Screening fifth forces in Generalized Proca Theories

Ying-li Zhang (章颖理)
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A. De Felice, L. Heisenberg, R. Kase, S. Tsujikawa, YZ and G. Zhao, Phys. Rev. D 93 (2016) 104016

Content

1. Generalized Proca Theory

- Explanation of late-time cosmic acceleration: Dark Energy or Modified Gravity Theories?
- Higher-dimensional frameworks: e.g. Dvali-Gabadadze-Porrati (DGP) model;
- Vainshtein mechanism: nonlinear interactions of helicity-0 mode;
- Galileon, covariant Galileon and Horndeski;
- 2013: No-go theorem for p-form Galileon with gauge invariance…

However, this is not the case for massive spin-1 fields. Using the technique developed in Horndeski, the Lagrangian is expressed as

$$\tilde{\mathcal{L}}_{i+2} = -\frac{1}{(4-i)!} G_{i+2}(X) \mathcal{E}_{\alpha_1 \cdots \alpha_i \gamma_{i+1 \cdots 4}} \mathcal{E}^{\beta_1 \cdots \beta_i \gamma_{i+1 \cdots 4}} \nabla_{\beta_1} A^{\alpha_1} \cdots \nabla_{\beta_i} A^{\alpha_i} \,,$$

Moreover, we the Horndeski form can be further extended to include

$$Y = A^\mu A^\nu F_\mu{}^\alpha F_{\nu\alpha}$$

$$L^{\mu\nu\alpha\beta} = \frac{1}{4} \epsilon^{\mu\nu\rho\sigma} \epsilon^{\alpha\beta\gamma\delta} R_{\rho\sigma\gamma\delta}$$

$$\tilde{F}^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta}$$

so that the whole Lagrangian is expressed as

$$S = \int d^4x \sqrt{-g}\,(\mathcal{L} + \mathcal{L}_M)\ , \qquad \mathcal{L} = \sum_{i=2}^{6} \mathcal{L}_i\ ,$$

where

$$\begin{aligned}
\mathcal{L}_2 &= G_2(X, F, Y)\,, \\
\mathcal{L}_3 &= G_3(X)\nabla_\mu A^\mu\,, \\
\mathcal{L}_4 &= G_4(X)R + G_{4,X}(X)\left[(\nabla_\mu A^\mu)^2 - \nabla_\rho A_\sigma \nabla^\sigma A^\rho\right]\ , \\
\mathcal{L}_5 &= G_5(X)G_{\mu\nu}\nabla^\mu A^\nu - \frac{1}{6}G_{5,X}(X)[(\nabla_\mu A^\mu)^3 - 3\nabla_\mu A^\mu \nabla_\rho A_\sigma \nabla^\sigma A^\rho + 2\nabla_\rho A_\sigma \nabla^\gamma A^\rho \nabla^\sigma A_\gamma] \\
&\quad - g_5(X)\tilde{F}^{\alpha\mu}\tilde{F}^{\beta}{}_{\mu}\nabla_\alpha A_\beta\,, \\
\mathcal{L}_6 &= G_6(X)L^{\mu\nu\alpha\beta}\nabla_\mu A_\nu \nabla_\alpha A_\beta + \frac{1}{2}G_{6,X}(X)\tilde{F}^{\alpha\beta}\tilde{F}^{\mu\nu}\nabla_\alpha A_\mu \nabla_\beta A_\nu\,,
\end{aligned}$$

2. Spherical Symmetric Solutions

A. De Felice, L. Heisenberg, R. Kase, S. Tsujikawa, YZ and G. Zhao, Phys. Rev. D 93 (2016) 104016 [arXiv:1602.00371 [gr-qc]]

- In this theory, for the vector field, besides two transverse modes, there appears the propagation of longitudinal mode, which would mediate the fifth force

Screened or not?

We consider the spherically symmetric and static background

$$ds^2 = -e^{2\Psi(r)}dt^2 + e^{2\Phi(r)}dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)$$

with the energy-momentum tensor

$$T^\mu_\nu = \mathrm{diag}(-\rho_m, P_m, P_m, P_m)$$

$$\Longrightarrow \quad P_m' + \Psi'(\rho_m + P_m) = 0$$

We take the form $\boxed{A^\mu = (\phi, A^i)}$

$$A_i = A_i^{(T)} + \nabla_i\chi$$

Let use firstly solve $A_i^{(T)}$

spherically symmetric $\Longrightarrow$ $A_2^{(T)} = A_3^{(T)} = 0$

Using the traceless condition

$$\nabla^i A_i^{(T)} = 0 \Longrightarrow A_1^{(T)\prime} + \frac{2}{r} A_1^{(T)} - \Phi' A_1^{(T)} = 0$$

$$\Downarrow$$

regularity at r=0

$$C = 0 \Longleftarrow A_1^{(T)} = C\,\frac{e^{\Phi}}{r^2}$$

So $A_1^{(T)} = 0$, we only need focus on the longitudinal mode

$$A^{\mu} = \left(\phi(r), e^{-2\Phi}\chi'(r), 0, 0\right)$$

For the consistency with local gravity experiments within the solar system, we require that the gravitational potentials need to be close to those in GR without the vector field

$$\frac{2M_{\rm pl}^2}{r}\Phi_{\rm GR}' - \frac{M_{\rm pl}^2}{r^2}\left(1-e^{2\Phi_{\rm GR}}\right) = e^{2\Phi_{\rm GR}}\rho_m\,,$$

$$\frac{2M_{\rm pl}^2}{r}\Psi_{\rm GR}' + \frac{M_{\rm pl}^2}{r^2}\left(1-e^{2\Phi_{\rm GR}}\right) = e^{2\Phi_{\rm GR}}P_m\,,$$

Assumption: $\rho_m(r<r_*)=\rho_0, \quad \rho_m(r>r_*)=0$

Choosing appropriate boundary conditions, the equations above can be solved as

$$r < r_* \, , \quad e^{\Psi_{\rm GR}} = \frac{3}{2}\sqrt{1 - \frac{\rho_0 r_*^2}{3M_{\rm pl}^2}} - \frac{1}{2}\sqrt{1 - \frac{\rho_0 r^2}{3M_{\rm pl}^2}} \, , \qquad e^{\Phi_{\rm GR}} = \left(1 - \frac{\rho_0 r^2}{3M_{\rm pl}^2}\right)^{-1/2} \, ,$$

$$r > r_* \, , \quad e^{\Psi_{\rm GR}} = \left(1 - \frac{\rho_0 r_*^3}{3M_{\rm pl}^2 r}\right)^{1/2} \, , \qquad e^{\Phi_{\rm GR}} = \left(1 - \frac{\rho_0 r_*^3}{3M_{\rm pl}^2 r}\right)^{-1/2} \, ,$$

$$\Rightarrow \begin{cases} r < r_* \, , \quad \Psi_{\rm GR} \simeq \frac{\rho_0}{12M_{\rm pl}^2}\left(r^2 - 3r_*^2\right) \, , \qquad \Phi_{\rm GR} \simeq \frac{\rho_0 r^2}{6M_{\rm pl}^2} \, , \\ r > r_* \, , \quad \Psi_{\rm GR} \simeq -\frac{\rho_0 r_*^3}{6M_{\rm pl}^2 r} \, , \qquad \Phi_{\rm GR} \simeq \frac{\rho_0 r_*^3}{6M_{\rm pl}^2 r} \, , \end{cases}$$

We will insert these into EOM and solve for χ

3. A concrete model with $\mathcal{L}_3$

- We consider the case where the cubic Lagrangian contributes while $\mathcal{L}_4 = M_{\rm pl}^2/2$ corresponds to GR.
- Meanwhile, we choose simple functions

$$G_2(X) = m^2 X\,, \qquad G_3(X) = \beta_3 X$$

- Assumption: Profile

$$\phi(r) = \phi_0 + f(r)\,, \qquad |f(r)| \ll |\phi_0|$$

i.e. the scalar potential does not vary much in the presence of the non-linear interaction.

In the following, we separate into two cases.

(I). $r < r_*$

In this case, inserting $\Psi_{\rm GR}(r < r_*)\,, \quad \Phi_{\rm GR}(r < r_*)$ into equations and expanding the exponential terms to the first order in $\Psi_{\rm GR}, \Phi_{\rm GR}$

0-com: $$\frac{d}{dr}(r^2\phi') - m^2 r^2 \phi - \beta_3 \phi \, \frac{d}{dr}(r^2 \chi') + \frac{\rho_0}{6M_{\rm pl}^2}\left[6\phi + r\left(\phi' + \beta_3 \chi' \phi\right)\right] r^2 \simeq 0\,,$$

1-com: $$m^2 \chi' + \beta_3 \left(\phi\phi' + \frac{2}{r}\chi'^2 + \frac{\rho_0 \phi^2}{6M_{\rm pl}^2} r\right) \simeq 0$$

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1-com: $$m^2\chi' + \beta_3\left(\phi\phi' + \frac{2}{r}\chi'^2 + \frac{\rho_0\phi^2}{6M_{\rm pl}^2} r\right) \simeq 0$$

$$\chi' = \frac{m^2 r}{4\beta_3}\left[-1 + \sqrt{1 - \frac{8\beta_3^2}{m^4 r}\left(\phi\phi' + \frac{\rho_0\phi^2}{6M_{\rm pl}^2} r\right)}\right]$$

$\mathrm{GR}: \beta_3/m^2 \to 0, \quad \chi' \to 0$

For our interest, we consider the massless limit with nonvanishing cubic term, i.e. $m^2/\beta_3 \to 0$

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In this case, inserting $\Psi_{\rm GR}(r<r_*)\,,\quad \Phi_{\rm GR}(r<r_*)$ into equations and expanding the exponential terms to the first order in $\Psi_{\rm GR}, \Phi_{\rm GR}$

0-com: $$\frac{d}{dr}(r^2\phi') - m^2 r^2\phi - \beta_3\phi\,\frac{d}{dr}(r^2\chi') + \frac{\rho_0}{6M_{\rm pl}^2}\left[6\phi + r\left(\phi' + \beta_3\chi'\phi\right)\right] r^2 \simeq 0\,,$$

1-com: $$m^2\chi' + \beta_3\left(\phi\phi' + \frac{2}{r}\chi'^2 + \frac{\rho_0\phi^2}{6M_{\rm pl}^2}r\right) \simeq 0$$

$$\chi' = \frac{m^2 r}{4\beta_3}\left[-1 + \sqrt{1 - \frac{8\beta_3^2}{m^4 r}\left(\phi\phi' + \frac{\rho_0\phi^2}{6M_{\rm pl}^2}r\right)}\right]$$

$${\rm GR}: \beta_3/m^2 \to 0, \quad \chi' \to 0$$

For our interest, we consider the massless limit with nonvanishing cubic term, i.e. $m^2/\beta_3 \to 0$

Then we have $\chi' = \sqrt{-\frac{r}{2}\left(\phi\phi' + \frac{\rho_0\phi^2}{6M_{\rm pl}^2}r\right)}$

Assumption: the scalar potential is slowly varying, i.e.

$$\phi(r) = \phi_0 + f(r)\,, \qquad |f(r)| \ll |\phi_0|\,,$$

neglecting $r\left(\phi' + \beta_3\chi'\phi\right)$, the 0-com can be integrated once with $\phi'(0) = 0$

$$f' - \beta_3\phi_0^{3/2}\sqrt{-\frac{r}{2}\left(f' + \frac{\rho_0\phi_0 r}{6M_{\rm pl}^2}\right)} = -\frac{\rho_0\phi_0}{3M_{\rm pl}^2}r$$

A specific solution is $f(r) = -Br^2$

$$B = \frac{\rho_0\phi_0}{6M_{\rm pl}^2}\mathcal{F}(s_{\beta_3})$$

$$\mathcal{F}(s_{\beta_3}) \equiv (1+s_{\beta_3})\left(1 - \sqrt{\frac{s_{\beta_3}}{1+s_{\beta_3}}}\right)$$

$$s_{\beta_3} \equiv \frac{3(\beta_3\phi_0 M_{\rm pl})^2}{4\rho_0}$$

Hence, the analytic solutions are

$$\begin{cases} \phi(r) = \phi_0 \left[1 - \mathcal{F}(s_{\beta_3}) \frac{\rho_0}{6M_{\rm pl}^2} r^2\right] \\ \chi'(r) = \sqrt{\frac{\rho_0 \phi_0^2}{6M_{\rm pl}^2} \left[\mathcal{F}(s_{\beta_3}) - \frac{1}{2}\right]} r \end{cases}$$

⇓

$$\begin{cases} s_{\beta_3} \ll 1, \quad \phi(r) \simeq \phi_0 \left(1 - \frac{\rho_0}{6M_{\rm pl}^2} r^2\right), \quad \chi'(r) \simeq \sqrt{\frac{\rho_0 \phi_0^2}{12M_{\rm pl}^2}} r \\ s_{\beta_3} \gg 1, \quad \phi(r) \simeq \phi_0 \left(1 - \frac{\rho_0}{12M_{\rm pl}^2} r^2\right), \quad \chi'(r) \simeq \frac{\rho_0}{6\beta_3 M_{\rm pl}^2} r \end{cases}$$

↑

$s_{\beta_3}^{-1/2}$ times smaller

⇓

when β_3 is large, screening effect is efficient to suppress longitudinal mode

(II). $r > r_*$

In this case, inserting $\Psi_{\rm GR}(r > r_*)\,,\quad \Phi_{\rm GR}(r > r_*)$ into equations and expanding the exponential terms to the first order in $\Psi_{\rm GR}, \Phi_{\rm GR}$

0-com: $$\frac{d}{dr}(r^2\phi') - m^2 r^2 \phi - \beta_3 \phi \frac{d}{dr}(r^2\chi') + \frac{\rho_0 r_*^3}{9M_{\rm pl}^4 r^2}\left[\rho_0 r_*^3 \phi + 3M_{\rm pl}^2 r^2 (2\phi' - \beta_3 \chi' \phi)\right] \simeq 0$$

1-com: $$m^2\chi' + \beta_3\left(\phi\phi' + \frac{2}{r}\chi'^2 + \frac{\rho_0 r_*^3 \phi^2}{6M_{\rm pl}^2 r^2}\right) \simeq 0$$

$m \to 0$

$$\chi' = \sqrt{-\frac{r}{2}\left(\phi\phi' + \frac{\rho_0 r_*^3 \phi^2}{6M_{\rm pl}^2 r^2}\right)}$$

matching $r < r_*$

Then the 0-com becomes: $$r^2\phi' - \beta_3 \phi_0^{3/2} r^2 \sqrt{-\frac{r}{2}\left(\phi' + \frac{\rho_0 \phi_0 r_*^3}{6M_{\rm pl}^2 r^2}\right)} \simeq -\frac{\rho_0 \phi_0 r_*^3}{3M_{\rm pl}^2}$$

So we can solve this algebraic equation to find ϕ'

$$\begin{cases} \phi'(r) = -\dfrac{\rho_0 \phi_0 r_*^3}{3M_{\rm pl}^2 r^2}\mathcal{F}(\xi)\,, \qquad \xi \equiv s_{\beta_3}\dfrac{r^3}{r_*^3} \\ \chi'(r) = \sqrt{\dfrac{\rho_0 r_*^3 \phi_0^2}{6M_{\rm pl}^2 r}\left[\mathcal{F}(\xi) - \dfrac{1}{2}\right]} \end{cases}$$

(a). $s_{\beta_3} \gg 1 \Rightarrow \xi \gg 1$

$$\phi'(r) \simeq -\frac{\rho_0 \phi_0 r_*^3}{6M_{\rm pl}^2 r^2}\,, \qquad \chi'(r) \simeq \frac{\rho_0 r_*^3}{6\beta_3 M_{\rm pl}^2 r^2}$$

(b). $s_{\beta_3} \lesssim 1$

$$\xi = 1 \quad \Rightarrow \quad r_V = \frac{r_*}{s_{\beta_3}^{1/3}} \quad \Rightarrow \quad \xi = \frac{r^3}{r_V^3}$$

(b-1). $r_* < r \ll r_V \quad \Rightarrow \quad \xi \ll 1$

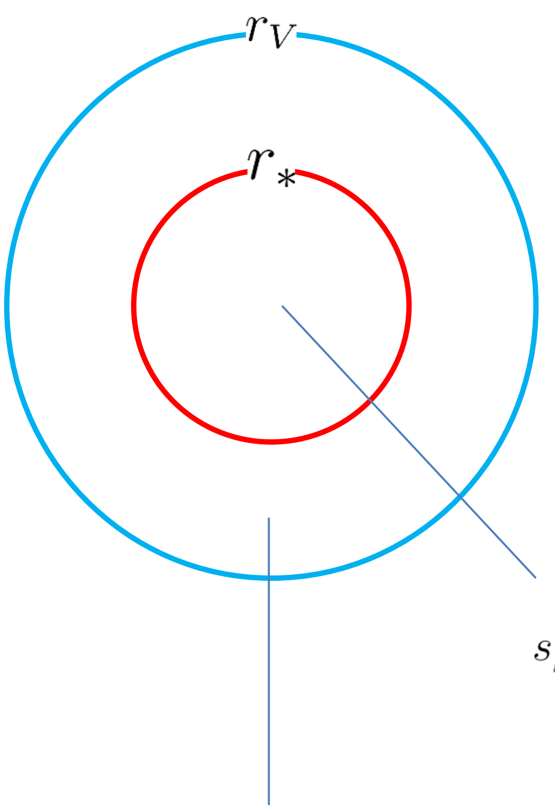
$$\phi'(r) \simeq -\frac{\rho_0 \phi_0 r_*^3}{3M_{\rm pl}^2 r^2}\,, \qquad \chi'(r) \simeq \sqrt{\frac{\rho_0 r_*^3 \phi_0^2}{12 M_{\rm pl}^2 r}}$$

(b-2). $r \gg r_V \quad \Rightarrow \quad \xi \gg 1$ In this case, $\mathcal{F}$ is close to ½, χ' suppressed in amplitude!!!

$$s_{\beta_3} \lesssim 1$$

$$\mathcal{F} \to \frac{1}{2}$$ Suppressed outside Vainshtein radius

$$\phi'(r) = -\frac{\rho_0 \phi_0 r_*^3}{3M_{\rm pl}^2 r^2} \mathcal{F}(\xi)\,, \qquad \chi'(r) = \sqrt{\frac{\rho_0 r_*^3 \phi_0^2}{6M_{\rm pl}^2 r}\left[\mathcal{F}(\xi) - \frac{1}{2}\right]}$$

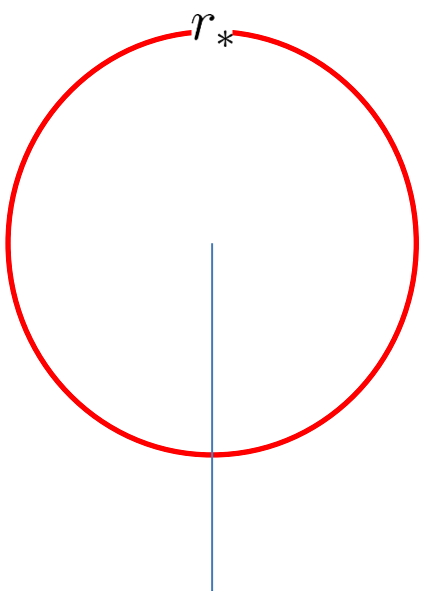


$$s_{\beta_3} \ll 1, \qquad \phi(r) \simeq \phi_0 \left(1 - \frac{\rho_0}{6M_{\rm pl}^2} r^2\right)\,, \qquad \chi'(r) \simeq \sqrt{\frac{\rho_0 \phi_0^2}{12 M_{\rm pl}^2} r}$$

$$\phi'(r) \simeq -\frac{\rho_0 \phi_0 r_*^3}{3M_{\rm pl}^2 r^2}\,, \qquad \chi'(r) \simeq \sqrt{\frac{\rho_0 r_*^3 \phi_0^2}{12 M_{\rm pl}^2 r}}$$

$$s_{\beta_3} \gg 1$$

New feature: Longitudinal mode suppressed outside Vainshtein radius



$$\phi'(r) \simeq -\frac{\rho_0 \phi_0 r_*^3}{6 M_{\rm pl}^2 r^2}\,, \qquad \chi'(r) \simeq \frac{\rho_0 r_*^3}{6 \beta_3 M_{\rm pl}^2 r^2}$$

Suppressed both inside and outside Vainshtein radius

$$\phi(r) \simeq \phi_0 \left(1 - \frac{\rho_0}{12 M_{\rm pl}^2} r^2\right)\,, \qquad \chi'(r) \simeq \frac{\rho_0}{6 \beta_3 M_{\rm pl}^2} r$$

4. Corrections to gravitational potentials

Inserting solutions for $\phi, \ \chi$ into 00 and 11 component of Einstein equations, we can write as:

$$\frac{2M_{\rm pl}^2}{r}\Phi' - \frac{M_{\rm pl}^2}{r^2}\left(1 - e^{2\Phi}\right) = e^{2\Phi}\rho_m + \Delta_\Phi \,,$$

$$\frac{2M_{\rm pl}^2}{r}\Psi' + \frac{M_{\rm pl}^2}{r^2}\left(1 - e^{2\Phi}\right) = e^{2\Phi}P_m + \Delta_\Psi \,,$$

We are interested in the case $r > r_*$. In the following, we separate into two cases:

(1). $s_{\beta_3} \gg 1$

Inserting solutions for $\phi'(r)\,,\ \chi'(r)$, we obtain

$$\Delta_\Phi \simeq \frac{5\Phi_*^2\phi_0^2 r_*^2}{72 r^4}\,, \qquad \Delta_\Psi \simeq -\frac{\Phi_*^2\phi_0^2 r_*^2}{72 r^4}\,,$$

$$\Phi_* \equiv \frac{\rho_0 r_*^2}{M_{\rm pl}^2} \ll 1$$

$$\Phi(r) \simeq \frac{\Phi_* r_*}{6r}\left[1 - \frac{5\Phi_*}{24}\left(\frac{\phi_0}{M_{\rm pl}}\right)^2 \frac{r_*}{r}\right]\,, \qquad \Psi(r) \simeq -\frac{\Phi_* r_*}{6r}\left[1 - \frac{\Phi_*}{8}\left(\frac{\phi_0}{M_{\rm pl}}\right)^2 \frac{r_*}{r}\right]\,.$$

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< 1

$$\gamma \equiv -\Phi/\Psi \simeq 1 - \frac{\Phi_*}{12}\left(\frac{\phi_0}{M_{\rm pl}}\right)^2 \frac{r_*}{r}$$

(1). $s_{\beta_3} \gg 1$

Inserting solutions for $\phi'(r)\,,\ \chi'(r)$, we obtain

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< 1

$$\gamma \equiv -\Phi/\Psi \simeq 1 - \frac{\Phi_*}{12}\left(\frac{\phi_0}{M_{\rm pl}}\right)^2 \frac{r_*}{r}$$

$|\gamma - 1| < 2.3 \times 10^{-5}$

$\Phi_* \simeq 10^{-6}$

$$|\gamma - 1| \simeq 10^{-7}(\phi_0/M_{\rm pl})^2 (r_*/r)$$

(2). $s_{\beta_3} < 1$

(a). $r_* < r < r_V$

$$\Delta_\Phi \simeq \frac{\sqrt{3\Phi_* r_*}\beta_3 \phi_0^3}{4r^{3/2}}, \qquad \Delta_\Psi \simeq \frac{\sqrt{3}(\Phi_* r_*)^{5/2}\beta_3 \phi_0^3}{432 r^{7/2}},$$

$$\Phi(r) \simeq \frac{\Phi_* r_*}{6r}\left[1 + \sqrt{s_{\beta_3}}\left(\frac{\phi_0}{M_{\rm pl}}\right)^2\left(\frac{r}{r_*}\right)^{3/2}\right], \qquad \Psi(r) \simeq -\frac{\Phi_* r_*}{6r}\left[1 - 2\sqrt{s_{\beta_3}}\left(\frac{\phi_0}{M_{\rm pl}}\right)^2\left(\frac{r}{r_*}\right)^{3/2}\right],$$

$$\left(\frac{\phi_0}{M_{\rm pl}}\right)^2\left(\frac{r}{r_*}\right)^{3/2} < 1$$

$$\gamma \simeq 1 + 3\sqrt{s_{\beta_3}}\left(\frac{\phi_0}{M_{\rm pl}}\right)^2\left(\frac{r}{r_*}\right)^{3/2} \quad \Longrightarrow \quad |\gamma - 1|_{\rm max} \simeq 3(\phi_0/M_{\rm pl})^2$$

(2). $s_{\beta_3} < 1$

(a). $r_* < r < r_V$

$$\Delta_\Phi \simeq \frac{\sqrt{3\Phi_* r_*}\beta_3\phi_0^3}{4r^{3/2}}\,, \qquad \Delta_\Psi \simeq \frac{\sqrt{3}(\Phi_* r_*)^{5/2}\beta_3\phi_0^3}{432 r^{7/2}}\,,$$

$$\Phi(r) \simeq \frac{\Phi_* r_*}{6r}\left[1+\sqrt{s_{\beta_3}}\left(\frac{\phi_0}{M_{\rm pl}}\right)^2\left(\frac{r}{r_*}\right)^{3/2}\right]\,, \qquad \Psi(r) \simeq -\frac{\Phi_* r_*}{6r}\left[1-2\sqrt{s_{\beta_3}}\left(\frac{\phi_0}{M_{\rm pl}}\right)^2\left(\frac{r}{r_*}\right)^{3/2}\right]\,,$$

$$\left(\frac{\phi_0}{M_{\rm pl}}\right)^2\left(\frac{r}{r_*}\right)^{3/2} < 1$$

$$\gamma \simeq 1+3\sqrt{s_{\beta_3}}\left(\frac{\phi_0}{M_{\rm pl}}\right)^2\left(\frac{r}{r_*}\right)^{3/2} \quad \Rightarrow \quad |\gamma-1|_{\rm max} \simeq 3(\phi_0/M_{\rm pl})^2$$

$|\gamma - 1| < 2.3\times 10^{-5}$

$$\phi_0 \lesssim 3\times 10^{-3} M_{\rm pl}$$

(b). $r > r_V$

$|\gamma - 1|$ begins to decrease.

Hence, when $s_{\beta_3} \gg 1$, longitudinal mode suppressed by Vainshtein mechanism. For $s_{\beta_3} < 1$, screening mechanism only works outside r_V.

$$\sqrt{s_{\beta_3}} \simeq 2.5 \times 10^{45}\, \beta_3 \frac{\phi_0}{M_{\rm pl}} \sqrt{\frac{1\ {\rm g/cm^3}}{\rho_0}} \approx 10^{44} \beta_3 \phi_0 / M_{\rm pl}$$

$$\rho_0 \approx 100\ {\rm g/cm^3}$$

Principally speaking, large s_{β_3} should be easily achieved.

5. Conclusion and Outlook

- At cubic order, provided that the coefficients of nonlinear operators are not too small, the Vainshtein mechanism works well so that the longitudinal mode can be efficiently screened;
- Many phenomenological aspects have been investigated in other references (Cosmology, GLPV extension and further extension…);
- Inflationary scenario?

Thank you for your attendance!

Appendix. Minkowski case for screening mechanism

Now we set the following forms:

$$f_2(X) = f_3(X) = X\,, \qquad f_4 = f_5 = 0, \qquad \beta_2 \equiv m^2$$

Then the EOM reduce to

0-com: $$\vec{\nabla}^2\phi - (m^2 + \beta_3\partial_i A^i)\phi + \rho = 0\,,$$

i-com: $$\partial^i\partial_j A^j - \vec{\nabla}^2 A^i + m^2 A^i + \beta_3\left(A^i\partial_j A^j - A_\lambda\partial^i A^\lambda\right) = 0$$

The vector can be decomposed into transverse and longitudinal modes:

$$A_i = A_i^{(T)} + \partial_i\chi\,, \qquad \partial^i A_i^{(T)} = 0$$

In the following, we study the longitudinal modes and neglect the transverse modes:

$$A_i^{(T)} = 0$$

Let us work in the spherical symmetric coordinate

$$ds^2 = -dt^2 + dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2)$$

Consider the trace of i-component.

$$\frac{d}{dr}\left(m^2r^2\chi' + \beta_3 r^2\phi\phi' + 2\beta_3 r\chi'^2\right) = 0$$

⇓ $\chi'(\infty) = \phi'(\infty) = 0$

$$m^2\chi' + \beta_3\phi\phi' + 2\beta_3\frac{\chi'^2}{r} = 0$$

⇓ $\chi'(\infty) = 0$

$$\chi' = \alpha r\,, \qquad \alpha = -\frac{m^2}{4\beta_3}\left[1 - \sqrt{1 - \frac{8\beta_3^2\phi\phi'}{m^4 r}}\right]$$

⇒ $m \to 0$

$$\chi' = \sqrt{-\frac{1}{2}\phi\phi' r}$$

Assume the profile

$$\phi(r) = \phi_0 + f(r)\,, \qquad |f(r)| \ll |\phi_0|$$

i.e. the scalar potential does not vary much in the presence of the non-linear interaction. Now we discuss the 0-component. It is expressed as

$$\frac{d}{dr}\left(r^2\phi'\right) - m^2 r^2 \phi + \rho r^2 - \beta_3 \phi \frac{d}{dr}\left(r^2\chi'\right) = 0$$

In the following, we consider the massless case $m = 0$

Let us separate into two cases.

- Case 1: around $r = 0$

assume $\rho(r) = \rho_0 = const$, using the approximation $\phi(r) \approx \phi_0\,, \qquad \phi'(r) = f'(r)\,,$ and neglecting the term $m^2 r^2 \phi$ then the 0-component can be integrated as

$$-r^2 f'(r) + \frac{\beta_3}{\sqrt{2}}\phi_0^{3/2} r^{5/2}\sqrt{-f'(r)} = \frac{1}{3}\rho_0 r^3 + C_1$$

$\phi'(0) = 0$ $\Downarrow$

$$-f'(r) + \frac{\beta_3}{\sqrt{2}}\phi_0^{3/2}\sqrt{-r f'(r)} = \frac{1}{3}\rho_0 r$$

$\Downarrow$

Particular solution: $f'(r) \propto -r \Longrightarrow$

$$\phi(r) = \phi_0 - Br^2\,,$$

$$B = \frac{\beta_3^2\phi_0^3}{16}\left[\sqrt{1 + \frac{8\rho_0}{3\beta_3^2\phi_0^3}} - 1\right]^2$$

- Case 2: $r > r_*$

  r_* is the scale at which the density $\rho(r)$ starts to decrease significantly.
  Neglecting ρr^2, taking the same assumption as in case 1, then the 0-component becomes

$$-r^2 f'(r) + \frac{\beta_3}{\sqrt{2}} \phi_0^{3/2} r^{5/2} \sqrt{-f'(r)} = C_2 = \frac{1}{3}\rho_0 r_*^3$$

match case 1

$$\phi'(r) = -\frac{\beta_3^2 \phi_0^3 r}{8} \left[\sqrt{1 + \frac{8\rho_0 r_*^3}{3\beta_3^2 \phi_0^3 r^3}} - 1 \right]^2$$