# Revisiting CDM isocurvature perturbations in curvaton scenario

Naoya Kitajima

Asia Pacific Center for Theoretical Physics

NK, D. Langlois, T. Takahashi, S. Yokoyama in prep

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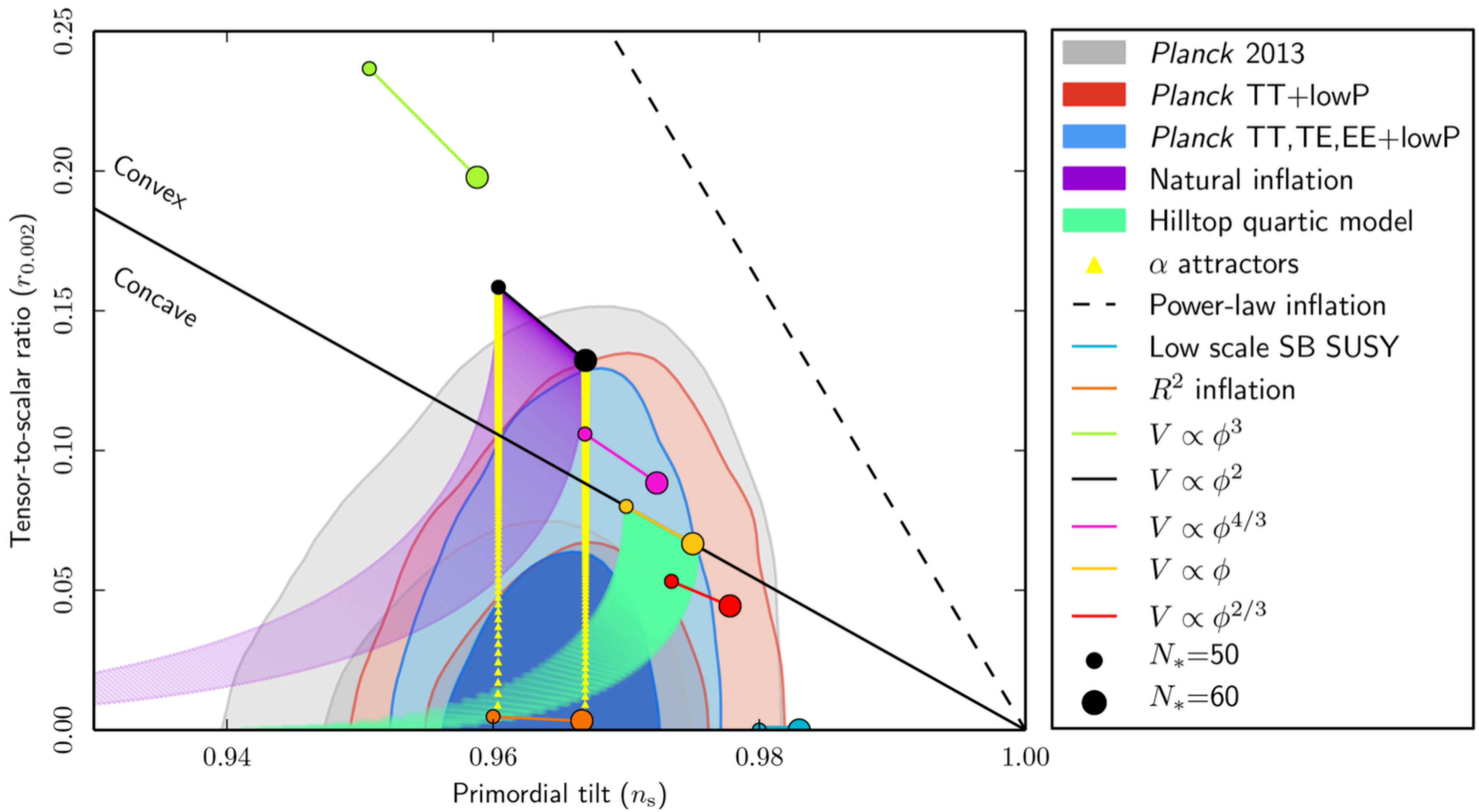
# Single field inflation models are tightly constrained

$$n_s = 0.968 \pm 0.006 \quad (68\% \text{ CL}), \quad r < 0.12 \quad (95\% \text{ CL})$$

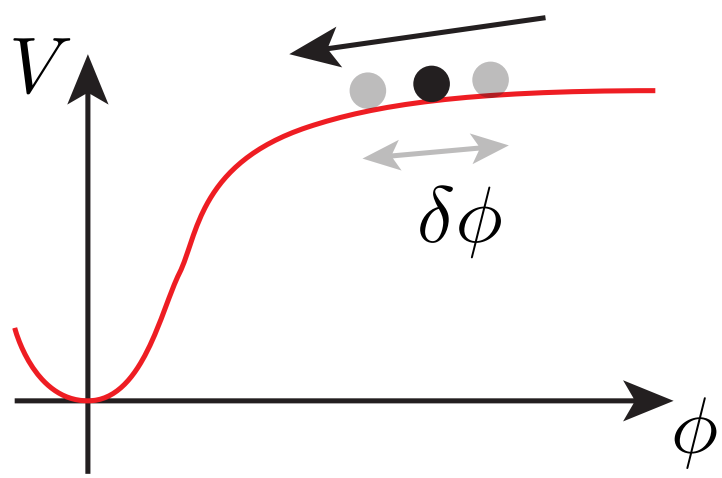


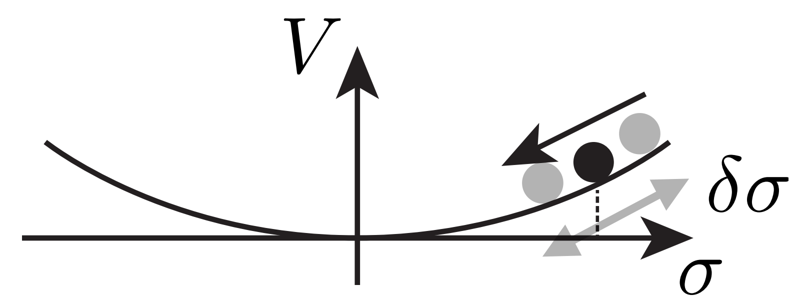

Small field model or $R^2$ model seem good..

# (One of) the next-to-simplest setup(s)

inflaton + extra scalar field

"Curvaton" : $\sigma$




Enqvist & Sloth / Lyth & Wands / Moroi & Takahashi (2001)

- light & subdominant during inflation

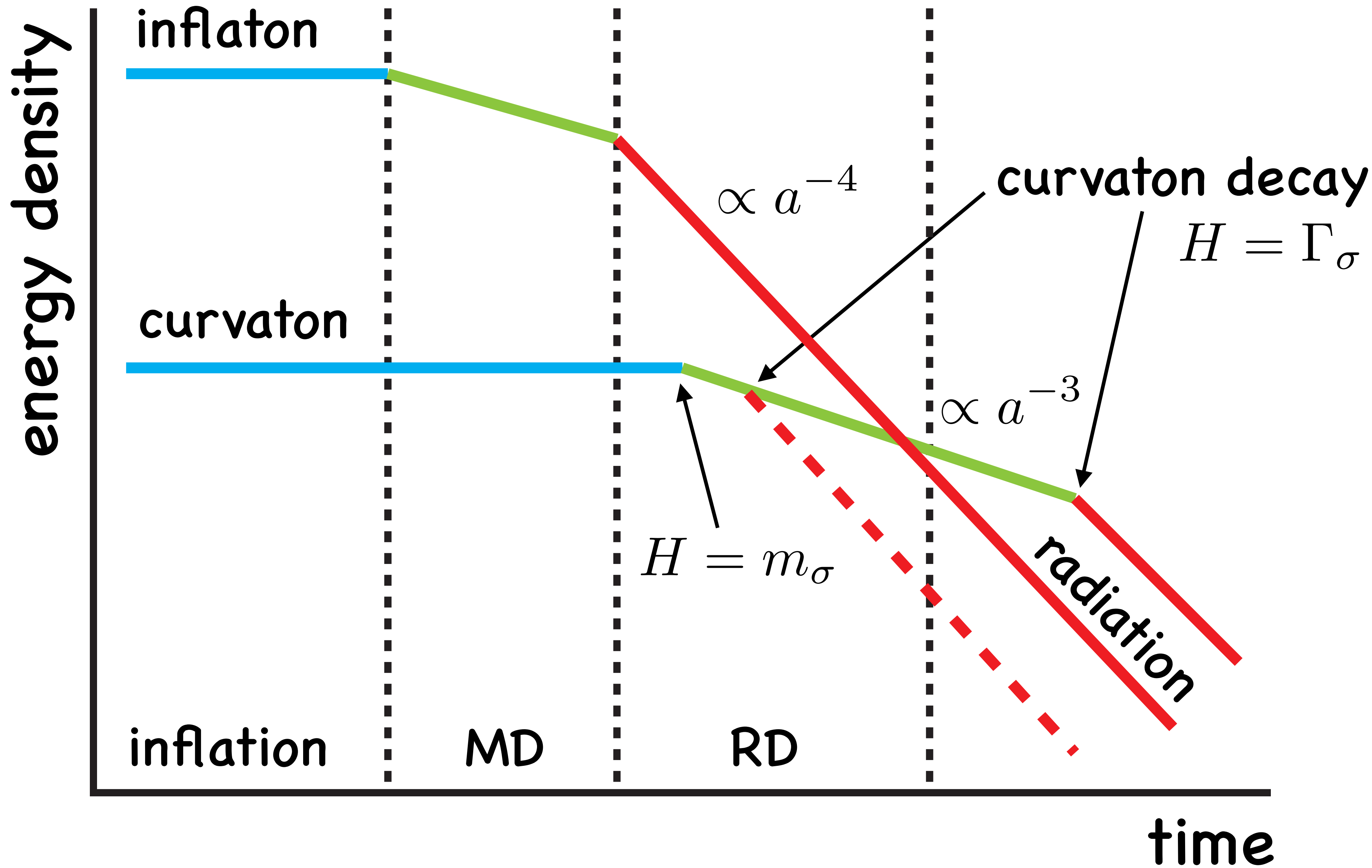
$m_\sigma \ll H_{\rm inf}, \ \rho_\sigma \ll \rho_\phi$

=> acquires quantum fluctuations

multiple origin for perturbations

- decays after inflation

# Cosmic history in curvaton scenario




- Large non-gaussianity
- Isocurvature perturbations

## - Non-gaussianity

$$\zeta = \zeta_G + \frac{3}{5} f_{\rm NL} (\zeta_G^2 - \langle \zeta_G^2 \rangle)$$

$$f_{\rm NL}^{\rm (local)} = 0.8 \pm 5.0$$ **(68%CL, Planck Collaboration 2015)**

**Curvaton model:** $$f_{\rm NL} = \frac{5}{4r_s} - \frac{5}{3} - \frac{5r_s}{6} \qquad r_s = \frac{3\Omega_{\sigma,{\rm dec}}}{3\Omega_{\sigma,{\rm dec}} + 4\Omega_{r,{\rm dec}}}$$

$$\Omega_{\sigma,{\rm dec}} = r_s = 1 \;\; \Rightarrow \;\; f_{\rm NL} = -\frac{5}{4}$$

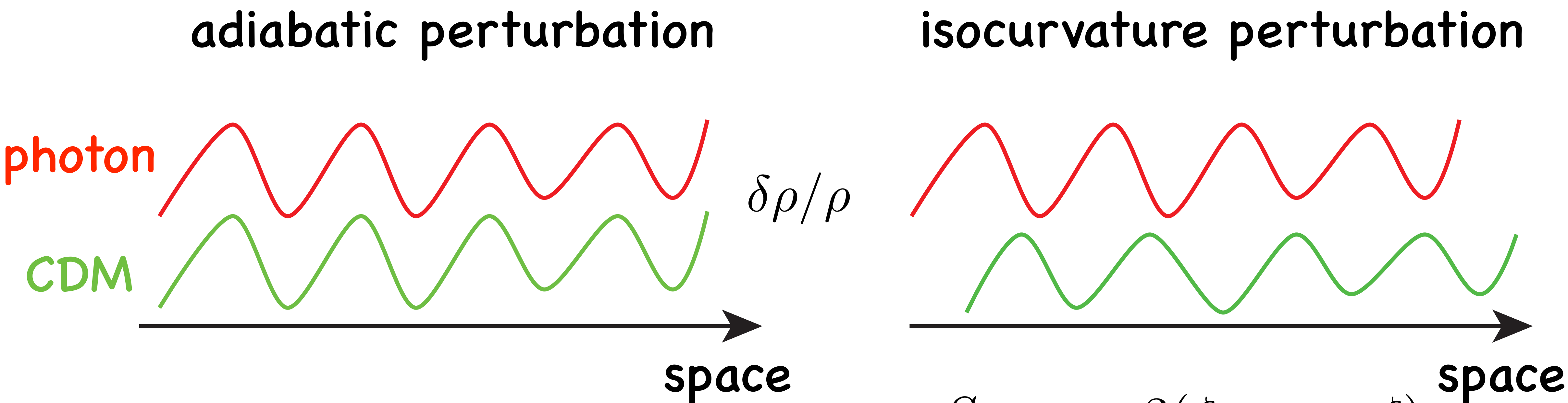
$$\Omega_{\sigma,{\rm dec}} \sim r_s \ll 1 \;\; \Rightarrow \;\; f_{\rm NL} \simeq \frac{5}{4r_s} \gg 1$$

! severely constrained

$$r_s \gtrsim 0.2$$

**Curvaton should dominate the universe before decay in order not to give too large $f_{\rm NL}$**

# - Isocurvature perturbations

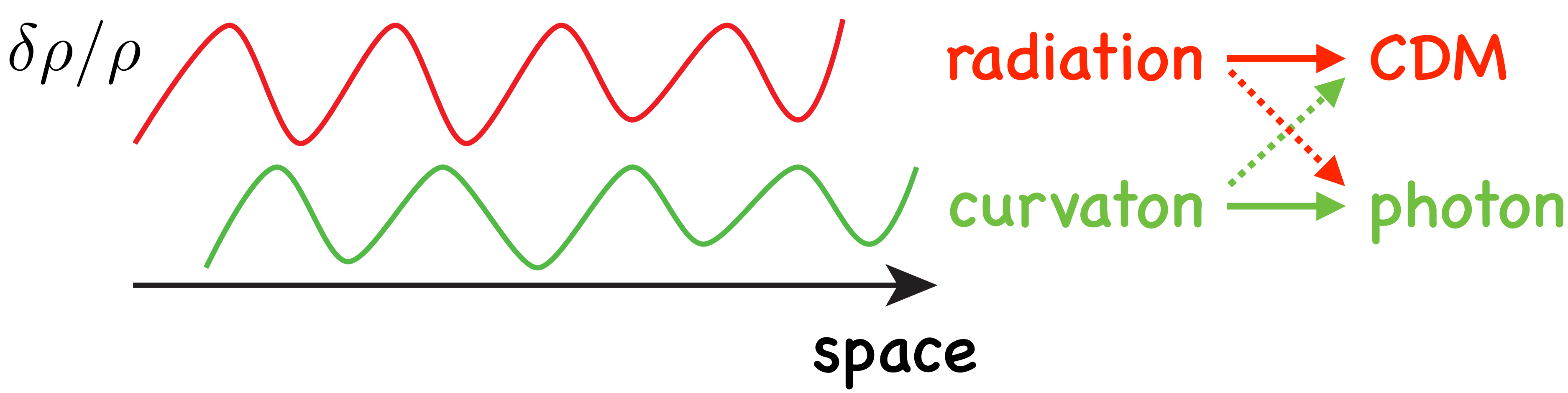



$$S_{\rm CDM} = 3(\zeta_{\rm CDM} - \zeta)$$

"intrinsic" such as axion

↕

Curvaton has "Residual" CDM isocurvature perturbation



# Constraint on isocurvature perturbations

**Planck 2015 constraint :**

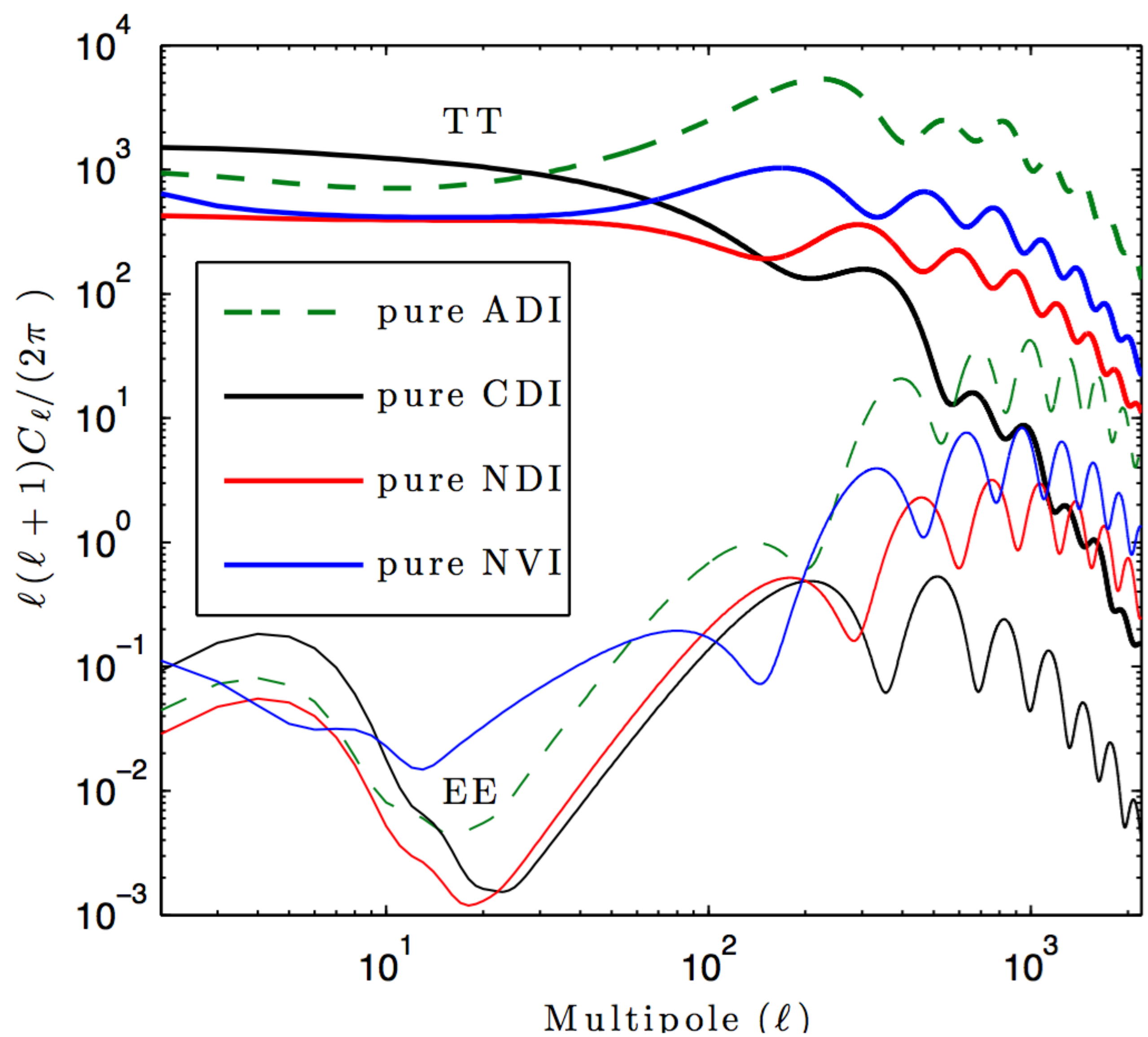
$$\beta_{\rm iso}(k_0) < 0.035 \ (\text{uncorrelated}), \ \ \beta_{\rm iso}(k_0) < 0.013 \ (\text{correlated})$$

**[95% CL]**

$$\beta_{\rm iso} = \frac{\mathcal{P}_S}{\mathcal{P}_\zeta + \mathcal{P}_S}$$

$$|S/\zeta| \lesssim 0.1$$

**curvaton model is constrained**




Planck collaboration 1502.02114

CDM isocurvature perturbation in curvaton scenario

# I. WIMP dark matter

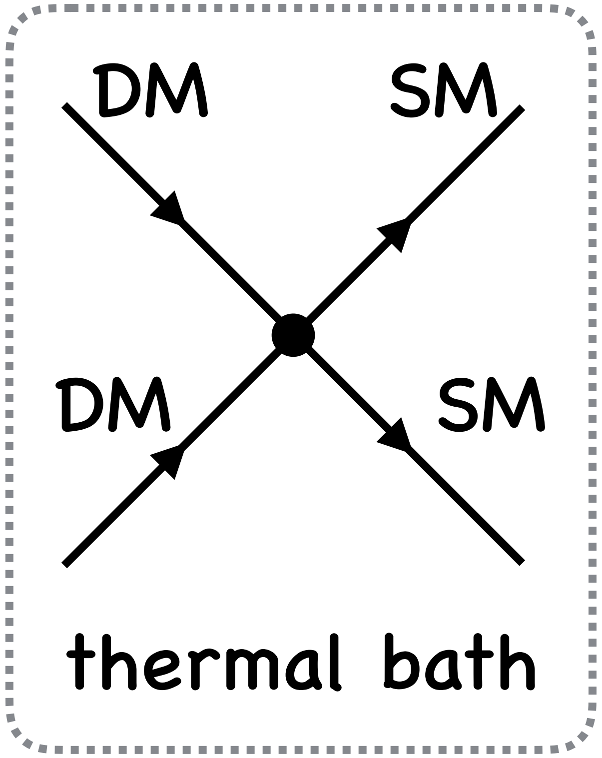
Weakly Interacting Massive Particle

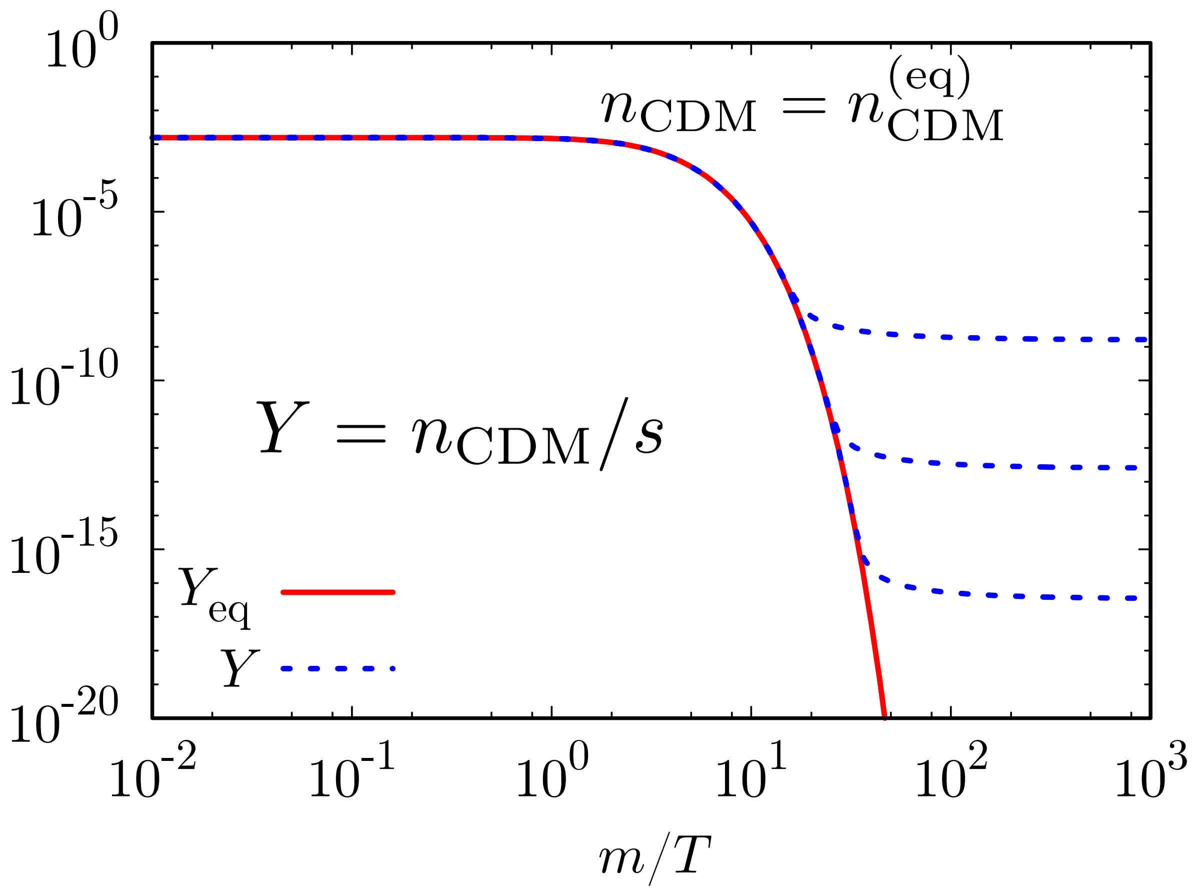
# Thermally produced WIMP

creation/annihilation rate for DM : $\Gamma_{\rm ann} = \langle \sigma_{\rm ann} v \rangle n_{\rm CDM}$

$\Gamma_{\rm ann} > H$ — DM is in thermal equilibrium

$\Gamma_{\rm ann} < H$ — DM is decoupled from thermal bath



# - “Sudden” freeze-out formalism -

to calculate isocurvature perturbation

CDM freezes out suddenly at

$$H = \Gamma_{\rm ann} = \langle \sigma_{\rm ann} v \rangle n_{\rm CDM} \quad \text{at} \quad N = N_{\rm fr}$$

| before freeze-out | after freeze-out |
| --- | --- |
| $n_{\rm CDM} = n_{\rm CDM}^{\rm (eq)}(T)$ | $n_{\rm CDM} = n_{\rm CDM}^{\rm (eq)}(T_{\rm fr}) e^{-3(N-N_{\rm fr})}$ |

$$n^{\rm (eq)} = \frac{g_*}{2\pi^2} \int_m^\infty \sqrt{E^2 - m^2} \frac{E dE}{e^{E/T} \pm 1}$$

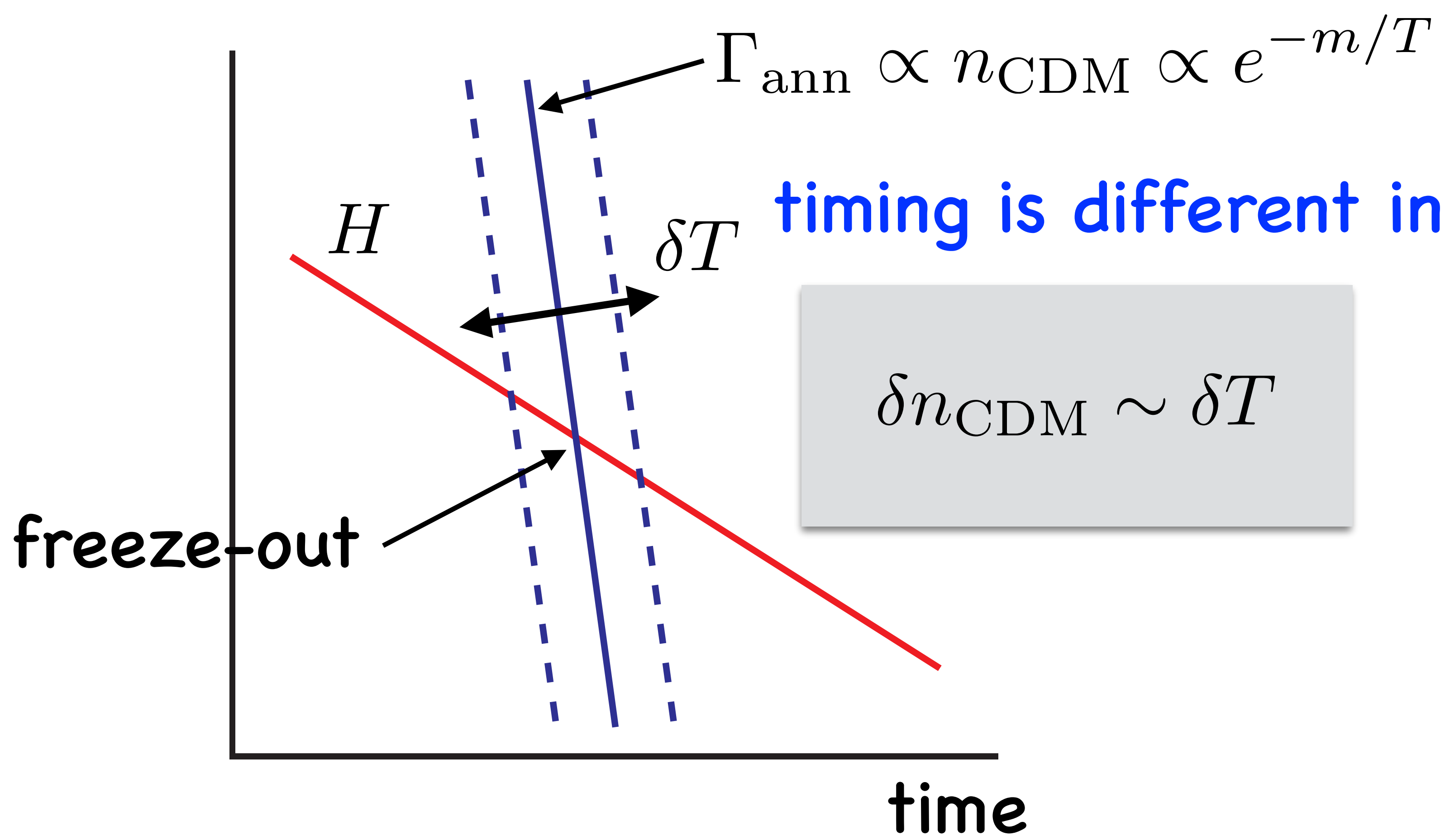
non-relativistic limit — $n^{\rm (eq)} \simeq g_* \left( \frac{mT}{2\pi} \right)^{3/2} e^{-m/T}$

# “Residual” CDM isocurvature perturbation

$$S_{\rm CDM} = 3(\zeta_{\rm CDM} - \zeta) = \frac{\delta(n_{\rm CDM}/s)}{n_{\rm CDM}/s}$$

$$\zeta = \frac{r_s}{3} S_i, \quad r_s = \frac{3\Omega_{\sigma,\rm dec}}{3\Omega_{\sigma,\rm dec} + 4\Omega_{r,\rm dec}}$$

DM is non-relativistic at freeze-out : $m/T_{\rm fr} \sim 20$



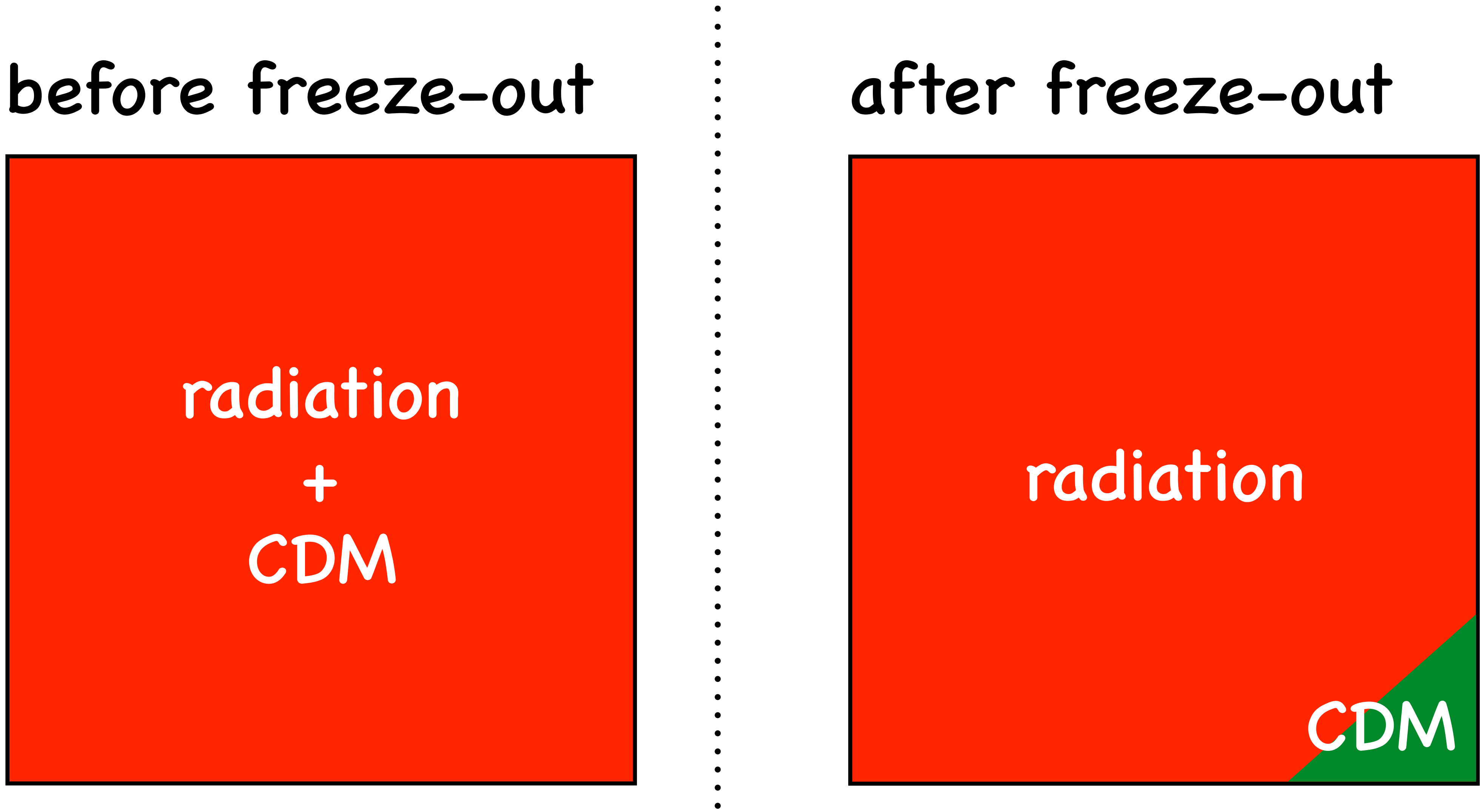

timing is different in each patch

$$\delta n_{\rm CDM} \sim \delta T$$

# case 1 — CDM freeze-out after curvaton decay

$$H = \Gamma_{\rm ann} \;\Rightarrow\; 3M_P^2\Gamma_{\rm ann}^2 = \rho_r(N_{\rm fr})$$ - single component

$$\zeta_{\rm CDM} = \zeta \;\Rightarrow\; S_{\rm CDM} = 0$$



## case 2 — CDM freeze-out **before** curvaton decay

$$\rho_r + \rho_\sigma = 3M_P^2\Gamma_{\rm ann}^2$$

radiation-curvaton fluid
(2-component)

$$S_{\rm CDM}/\zeta = 3\left[\frac{\Omega_{\sigma,\rm fr}}{r_s}\left(\frac{m}{T_{\rm fr}} - \frac{3}{2}\right)\frac{1}{2(\alpha_{\Lambda,\rm fr} - 2) + \Omega_{\sigma,\rm fr}} - 1\right]$$

$$\alpha_{\Lambda,\rm fr} \equiv \left.\frac{d\ln\Gamma_{\rm ann}}{d\ln T}\right|_{\rm fr}$$

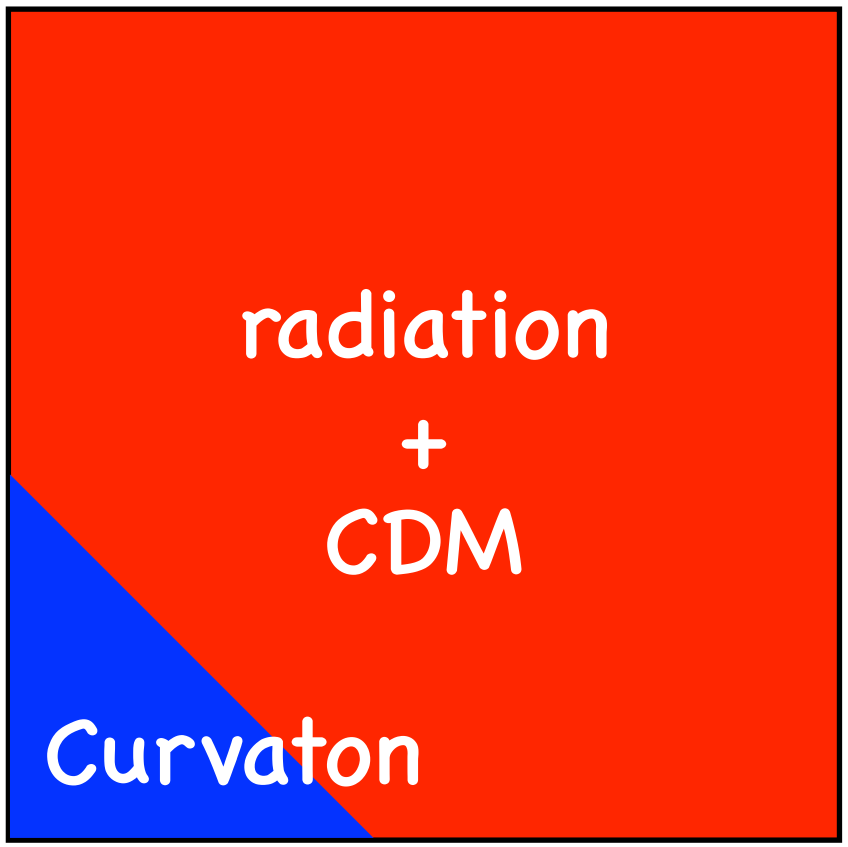
NK, Langlois, Takahashi, Yokoyama in prep
(consistent with Lyth, Wands astro-ph/0306500)

non-relativistic limit :

$$\alpha_{\Lambda,\rm fr} \approx \frac{m}{T_{\rm fr}} \gg 1 \;\Rightarrow\; S_{\rm CDM}/\zeta \approx 3\left(\frac{\Omega_{\sigma,\rm fr}}{2r_s} - 1\right)$$

- Case A

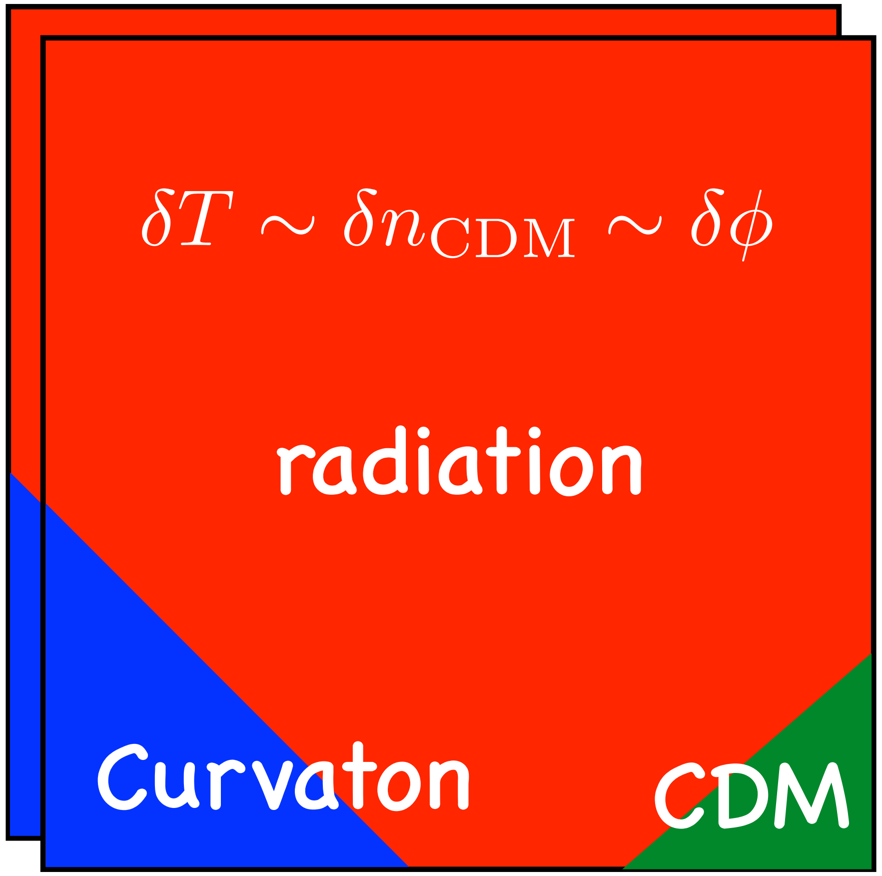
before CDM freeze-out




$\Omega_{\sigma,\mathrm{fr}} \ll 1$

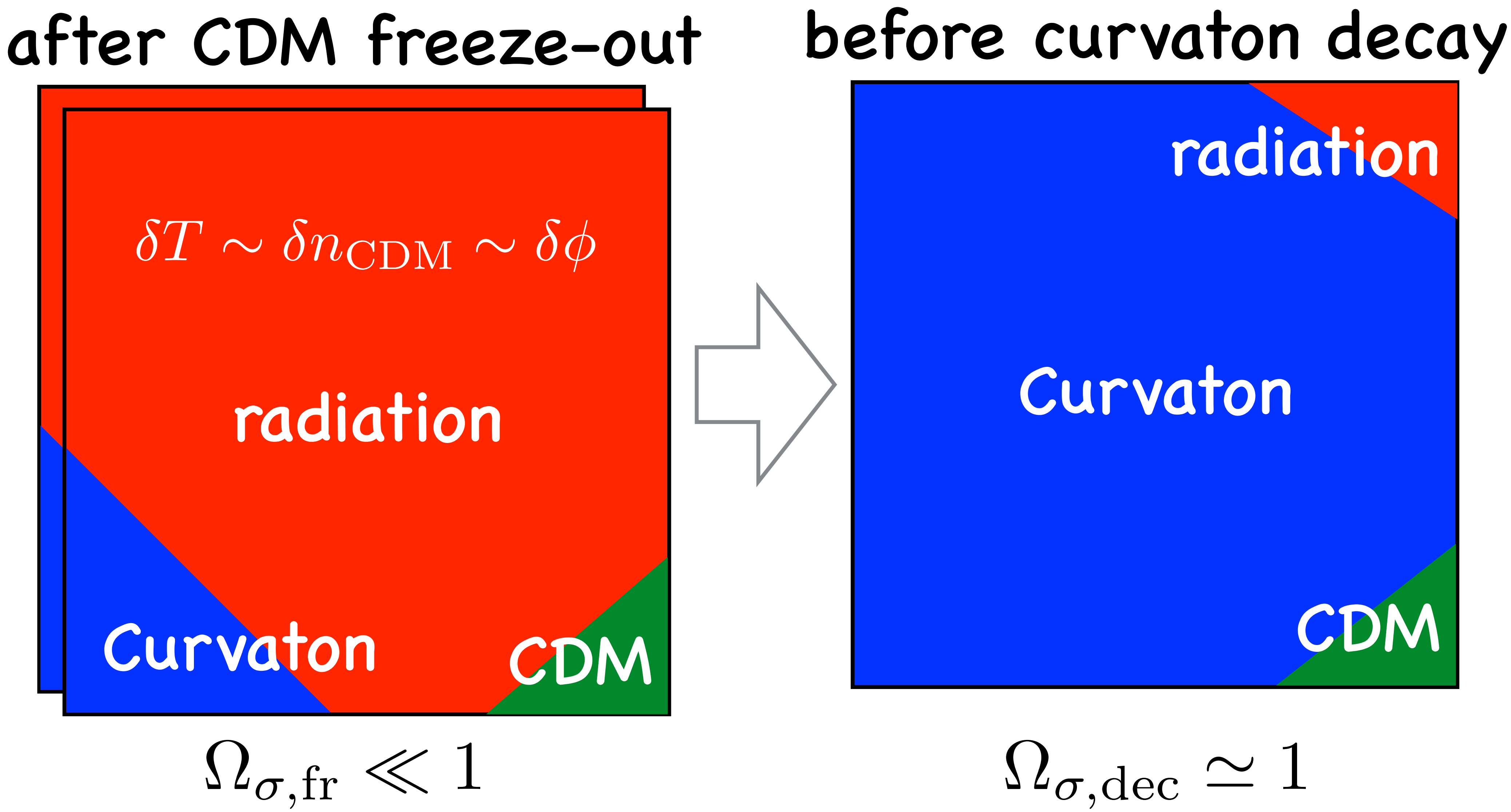
- Case A

after CDM freeze-out



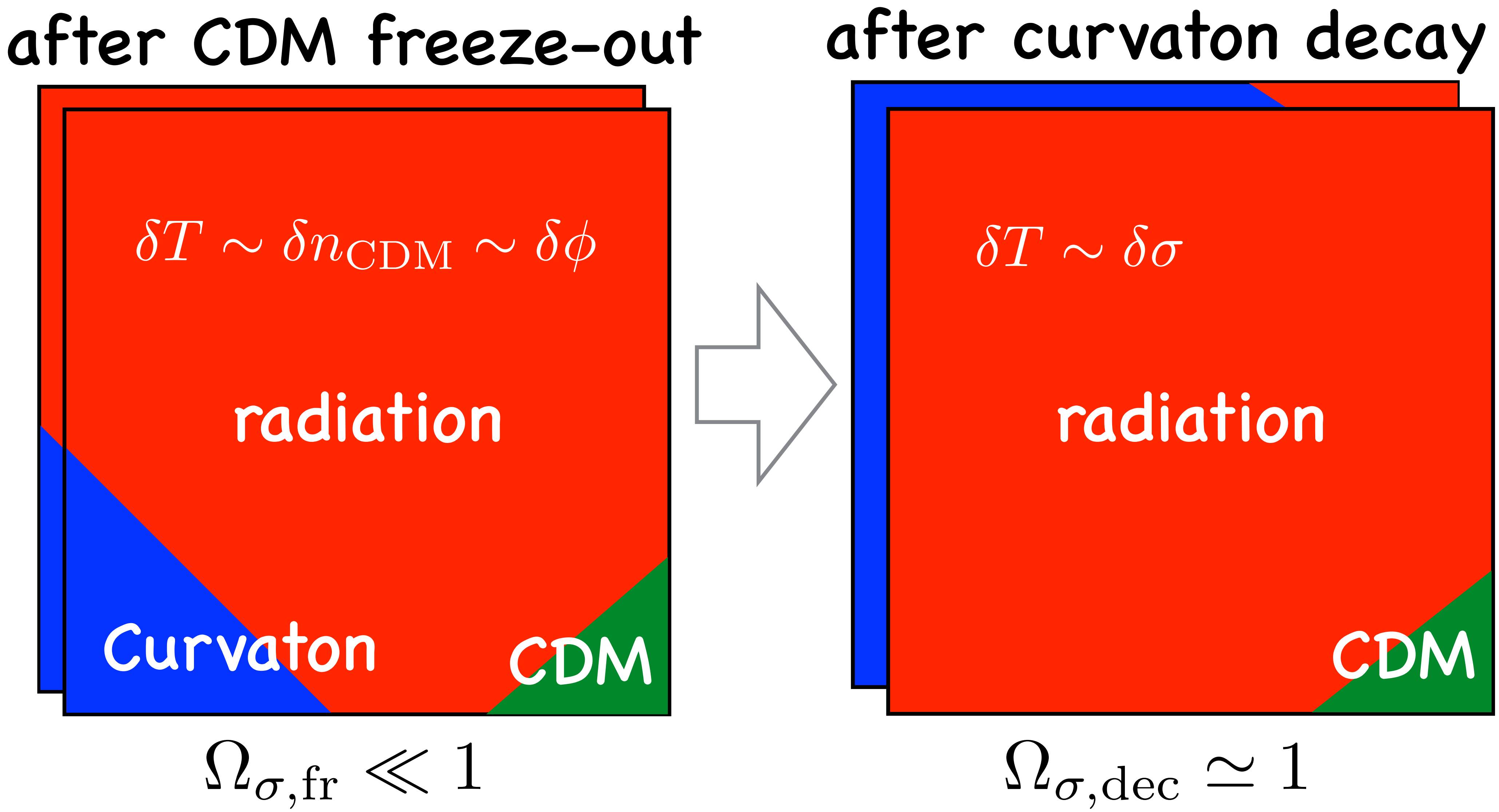

$\Omega_{\sigma,\rm fr} \ll 1$

## - Case A



after CDM freeze-out
before curvaton decay
$\delta T \sim \delta n_{\rm CDM} \sim \delta\phi$
radiation
Curvaton
CDM
radiation
Curvaton
CDM
$\Omega_{\sigma,\rm fr} \ll 1$
$\Omega_{\sigma,\rm dec} \simeq 1$

## - Case A

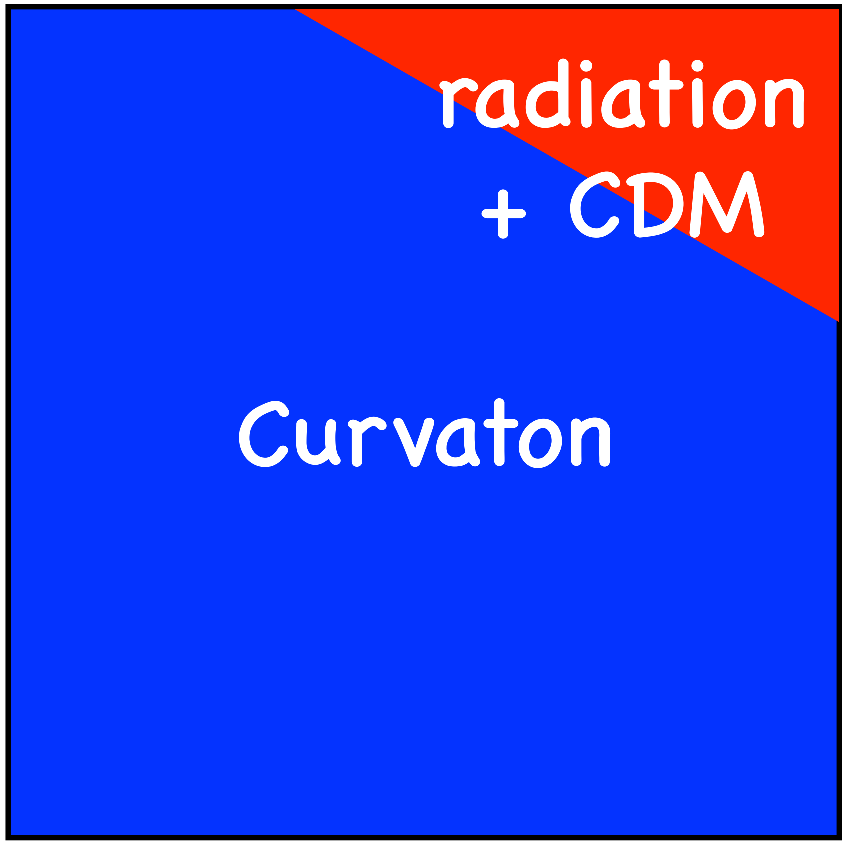



$$S_{\rm CDM}/\zeta \approx 3\left(\frac{\Omega_{\sigma,{\rm fr}}}{2r_s} - 1\right) \simeq -3$$

ruled out!

- Case B

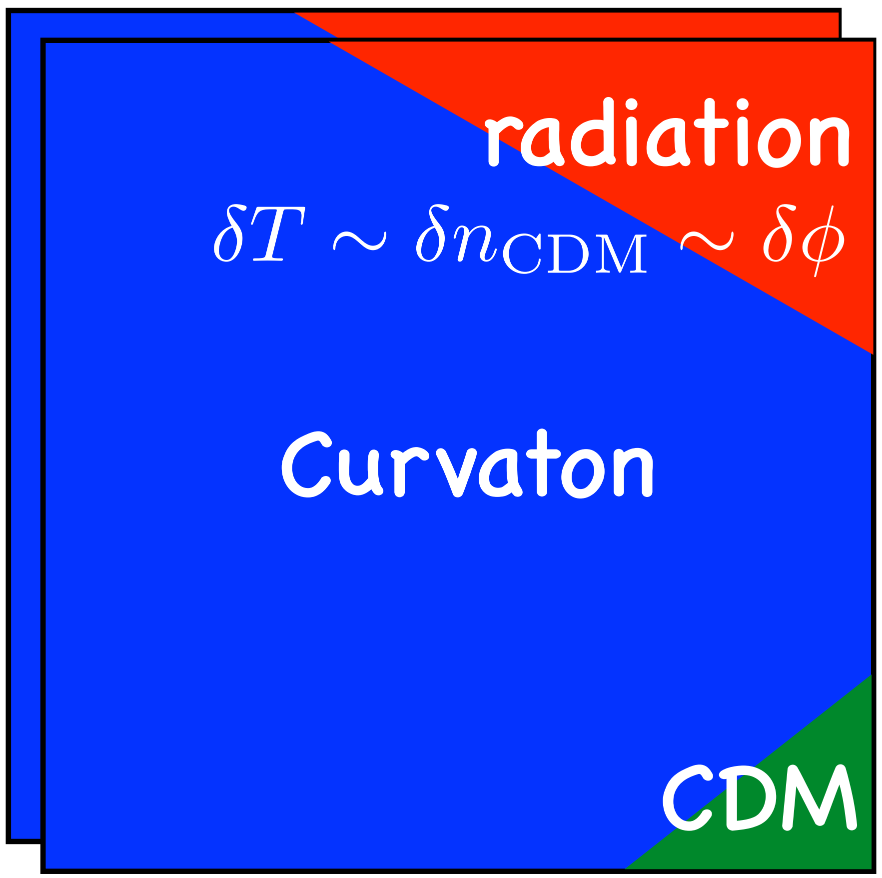
before CDM freeze-out




$$\Omega_{\sigma,\mathrm{fr}} \simeq 1$$

- Case B

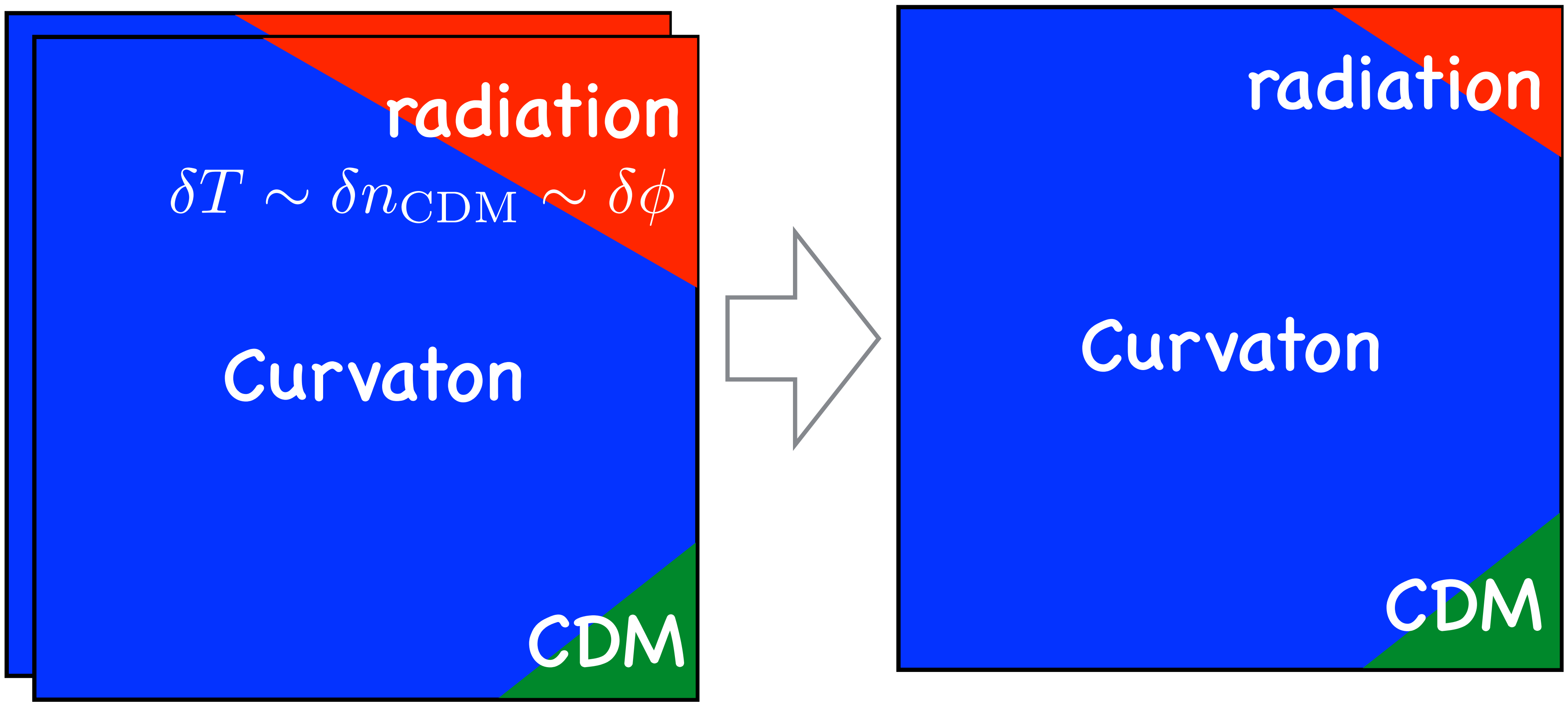
before CDM freeze-out




$\Omega_{\sigma,\rm fr} \simeq 1$

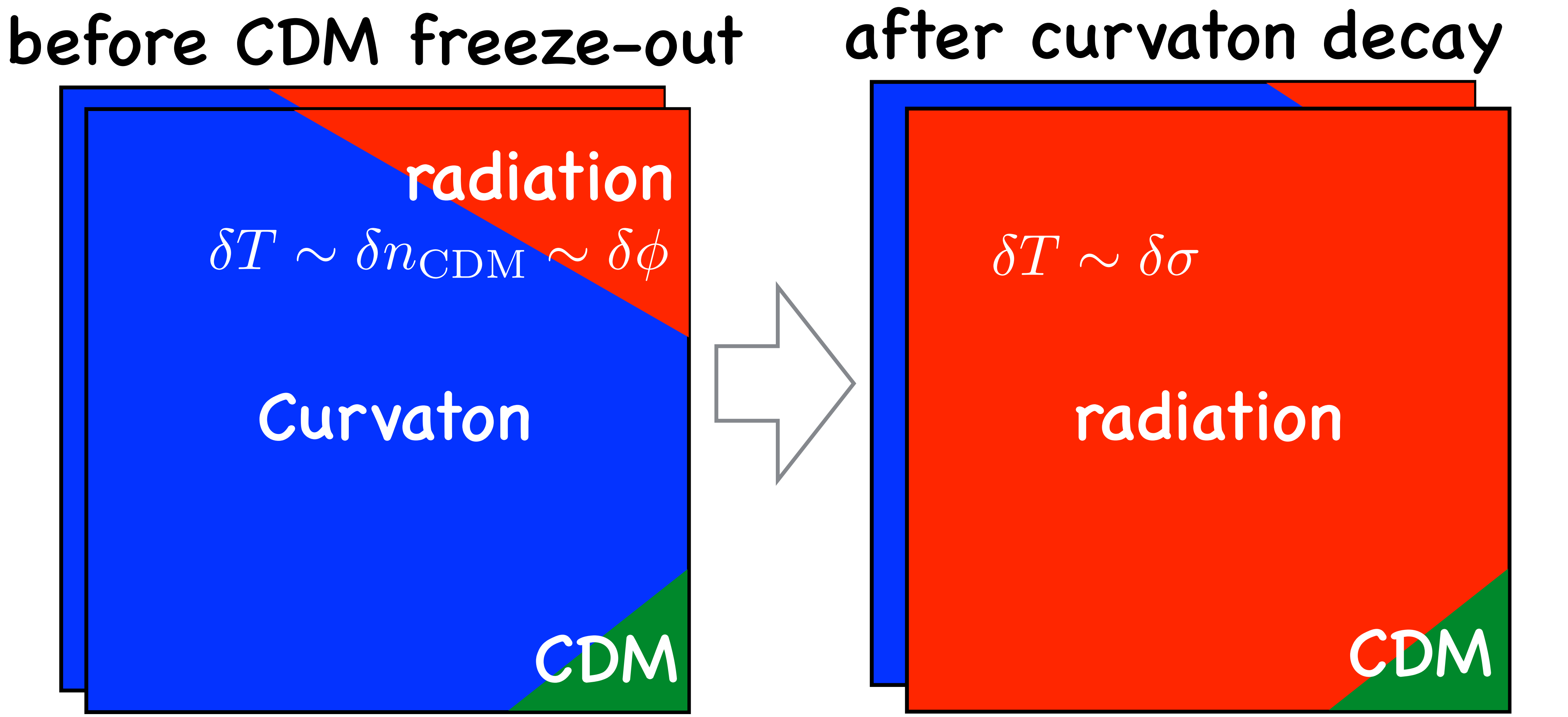
- Case B

before CDM freeze-out | before curvaton decay




$\Omega_{\sigma,\rm fr} \simeq 1$ | $\Omega_{\sigma,\rm dec} \simeq 1$

- Case B

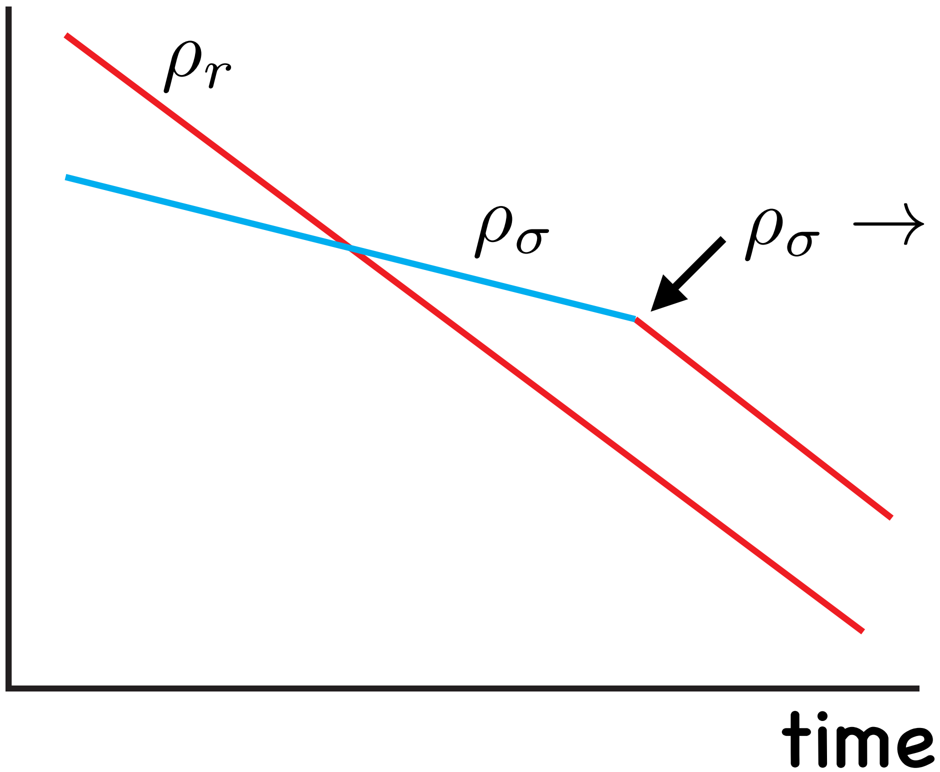



$\Omega_{\sigma,\rm fr} \simeq 1$

$\Omega_{\sigma,\rm dec} \simeq 1 \quad r_s \sim 1$

$$S_{\rm CDM}/\zeta \approx 3\left(\frac{\Omega_{\sigma,\rm fr}}{2r_s} - 1\right) \simeq -\frac{3}{2}$$ ruled out??

## sudden decay approximation




$$\dot{\rho}_r + 4H\rho_r = 0$$

$$\dot{\rho}_\sigma + 3H\rho_\sigma = 0$$

until decay

it is not valid in some cases
e.g. T-dependent decay rate

NK, Langlois, Takahashi, Takesako and Yokoyama 1407.5148

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## More realistic analysis

(numerical analysis)

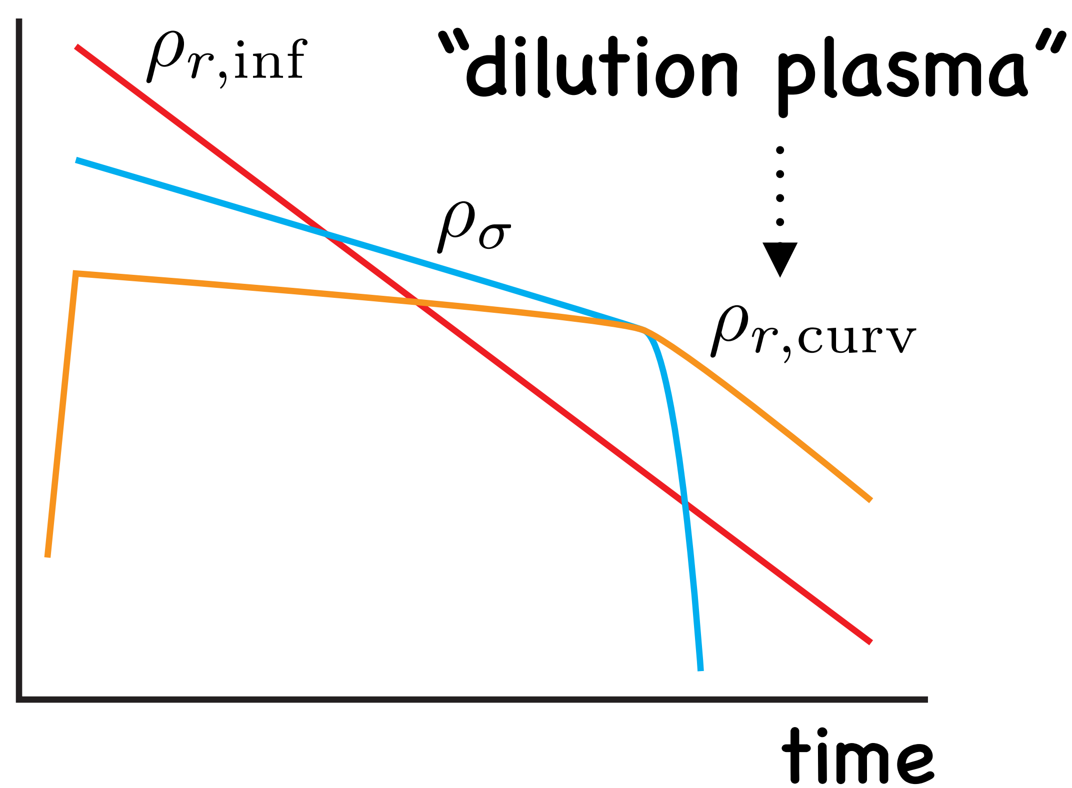
$$\dot{\rho}_\sigma + 3H\rho_\sigma = -\Gamma\rho_\sigma$$

$$\dot{\rho}_r + 4H\rho_r = \Gamma\rho_\sigma$$

$\rho_r \to$

$$\dot{\rho}_{r,\mathrm{inf}} + 4H\rho_{r,\mathrm{inf}} = 0$$

$$\dot{\rho}_{r,\mathrm{curv}} + 4H\rho_{r,\mathrm{curv}} = \Gamma_\sigma\rho_\sigma$$

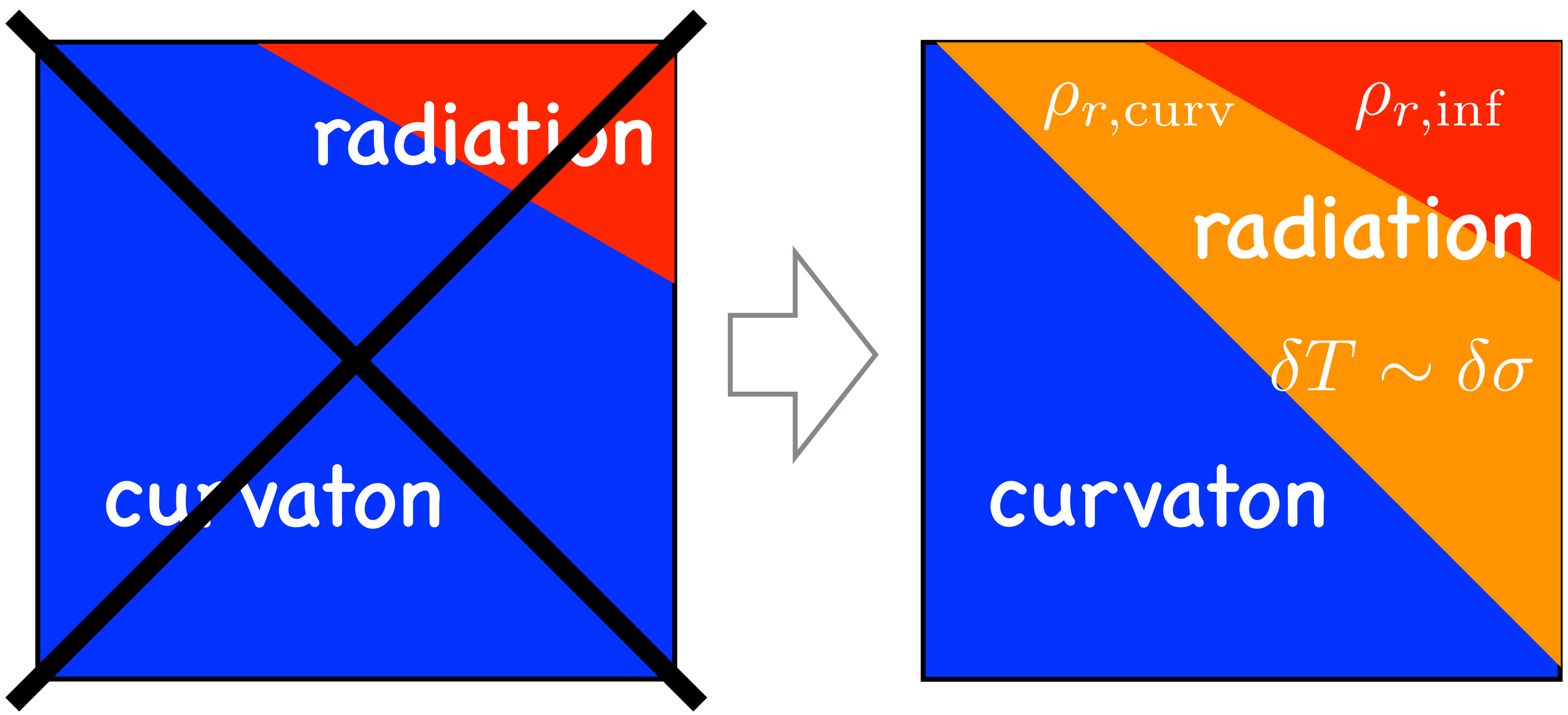


**w/o dilution plasma :** $\rho_r = \rho_{r,\mathrm{inf}}, \quad T = \left[\frac{30}{\pi^2 g_*}\rho_{r,\mathrm{inf}}\right]^{1/4}$

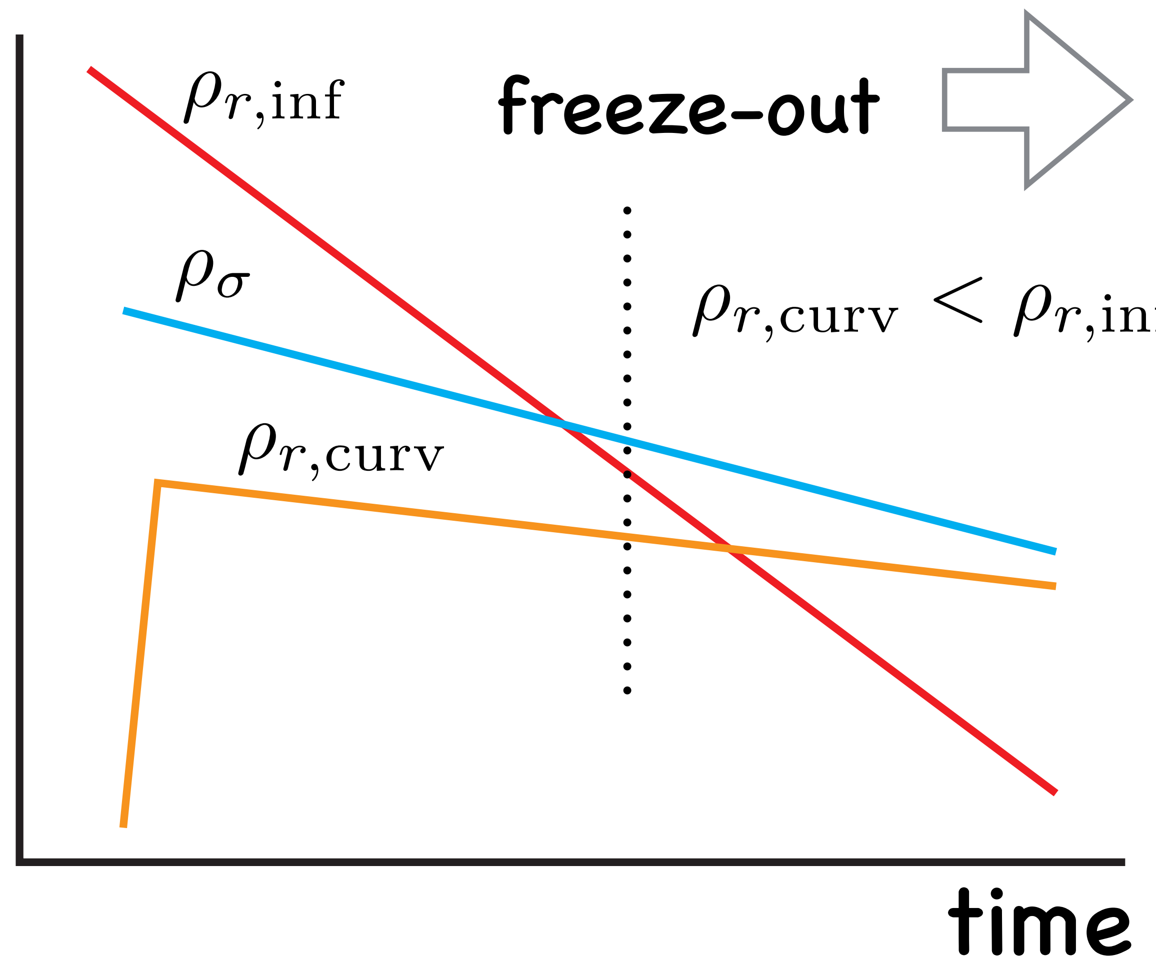
**w/ dilution plasma**

$$\rho_r = \rho_{r,\mathrm{inf}} + \rho_{r,\mathrm{curv}}, \quad T = \left[\frac{30}{\pi^2 g_*}(\rho_{r,\mathrm{inf}} + \rho_{r,\mathrm{curv}})\right]^{1/4}$$

$\rho_{r,\mathrm{curv}} \propto a^{-3/2}$ **see e.g. Kolb & Turner**

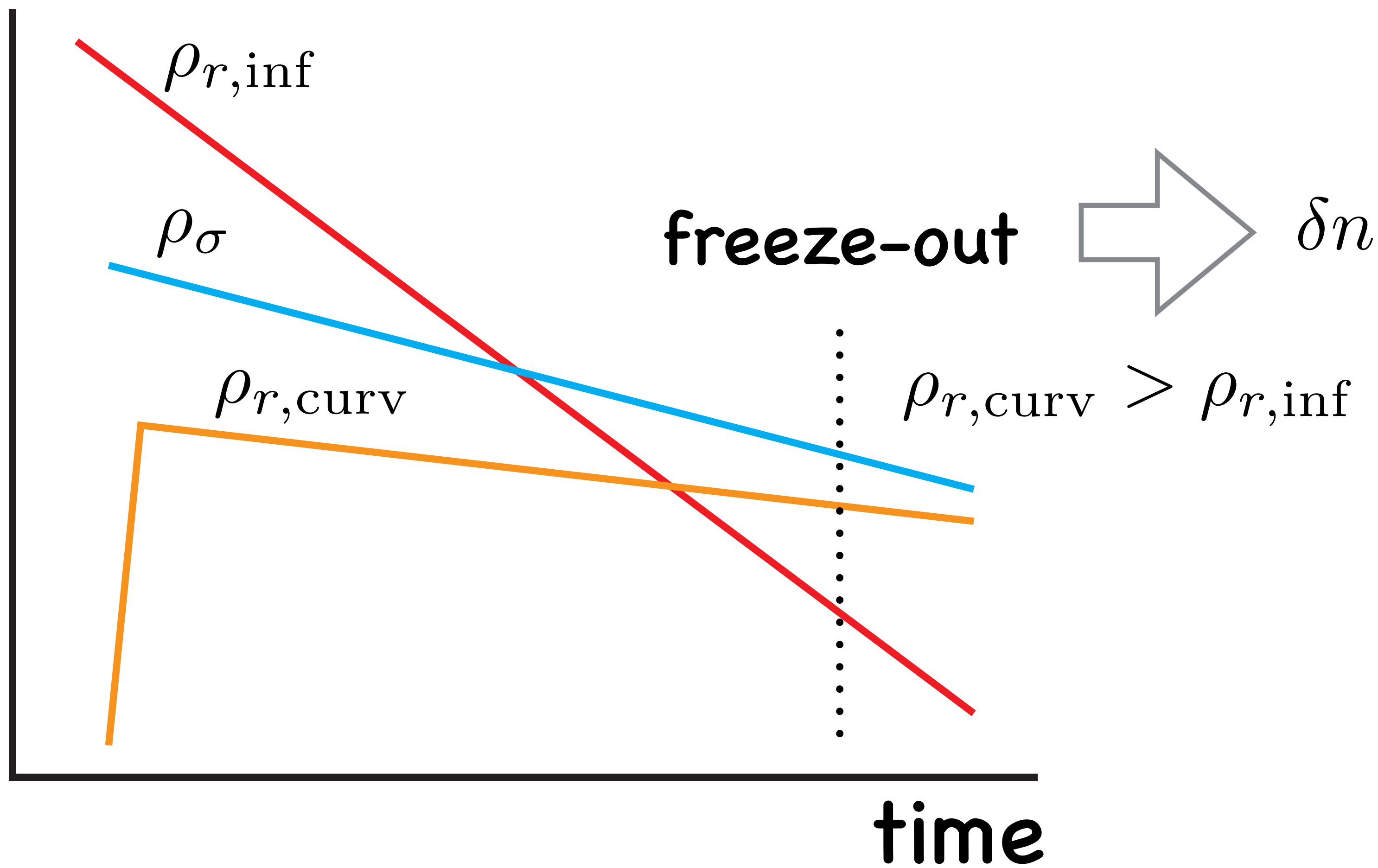


# Primordial radiation vs dilution plasma




$$\delta n_{\mathrm{CDM}} \sim \delta \rho_{r,\mathrm{inf}} \sim \delta \phi$$

# Primordial radiation vs dilution plasma




$$\delta n_{\mathrm{CDM}} \sim \delta\rho_{r,\mathrm{curv}} \sim \delta\sigma$$

$$\rho_{r,\mathrm{inf}} \gg \rho_{r,\mathrm{curv}} \;\Rightarrow\; S_{\mathrm{CDM}}/\zeta \simeq 0$$

# Numerical calculations

$$H^2 = \frac{1}{3M_P^2}(\rho_r + \rho_\sigma + \rho_{\rm CDM}) \simeq \frac{1}{3M_P^2}(\rho_r + \rho_\sigma)$$

$$\dot{\rho}_r + 4H\rho_r = \Gamma\rho_\sigma$$ (+ CDM annihilation)

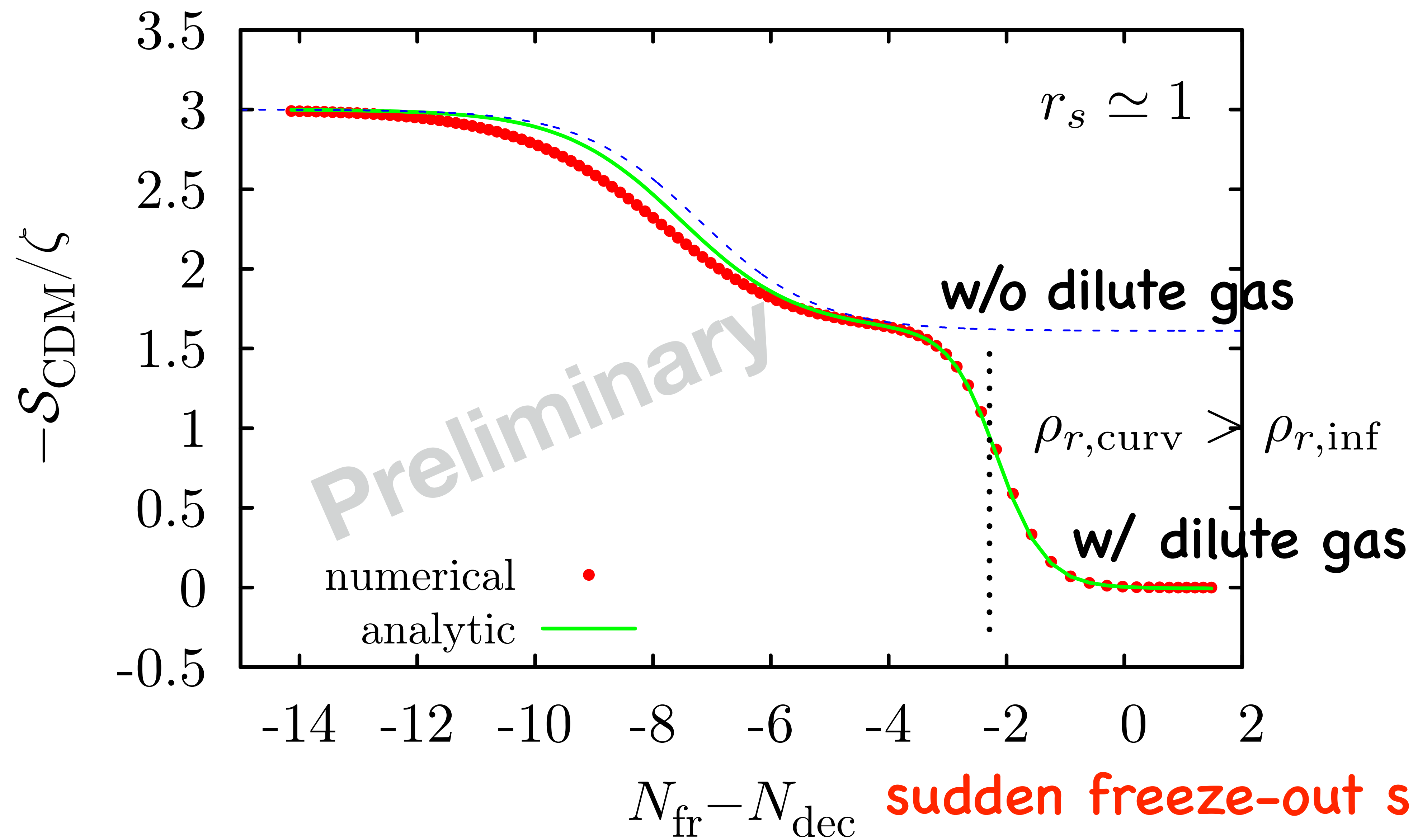
$$\dot{\rho}_\sigma + 3H\rho_\sigma = -\Gamma\rho_\sigma$$

$$\dot{n}_{\rm CDM} + 3Hn_{\rm CDM} = -\langle\sigma_{\rm ann}v\rangle\left(n_{\rm CDM}^2 + (n_{\rm CDM}^{\rm (eq)})^2\right)$$

$$N = \int H dt, \quad \zeta_i = \delta N + \frac{1}{3(1+w_i)}\ln\left(\frac{\rho_i}{\bar{\rho}_i}\right)$$

## — Analytic & Numerical results —

NK, Langlois, Takahashi, Yokoyama in prep

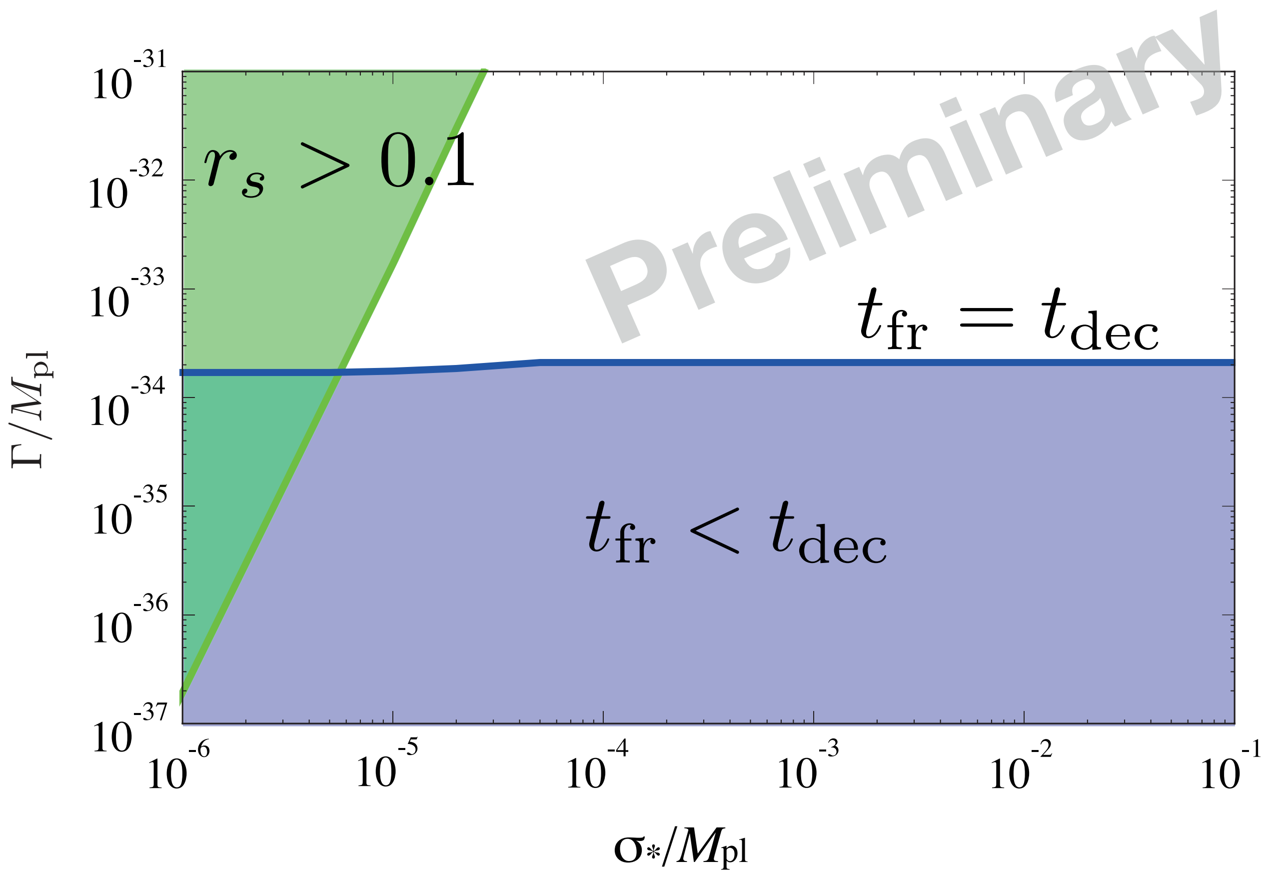


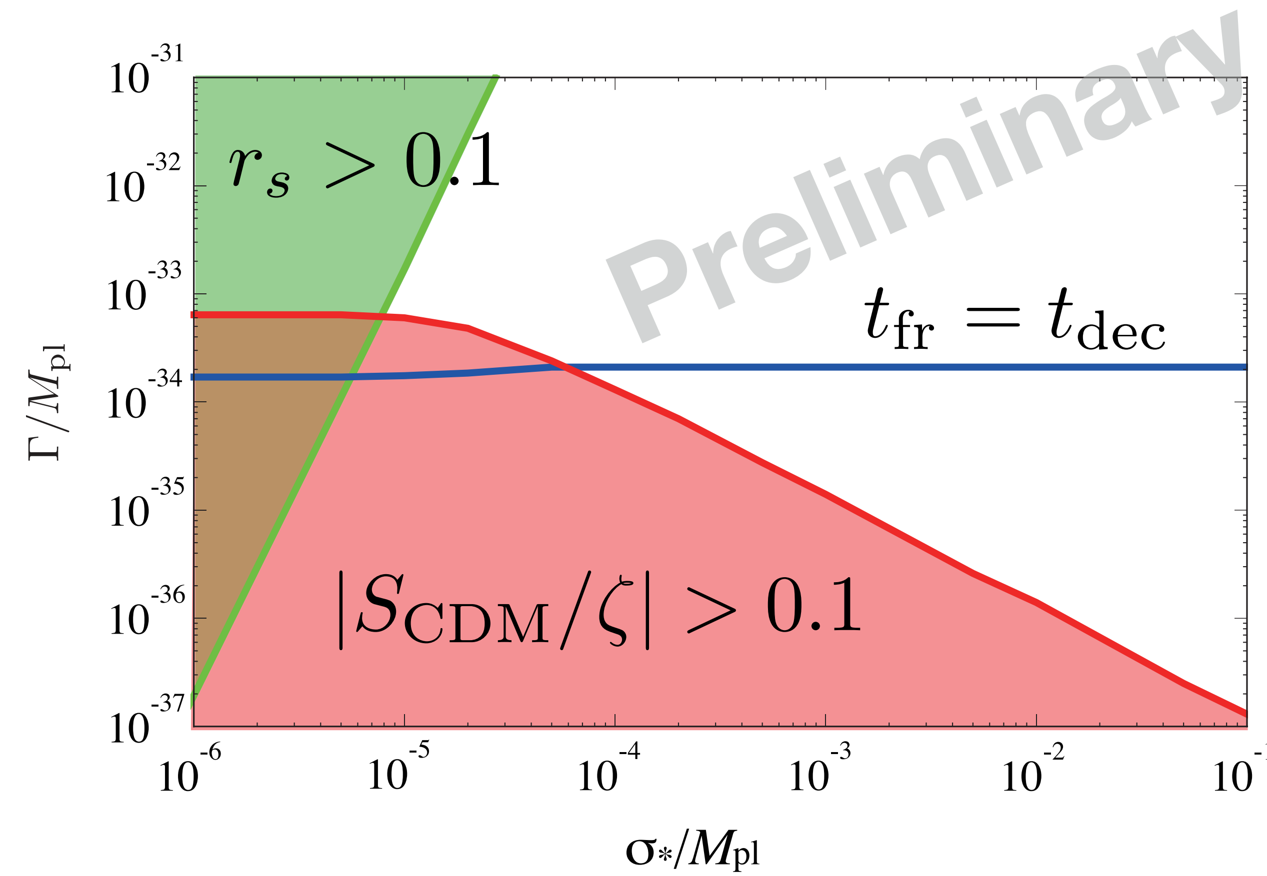

sudden freeze-out seems OK!

Isocurvature vanishes if axion starts to oscillate
after dilute gas exceeds primordial radiation

# Allowed parameter region

NK, Langlois, Takahashi, Yokoyama in prep




w/o dilution plasma

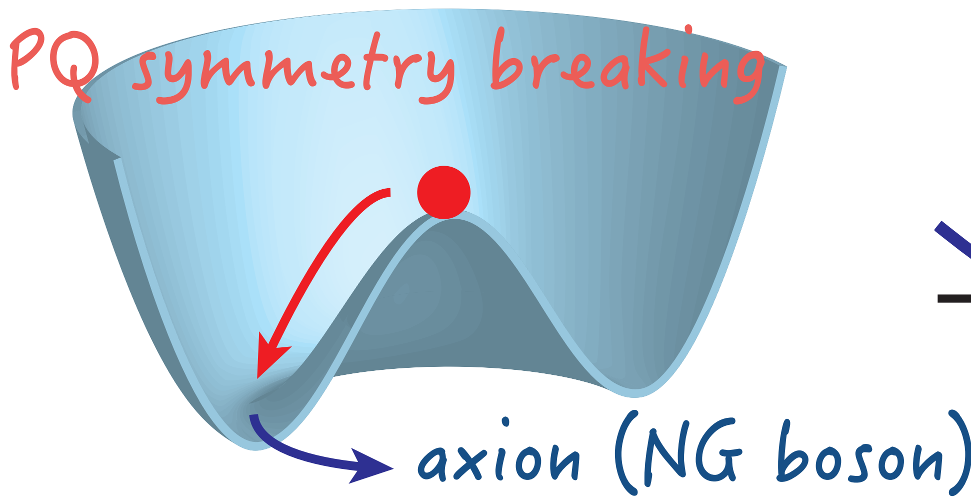
w/ dilution plasma

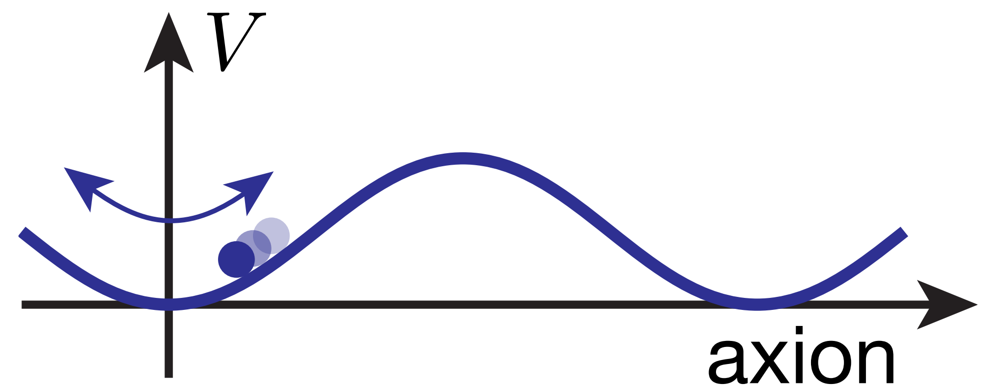
Allowed region is broadened for $r_s \sim 1$

CDM isocurvature perturbation in curvaton scenario

# II. Axion dark matter

# — Axion dark matter —




**axion mass :** $m_a(T) \simeq \begin{cases} m_a (T_{\rm cr}/T)^{\beta} & \text{for} \quad T > T_{\rm cr} \\ m_a & \text{for} \quad T < T_{\rm cr} \end{cases}$

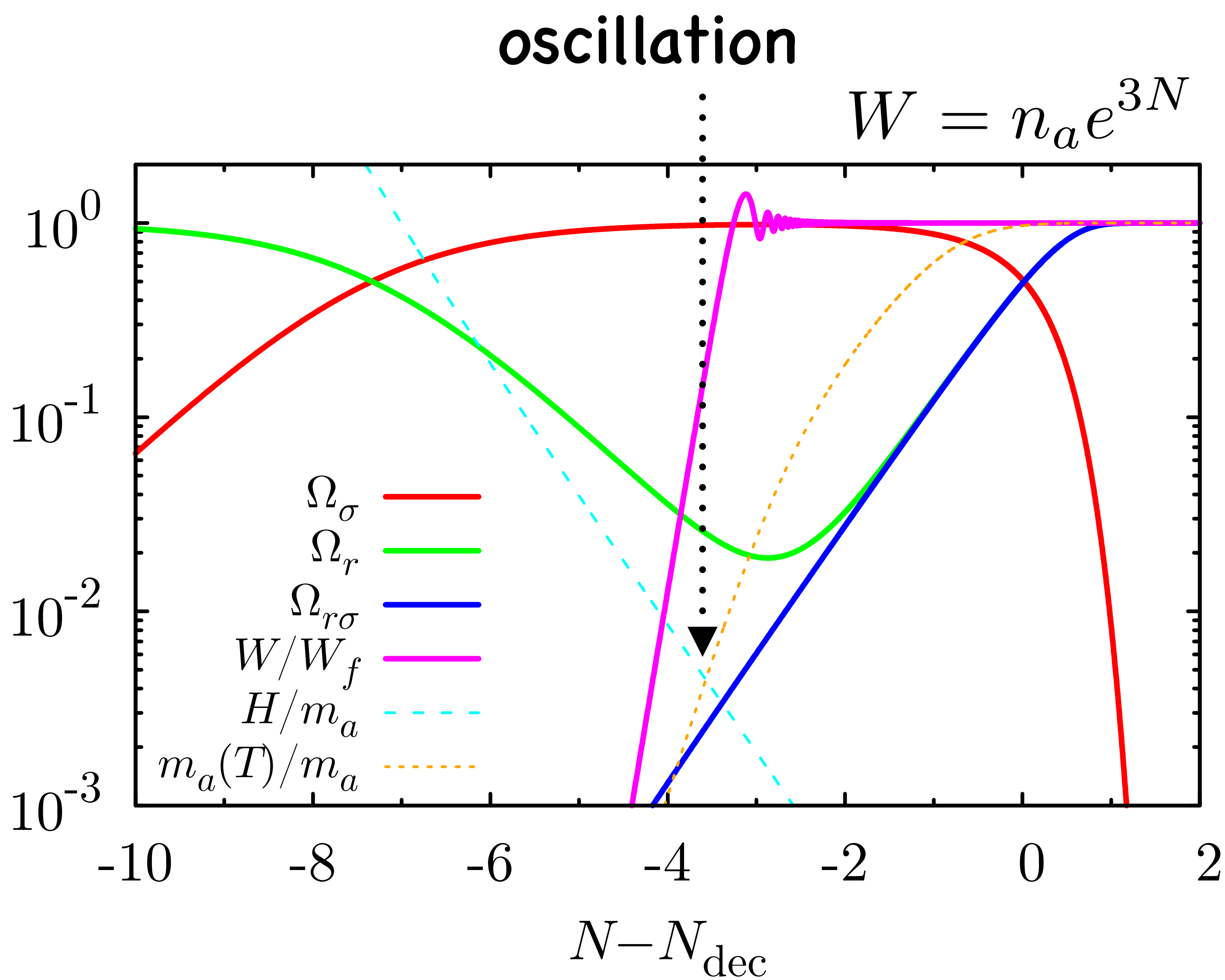
**QCD axion :**

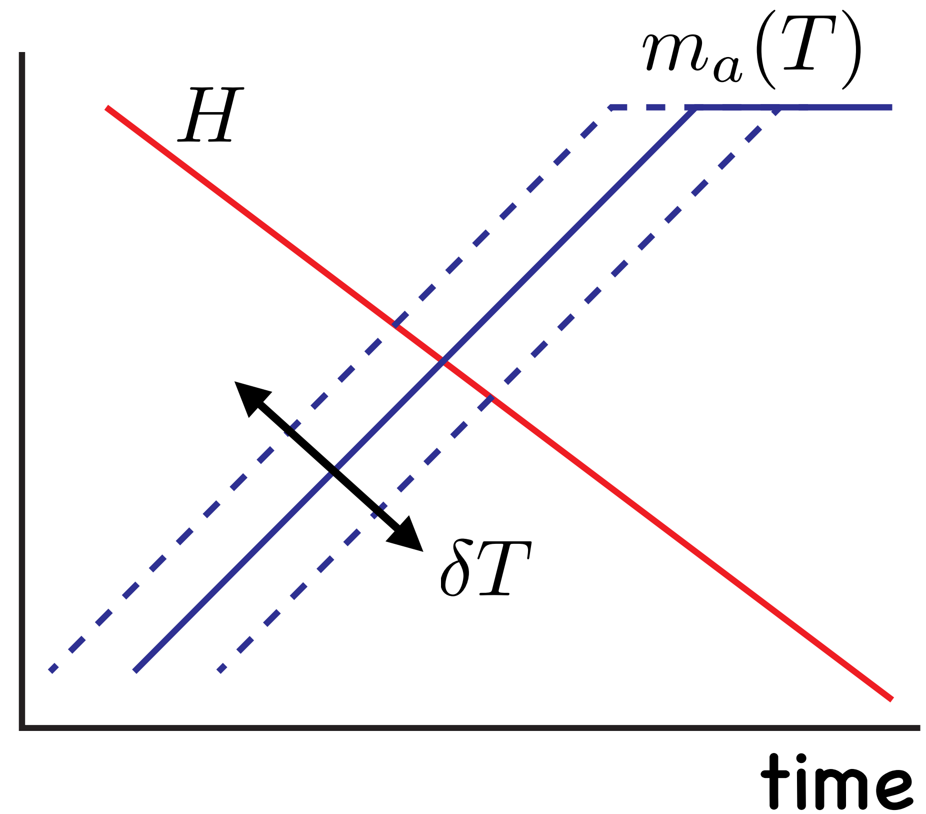
$$m_a(T) \simeq \begin{cases} 4.05 \times 10^{-4} \dfrac{\Lambda_{\rm QCD}^2}{F_a} \left( \dfrac{T}{\Lambda_{\rm QCD}} \right)^{-3.34} & T > 0.26 \Lambda_{\rm QCD} \\ 3.82 \times 10^{-2} \dfrac{\Lambda_{\rm QCD}^2}{F_a} & T < 0.26 \Lambda_{\rm QCD} \end{cases}$$

$$F_a = 10^9 - 10^{12} \ \text{GeV}, \ \Lambda_{\rm QCD} \simeq 400 \ \text{MeV}$$

# "Sudden" beginning of oscillation at $H = m_a(T_{\rm osc})$

$$\left.\frac{n_a}{s}\right|_{t>t_{\rm osc}} = \frac{n_a(T_{\rm osc})}{s(T_{\rm osc})}$$




$$\delta n_a \sim \delta T$$

## case 1 — Axion oscillation **after** curvaton decay

$$H = m_a(T_{\rm osc}) \;\Rightarrow\; 3M_P^2 m_a^2(T_{\rm osc}) = \rho_r(N_{\rm osc})$$ - single component

$$\zeta_{\rm CDM} = \zeta \;\Rightarrow\; S_{\rm CDM} = 0$$

## case 2 — Axion oscillation **before** curvaton decay

$$\rho_r + \rho_\sigma = 3M_P^2 m_a^2(T_{\rm osc})$$

$$S_{\rm CDM}/\zeta = 3\left(\frac{\Omega_{\sigma,{\rm osc}}}{r_s}\frac{3+\beta}{4+2\beta-\Omega_{\sigma,{\rm osc}}} - 1\right)$$

(consistent with Lyth, Wands astro-ph/0306500)

# Numerical calculations

$$H^2 = \frac{1}{3M_P^2}(\rho_r + \rho_\sigma + \rho_{\rm CDM}) \simeq \frac{1}{3M_P^2}(\rho_r + \rho_\sigma)$$

$$\dot{\rho}_r + 4H\rho_r = \Gamma\rho_\sigma$$

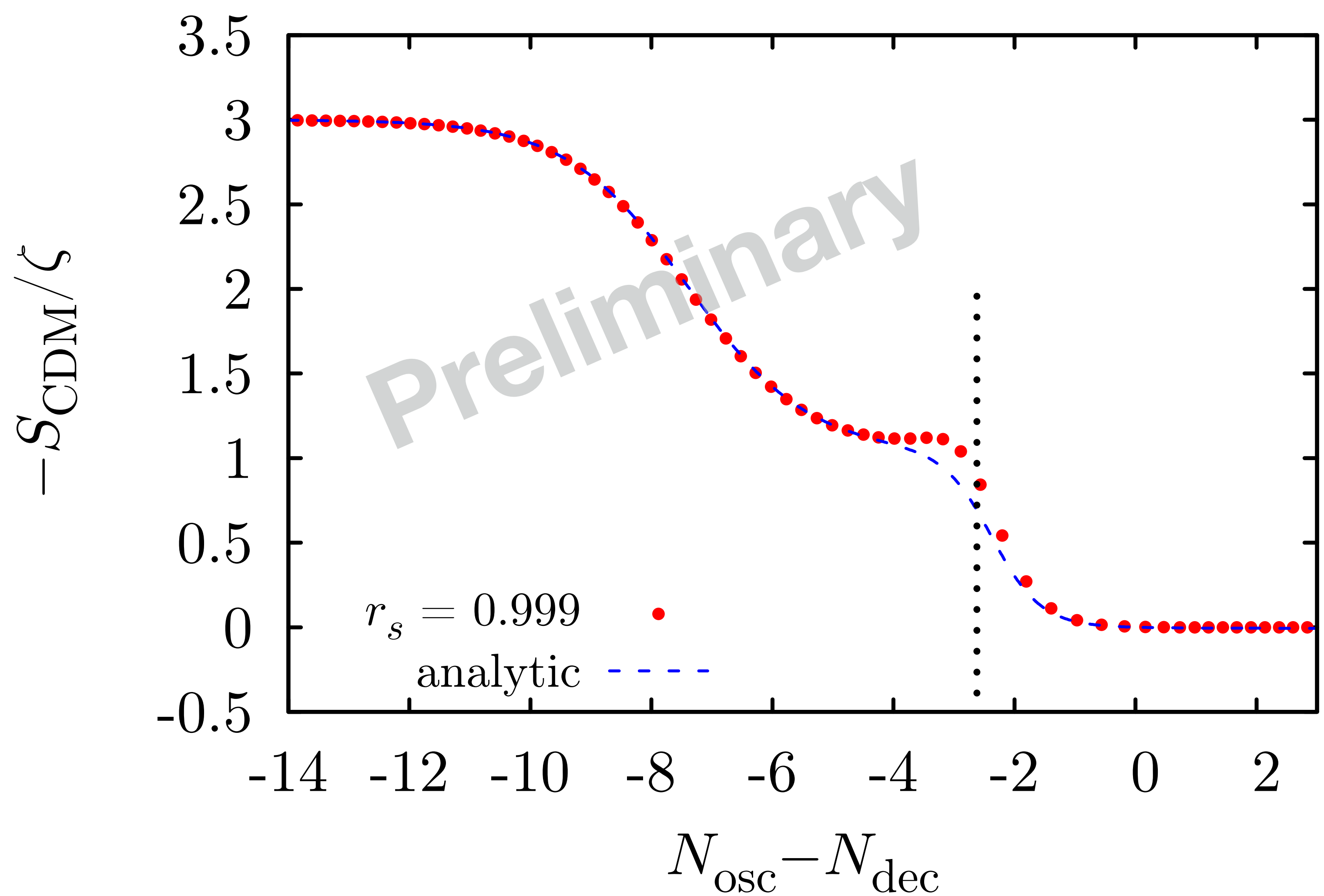
$$\dot{\rho}_\sigma + 3H\rho_\sigma = -\Gamma\rho_\sigma$$

$$\ddot{a} + 3H\dot{a} + m_a^2(T)a = 0$$ **(axion)**

$$N = \int H dt, \quad \zeta_i = \delta N + \frac{1}{3(1+w_i)} \ln\left(\frac{\rho_i}{\bar{\rho}_i}\right)$$

# analytic & numerical results

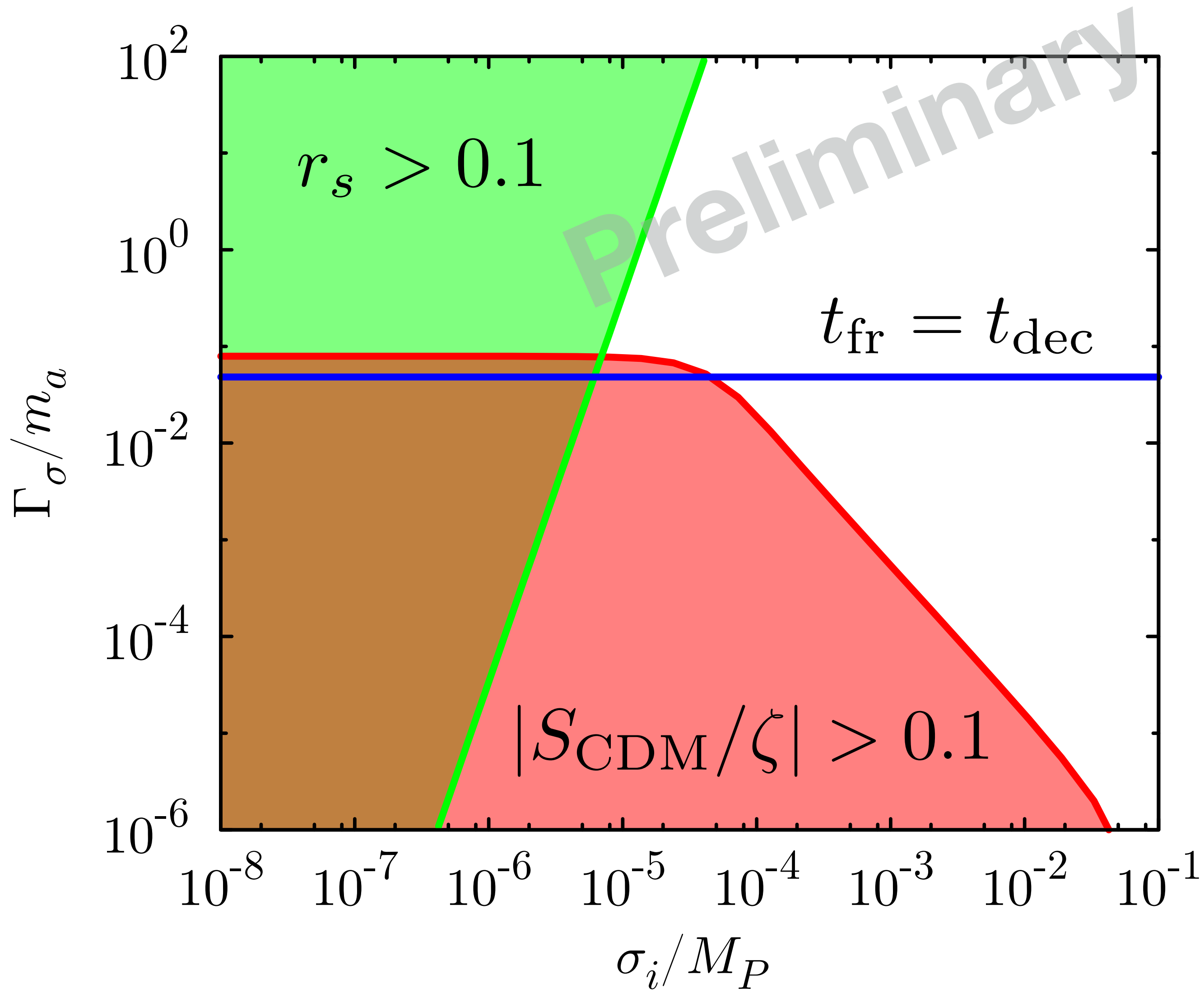
NK, Langlois, Takahashi, Yokoyama in prep




Isocurvature vanishes if axion starts to oscillate after dilute gas exceeds primordial radiation

# Allowed parameter region

NK, Langlois, Takahashi, Yokoyama in prep




**Allowed region is broadened for $r_s \sim 1$**

# — Summary —

We revisited isocurvature perturbations in curvaton model. "Residual" CDM isocurvature perturbations can be produced in this model In previous thought, it becomes large ($S/\zeta$ = -3 — -3/2) and it is already ruled out by observations unless CDM produced after curvaton decay.

Detailed analysis beyond sudden decay approximation (including dilution plasma from gradual decay of the curvaton) shows that CDM isocurvature can be suppressed even if curvaton decays after CDM production (freeze-out of WIMP/ beginning of axion oscillation).

constraint from isocurvature is relaxed

good news or bad news?