The 3rd Korea-Japan Workshop on Dark Energy

Redshift Space Distortion as a Standard Ruler

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OUTLINE

- Power spectrum in redshift space
  - Fingers of God and Kaiser effect (Redshift space distortion)
- Monopole and Quadrupole in linear theory
- Redshift space perturbed Power spectrum
- Monopole and Quadrupole in quasi-linear theory
- Dark energy dependence on quadrupole

Power Spectrum in Redshift Space

- Kaiser effect (RSD) : due to coherent peculiar velocities towards the central mass (large scale effect) (*refer Matsubara's talk*)
- Fingers of God effect : random velocity dispersions in galaxy clusters (affects only redshift not position, stretching only radially, small scale effect)
- Redshift space linear power spectrum

Real space matter power spectrum (what we calculate)

$$\mathrm{P}_{\mathrm{s}}^{(\mathrm{L})}(k,\mu,z) = \mathrm{D}_{\mathrm{FoG}}^{2}(k,\mu,z)\, b(k,z)^2 (1+\beta\mu^2)^2 \mathrm{P}^{(\mathrm{L})}(k,\mu,z)$$

where $\beta(k,z) = \frac{f(z)}{b(k,z)}$, $f(z) = \frac{d\ln \mathrm{D}_1}{d\ln a}$, $\delta^{(1)}(k,z) = \mathrm{D}_1(z)\delta_1(k)$

RSD and Fingers of God

- Redshift space distortion (RSD) is measured using (*refer Okumura's talk*)

$$\text{where } \beta(k,z) = \frac{f(z)}{b(k,z)},\ f(z) = \frac{d\ln \mathrm{D}_1}{d\ln a},\ \delta^{(1)}(k,z) = \mathrm{D}_1(z)\delta_1(k)$$

- Lorentzian and Gaussian forms

$$\mathrm{D}_{\mathrm{FoG}}^{\mathrm{Lor}}(k,\mu,z,\sigma) = \frac{1}{1+0.5[k\mu\sigma(z)D(z)f(z)]^2}$$

$$\mathrm{D}_{\mathrm{FoG}}^{\mathrm{Gau}}(k,\mu,z,\sigma) = \exp\left[-0.5[k\mu\sigma(z)D(z)f(z)]^2\right]$$

- velocity dispersion

$$\sigma^2(z) = \frac{4\pi}{3}\int \frac{dq}{(2\pi)^3}\mathrm{P}_{\theta\theta}(q,z)$$

Monopole and Quadrupole in Linear Theory

- Decompose to angular dependence (multipoles)

$$P(k,\mu,z) = \sum_{l=0,2,4,\cdots} P_l(k,z)\mathcal{L}_l(\mu)$$

$$P_l(k,z) = \frac{2l+1}{2}\int_{-1}^{1} d\mu P(k)\mathcal{L}_l(\mu)$$

- Monopole : angular averaged redshift space power spectrum

$$P_0(k,z) = (1+\frac{2}{3}f+\frac{1}{5}f^2)P_L(k,z)$$

Positive

- Quadrupole : leading anisotropic contribution of redshift space power spectrum

$$P_2(k,z) = (\frac{4}{3}f+\frac{4}{7}f^2)P_L(k,z)$$

Positive

Perturbed Redshift Space Power Spectrum

- Perturbed redshift space power spectrum can be calculated from SPT / LPT / iPT (*refer Matsubara's talk and Gong's talk for GR*)

$$\begin{aligned} P_s^{(\mathrm{Scoccimarro})}(k,\mu,z) &= \Big(b^2(k,z)P_{\delta\delta}(k,z) + 2b(k,z)f(z)\mu^2 P_{\delta\theta}(k,z) + f(z)^2\mu^4 P_{\theta\theta}(k,z)\Big) \\ &= b^2(k)\Big(P_{\delta\delta}(k) + 2\beta(k,z)\mu^2 P_{\delta\theta}(k) + \beta^2(k,z)\mu^4 P_{\theta\theta}\Big) \qquad (\end{aligned}$$

$$\begin{aligned} P_s^{(\mathrm{Matsubara})}(k,\mu,z) &= P_{s,11}(k,\mu,z) + P_{s,22}(k,\mu,z) + 2P_{s,13}(k,\mu,z) \\ &= \Big[K_1^{(s)}(k,\mu,z)\Big]^2 P_L(k,z) + 2\int \frac{d^3q}{(2\pi)^3} P_L(q,z) P_L(|k-q|,z)\Big[K_2^{(s)}(\vec{q},\vec{k}-\vec{q},\mu,z)\Big]^2 \\ &\quad +6\Big[K_1^{(s)}(k)\Big] P_L(k,z)\int \frac{d^3q}{(2\pi)^3} P_L(q,z)\Big[K_3^{(s)}(\vec{q},-\vec{q},\vec{k},\mu,z)\Big] \qquad (2) \end{aligned}$$

$$P_s^{(\mathrm{TNS})}(k,\mu,z) = P_s^{(\mathrm{Scoccimarro})}(k,\mu,z) + A(k,\mu,b,z) + B(k,\mu,b,z)$$

Taruya, Nichimichi, Saito

MONOPOLE AND QUADRUPOLE IN QUASI-LINEAR THEORY

- Monopole

$$\begin{aligned}
P_0(k,z) &= (\mathrm{P}_L + \mathrm{A}_{00}\mathrm{P}_{22} + \mathrm{B}_{00}\mathrm{P}_{13}) + \left(\frac{2}{3}\mathrm{P}_L + \frac{1}{3}\mathrm{A}_{11}\mathrm{P}_{22} + \frac{1}{3}(\mathrm{B}_{00} + \mathrm{B}_{11})\mathrm{P}_{13}\right) f \\
&+ \left(\frac{1}{5}\mathrm{P}_L + \left(\frac{1}{3}\mathrm{A}_{12} + \frac{1}{5}\mathrm{A}_{22}\right)\mathrm{P}_{22} + \left(\frac{1}{3}\mathrm{B}_{12} + \frac{1}{5}(\mathrm{B}_{11} + \mathrm{B}_{22})\right)\mathrm{P}_{13}\right) f^2 \\
&+ \left(\left(\frac{1}{5}\mathrm{A}_{23} + \frac{1}{7}\mathrm{A}_{33}\right)\mathrm{P}_{22} + \left(\frac{1}{7}\mathrm{B}_{22} + \frac{1}{5}(\mathrm{B}_{12} + \mathrm{B}_{23})\right)\mathrm{P}_{13}\right) f^3 \\
&+ \left(\left(\frac{1}{5}\mathrm{A}_{24} + \frac{1}{7}\mathrm{A}_{34} + \frac{1}{9}\mathrm{A}_{44}\right)\mathrm{P}_{22} + \frac{1}{7}\mathrm{B}_{23}\mathrm{P}_{13}\right) f^4
\end{aligned}$$

Positive

Monopole and Quadrupole in Quasi-linear Theory

- Quadrupole

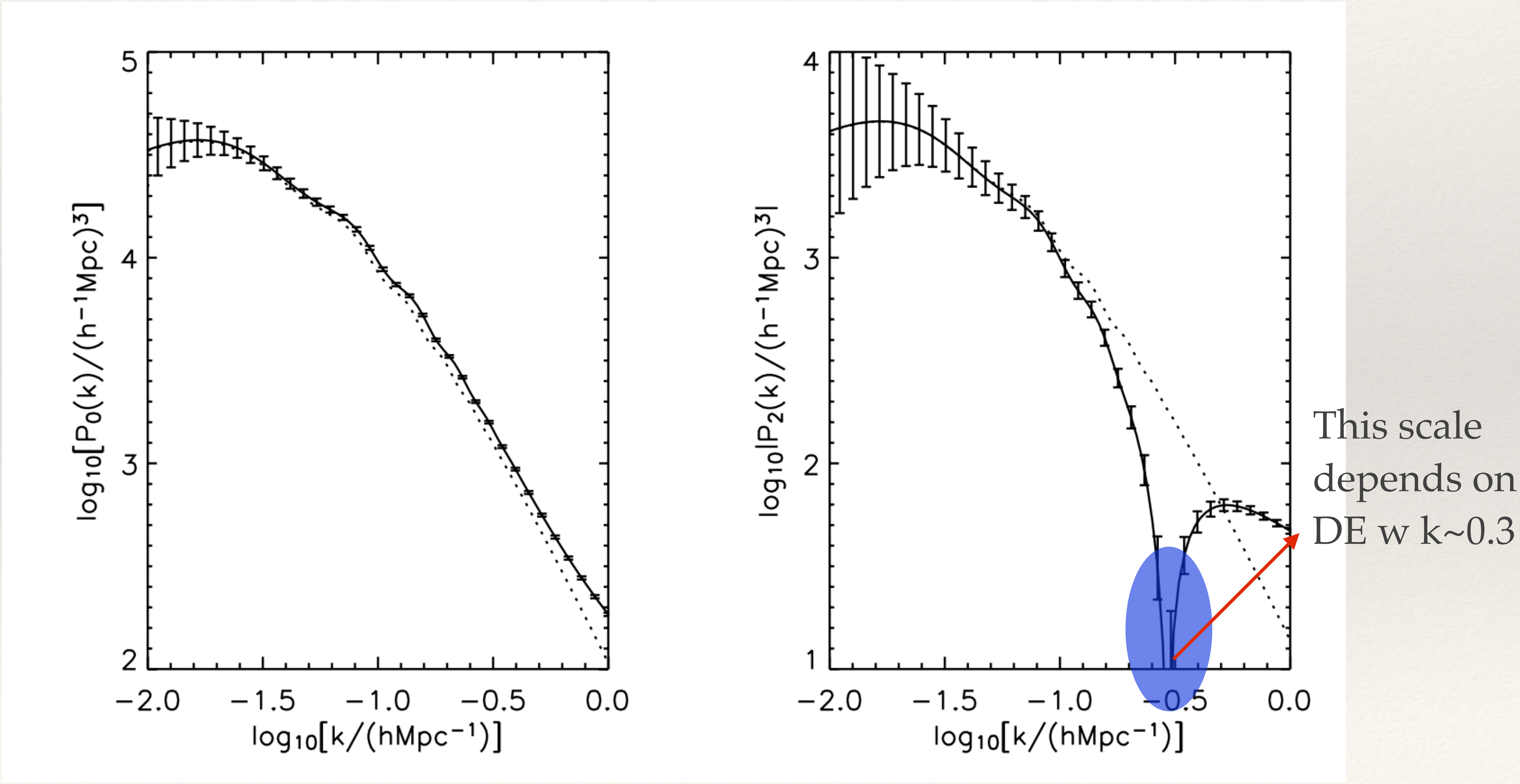
$$
\begin{aligned}
P_2(k,z) \;=\; & \left(\frac{4}{3}\mathrm{P}_L + \frac{2}{3}\mathrm{A}_{11}\mathrm{P}_{22} + \frac{2}{3}(\mathrm{B}_{00}+\mathrm{B}_{11})\mathrm{P}_{13}\right) f \\
& + \left(\frac{4}{7}\mathrm{P}_L + \left(\frac{2}{3}\mathrm{A}_{12} + \frac{4}{7}\mathrm{A}_{22}\right)\mathrm{P}_{22} + \left(\frac{4}{7}\mathrm{B}_{11} + \frac{2}{3}\mathrm{B}_{12} + \frac{4}{7}\mathrm{B}_{22}\right)\mathrm{P}_{13}\right) f^2 \\
& + \left(\left(\frac{4}{7}\mathrm{A}_{23} + \frac{10}{21}\mathrm{A}_{33}\right)\mathrm{P}_{22} + \left(\frac{4}{7}\mathrm{B}_{12} + \frac{10}{21}\mathrm{B}_{22} + \frac{4}{7}\mathrm{B}_{23}\right)\mathrm{P}_{13}\right) f^3 \\
& + \left(\left(\frac{4}{7}\mathrm{A}_{24} + \frac{10}{21}\mathrm{A}_{34} + \frac{40}{99}\mathrm{A}_{44}\right)\mathrm{P}_{22} + \frac{10}{21}\mathrm{B}_{23}\mathrm{P}_{13}\right) f^4
\end{aligned}
$$

This term change signs at the specific scale k for the different DE models

Positive ⟶ Negative as k ↑

Monopole and Quadrupole in Quasi-linear Theory

- Monopole and quadrupole *Yamamoto, Bassett, Nishioka 05*



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Dark Energy Dependence on Quadrupole

$$
\begin{aligned}
P_s^{(\mathrm{Matsubara})}(k,\mu,z) &= P_{s,11}(k,\mu,z) + P_{s,22}(k,\mu,z) + 2P_{s,13}(k,\mu,z) \\
&= \left[K_1^{(s)}(k,\mu,z)\right]^2 P_L(k,z) + 2\int \frac{d^3q}{(2\pi)^3} P_L(q,z) P_L(|k-q|,z) \left[K_2^{(s)}(\vec{q},\vec{k}-\vec{q},\mu,z)\right]^2 \\
&\quad +6\left[K_1^{(s)}(k)\right] P_L(k,z) \int \frac{d^3q}{(2\pi)^3} P_L(q,z) \left[K_3^{(s)}(\vec{q},-\vec{q},\vec{k},\mu,z)\right] \qquad (2)
\end{aligned}
$$

- P_{11} : $K_1^{(s)}(k,\mu,z) = 1 + \frac{f(w,z)}{b}\mu^2$ $\quad P_L(k,z) = D_1(w,z)^2 P_L(k,w)$

- P_{22} : $\left(K_2^{(s)}(\vec{q},\vec{k}-\vec{q})\right)^2 = \left(F_2 + f\mu^2 G_2 + \frac{fk\mu}{2}\left(\frac{q_z}{q^2} + \frac{k_z - q_z}{|\vec{k}-\vec{q}|^2}\right) + \frac{(fk\mu)^2}{2}\left(\frac{q_z(k_z-q_z)}{q^2|\vec{k}-\vec{q}|^2}\right)\right)^2$

$$
F_2(a,\vec{q},\vec{k}-\vec{q}) = \frac{2c_{22}(r-x) + c_{21}\left(r+x-2rx^2\right)}{2r\left(1+r^2-2rx\right)}
$$

$$
G_2(a,\vec{q},\vec{k}-\vec{q}) = -\frac{2c_{\theta 22}(r-x) + c_{\theta 21}\left(r+x-2rx^2\right)}{2r\left(1+r^2-2rx\right)}
$$

Ex : $\frac{A_{00}}{(1+r^2-2rx)^2} = 2r^2F_2^2 = \frac{(7x+r(3-10x^2))^2}{98(1+r^2-2rx)^2}$

DE dependence on $c_{21}(z,w)$, $c_{22}(z,w)$, etc
Usually, one considers w dependence on D_1 only

Improvement by Including T-dependence on F & G

- Comparison on the matter power spectrums with and without EdS assumption in the kernels (*SL, Park, Biern 14*)

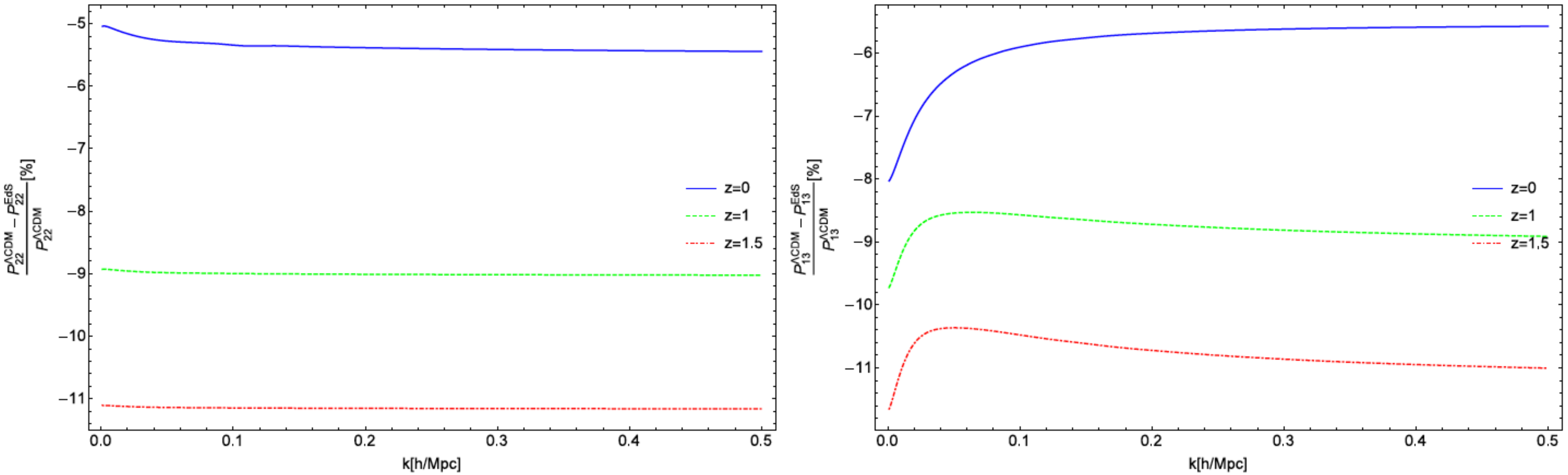


FIG. 2: Errors in P_{22} and P_{13} a) Differences between the correct P_{22} and the one with EdS assumption at the different epoches. The solid, dashed, and dotdashed lines correspond to $z = 0$, 1.0, and 1.5, respectively. a) Differences between the correct P_{13} and the EdS assumed P_{13} at different epoches.

Improvement by Including T-dependence on F & G

- Comparison on the matter power spectrums with and without EdS assumption in the kernels

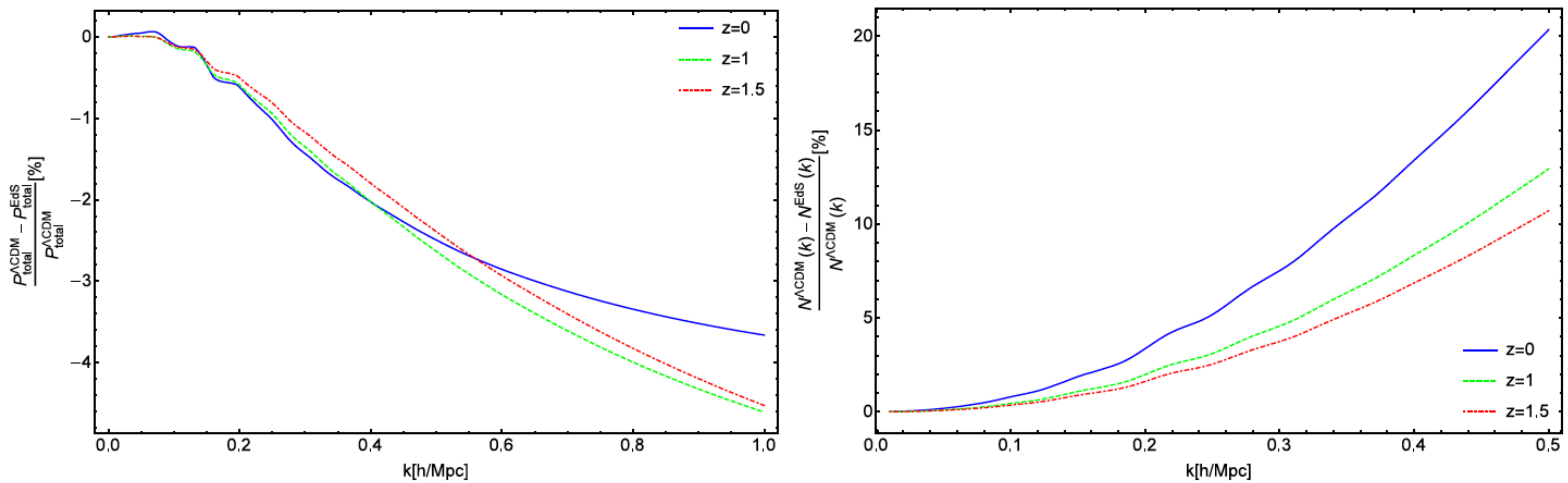
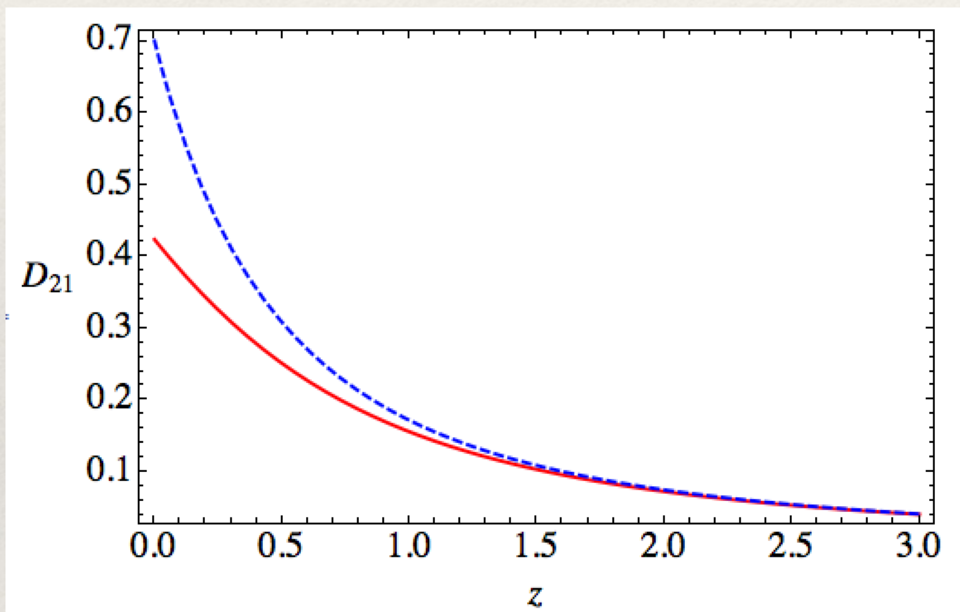
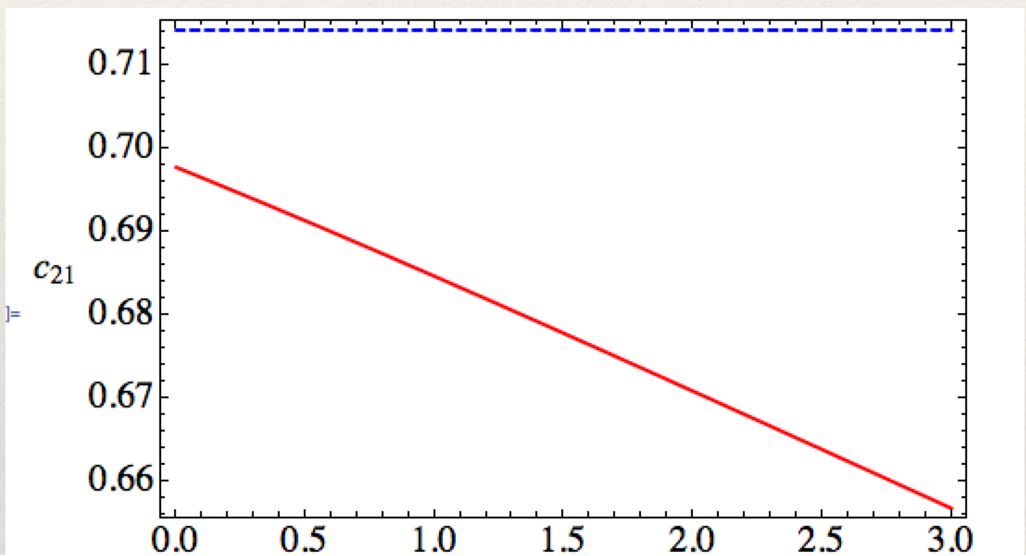


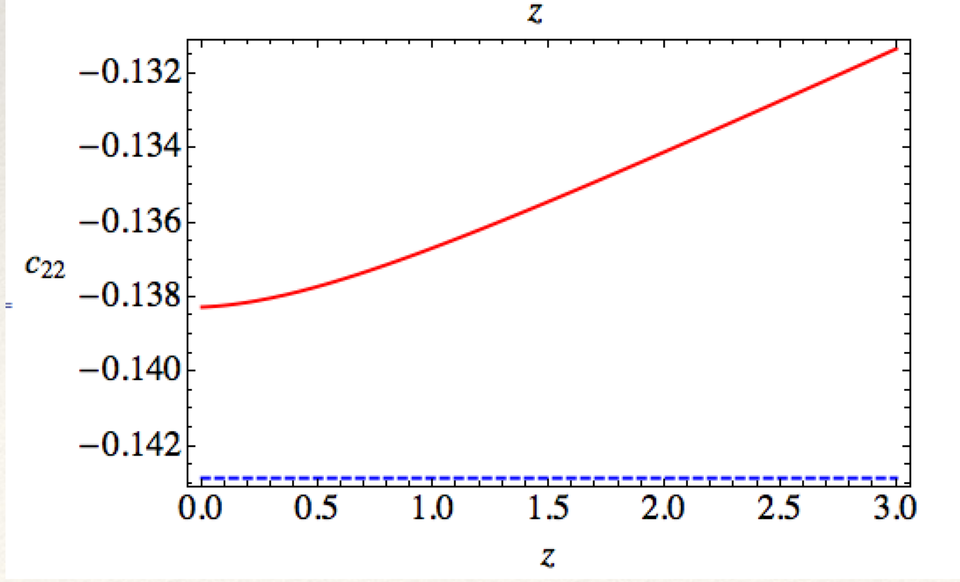
FIG. 3: Errors in $P_{\rm total}$ and N a) Differences between the correct $P_{\rm total}$ and the one with $\lambda = 1$ (EdS) assumption at the different epoches. The solid, dashed, and dotdashed lines correspond to $z = 0$, 0.5, and 1, respectively. a) Differences between the correct P_{NL} and the $\lambda = 1$ (EdS) assumed P_{NL} at different epoches.

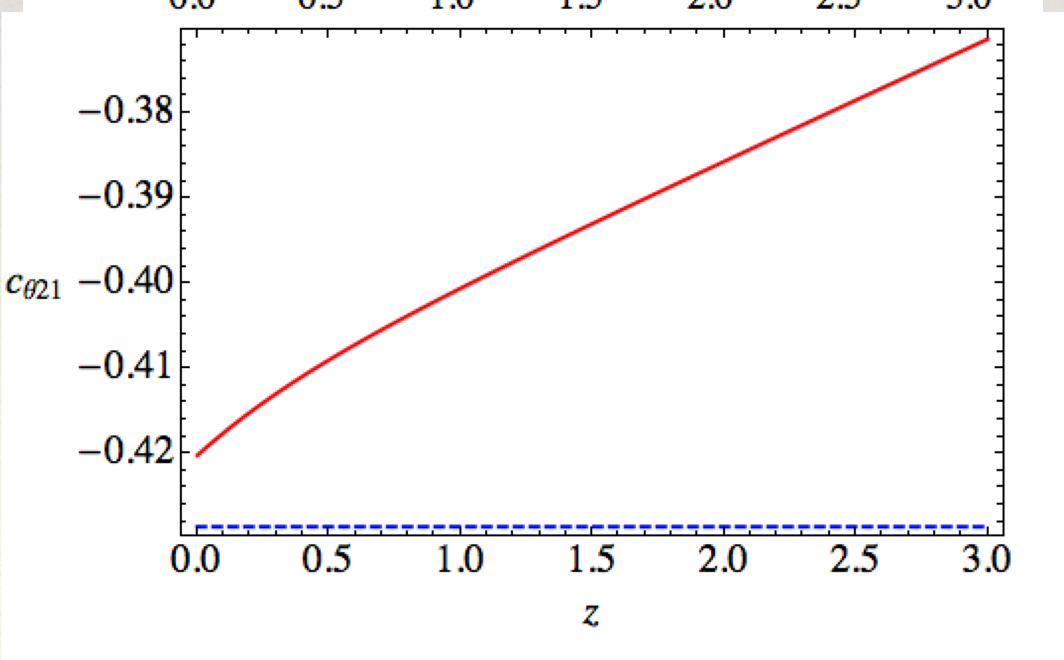
Improvement by Including T-dependence on F & G

- As z increases, D approaches to that of EdS. But F & G deviate more from those of EdS.









Continuation

- How sensitive the flipping scale on DE e.o.s
- Comparison with other sensitivities (Ω_m, b, etc)
- RS effect has similar properties but with very low S/N (*SL, 15*)

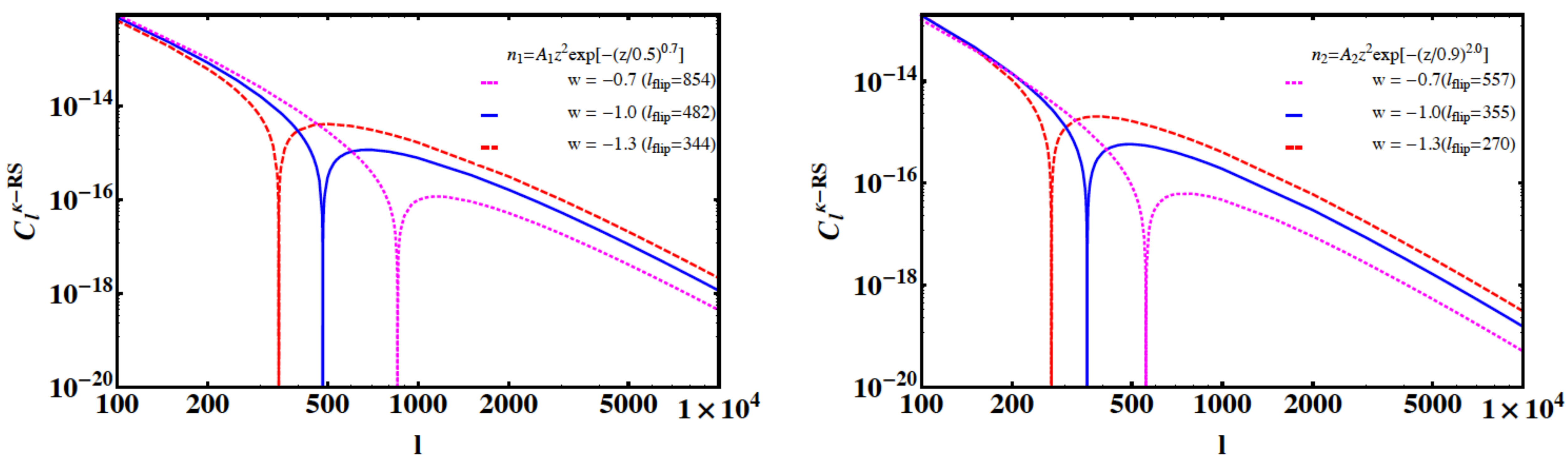


FIG. 1: CMB-WL cross-correlation spectra, $C_l^{RS-\kappa}$ of different dark energy models for different WL surveys ($n_{1(2)}$).

THANKS !

ありがとうございます!

감사합니다!