Time delay estimation of strong lens systems

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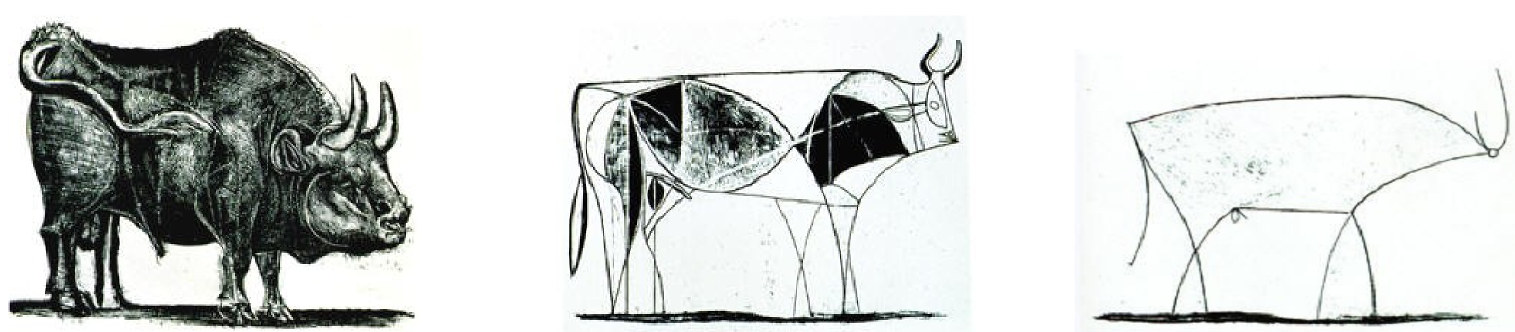


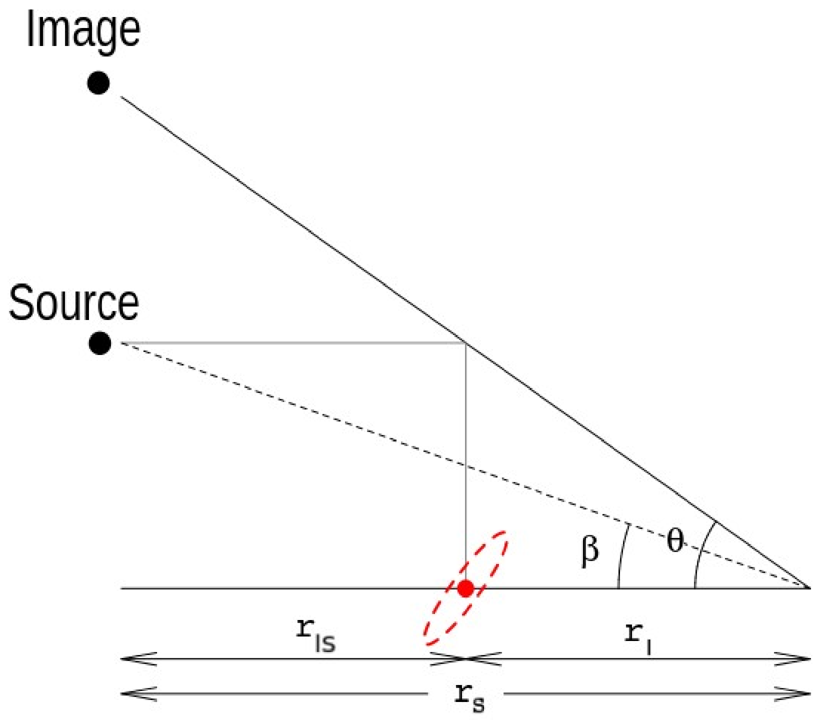
Bull pictures courtesy of “Pablo Picasso”

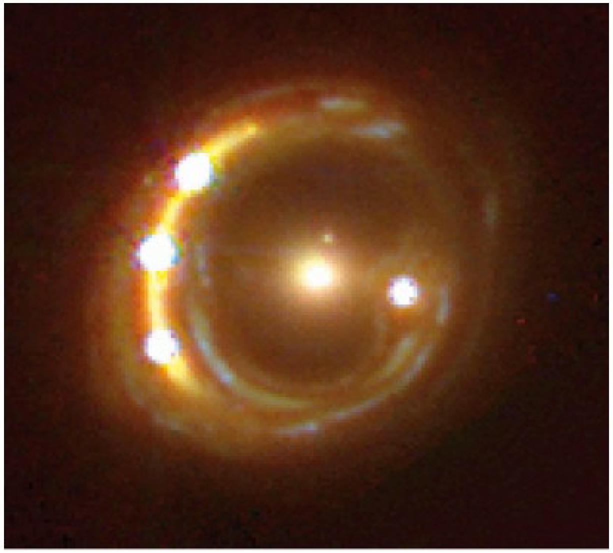
All models are false, some are useful. (George E. P. Box)

Outline

- An introduction to strong gravitational lensing and time delay
- Strong lens time delay challenge II (TDC1):
  - Characteristics of simulated light curves
  - Our algorithm: (smoothing, cross-correlation)
  - Performance of methodology
- Time delay estimation of SDSS J1001+5027:
  - Characteristics of light curves
  - Our improved algorithm: (smoothing, weighted cross-correlation, MSE, …)
  - Error estimation
  - Comparing estimated time delays with the results of other groups

Strong gravitational lensing





HST ACS image of RXJ1131-1231

Time delay: $$\Delta t(\vec{\theta}, \vec{\beta}) = \frac{r_l r_s}{r_{ls}}(1 + z_l)\phi(\vec{\theta}, \vec{\beta})$$

Fermat potential: $$\phi(\vec{\theta}, \vec{\beta}) = \frac{(\vec{\theta} - \vec{\beta})^2}{2} - \psi(\vec{\theta})$$

geometric delay

lensing potential delay

(Linder 2011, Tewes 2012)

Strong lensing surveys

- Recent survey:
  COSMOGRAIL: the COSmological MOnitoring of GRAvItational Lenses
  (http://www.cosmograil.org)

- Future survey:
  LSST: the Large Synoptic Survey Telescope (LSST) with 10 years observation will be expected to monitor several thousand time delay lens systems.

- Need to design the fast and reliable algorithms for the time delay estimation.

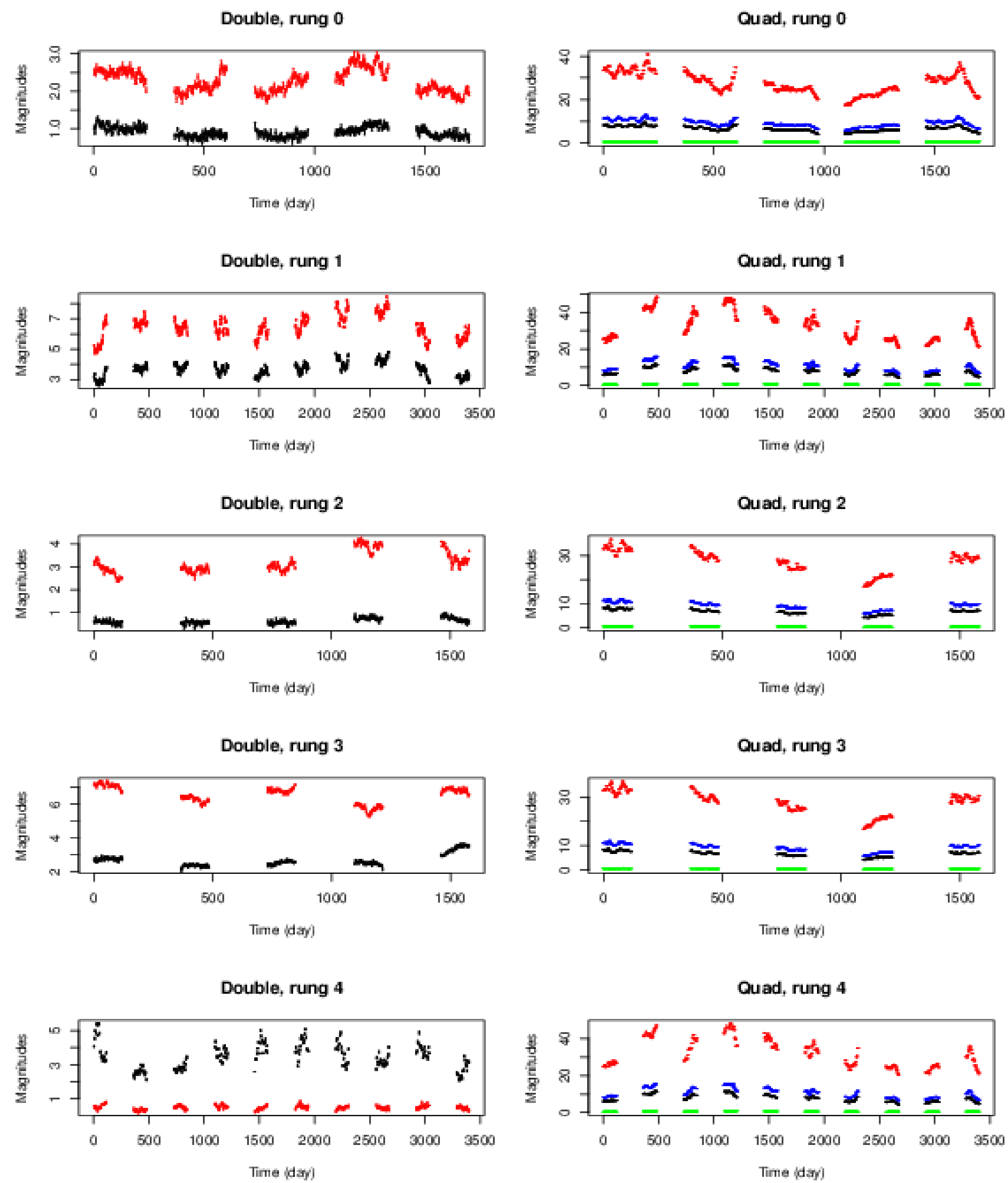
- Strong Lens Time Delay Challenge:
  TDC0
  TDC1
  TDC2 coming soon!

Strong Lens
Time Delay Challenge II

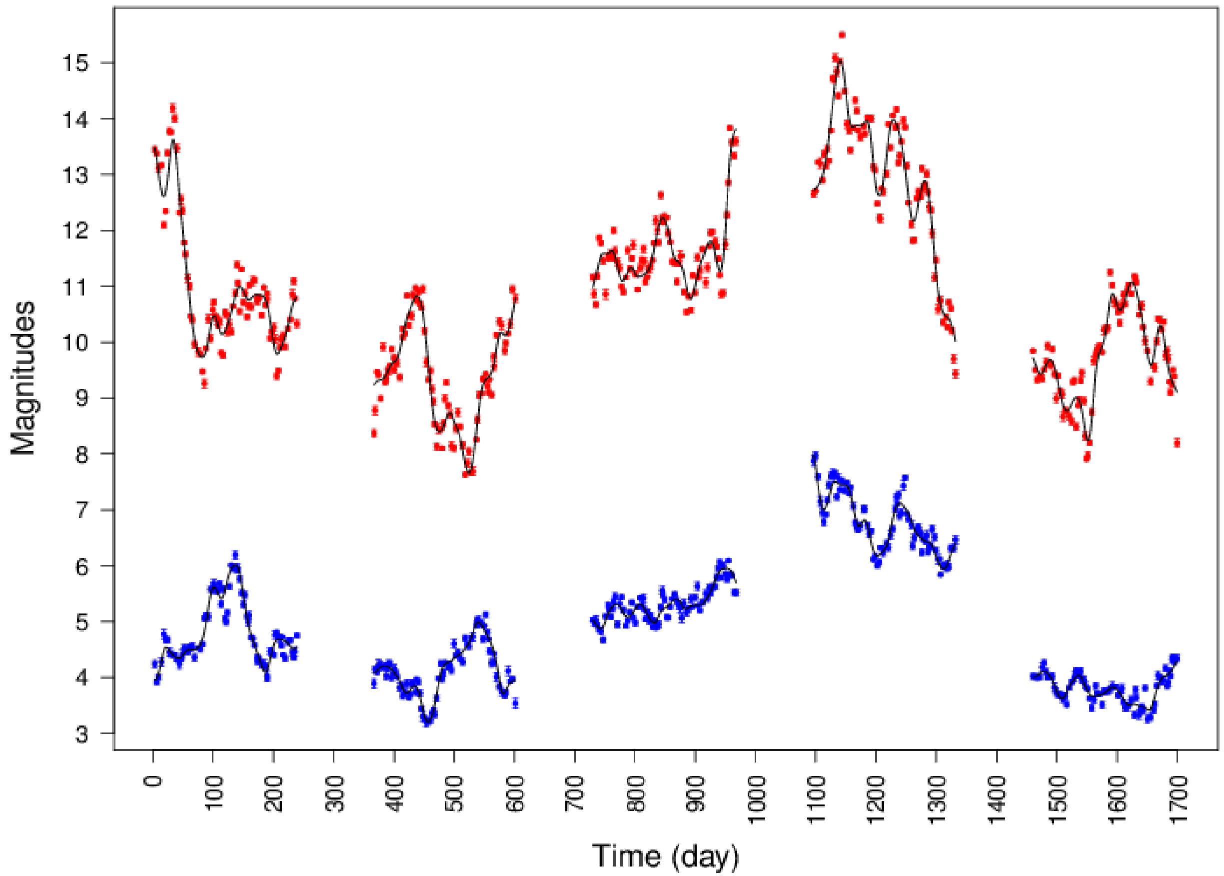
Strong Lens Time Delay Challenge: simulated data

The TDC1 simulated data is provided in five different categories (rungs)

Each rung contains the light curves of 720 Double and 152 Quad image systems.



Methodology: smoothing

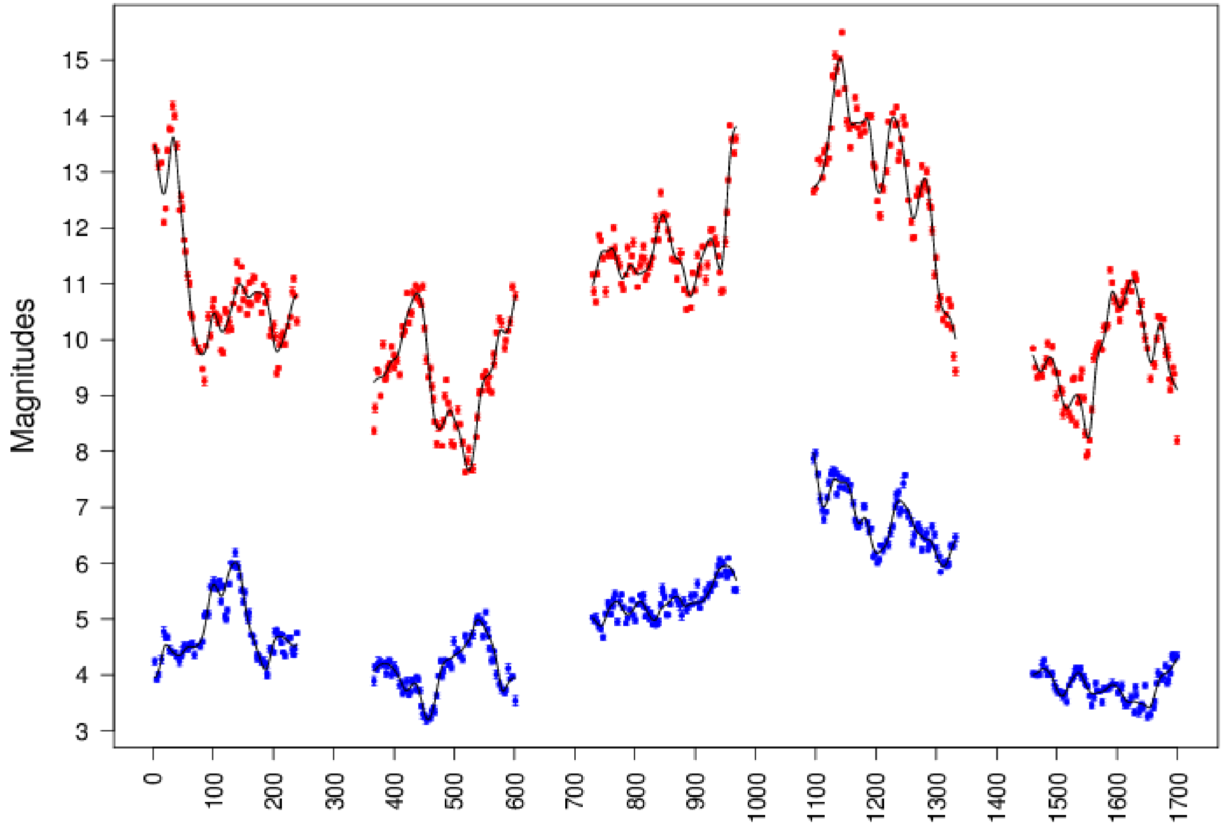


$$A^s(t) = A^g(t) + N(t) \sum_i \frac{A^d(t_i) - A^g(t_i)}{\sigma_d^2(t_i)} \times exp\left[-\frac{(t_i - t)^2}{2\Delta^2}\right]$$

where $$N(t)^{-1} = \sum_i exp\left[-\frac{(t_i - t)^2}{2\Delta^2}\right] \frac{1}{\sigma_d^2(t_i)}$$

(Shafieloo et al. 2006, Shafieloo 2007, Shafieloo & Clarkson 2010)

Methodology: cross-correlation

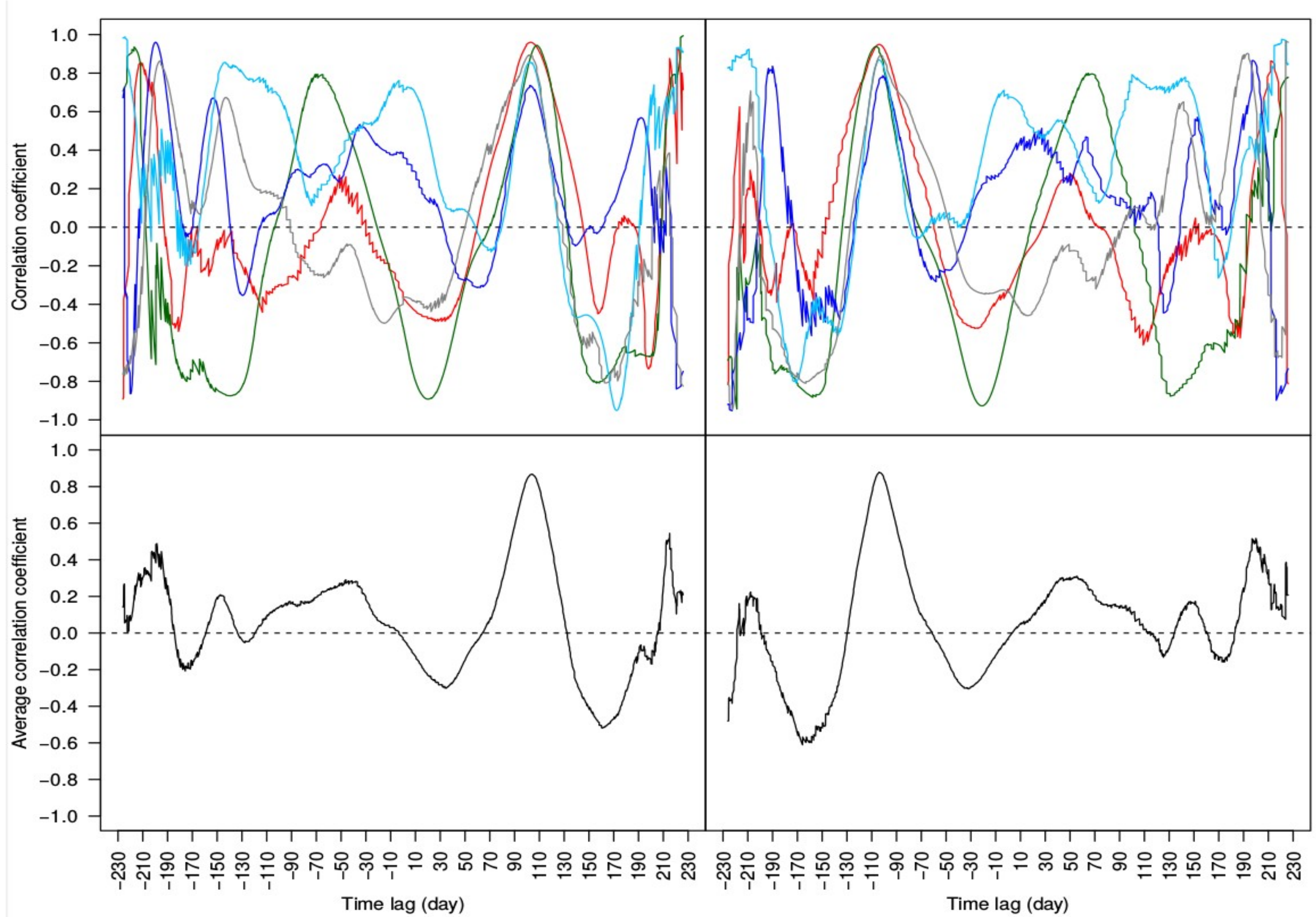


$$\rho_{XY} = \frac{\operatorname{cov}(X,Y)}{\sigma_X \sigma_Y} \Rightarrow r_{xy} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2 \sum_{i=1}^{n}(y_i - \bar{y})^2}},$$

$$\left.\begin{aligned} X' &= a_x X + b_x \\ Y' &= a_y Y + b_y \end{aligned}\right\} \Rightarrow \rho_{XY} = \rho_{X'Y'}$$

(Peterson 2001, Wasserman 2004)

Methodology: time delay estimation



Correlation > 0.6

$$|\tilde{\Delta t}_{A_1 A_2}| = \frac{|\tilde{\Delta t}_{A_1;A_2^s}| + |\tilde{\Delta t}_{A_2;A_1^s}|}{2} \qquad \sigma^{ini}_{A_1 A_2} = \sqrt{2} \times \frac{\left| |\tilde{\Delta t}_{A_1;A_2^s}| - |\tilde{\Delta t}_{A_2;A_1^s}| \right|}{2}$$

(Aghamousa, Shafieloo, APJ, 2015)

Methodology: error estimation, using Quad systems

The light curves of a Quad image are labeled A1 , A2 , B1 and B2. For every Quad system we should have:

$$\underbrace{\tilde{\Delta} t_{A_1A_2} - (\tilde{\Delta} t_{A_1B_1} + \tilde{\Delta} t_{B_1A_2})}_{T_{dif}} \pm \underbrace{\sqrt{(\sigma^{ini}_{\tilde{\Delta} t_{A_1A_2}})^2 + (\sigma^{ini}_{\tilde{\Delta} t_{A_1B_1}})^2 + (\sigma^{ini}_{\tilde{\Delta} t_{B_1A_2}})^2}}_{\sigma_{T_{dif}}} \equiv 0$$

$$T_{dif} \pm \sigma_{T_{dif}} \equiv 0$$

$$|T_{dif}| \leqslant \sigma_{T_{dif}}$$

Yes → We can assume that all time delays and their corresponding errors are estimated consistently.

No → $\sigma^2_{ec} = |T_{dif}|^2 - \sigma^2_{T_{dif}}$ → $\sigma^{new}_{\tilde{\Delta} t_{A_1A_2}} = \sqrt{(\sigma^{ini}_{\tilde{\Delta} t_{A_1A_2}})^2 + \frac{\alpha}{3}\sigma^2_{ec}}$

(Aghamousa, Shafieloo, APJ, 2015)

Methodology: error estimation, using Quad systems

Error profile for Double systems

For **Rung 0**,

if $|\tilde{\Delta t}| \leqslant 20 \Rightarrow \sigma_R = 0.06 \times |\tilde{\Delta t}|$, if $|\tilde{\Delta t}| > 20 \Rightarrow \sigma_R = 1.2$

For **Rung 1**,

if $|\tilde{\Delta t}| \leqslant 20 \Rightarrow \sigma_R = 0.06 \times |\tilde{\Delta t}|$, if $|\tilde{\Delta t}| > 20 \Rightarrow \sigma_R = 1.3$

For **Rung 2**,

if $|\tilde{\Delta t}| \leqslant 30 \Rightarrow \sigma_R = 0.07 \times |\tilde{\Delta t}|$, if $|\tilde{\Delta t}| > 30 \Rightarrow \sigma_R = 1.3$

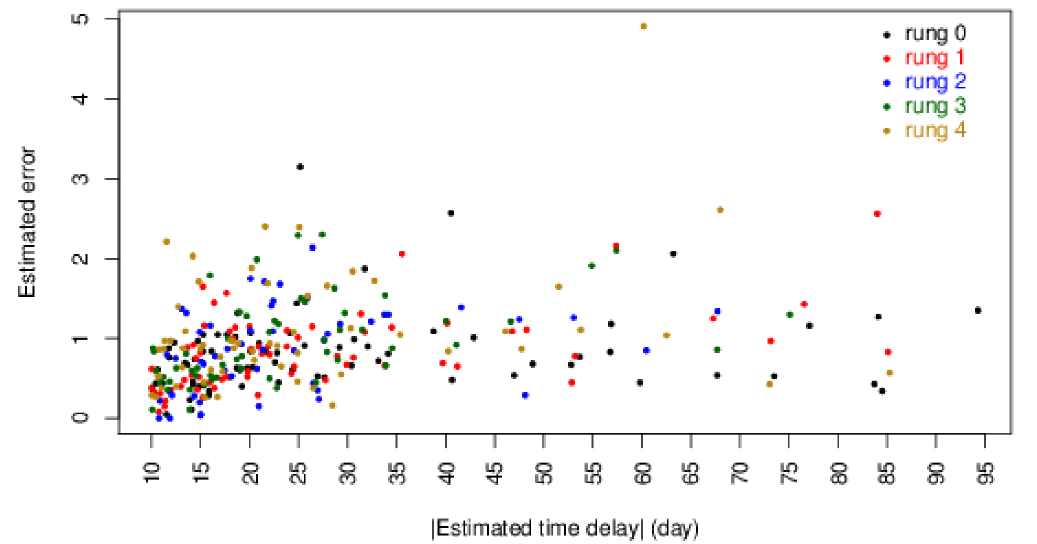
For **Rung 3**,

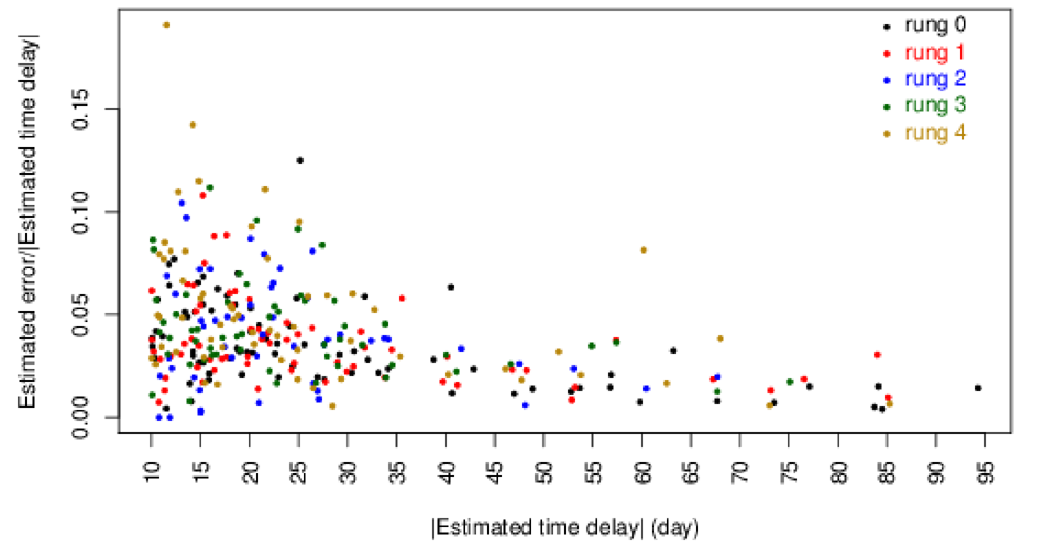
if $|\tilde{\Delta t}| \leqslant 30 \Rightarrow \sigma_R = 0.08 \times |\tilde{\Delta t}|$, if $|\tilde{\Delta t}| > 30 \Rightarrow \sigma_R = 1.5$

For **Rung 4**,

if $|\tilde{\Delta t}| \leqslant 25 \Rightarrow \sigma_R = 0.08 \times |\tilde{\Delta t}|$, if $|\tilde{\Delta t}| > 25 \Rightarrow \sigma_R = 1.5$

$$\sigma_{\tilde{\Delta t}_{A_1 A_2}} = \sqrt{(\sigma^{ini}_{\tilde{\Delta t}_{A_1 A_2}})^2 + \sigma_R^2}$$





(Aghamousa, Shafieloo, APJ, 2015)

Results: TDC1 paper

STRONG LENS TIME DELAY CHALLENGE: II. RESULTS OF TDC1

Kai Liao[1,2*], Tommaso Treu[2*], Phil Marshall[3], Christopher D. Fassnacht[4], Nick Rumbaugh[4], Gregory Dobler[5,20], Amir Aghamousa[9], Vivien Bonvin[13], Frederic Courbin[13], Alireza Hojjati[6,7], Neal Jackson[12], Vinay Kashyap[17], S. Rathna Kumar[14], Eric Linder[8,18], Kaisey Mandel[17], Xiao-Li Meng[15], Georges Meylan[13], Leonidas A.Moustakas[11], Tushar P. Prabhu[14], Andrew Romero-Wolf[11], Arman Shafieloo[9,10], Aneta Siemiginowska[17], Chelliah S. Stalin[14], Hyungsuk Tak[15], Malte Tewes[19], and David van Dyk[16]

Draft version September 5, 2014

ABSTRACT

We present the results of the first strong lens time delay challenge. The motivation, experimental design, and entry level challenge are described in a companion paper. This paper presents the main challenge, TDC1, which consisted in analyzing thousands of simulated light curves blindly. The observational properties of the light curves cover the range in quality obtained for current targeted efforts (e.g. COSMOGRAIL) and expected from future synoptic surveys (e.g. LSST), and include "evilness" in the form of simulated systematic errors. 7 teams participated in TDC1, submitting results from 78 different method variants. After a describing each method, we compute and analyze basic statistics measuring accuracy (or bias) A, goodness of fit χ^2, precision P, and success rate f. For some methods we identify outliers as an important issue. Other methods show that outliers can be controlled via visual inspection or conservative quality control. Several methods are competitive, i.e. give $|A| < 0.03$, $P < 0.03$, and $\chi^2 < 1.5$, with some of the methods already reaching sub-percent accuracy. The fraction of light curves yielding a time delay measurement is typically in the range f =20–40%. It depends strongly on the quality of the data: COSMOGRAIL-quality cadence and light curve lengths yield significantly higher f than does sparser sampling. We estimate that LSST should provide around 400 robust time-delay measurements, each with $P < 0.03$ and $|A| < 0.01$, comparable to current lens modeling uncertainties. In terms of observing strategies, we find that A and f depend mostly on season length, while P depends mostly on cadence and campaign duration.

Subject headings: gravitational lensing — methods: data analysis

$$f \equiv \frac{N_{submitted}}{N}$$

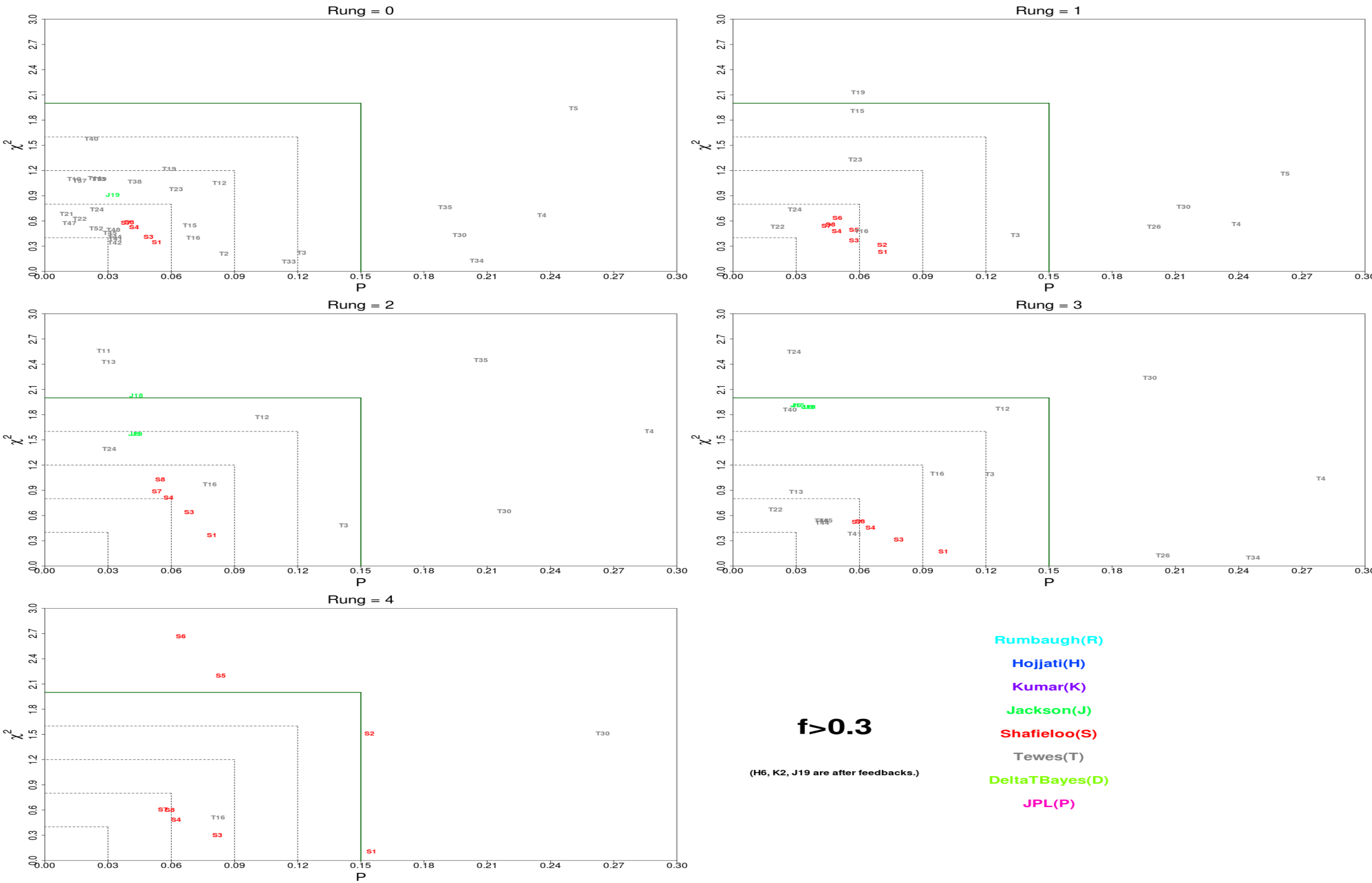
$$P = \frac{1}{fN}\sum_i \left(\frac{\sigma_i}{|\Delta t_i|}\right)$$

$$A = \frac{1}{fN}\sum_i \left(\frac{|\tilde{\Delta t_i}| - |\Delta t_i|}{|\Delta t_i|}\right)$$

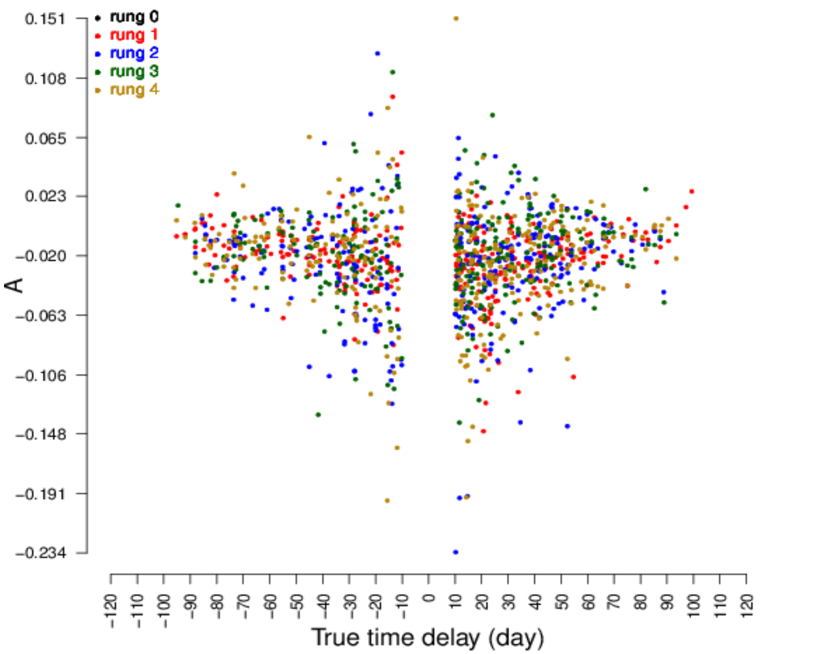
$$\chi^2 = \frac{1}{fN}\sum_i \left(\frac{\tilde{\Delta t_i} - \Delta t_i}{\sigma_i}\right)^2$$

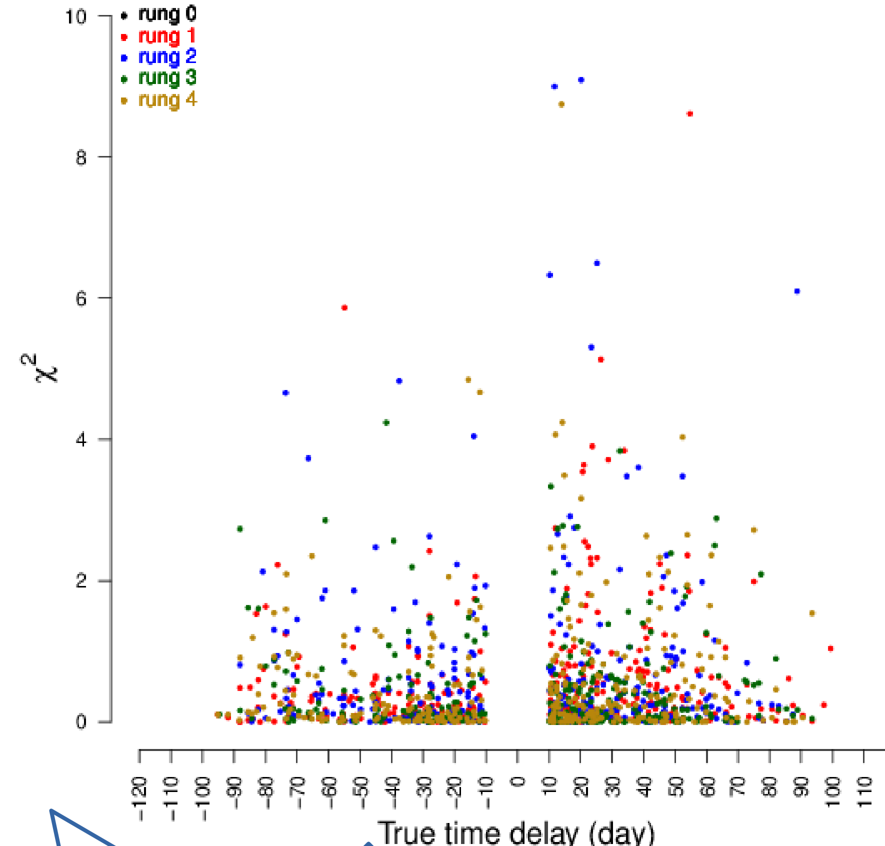
Rung	f	χ^2	P	A
0	0.529	0.579	0.038	-0.018
1	0.366	0.543	0.044	-0.022
2	0.350	0.885	0.053	-0.025
3	0.337	0.524	0.059	-0.021
4	0.346	0.608	0.056	-0.024

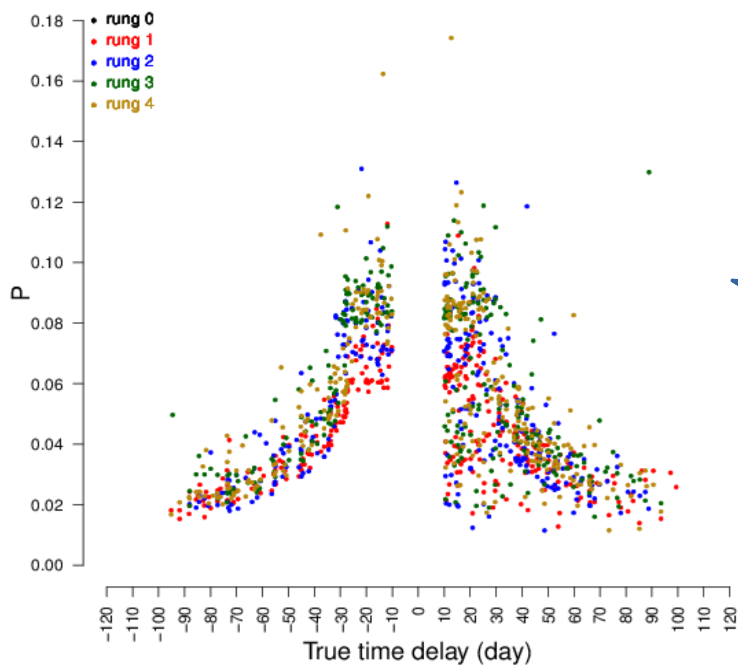
Results: estimated vs true time delay



Results: histogram of metrics







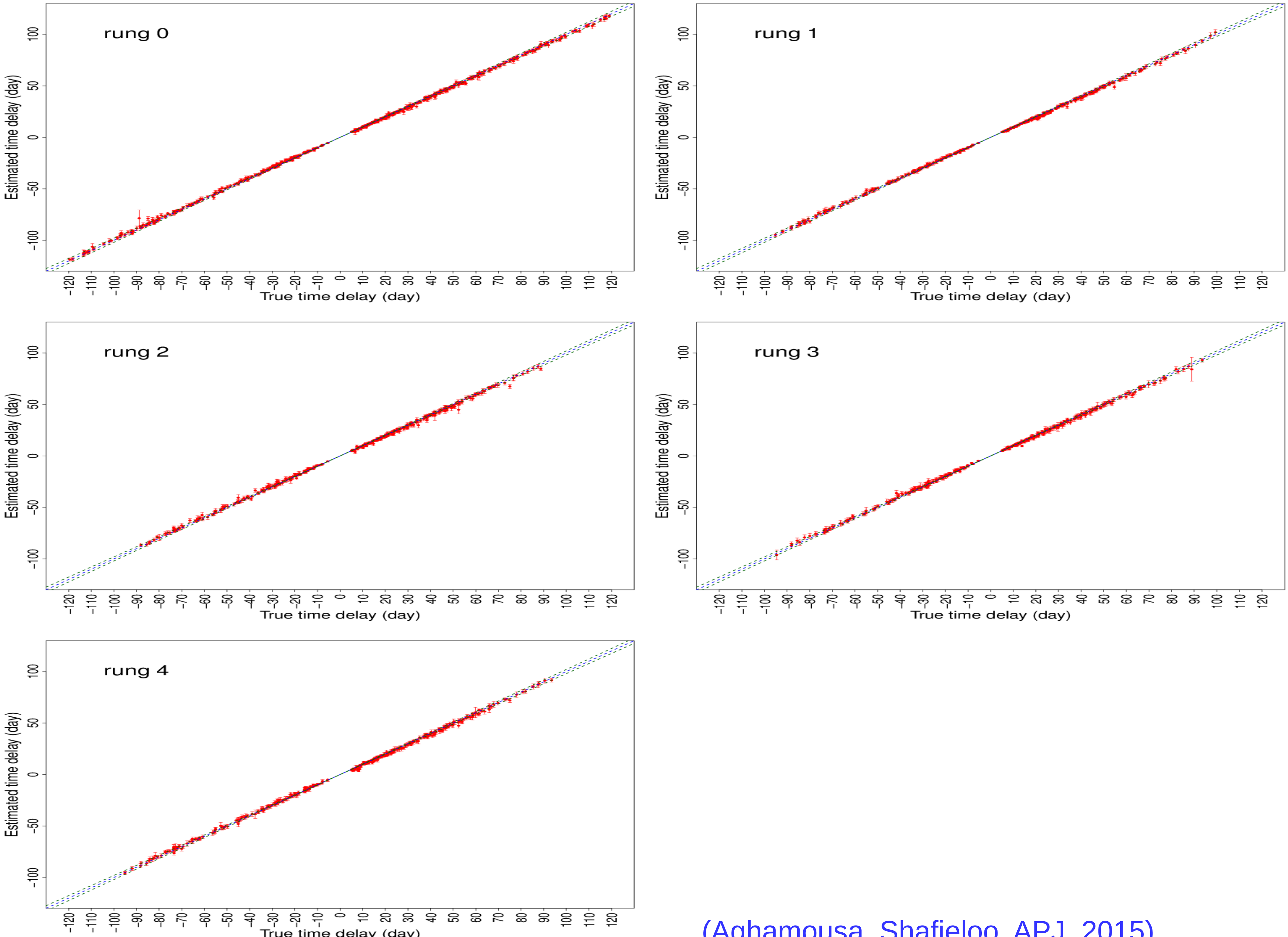
there are only 5 items with

$\chi^2 \geq 10$

out of few thousand entries

(Aghamousa, Shafieloo, APJ, 2015)

Results: estimated vs true time delay



(Aghamousa, Shafieloo, APJ, 2015)

Results: calibration

Rung	f	χ^2	P	A
0	0.529	0.579	0.038	-0.018
1	0.366	0.543	0.044	-0.022
2	0.350	0.885	0.053	-0.025
3	0.337	0.524	0.059	-0.021
4	0.346	0.608	0.056	-0.024

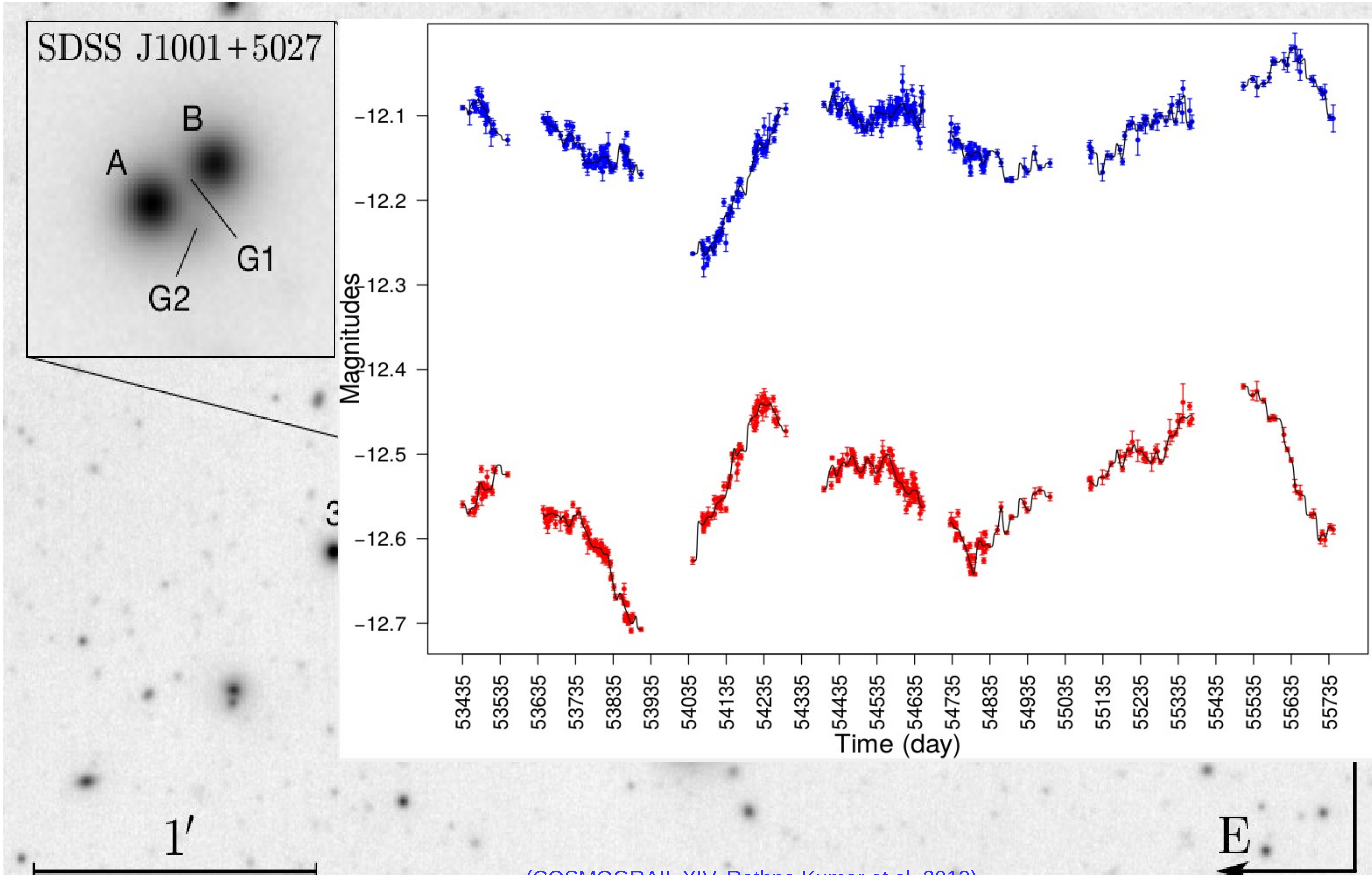
$$\tilde{\Delta t_i} + 0.5 \quad \text{and} \quad \sigma_i/\sqrt{2}$$

Rung	f	χ^2	P	A
0	0.529	0.792	0.027	-0.0014
1	0.366	0.660	0.031	-0.0036
2	0.350	1.439	0.038	-0.0058
3	0.337	0.766	0.041	-0.0010
4	0.346	0.868	0.040	-0.0048

(Aghamousa, Shafieloo, APJ, 2015)

Time delay estimation of SDSS J1001+5027

Light curves



(COSMOGRAIL XIV, Rathna Kumar et al. 2013)

Methodology

algorithm

$$A^s(t) = A^g(t) + N(t)\sum_i \frac{A^d(t_i) - A^g(t_i)}{\sigma_d^2(t_i)} \times exp\left[-\frac{(t_i - t)^2}{2\Delta^2}\right]$$

Weighted Correlation:

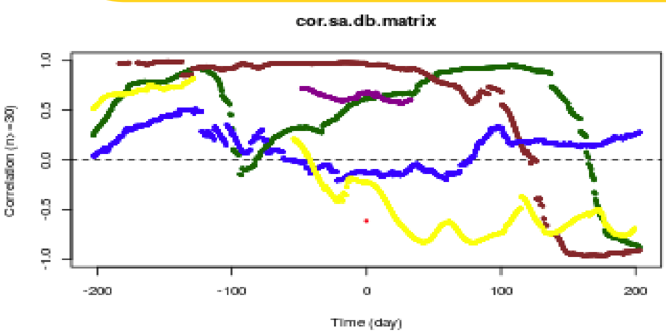
$$r_w = \frac{\sum_{i=1}^{n} w_i(x_i - \bar{x}_w)(y_i - \bar{y}_w)}{\sqrt{\sum_{i=1}^{n} w_i(x_i - \bar{x}_w)^2 \sum_{i=1}^{n} w_i(y_i - \bar{y}_w)^2}}$$

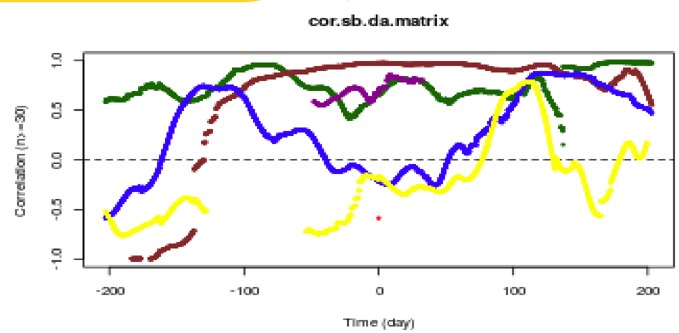
$$\bar{x}_w = \sum_{i=1}^{n} w_i x_i, \quad \bar{y}_w = \sum_{i=1}^{n} w_i y_i \qquad w_i = \frac{1}{\sigma_i^2}$$

$\Delta = ?$

$n = ?$

Mean or Mirror estimator?

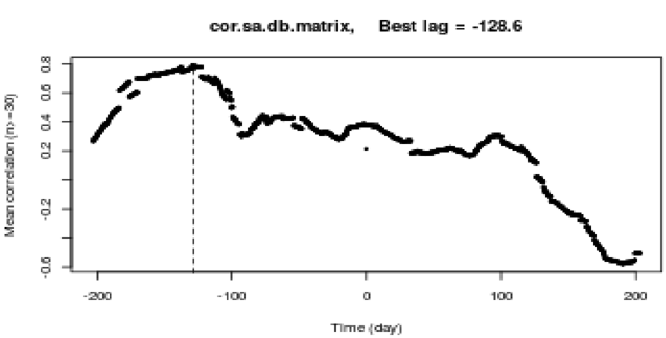


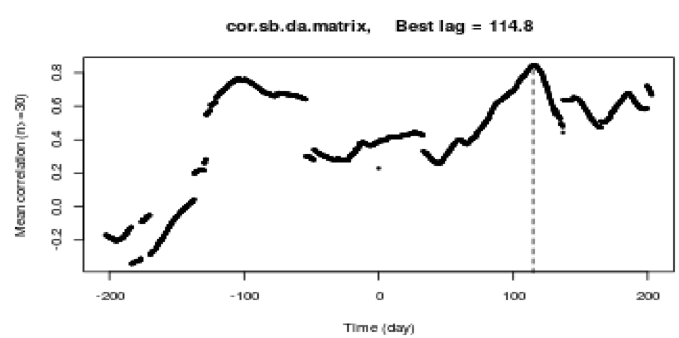


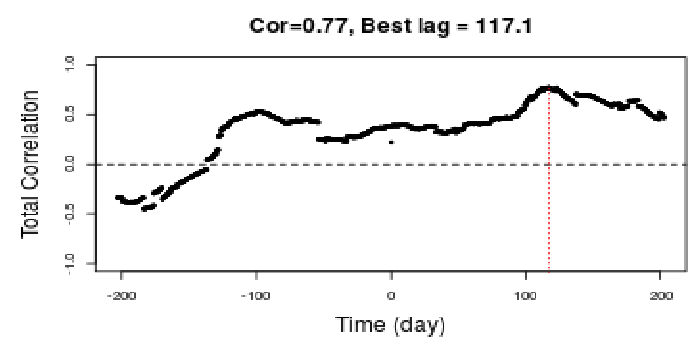
Weighted Correlation

Delta = 8, n>30

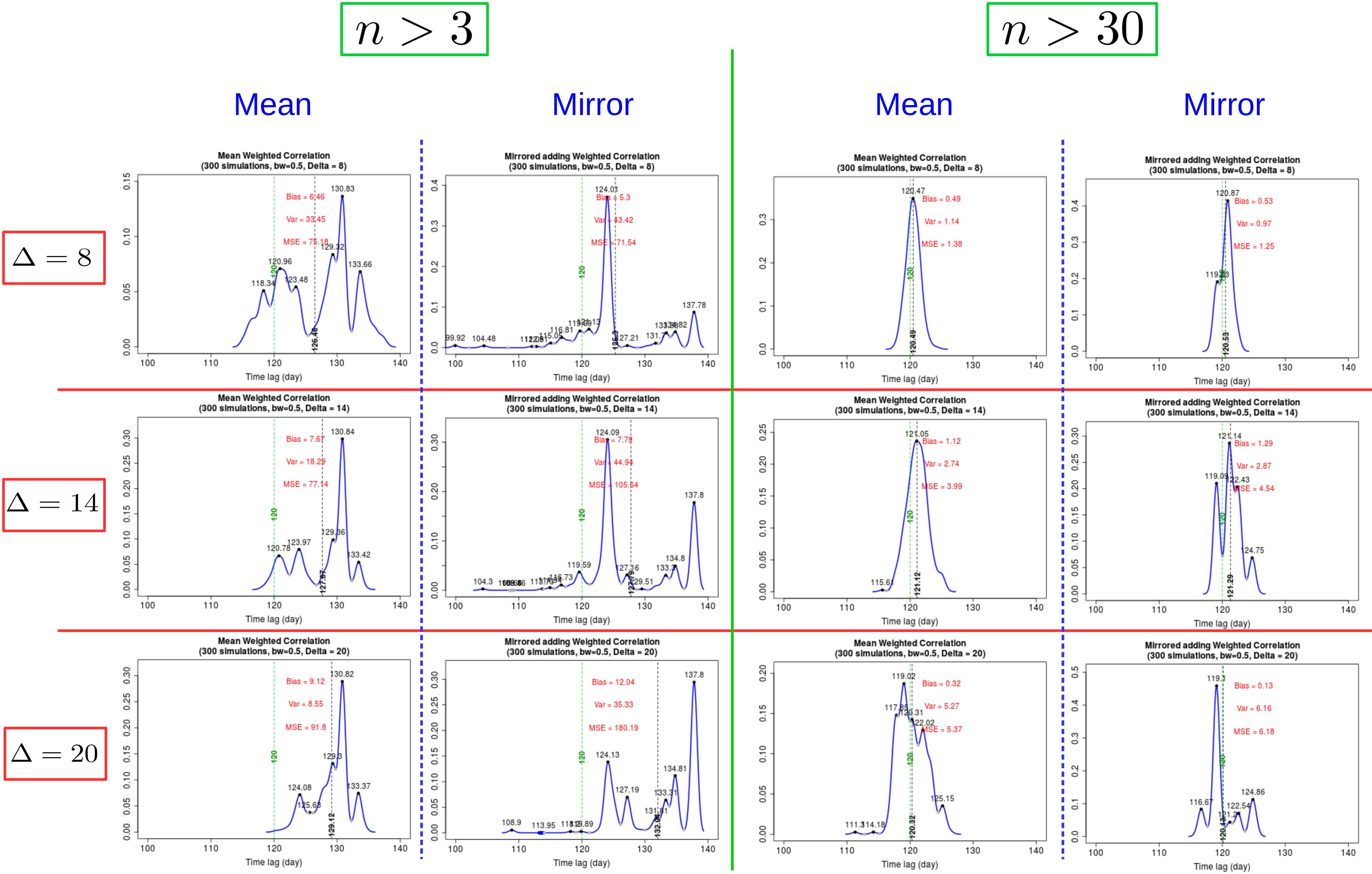
Mean = 121.7 Mirror = 117.1







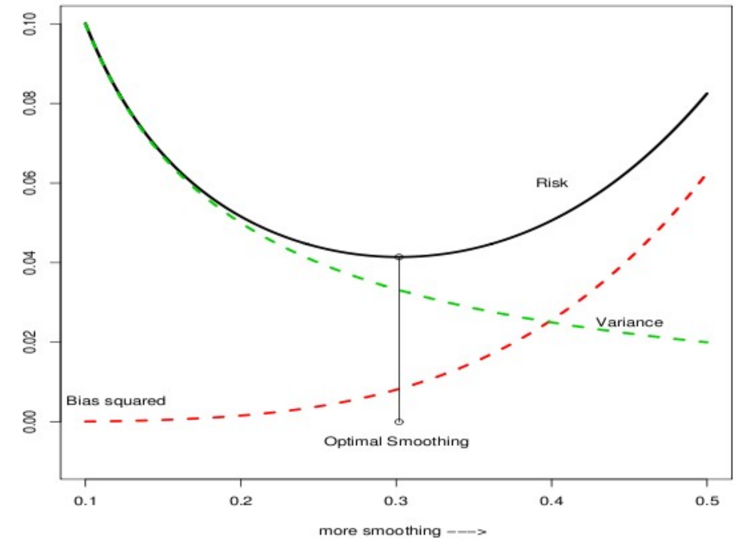
Simulation



n > 3
n > 30
Mean
Mirror
Mean
Mirror
Δ = 8
Δ = 14
Δ = 20
Mean Weighted Correlation (300 simulations, bw=0.5, Delta = 8)
Bias = 6.46
Var = 33.45
MSE = 75.18
Mirrored adding Weighted Correlation (300 simulations, bw=0.5, Delta = 8)
Bias = 5.3
Var = 43.42
MSE = 71.54
Mean Weighted Correlation (300 simulations, bw=0.5, Delta = 8)
Bias = 0.49
Var = 1.14
MSE = 1.38
Mirrored adding Weighted Correlation (300 simulations, bw=0.5, Delta = 8)
Bias = 0.53
Var = 0.97
MSE = 1.25
Mean Weighted Correlation (300 simulations, bw=0.5, Delta = 14)
Bias = 7.67
Var = 18.29
MSE = 77.14
Mirrored adding Weighted Correlation (300 simulations, bw=0.5, Delta = 14)
Bias = 7.79
Var = 44.94
MSE = 105.64
Mean Weighted Correlation (300 simulations, bw=0.5, Delta = 14)
Bias = 1.12
Var = 2.74
MSE = 3.99
Mirrored adding Weighted Correlation (300 simulations, bw=0.5, Delta = 14)
Bias = 1.29
Var = 2.87
MSE = 4.54
Mean Weighted Correlation (300 simulations, bw=0.5, Delta = 20)
Bias = 9.12
Var = 8.55
MSE = 91.8
Mirrored adding Weighted Correlation (300 simulations, bw=0.5, Delta = 20)
Bias = 12.04
Var = 35.33
MSE = 180.19
Mean Weighted Correlation (300 simulations, bw=0.5, Delta = 20)
Bias = 0.32
Var = 5.27
MSE = 5.37
Mirrored adding Weighted Correlation (300 simulations, bw=0.5, Delta = 20)
Bias = 0.13
Var = 6.16
MSE = 6.18
Time lag (day)

Mean Squared Error (MSE)

$\hat{\theta}_n$ is the **estimator** of θ **unknown parameter/value**.



Bias:

$$\text{Bias}(\hat{\theta}_n) = \mathbb{E}(\hat{\theta}_n) - \theta$$

Variance:

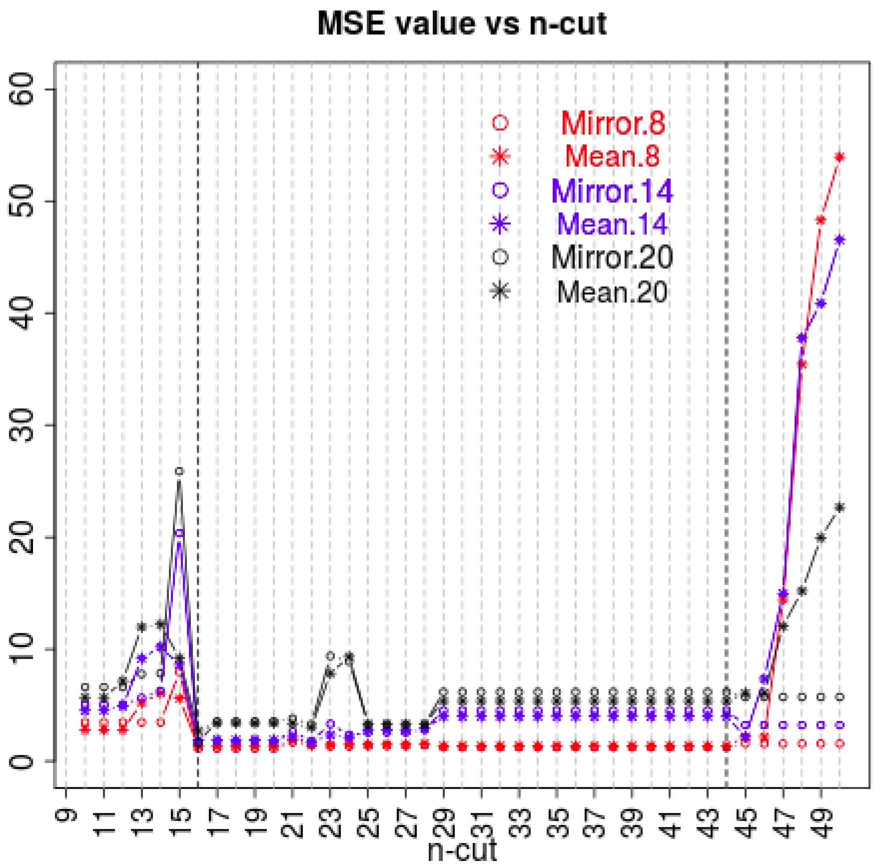
$$\text{Var}(\hat{\theta}_n) = \mathbb{E}\left[(\hat{\theta}_n - \bar{\theta}_n)^2\right] \qquad \text{where} \quad \theta_n = \mathbb{E}(\theta_n)$$

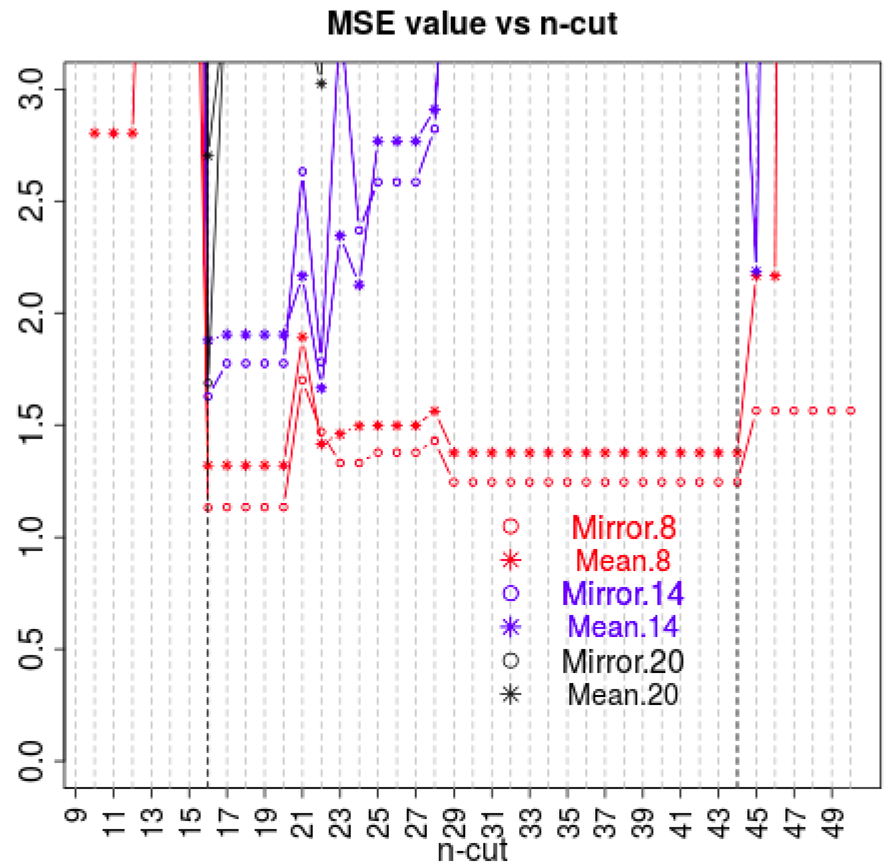
Mean Squared Error (MSE)

$$\text{MSE}(\hat{\theta}_n) = \mathbb{E}\left[(\hat{\theta}_n - \theta)^2\right] = \cdots = \Big(\mathbb{E}(\hat{\theta}_n) - \theta\Big)^2 + \mathbb{E}\left[(\hat{\theta}_n - \bar{\theta}_n)^2\right]$$

$$\text{MSE} = \text{Bias}^2 + \text{Var}$$

Optimum estimator





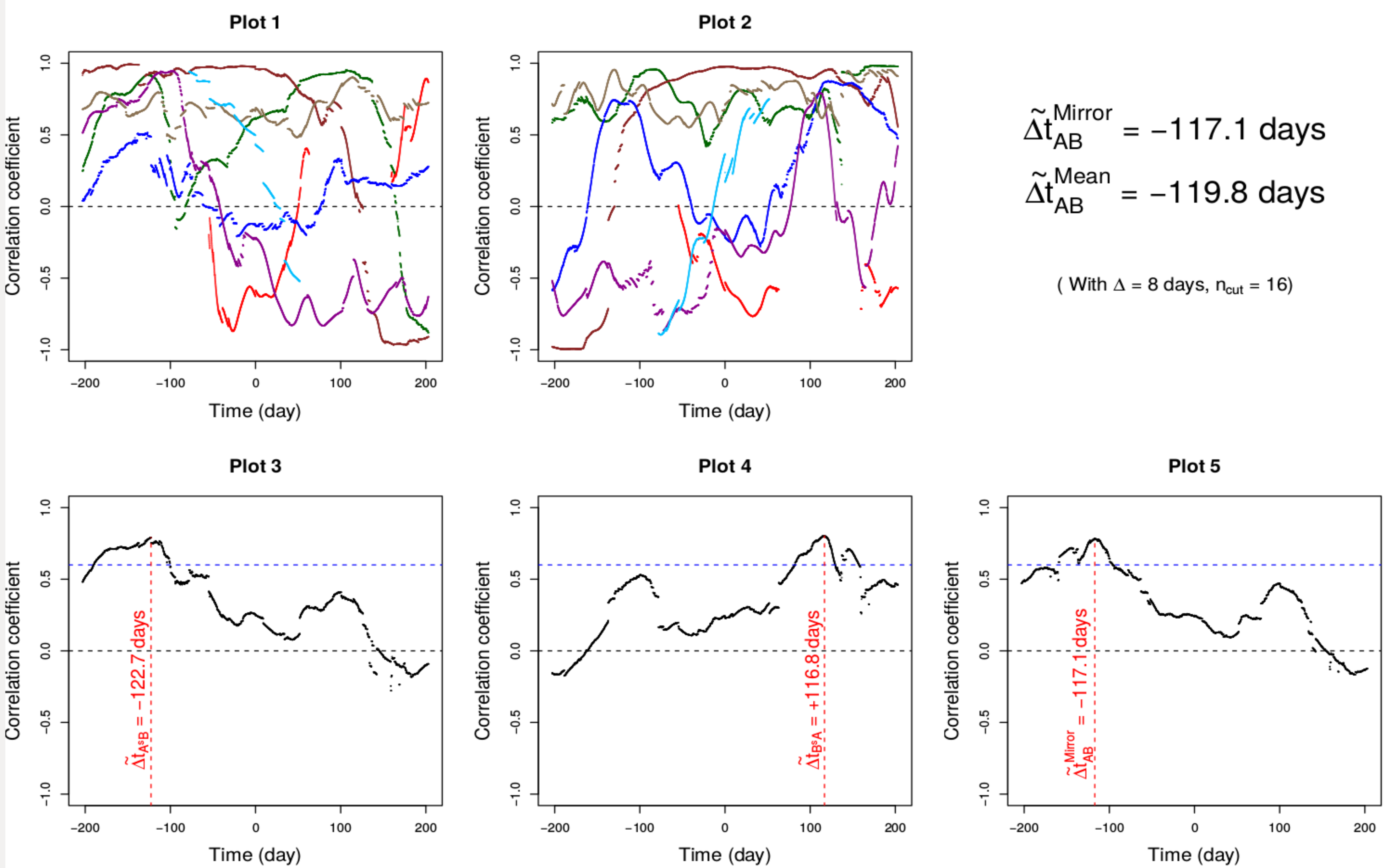
Mean & Mirror

$$n_{cut} = 16$$

$$\Delta = 8$$

(Aghamousa, Shafieloo, arXiv:1603.06331, 2016)

Time delay estimation



$$\tilde{\Delta t}_{AB}^{Mirror} = -117.1 \text{ days}$$

$$\tilde{\Delta t}_{AB}^{Mean} = -119.8 \text{ days}$$

(With Δ = 8 days, n_{cut} = 16)

(Aghamousa, Shafieloo, arXiv:1603.06331, 2016)

Error estimation

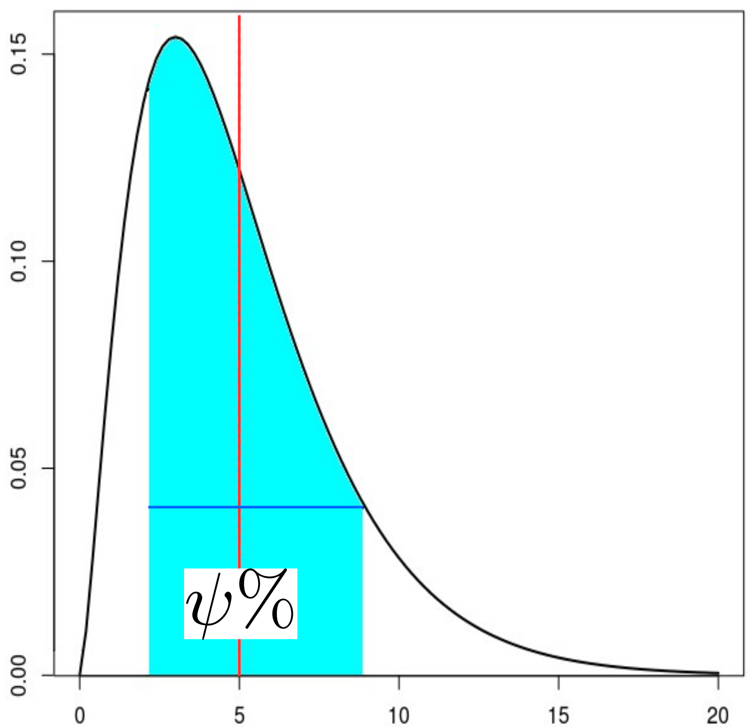
- **Confidence set.** $$\mathbb{P}(a \leq \theta \leq b) = (1-\alpha) \equiv \psi\%$$
  - To find a confidence interval we need the **probability distribution of estimator** which can be known from **statistical properties of estimator** or can be achieved by **simulation**.
  - In some cases we know the probability distribution of a quantity which has a relation to estimator.

$$\mathbb{P}\left(\mathbb{E}(\hat{\theta}_n) - l_1 \leq \quad \hat{\theta}_n \quad \leq \mathbb{E}(\hat{\theta}_n) + l_2\right) = (1-\alpha) \equiv \psi\%$$

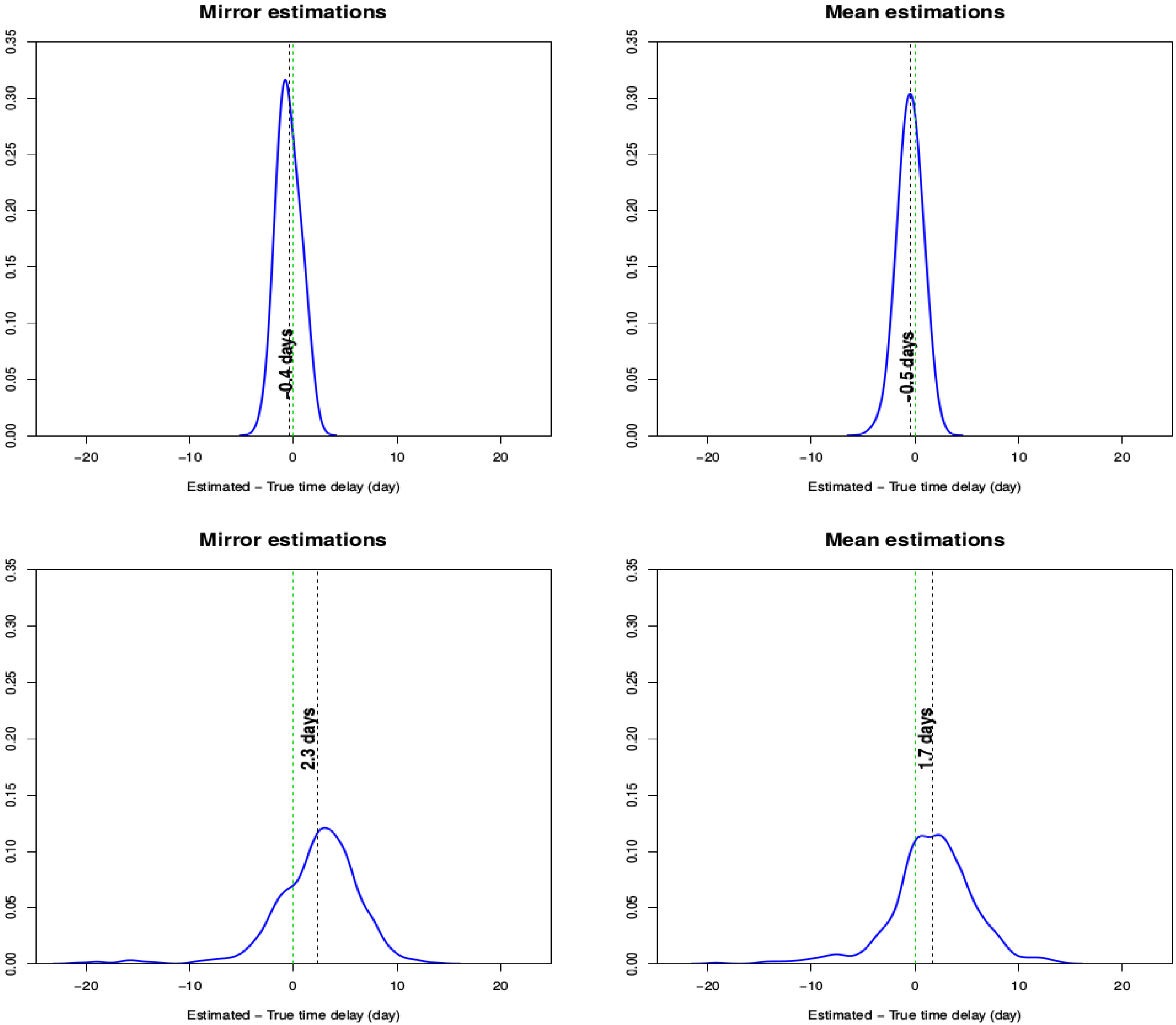
$$\mathbb{P}\left(\mathbb{E}(\hat{\theta}_n) - \theta) - l_1 \leq \hat{\theta}_n - \theta \leq (\mathbb{E}(\hat{\theta}_n) - \theta) + l_2\right)$$

$$\mathbb{P}\left(\underbrace{\hat{\theta} - l_2 - \overbrace{(\mathbb{E}(\hat{\theta}) - \theta)}^{\text{Bias}}}_{a} \leq \theta \leq \underbrace{\hat{\theta} + l_1 - \overbrace{(\mathbb{E}(\hat{\theta}) - \theta)}^{\text{Bias}}}_{b}\right)$$

$$\mathbb{P}(a \leq \theta \leq b) = (1-\alpha) \equiv \psi\%$$



Results



Using simple Monte Carlo simulations

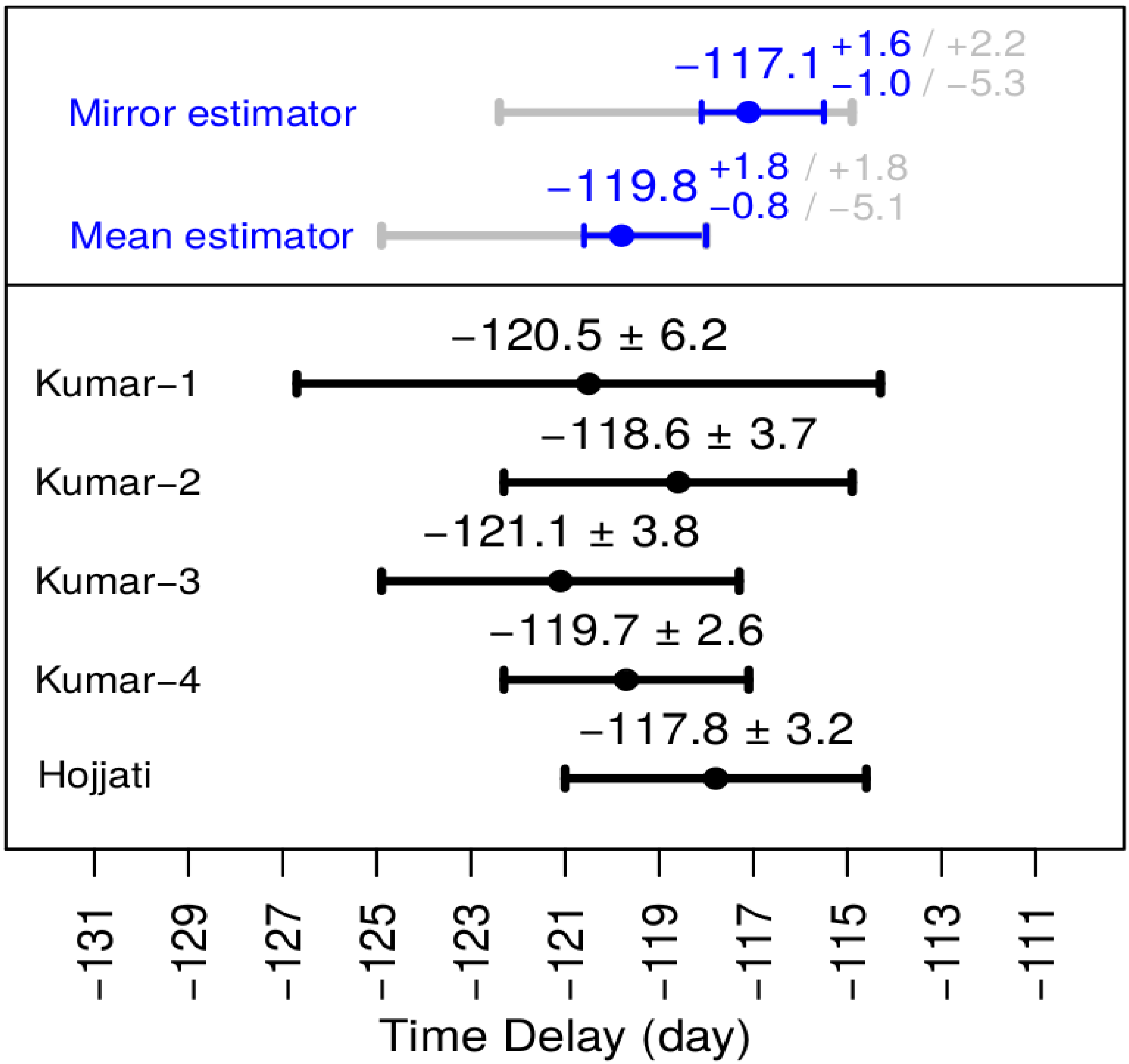
$$\tilde{\Delta t}_{AB}^{mirror} = -117.1^{+1.6}_{-1.0} \quad \text{days}$$
$$\tilde{\Delta t}_{AB}^{mean} = -119.8^{+1.8}_{-0.8} \quad \text{days}$$

Using simulations by COSMOGRAIL XIV (Rathna Kumar et al. 2013)

$$\tilde{\Delta t}_{AB}^{mirror} = -117.1^{+2.2}_{-5.3} \quad \text{days}$$
$$\tilde{\Delta t}_{AB}^{mean} = -119.8^{+1.8}_{-5.1} \quad \text{days}$$

(Aghamousa, Shafieloo, arXiv:1603.06331, 2016)

Results comparison



$$\Delta = 8$$

$$n_{cut} = 16 \quad n_{cut} = 4$$

$$\mathbb{P}(a \leq \theta \leq b) = (1 - \alpha) \equiv \psi\%$$

(Aghamousa, Shafieloo, arXiv:1603.06331, 2016)

Conclusion

- Strong gravitational lensing can be used as an independent way to estimate the Hubble parameter and breaking some degeneracies in cosmology.
- The time delay estimation has a crucial role in this story and we need to design robust and precise algorithm.
- Strong Lens Time Delay Challenge II has been held based on some thousands simulated data sets and encouraged the researchers to propose new methods and algorithms to estimate associated time delays.
- We have designed an algorithm for time delay estimation based on the smoothing and cross correlation. Our algorithm showed outstanding results in the challenge.
- The Time delay estimation of SDSS J1001+5027: Applying improved algorithm on real data. Using weighted correlation and number of data criterion.
- We introduced two different time delay estimators. We selected the optimum estimator based on minimum MSE which is calculated by simulation.
- We elaborated a pedagogical explanation for error estimation based on statistical characteristics of employed estimator.
- Finally our estimators result two time delay values for SDSS J1001+5027 system which are consistent to each other.
- In comparison with other estimations reported by different researchers, our estimations are consistent with smaller errors.

Thank you!