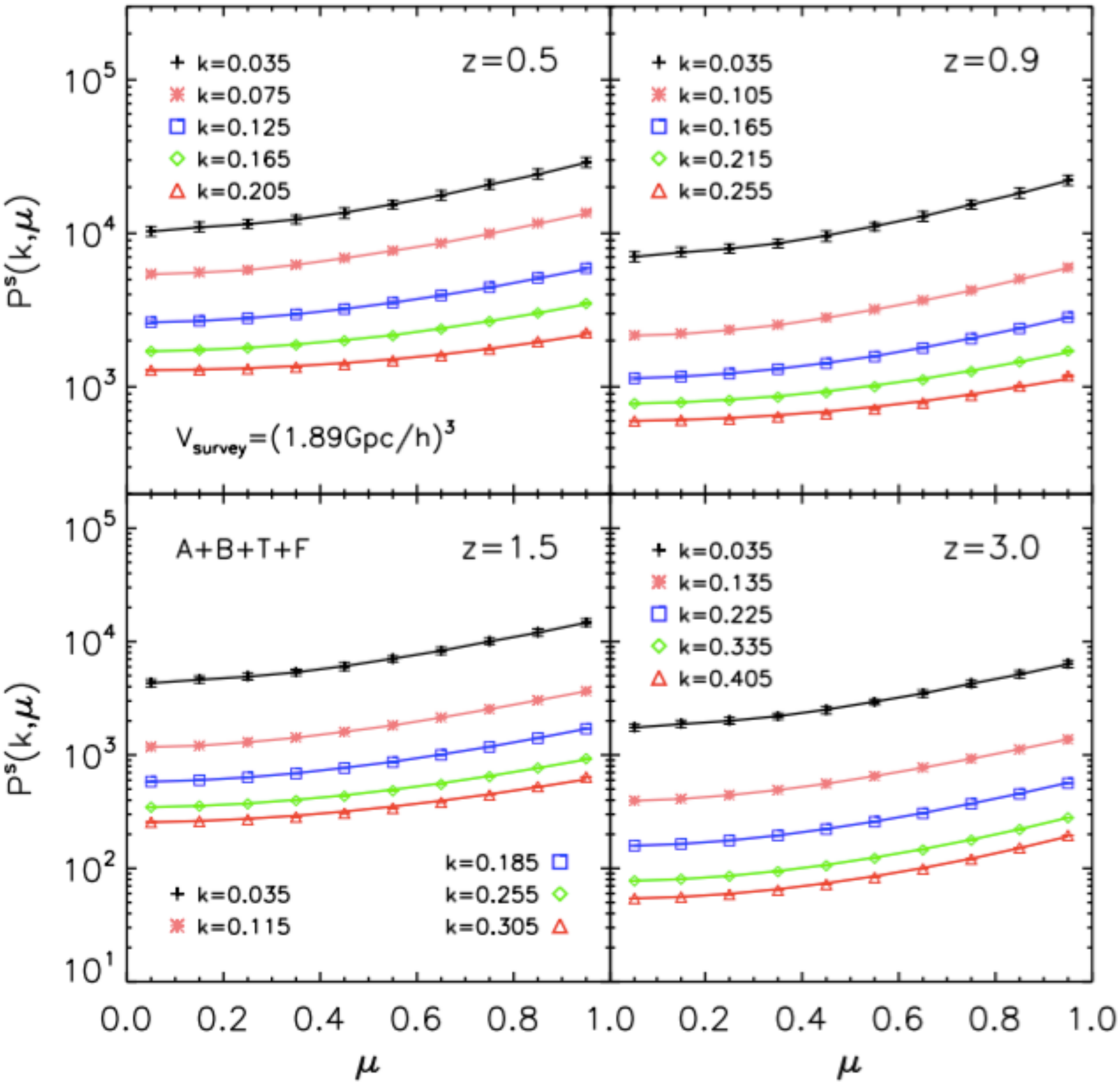
1. Multipole test:

\sigma_v as free parameter

$$P_\ell^{(S)}(k) = \frac{2\ell + 1}{2} \int_{-1}^{1} d\mu P^{(S)}(k, \mu) \mathcal{P}_\ell(\mu),$$



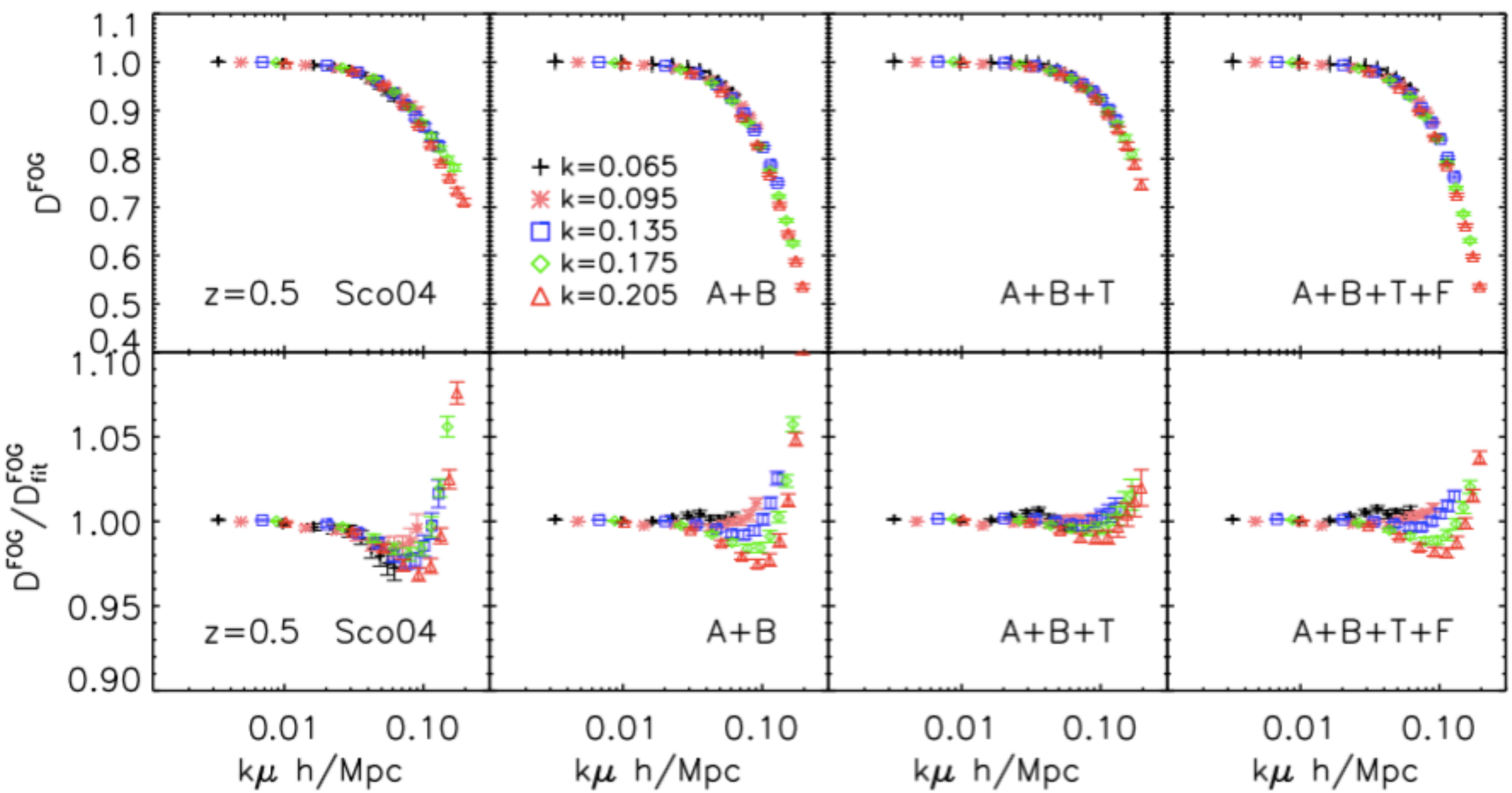
2. 2D power spectrum test:



3. FoG convergence test

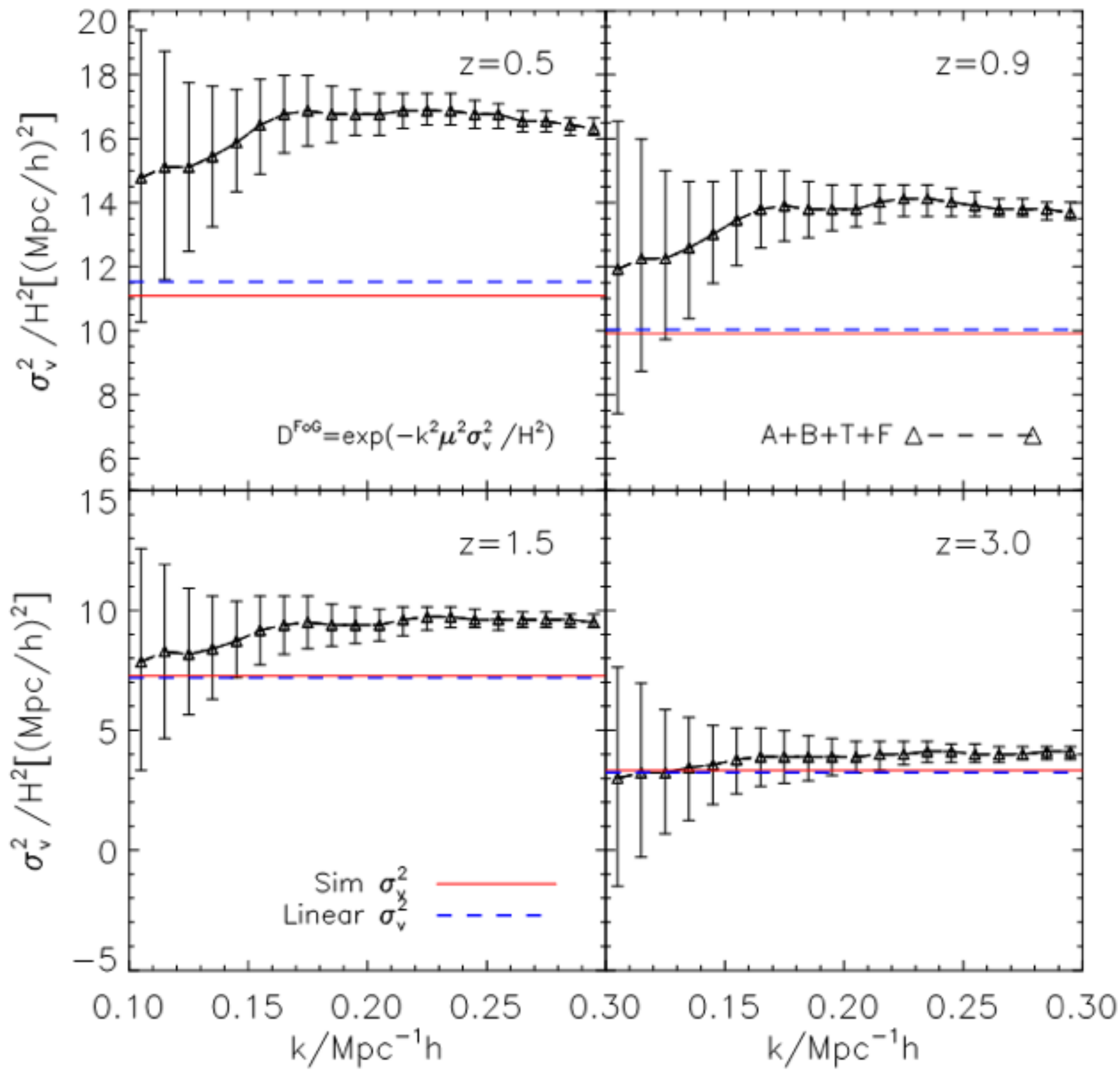
$$D^{\rm FoG} = \frac{P^{\rm (S)}_{\rm sim}(k,\mu)}{P_{\rm perturbed}(k,\mu)}$$

$$\begin{aligned} P^{\rm (S)}(k,\mu) &= D^{\rm FoG}(k\mu\sigma_z)P_{\rm perturbed}(k,\mu) \\ &= D^{\rm FoG}(k\mu\sigma_z)[P_{\delta\delta}+2\mu^2P_{\delta\Theta}+\mu^4P_{\Theta\Theta} \quad (15) \\ &\quad +A(k,\mu)+B(k,\mu)+T(k,\mu)+F(k,\mu)]. \end{aligned}$$



4. \Sigma_v consistency test

Check if σ_z is a constant over scales:



Question

why the measure and fitted \sigma_v is different ?

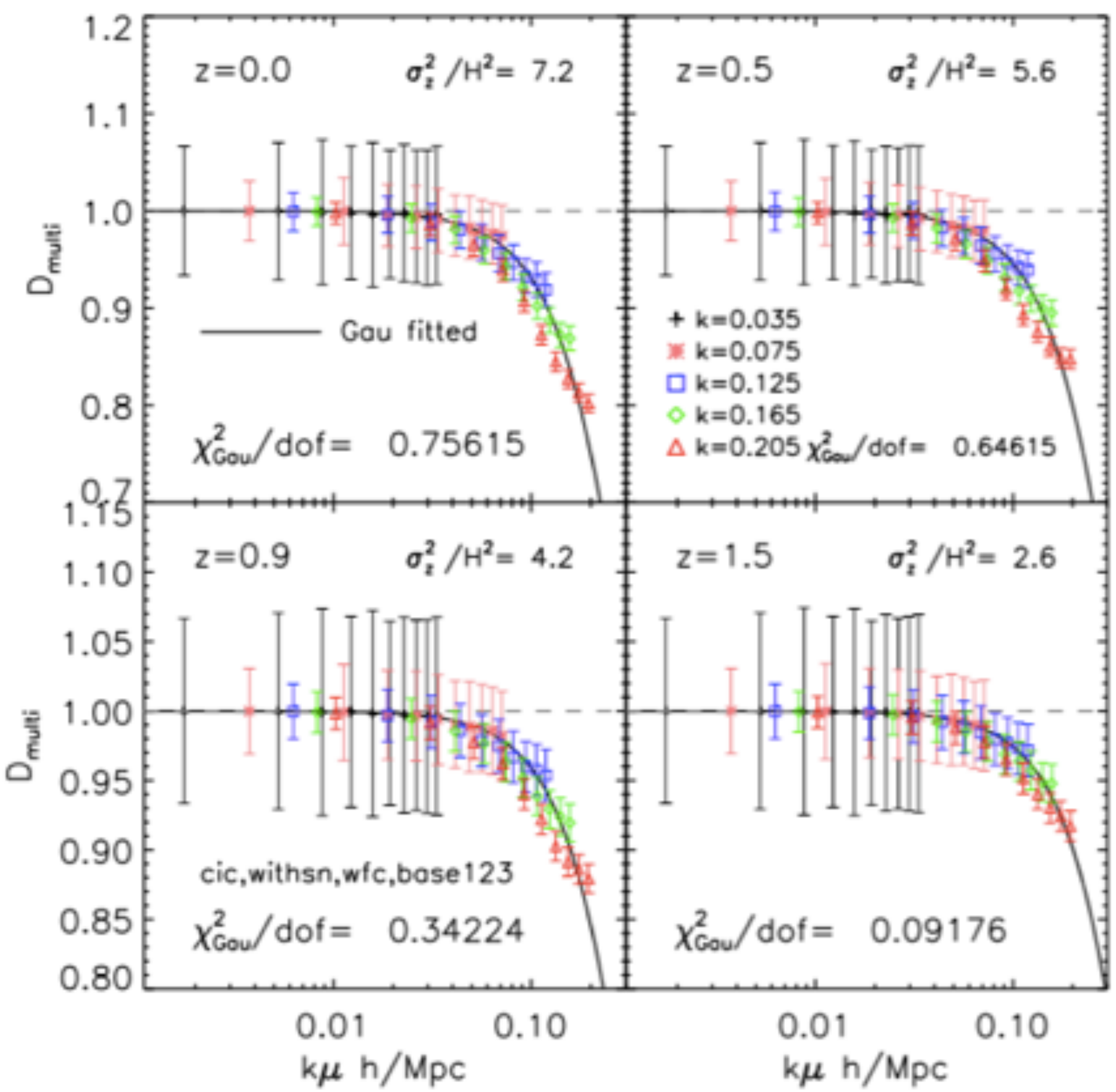
$$
\begin{aligned}
& D^{\mathrm{FoG}}_{\mathrm{local}}(k\mu, \boldsymbol{x}) \left[\langle e^{j_1 A_1} A_2 A_3\rangle_c + \langle e^{j_1 A_1} A_2\rangle_c \langle e^{j_1 A_1} A_3\rangle_c\right] \\
& \simeq j_1^0 \langle A_2 A_3\rangle_c + j_1^1 \langle A_1 A_2 A_3\rangle_c \\
& + j_1^2 \left\{ \langle A_1 A_2\rangle_c \langle A_1 A_3\rangle_c + \frac{1}{2} \langle A_1^2 A_2 A_3\rangle_c - \langle u_z u_z'\rangle_c \langle A_2 A_3\rangle_c \right\} \\
& + \mathcal{O}(j_1^3)\,, \qquad (14)
\end{aligned}
$$

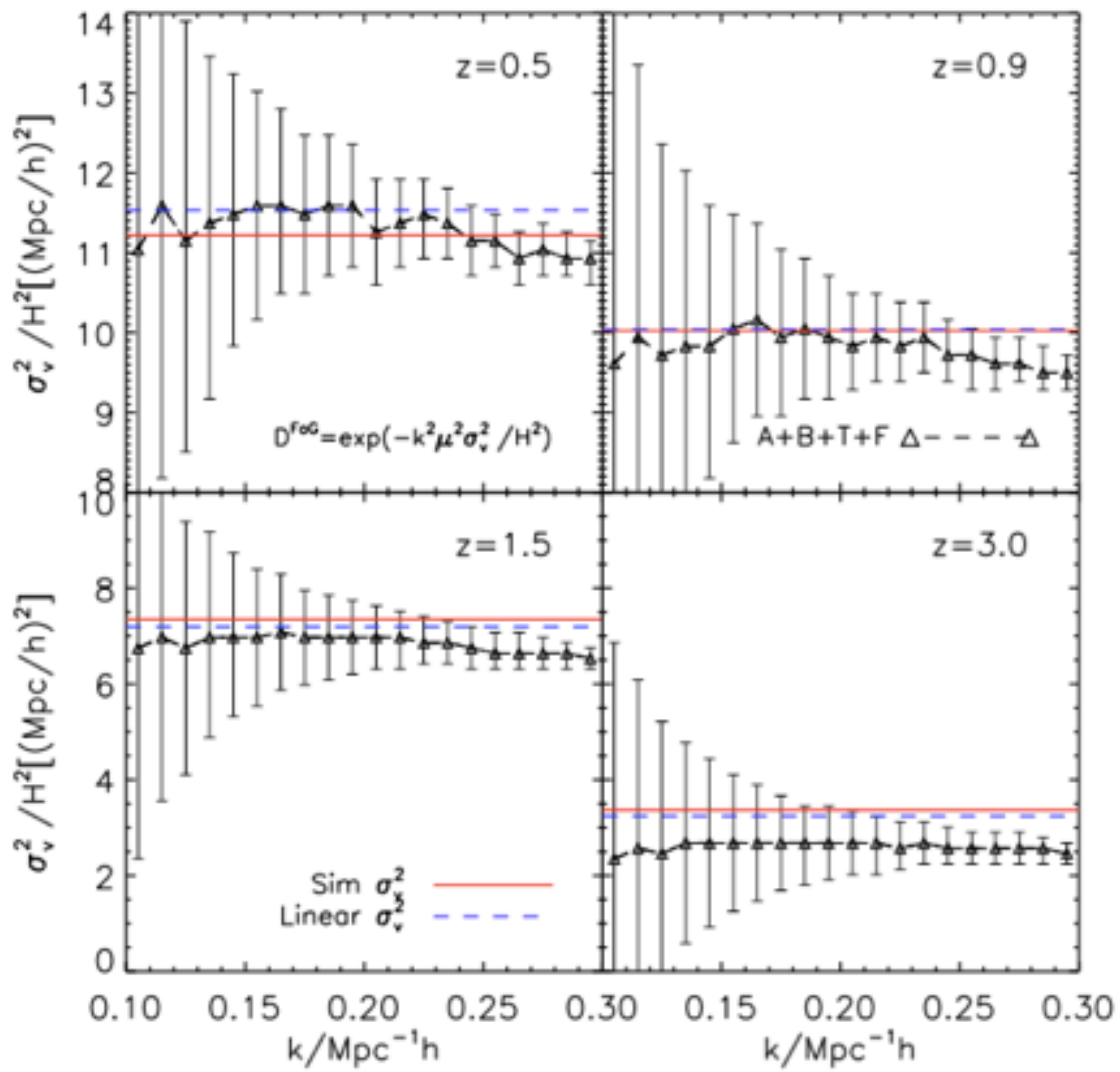
$$
\begin{aligned}
D^{\mathrm{FoG}}_{\mathrm{corr}}(k\mu, \boldsymbol{x}) &\equiv \exp\left\{ \sum_{n=1}^{\infty} j_1^{2n} \frac{\langle (u_z(\boldsymbol{r}) - u_z(\boldsymbol{r}'))^{2n}\rangle_c - \langle u_z(\boldsymbol{r})^{2n}\rangle_c - \langle u_z(\boldsymbol{r}')^{2n}\rangle_c}{(2n)!} \right\} \\
&= \exp\left\{ -j_1^2 \langle u_z(\boldsymbol{r}) u_z(\boldsymbol{r}')\rangle_c + \sum_{n=2}^{\infty} j_1^{2n} \frac{\langle (u_z(\boldsymbol{r}) - u_z(\boldsymbol{r}'))^{2n}\rangle_c - \langle u_z(\boldsymbol{r})^{2n}\rangle_c - \langle u_z(\boldsymbol{r}')^{2n}\rangle_c}{(2n)!} \right\}
\end{aligned}
$$

Answer:
multi-streaming effect?

$$P^s = \int d^3\boldsymbol{r} \exp(i\boldsymbol{k}\cdot\boldsymbol{r}) \times \left\langle (1+\delta_1)\exp(i\frac{k_z v_{1z}}{H})(1+\delta_2)\exp(-i\frac{k_z v_{2z}}{H}) \right\rangle. \quad (4)$$

$$D_{\rm multi} = \frac{\hat{P}^s}{\hat{P}^s_{\rm single}}.$$





Preliminary results

Discussions

1. RSD templates from N-body simulations?
2. Rescaling of the templates between different redshifts and cosmological parameters
3. Perturbative calculations
4. Bias issue — discussion tomorrow
5. Neutrino and MG