The Use of Disformal Transformation

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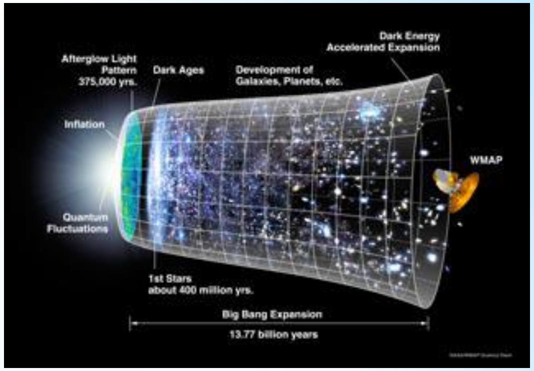
Big mystery in cosmology

Acceleration of cosmic expansion



- Inflation: early stage of the Universe

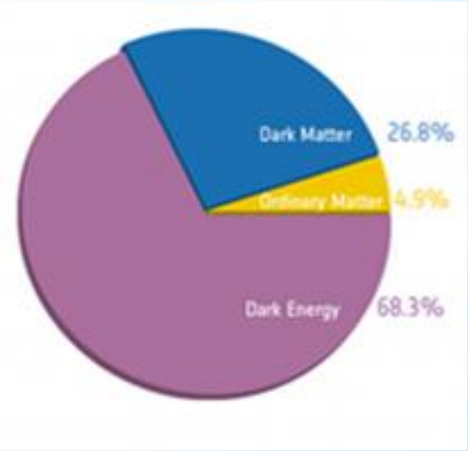
  What is an inflaton ?

- Present Acceleration

  cosmological constant

  $$\Lambda \sim 10^{-120} m_{PL}^2$$

  →
  - Dark Energy
  - Modified gravity



google scholar

"modified gravity" & "accelerating universe"

-1995	5
1996-2000	11
2001-2005	104
2006-2010	479
2011-2015	1020

Brans-Dicke theory
Scalar-tensor theory
Einstein-aether theory
TeVeS theory
f(R) gravity
Galileon, Horndeski theory
massive gravity, bigravity

General Relativity

How to analyze them

- Fundamental approach

  Unified theory/quantum correction/natural extension

  KKLT
  massive gravity

- Phenomenological approach

  Given back ground + possible extension

  PPN approach
  effective theory

- General relativistic approach

  description in the Einstein frame

→ ***present talk***

Toward the EH action

a gravitational theory with the action

$$S = \int d^D x \sqrt{-g} F(g^{\mu\nu}, R_{\mu\nu}, C^{\mu}{}_{\nu\rho,\sigma}, \psi)$$

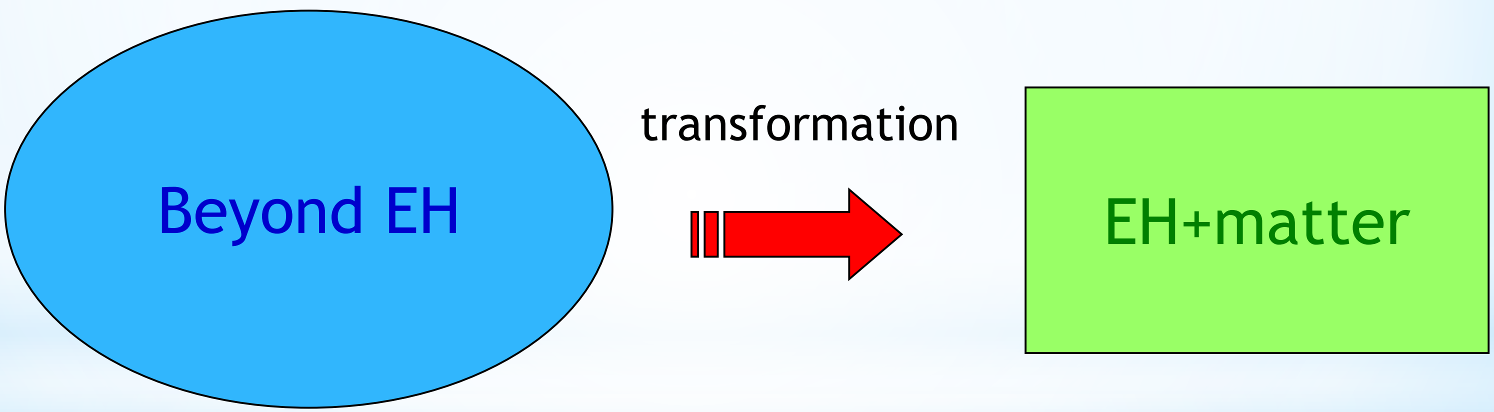
$F(\cdots)$ An arbitrary function

ψ matter fields including scalar fields

It may show some interesting properties of gravity
But it is too complicated to analyze it

Toward the EH action :

If we can find an equivalent gravitational theory only with the EH action by some transformation, it makes our discussion simpler.



Basic equations are very complicated

well known

1. A scalar-tensor type theory

KM (1989)

$$S = \int d^D x \sqrt{-g} \left[f(\phi) R - \frac{\epsilon_\phi}{2} (\nabla \phi)^2 - V(\phi) \right]$$

$$\hat{g}_{\mu\nu} = e^{2\omega} g_{\mu\nu}$$

a conformal transformation

$$\omega = \frac{1}{D-2} \ln(2\kappa^2 |f(\phi)|)$$

$$S = \int d^D x \sqrt{-\hat{g}} \left[\frac{1}{2\kappa^2} R(\hat{g}) - \frac{1}{2} (\nabla \sigma)^2 - U(\sigma) \right]$$

$$\kappa\sigma = \int d\phi \left[\frac{\epsilon_\phi (D-2) f(\phi) + 2(D-1)(f'(\phi))^2}{2(D-2) f^2(\phi)} \right]^{1/2}$$

$$U(\sigma) = \epsilon_f [2\kappa^2 |f(\phi)|]^{-D/(D-2)} V(\phi)$$

Higgs inflation

Bezrukov, Shaposhnikov (2008)

Spokoiny (1984); Salopek, Bond,B ardeen (1989); Futamase, KM (1989); Fakir, Unruh (1990)

Higgs field: +non-minimal coupling $(\xi < 0)$

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} R - \frac{1}{2} \xi \phi^2 R - \frac{1}{2} (\nabla \phi)^2 - V(\phi) \right]$$

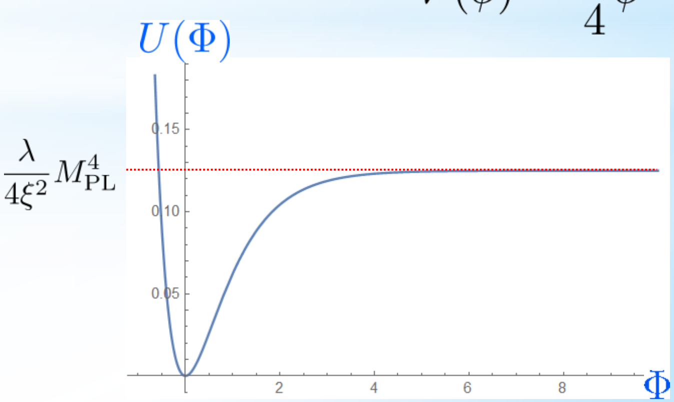
conformal transformation $\tilde{g}_{\mu\nu} = (1 - \xi\kappa^2\phi^2) g_{\mu\nu}$

$$S = \int d^4x \sqrt{-\tilde{g}} \left[\frac{1}{2\kappa^2} \tilde{R} - \frac{1}{2} (\tilde{\nabla} \Phi)^2 - U(\Phi) \right]$$

$$V(\phi) = \frac{\lambda}{4} \phi^4$$

$$\frac{d\Phi}{d\phi} = \frac{1}{\sqrt{(1 - \xi\kappa^2\phi^2)}}$$

$$U(\Phi) = \frac{1}{(1 - \xi\kappa^2\phi^2)^2} V(\phi)$$



2. F(R , ϕ) theory

$$S=\int d^Dx\sqrt{-g}\left[F(R,\phi)-\frac{\epsilon_\phi}{2}(\nabla\phi)^2\right]$$

KM (1989)

higher derivarives

Jakubiec, Kijowski (1987);
Magnano, Ferraris, Francaviglia, (1987);
Ferraris, Francaviglia, Magnano, (1988)

$$\hat{g}_{\mu\nu}=e^{2\omega}g_{\mu\nu}$$ a conformal transformation

$$\omega=\frac{1}{D-2}\ln\left[2\kappa^2\left|\frac{\partial F}{\partial R}\right|\right] \qquad \kappa\sigma=\sqrt{\frac{D-1}{D-2}}\ln\left[2\kappa^2\left|\frac{\partial F}{\partial R}\right|\right]$$

"new degree of freedom"

$$S=\int d^Dx\sqrt{-\hat{g}}\left[\frac{1}{2\kappa^2}R(\hat{g})-\frac{1}{2}(\hat{\nabla}\sigma)^2\right.$$
$$\left.-\frac{\epsilon_\phi\epsilon_F}{2}e^{-\sqrt{\frac{D-1}{D-2}}\kappa\sigma}(\hat{\nabla}\phi)^2-U(\phi,\sigma)\right]$$

$$U(\phi,\sigma)=\epsilon_F\left[2\kappa^2\left|\frac{\partial F}{\partial R}\right|\right]^{-D/(D-2)}\left(R\frac{\partial F}{\partial R}-F(R)\right)$$

A simple example

KM (1988)

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\left[R + \alpha R^2\right]$$

: Starobinski inflation

It contains higher derivatives

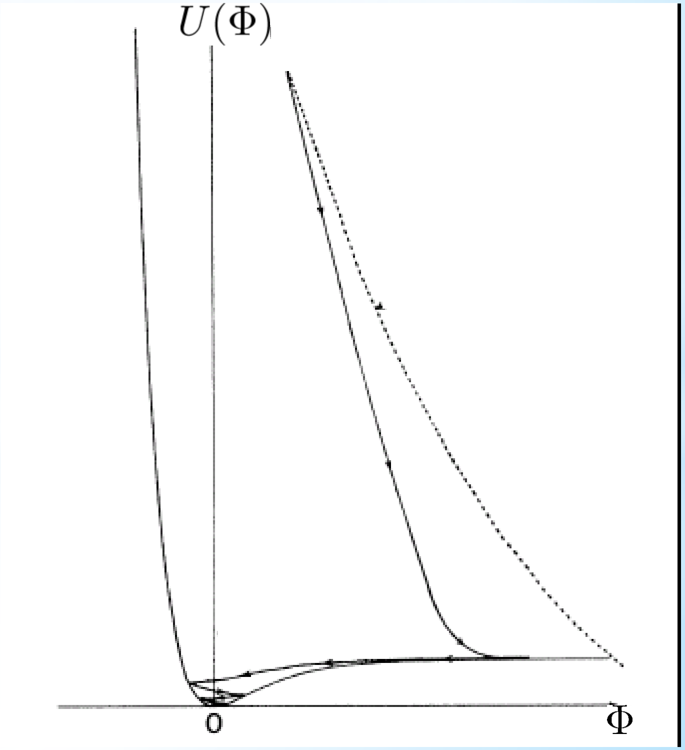
conformal transformation $\tilde{g}_{\mu\nu} = (1 + 2\alpha R) g_{\mu\nu}$

$$S = \int d^4x\sqrt{-\tilde{g}}\left[\frac{1}{2\kappa^2}\tilde{R} - \frac{1}{2}(\tilde{\nabla}\Phi)^2 - U(\Phi)\right]$$

GR + a scalar field with a potential U(Φ)

$$\kappa\Phi = \sqrt{\frac{3}{2}}\ln\left[1 + 2\alpha R\right]$$

$$U(\Phi) = \frac{1}{8\alpha}\left(1 - e^{-\sqrt{\frac{3}{2}}\kappa\Phi}\right)^2$$



It is easy to judge
whether inflation occurs or not

3. F($R_{\mu\nu}$) theory

Jakubiec, Kijowski , GRG 19 (1987) 719 ;
Magnano, Ferraris, Francaviglia, GRG 19 (1987) 465 ;
Ferraris, Francaviglia, Magnano, CQG. 5 (1988) L95

$$S = \int d^4x \sqrt{-g} F(g^{\mu\nu}, R_{\mu\nu})$$

$$\sqrt{-q} q^{\mu\nu} = 2\kappa^2 \sqrt{-g} \frac{\partial F}{\partial R_{\mu\nu}}$$

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-q} \Big[R(q, \partial q, \partial q^2) + q^{\mu\nu} (C^\rho{}_{\rho\sigma} C^\sigma{}_{\mu\nu} - C^\rho{}_{\sigma\mu} C^\sigma{}_{\rho\nu})$$

$$- q^{\mu\nu} \mathcal{R}_{\mu\nu} + \frac{\sqrt{-g}}{\sqrt{-q}} F(\mathcal{R}_{\mu\nu}(g,q), g^{\alpha\beta}) \Big] + S_{\text{matter}}(g^{\alpha\beta}, \psi)$$

$$C^\rho{}_{\mu\nu} = \frac{1}{2} g^{\rho\sigma} \left(\nabla^{(q)}_\mu g_{\nu\sigma} + \nabla^{(q)}_\nu g_{\mu\sigma} - \nabla^{(q)}_\sigma g_{\mu\nu} \right)$$

$$R_{\mu\nu} = \mathcal{R}_{\mu\nu}(g^{\alpha\beta}, q^{\gamma\delta})$$

The EH gravitational action + spin 2 field ($g^{\mu\nu}$) + other matter fields

new Higgs inflation   Germani, Kehagias (2010)

Higgs field: + derivative coupling

$$S=\int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}\left(R+\alpha G^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi\right)-\frac{1}{2}(\nabla\phi)^2-V(\phi)\right]$$

The EH gravitational action ($q^{\mu\nu}$) + spin 2 field ($g^{\mu\nu}$) + other matter fields

Behavior ?

The previous method may not work

Instead, we may use a disformal transformation

$$\tilde{g}_{\mu\nu}=\Omega^2\left(g_{\mu\nu}+u_\mu u_\nu\right)$$

u_μ: a timelike vector

$$u_\mu u^\mu=-\lambda^2$$

disformal transformation

$$\tilde{g}_{\mu\nu} = \Omega^2 \left(g_{\mu\nu} + u_\mu u_\nu\right) \qquad u_\mu u^\mu = -\lambda^2$$

$$\sqrt{-\tilde{g}} = \Omega^4 (1-\lambda^2)^{\frac{1}{2}} \sqrt{-g}$$

$$\tilde{g}^{\mu\nu} = \Omega^{-2} \left(g^{\mu\nu} - \frac{1}{1-\lambda^2} u^\mu u^\nu \right)$$

$$\tilde{\Gamma}^\mu_{\nu\rho} = \Gamma^\mu_{\nu\rho} + \gamma^\mu_{\nu\rho} \qquad \gamma^\mu_{\nu\rho} = f^\mu_{\nu\rho} + \omega^\mu_{\nu\rho}$$

$$f^\mu{}_{\rho\sigma} = \frac{1}{2} \left(g^{\mu\nu} - \frac{1}{1-\lambda^2} u^\mu u^\nu \right) \left[\nabla_\rho (u_\nu u_\sigma) + \nabla_\sigma (u_\nu u_\rho) - \nabla_\nu (u_\rho u_\sigma) \right]$$

$$\omega^\mu{}_{\rho\sigma} = \delta^\mu_\rho \nabla_\sigma \ln\Omega + \delta^\mu_\sigma \nabla_\rho \ln\Omega - \left(g^{\mu\nu} - \frac{1}{1-\lambda^2} u^\mu u^\nu \right) (g_{\rho\sigma} + u_\rho u_\sigma) \nabla_\nu \ln\Omega$$

$$\begin{aligned}\tilde{R} =& \Omega^{-2} \Big[\frac{2-\lambda^2}{2(1-\lambda^2)} R - \frac{1}{1-\lambda^2} \boxed{G_{\mu\nu} u^\mu u^\nu} \\ &+ \nabla_\mu (g^{\rho\sigma} \gamma^\mu_{\rho\sigma}) - \nabla^\rho \gamma^\mu{}_{\mu\rho} + g^{\rho\sigma} \gamma^\alpha_{\rho\sigma} \gamma^\mu{}_{\mu\alpha} - g^{\rho\sigma} \gamma^\beta{}_{\alpha\rho} \gamma^\alpha{}_{\beta\sigma} \\ &- \frac{1}{1-\lambda^2} u^\rho u^\sigma \left(\nabla_\mu \gamma^\mu_{\rho\sigma} - \nabla_\sigma \gamma^\mu{}_{\mu\rho} + \gamma^\mu{}_{\mu\alpha} \gamma^\alpha_{\rho\sigma} - \gamma^\beta{}_{\alpha\rho} \gamma^\alpha{}_{\beta\sigma} \right) \Big]\end{aligned}$$

$$
\begin{aligned}
\tilde{G}_{\mu\nu} &= \tilde{R}_{\mu\nu} - \frac{1}{2}\tilde{g}_{\mu\nu}\tilde{R} \\
&= G_{\mu\nu} + \frac{1}{2(1-\lambda^2)}(g_{\mu\nu} + u_\mu u_\nu)\boxed{u^\alpha u^\beta G_{\alpha\beta}} \\
&\quad - \frac{\lambda^2}{4(1-\lambda^2)}(g_{\mu\nu} + u_\mu u_\nu)\boxed{R} - \frac{1}{2}u_\mu u_\nu \boxed{R} \\
&\quad + \nabla_\rho \gamma^\rho_{\mu\nu} - \nabla_\nu \gamma^\rho{}_{\rho\mu} + \gamma^\rho{}_{\rho\sigma}\gamma^\sigma_{\mu\nu} - \gamma^\sigma{}_{\rho\mu}\gamma^\rho{}_{\sigma\nu} \\
&\quad - \frac{1}{2}(g_{\mu\nu} + u_\mu u_\nu)\Big[\nabla_\alpha(g^{\rho\sigma}\gamma^\alpha_{\rho\sigma}) - \nabla^\rho \gamma^\sigma{}_{\sigma\rho} + g^{\rho\sigma}\gamma^\alpha_{\rho\sigma}\gamma^\beta{}_{\beta\alpha} - g^{\rho\sigma}\gamma^\beta{}_{\alpha\rho}\gamma^\alpha{}_{\beta\sigma} \\
&\quad - \frac{1}{1-\lambda^2}u^\rho u^\sigma\left(\nabla_\alpha \gamma^\alpha_{\rho\sigma} - \nabla_\sigma \gamma^\alpha{}_{\alpha\rho} + \gamma^\beta{}_{\beta\alpha}\gamma^\alpha_{\rho\sigma} - \gamma^\beta{}_{\alpha\rho}\gamma^\alpha{}_{\beta\sigma}\right)\Big]
\end{aligned}
$$

new Higgs inflation

$$u_\mu = \beta \nabla_\mu \phi \qquad \lambda^2 = 2\beta^2 X \qquad X = -\frac{1}{2}(\nabla\phi)^2$$

$$\tilde{G}_{\mu\nu}\tilde{\nabla}^\mu\phi\tilde{\nabla}^\nu\phi$$

$$= \frac{1}{\beta^2\Omega^4(1-\lambda^2)^2}\Bigg[\left(1-\frac{\lambda^2}{2}\right)u^\mu u^\nu G_{\mu\nu} - \frac{\lambda^4}{4}R$$

$$+ u^\mu u^\nu \left(\nabla_\rho \gamma^\rho_{\mu\nu} - \nabla_\nu \gamma^\rho_{\rho\mu} + \gamma^\rho_{\rho\sigma}\gamma^\sigma_{\mu\nu} - \gamma^\sigma_{\rho\mu}\gamma^\rho_{\sigma\nu}\right)$$

$$+ \frac{\lambda^2}{2}\Big\{(1-\lambda^2)\left(\nabla_\alpha(g^{\rho\sigma}\gamma^\alpha_{\rho\sigma}) - \nabla^\rho\gamma^\sigma_{\sigma\rho} + (g^{\rho\sigma}\gamma^\alpha_{\rho\sigma})\gamma^\beta_{\beta\alpha} - g^{\rho\sigma}\gamma^\beta_{\rho\alpha}\gamma^\alpha_{\sigma\beta}\right)$$

$$- u^\rho u^\sigma \left(\nabla_\alpha \gamma^\alpha_{\rho\sigma} - \nabla_\sigma \gamma^\alpha_\alpha + \gamma^\beta_{\beta\alpha}\gamma^\alpha_{\rho\sigma} - \gamma^\beta_{\alpha\rho}\gamma^\alpha_{\beta\sigma}\right)\Big\}\Bigg]$$

disformal transformation

$$\tilde{g}_{\mu\nu} = \Omega^2\left(g_{\mu\nu} + \beta^2 \nabla_\mu\phi\nabla_\nu\phi\right)$$

$$\Omega^2 = \frac{(2-\lambda^2)}{2(1-\lambda^2)^{\frac{1}{2}}} \qquad \beta^2 = \alpha(1-\lambda^2)^{-\frac{1}{2}} \Longrightarrow \lambda^4(1-\lambda^2) = 4\alpha^2 X^2$$

The EH gravitational action + Higgs field ϕ with higher-derivatives

The higher-derivative terms are too complicated

It may be better to analyze it in the original frame

However, if we can **ignore the higher-derivative terms**,
The analysis in the disformal frame becomes easy

⇕

Germani, Martucci, Moyassari (2012)

Slow-rolling inflationary phase

$$\mathcal{L}_{\text{Higgs}} = -\sqrt{-\tilde{g}}\left[\left(\frac{1+\alpha V(\phi)}{2}\right)(\tilde{\nabla}\phi)^2 + V(\phi)\right] + \cdots$$

higher-derivative terms

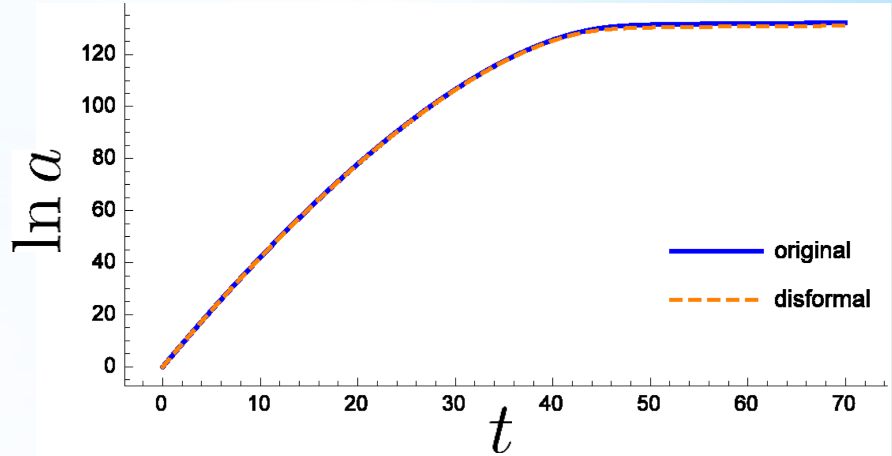
$$= -\sqrt{-\tilde{g}}\left[\frac{1}{2}(\tilde{\nabla}\Phi)^2 + U(\Phi)\right] + \cdots$$

$$U(\Phi) = \begin{cases} \frac{\lambda}{4}\Phi^4 & \Phi \ll \Phi_{cr} \\ 3\sqrt[3]{\frac{3\lambda}{4}} M_{\text{PL}}^4 \left(\frac{M}{M_{\text{PL}}}\right)^{4/3}\left(\frac{\Phi}{M_{\text{PL}}}\right)^{4/3} & \Phi \gg \Phi_{cr} \end{cases}$$

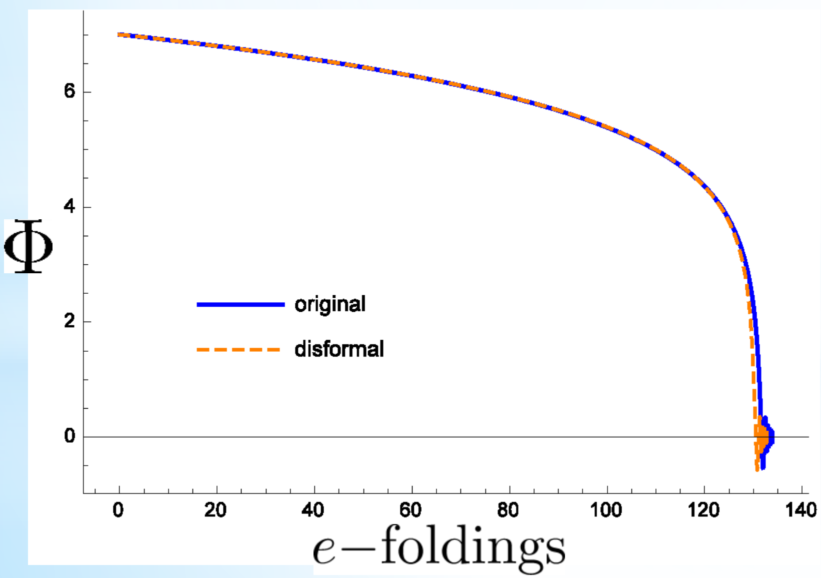
$$\alpha = \frac{1}{M^2 M_{\text{PL}}^2}$$

The original frame
vs
the disformal frame with trancation

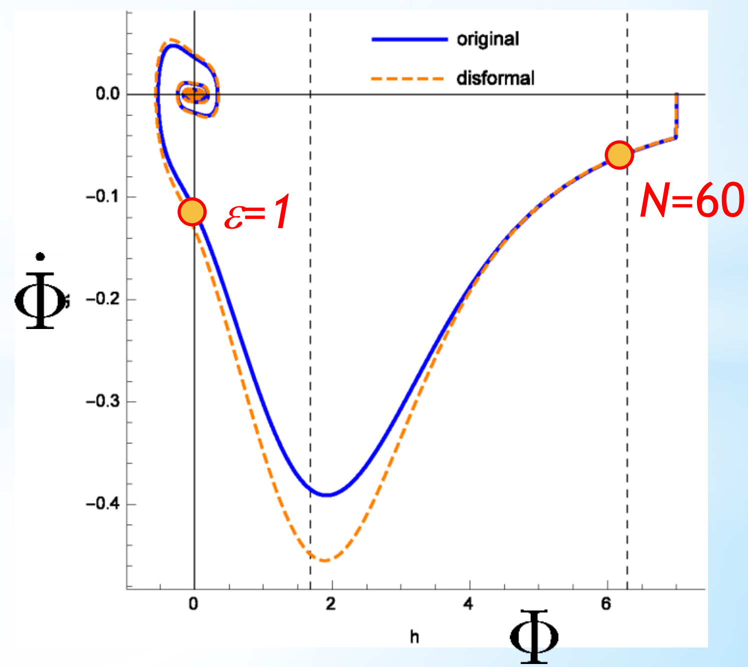
scale factor



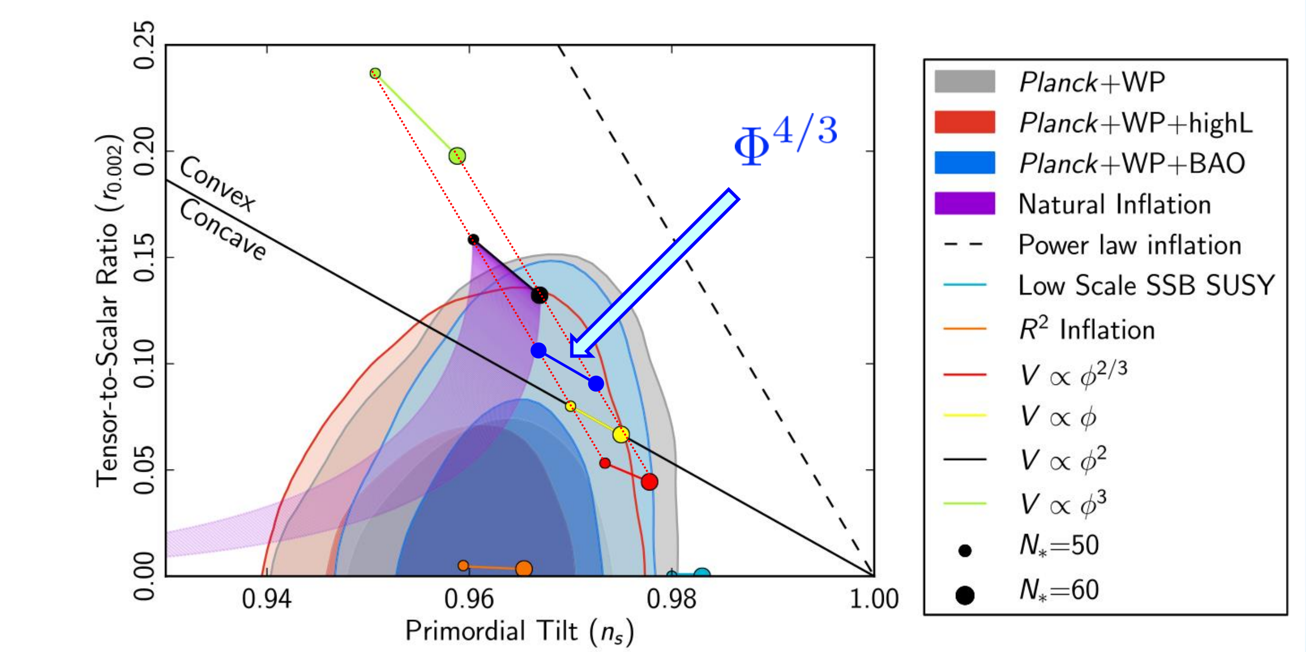
Higgs field



phase space of Higgs field



Observational constraint



K.N. Abazajian et al (2014)

Hybrid Higgs Inflation (conventional+new)

Easther, KM, Musoke, Sato (2016)

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}\left(R + \alpha G^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi\right) - \frac{1}{2}\xi\phi^2 R - \frac{1}{2}(\nabla\phi)^2 - V(\phi)\right]$$

Kamada et al (2012): generalized Higgs inflation

↓ disformal transformation

EH action +

$$\mathcal{L}_{\text{Higgs}} = -\sqrt{-g}\left[\left(\frac{(1-\xi\kappa^2\phi^2)+\alpha V(\phi)}{2(1-\xi\kappa^2\phi^2)^2}\right)(\nabla\phi)^2 + \frac{V(\phi)}{(1-\xi\kappa^2\phi^2)^2}\right] + \cdots$$

$$= -\sqrt{-g}\left[\frac{1}{2}(\nabla\Phi)^2 + U(\Phi)\right] + \cdots$$

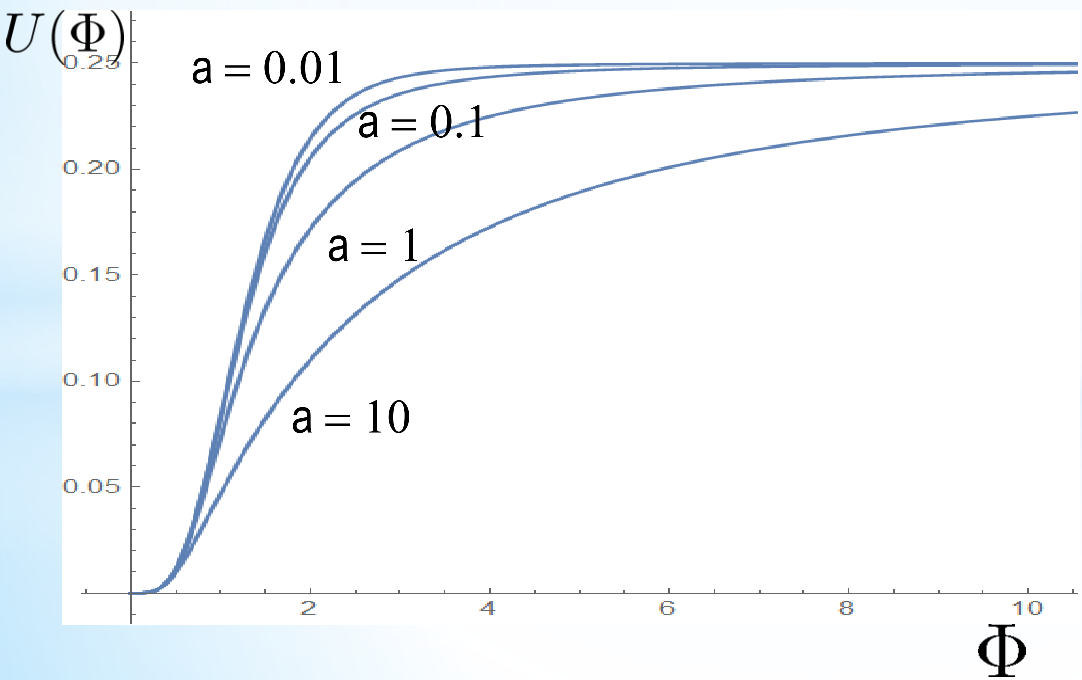
$$\Phi = \int \frac{\sqrt{(1-\xi\kappa^2\phi^2)+\alpha V(\phi)}}{(1-\xi\kappa^2\phi^2)}d\phi \qquad U(\Phi) = \frac{V(\phi)}{(1-\xi\kappa^2\phi^2)^2}$$

$$V = \frac{\lambda}{4}\phi^4 \qquad \alpha = \frac{1}{M^2 M_{\rm PL}^2}$$

$$a := \frac{\lambda}{4}\frac{M_{\rm PL}^2}{\xi^2 M^2}$$

$$U(\Phi) = \frac{V(\phi)}{(1-\xi\kappa^2\phi^2)^2}$$

$$= \frac{\lambda}{4\xi^2}M_{\rm PL}^4\left(1 - \frac{2a}{|\xi|}\frac{M_{\rm PL}^2}{\Phi^2} + \cdots - 8\exp\left[-2\frac{|\xi|^{1/2}\Phi}{M_{\rm PL}}\right] + \cdots\right)$$

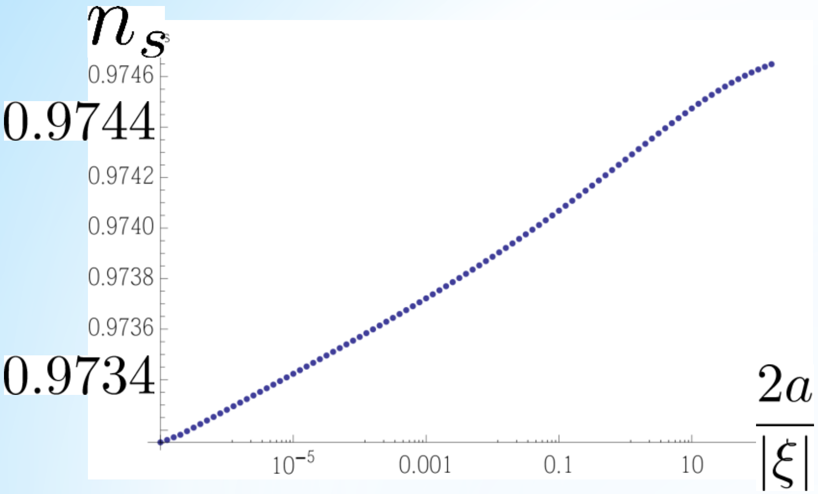


$a \ll 1$

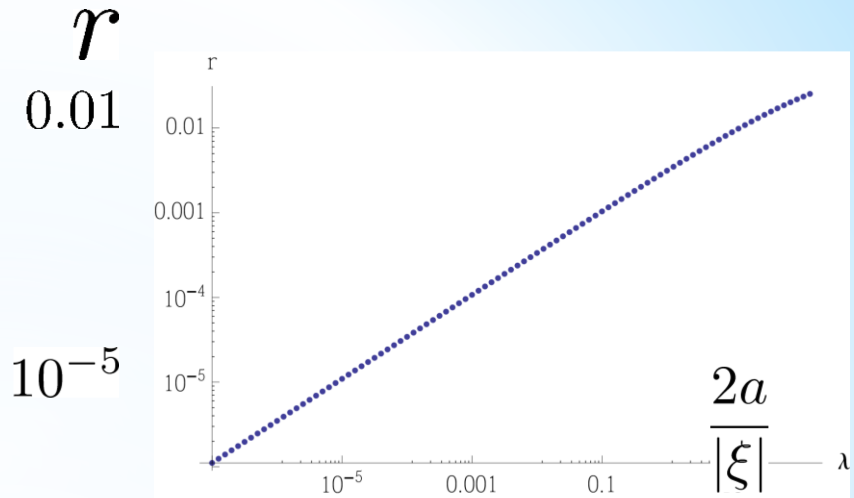
the original Higgs inflation

$a \sim O(1)$

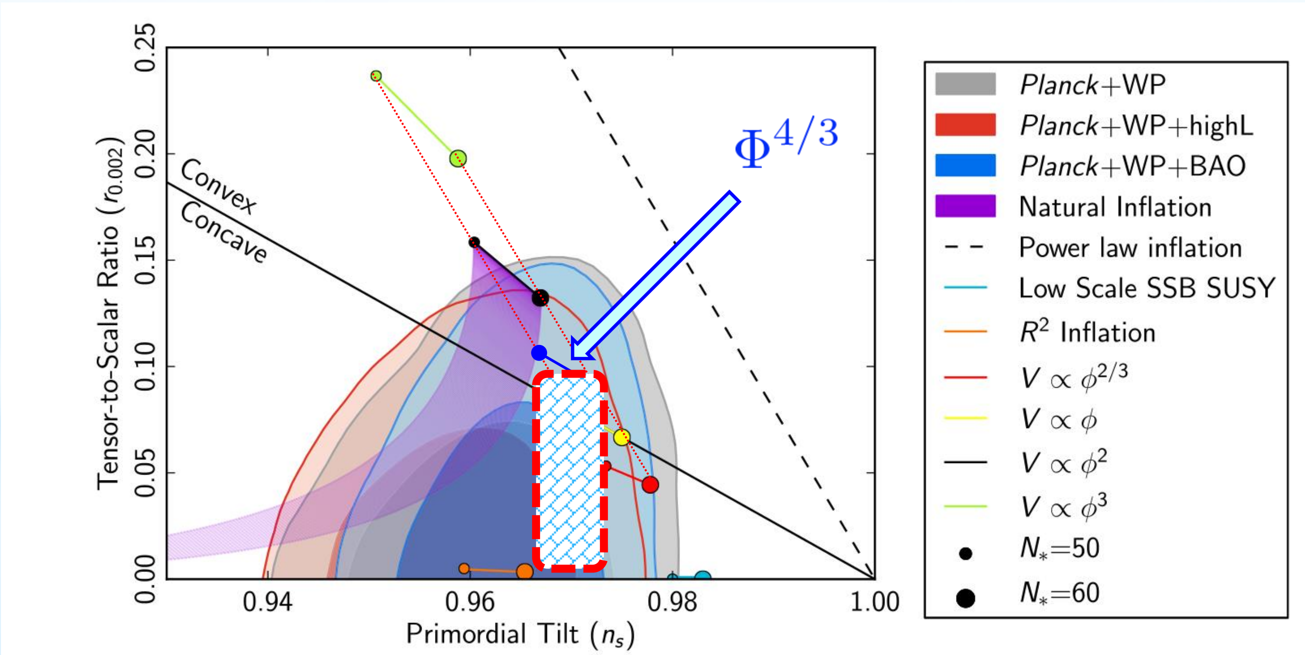
$U \propto 1 - c_0\Phi^{-2}$



n_s
0.9746
0.9744
0.9742
0.9740
0.9738
0.9736
0.9734
10^{-5}
0.001
0.1
10
$\frac{2a}{|\xi|}$



r
0.01
0.001
10^{-4}
10^{-5}
10^{-5}
0.001
0.1
$\frac{2a}{|\xi|}$
λ



Tensor-to-Scalar Ratio ($r_{0.002}$)
Primordial Tilt (n_s)
0.00
0.05
0.10
0.15
0.20
0.25
0.94
0.96
0.98
1.00
Convex
Concave
$\Phi^{4/3}$
Planck+WP
Planck+WP+highL
Planck+WP+BAO
Natural Inflation
Power law inflation
Low Scale SSB SUSY
R^2 Inflation
$V \propto \phi^{2/3}$
$V \propto \phi$
$V \propto \phi^2$
$V \propto \phi^3$
N_*=50
N_*=60

Other models ?

Galileon

generalized Galileon

⋮

disformal transformation →

EH action
with some potential

Analysis is simple

"Equivalence" between two theories
when we ignore the higher-derivative terms

Some remarks on disformal transformation

- causal structure

  conformal transformation : null $\longrightarrow$ null

  disformal transformation : null $\not\longrightarrow$ null

  A causal structure is changed

- coupling to matter fields

Black Holes in Horava-Lifshitz gravity (or Einstein-aether theory)

Misonoh, KM(2015)

Non-projectable HL gravity

$$S_{\rm HL} = \int dt d^3x N \sqrt{g_3} \left(\mathcal{L}_K + \mathcal{L}_P\right)$$

$$\mathcal{L}_K = \alpha\left(\mathcal{K}_{ij}\mathcal{K}^{ij} - \lambda\mathcal{K}^2\right)$$

$$\mathcal{L}_P = -\left(\mathcal{V}_{z=1} + \mathcal{V}_{z=2} + \mathcal{V}_{z=3}\right)$$

z : scaling parameter

$$\mathcal{V}_{z=1} := \gamma_0\mathcal{R} + \gamma_1\Phi_i\Phi^i$$

$$\mathcal{V}_{z=2} := \gamma_3(\Phi_i\Phi^i)^2 + \cdots + \gamma_6(\Phi_i\Phi^i)\mathcal{R} + \cdots + \gamma_{10}\mathcal{R}^2$$

$$\mathcal{V}_{z=3} := \gamma_{11}(\Phi_i\Phi^i)^3 + \cdots + \gamma_{36}\mathcal{R}^{ij}\mathcal{D}_i\mathcal{D}_j\mathcal{R}$$

z= 2,3 : higher spatial derivative terms

$$\Phi_i := \mathcal{D}_i \ln N$$

$$\mathcal{R}_{ij}, \mathcal{R} := \mathcal{R}^i_i$$

Einstein-aether theory

$$S_{æ} = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\,\mathcal{L}_{æ}$$

u^μ: aether field

$$u_\mu u^\mu = -1$$

$$\mathcal{L}_{æ} = R - M^{\mu\nu}{}_{\alpha\beta}\left(\nabla_\mu u^\alpha\right)\left(\nabla_\nu u^\beta\right)$$

$$M^{\mu\nu}{}_{\alpha\beta} := c_{13}\delta^\mu{}_\beta\delta^\nu{}_\alpha + c_2\delta^\mu{}_\alpha\delta^\nu{}_\beta - c_{14}u^\mu u^\nu g_{\alpha\beta}$$

$$c_{13} := c_1 + c_3, c_{14} := c_1 + c_4$$

IR limit (z=1) ⟺ **Einstein-aether theory (hypersurface orthogonal)**

$$u_\mu := \frac{\nabla_\mu\varphi}{\sqrt{-(\nabla^\alpha\varphi)(\nabla_\alpha\varphi)}}$$

Relation between the coupling constants in two theory

$$\alpha = \frac{1-c_{13}}{16\pi G}, \quad \lambda = \frac{1-c_2}{1-c_{13}},$$

$$\frac{\gamma_0}{\alpha} = -\frac{1}{1-c_{13}}, \quad \frac{\gamma_1}{\alpha} = -\frac{c_{14}}{1-c_{13}}$$

$$S_{æ+z=2} = \frac{1}{16\pi G} \int d^4x \sqrt{-g} \left[\mathcal{L}_{æ} + \mathcal{L}_{z=2} \right]$$

$$\mathcal{L}_{z=2} = -M_{\rm PL}^{-2} \left(\beta_1 \dot{u}^4 + \beta_2 \dot{u}^2 \mathcal{R} + g_2 \mathcal{R}^2 \right)$$

z=2 terms

Two propagating modes

graviton (helicity 2)

scalar-graviton (helicity 0)

dispersion relations

$$\omega_G^2 = \frac{1}{1 - c_{13}} k^2$$

$$\omega_S^2 = \frac{(c_{13} + c_2)(2 - c_{14})}{c_{14}(1 - c_{13})(2 + c_{13} + 3c_2)} k^2 + \frac{8(c_{13} + c_2) g_2}{2 + c_{13} + 3c_2} \left(\frac{k^2}{M_{\rm PL}} \right)^2$$

The propagation velocities are different

The invariance in the above model
under the following disformal transformation

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} + (1-\sigma)u_\mu u_\nu \,, \;\; \tilde{u}^\mu = \sigma^{-1/2} u^\mu$$

σ is a positive constant

$$\tilde{\gamma}_{\mu\nu} = \gamma_{\mu\nu} \,, \;\; \tilde{u}^\mu = \sigma^{-1/2} u^\mu \qquad \gamma_{\mu\nu} := g_{\mu\nu} + u_\mu u_\nu$$

The three metric is invariant

The aether field is scaled

⇨

The propagating speeds in IR limit are scaled as $\sigma^{-1/2}$
that of the scalar-graviton in UV limit is unchanged.

Black Hole ?

The metric horizon (null surface) is not an event horizon

IR limit (for low energy particles)

$$c_{\rm G}^2 \sim \frac{1}{1-c_{13}}\,,\ c_{\rm S}^2 \sim \frac{(c_{13}+c_2)(2-c_{14})}{c_{14}(1-c_{13})(2+c_{13}+3c_2)}$$

$$\tilde{c}_{\rm G}^2 = \sigma^{-1} c_{\rm G}^2\,,\ \tilde{c}_{\rm S}^2 = \sigma^{-1} c_{\rm S}^2 \Rightarrow \tilde{c}_{\rm G}^2 = 1 \text{ or } \tilde{c}_{\rm S}^2 = 1$$

UV limit (for high energy particles)

$$c_{\rm G}^2 \sim \frac{1}{1-c_{13}}\,,\ c_{\rm S}^2 \sim \frac{8(c_{13}+c_2)g_2}{2+c_{13}+3c_2}\left(\frac{k}{M_{\rm PL}}\right)^2 \to \infty$$

The ultimately excited scalar-graviton should propagate along the three dimensional spacelike hypersurface

universal horizon

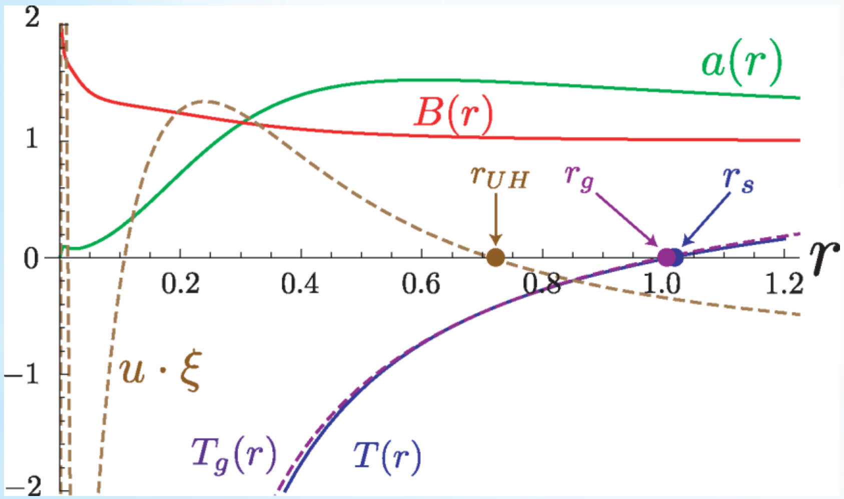
The hypersurface Σ parallel to the timelike Killing vector ξ

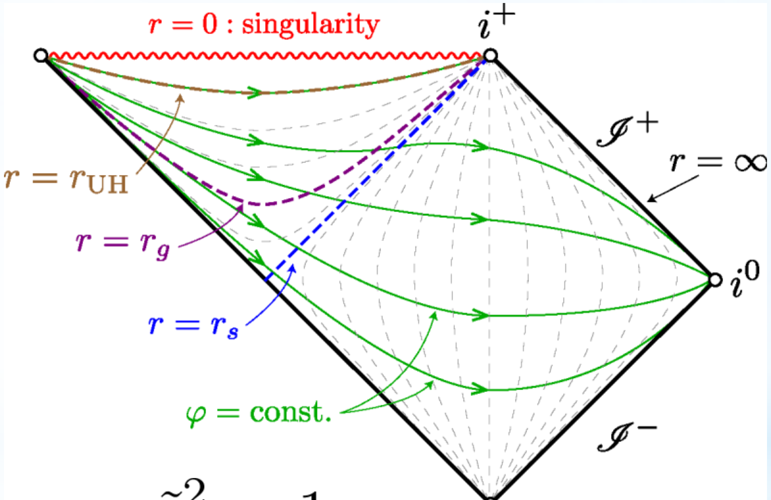
$$u\cdot\xi = 0$$

BH in the Einstein-aether theory

$$ds^2 = -T(r)dv^2 + 2B(r)dvdr + r^2 d\Omega^2$$

$$u^\mu = \left(a(r), b(r), 0, 0\right) \qquad b(r) = \frac{a(r)^2 T(r) - 1}{2a(r)B(r)}$$

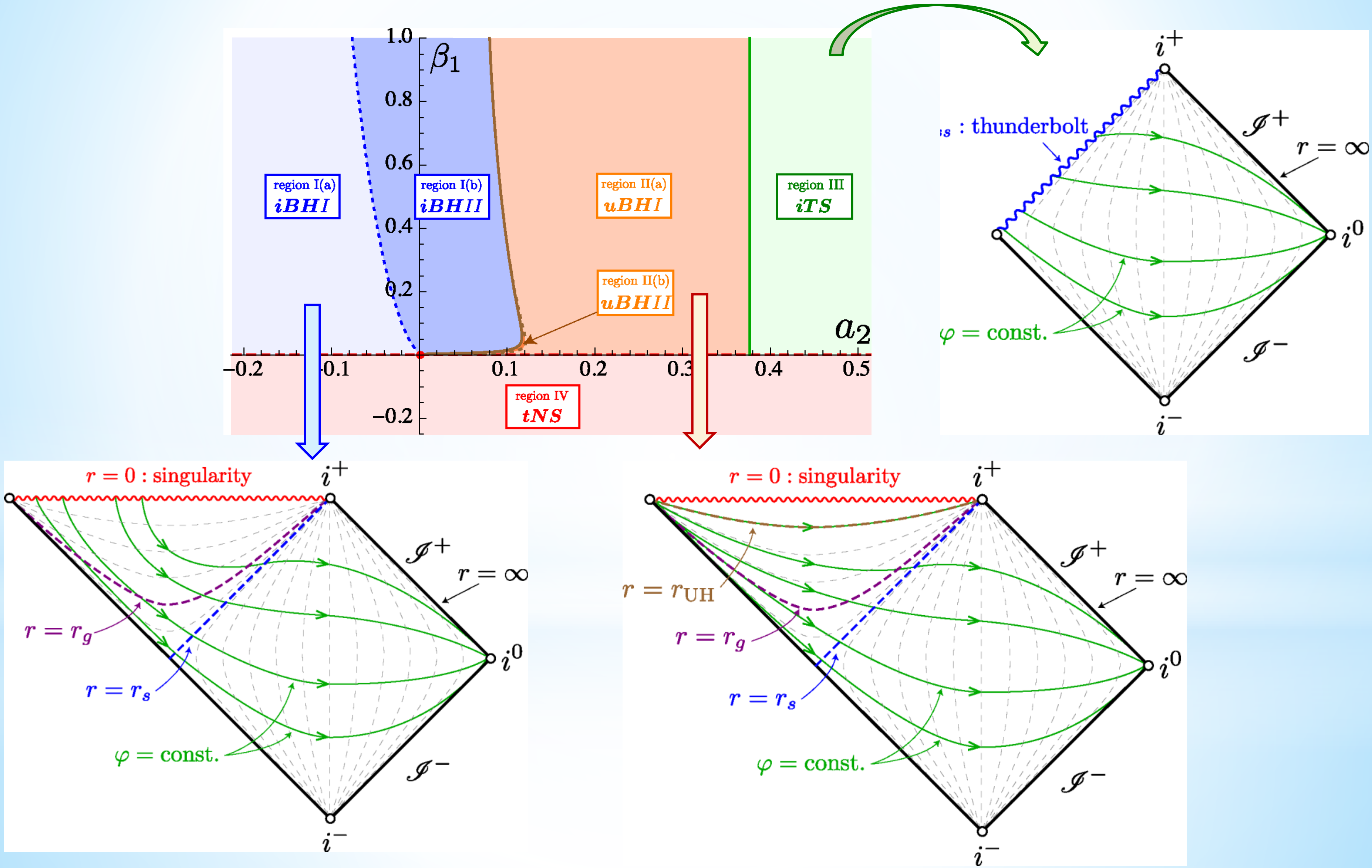




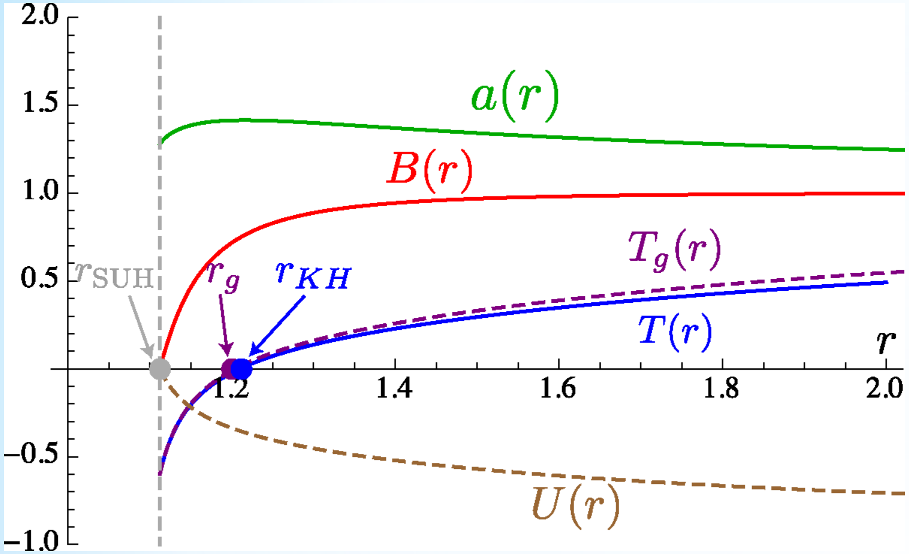
$\tilde{c}_S^2 = 1$

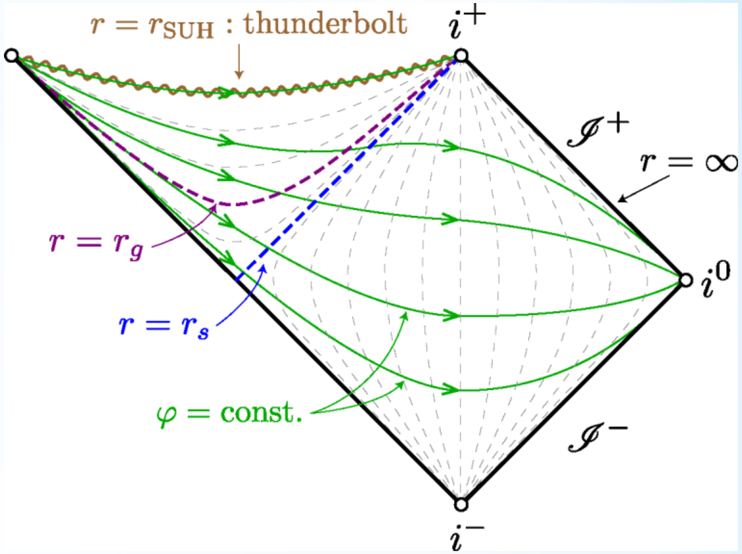
BH in HL gravity (z=2)

$$\mathcal{L}_{z=2} = -M_{\rm PL}^{-2}\beta_1 \dot{u}^4$$



$$\mathcal{L}_{z=2} = -M_{\rm PL}^{-2} g_2 \mathcal{R}^2$$





universal horizon : singular

outside : regular

⇨ No information from the singularity

Thunderbolt singularity

Coupling with perfect fluid

KM, Y. Fujii (09)

Modified gravity (e.g. scalar tensor theory)

MODEL
$$S_J = \int d^4x\sqrt{-g}\left[\frac{\xi}{2}\phi^2 R(g) - \frac{\epsilon}{2}(\nabla\phi)^2 - V(\phi)\right] + \int d^4x\sqrt{-g}L_{\mathrm{m}}(\psi, g)$$

conformal transformation

$g \rightarrow g \exp(2\zeta\kappa\sigma)$

$\zeta = \sqrt{\xi/(\varepsilon+6\xi)}$

Einstein gravity (g) + scalar field σ

$U = V \exp(-4\zeta\kappa\sigma)$

Dynamics without matter is well-known

But, coupling with matter is important

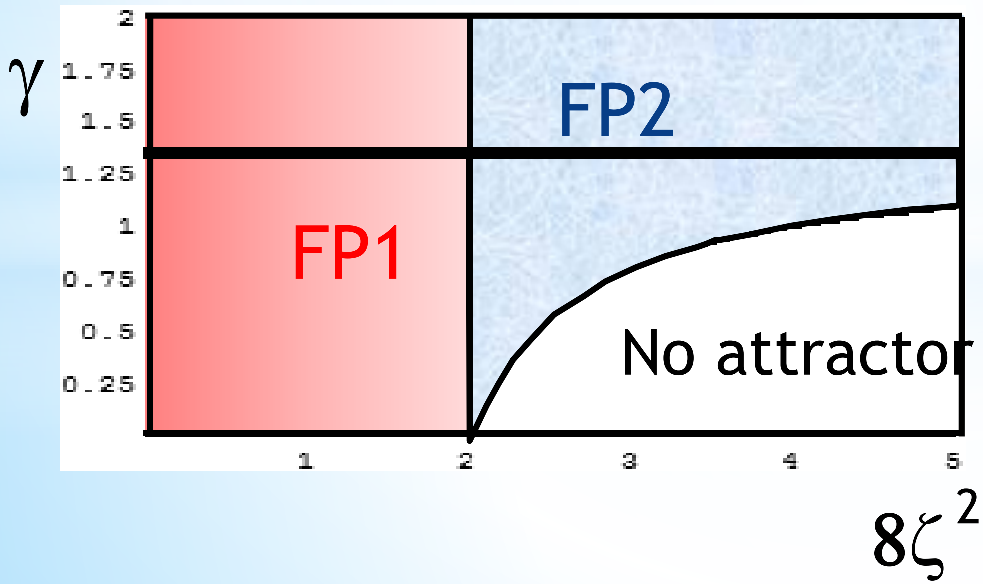
$V = V_0$ (constant)

$$H^2 + \frac{k}{a^2} = \frac{\kappa^2}{3}\left[\frac{1}{2}\left(\frac{d\sigma}{dt}\right)^2 + U + \rho\right]$$

$$\frac{d^2\sigma}{dt^2} + 3H\frac{d\sigma}{dt} + \frac{\partial U}{\partial \sigma} = \zeta\kappa(\rho - 3P)$$

$$\frac{d\rho}{dt} + 3\gamma H\rho = -\zeta\kappa(4-3\gamma)\frac{d\sigma}{dt}\rho$$

$$P = (\gamma - 1)\rho$$



Two fixed points

FP1 Scalar field dominant

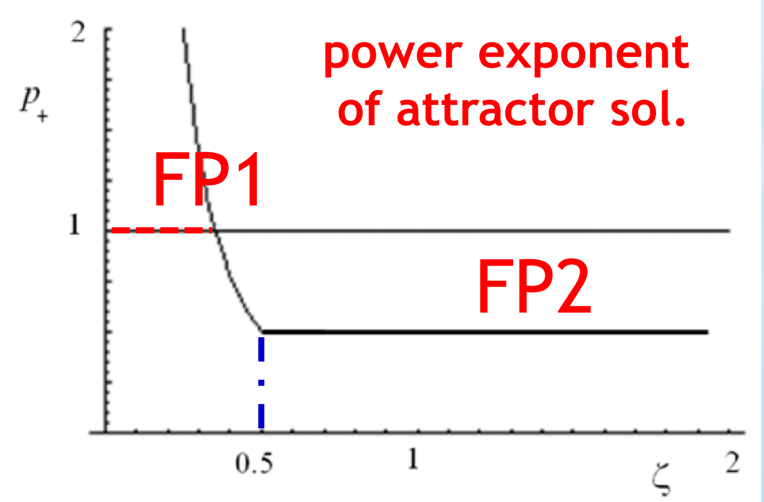
$$a \propto t^{\frac{1}{8\zeta^2}} \qquad \kappa\sigma = \frac{1}{2\zeta}\ln t + \text{const}$$

FP2 Scaling solution

$$\left(\frac{\rho}{V}\right)_2 = \frac{2(4\zeta^2 - 1)}{2 - \gamma - 2(4-3\gamma)\zeta^2} \quad \text{const}$$

$$a \propto t^{\frac{1}{2}} \qquad \kappa\sigma = \frac{1}{2\zeta}\ln t + \text{const}$$

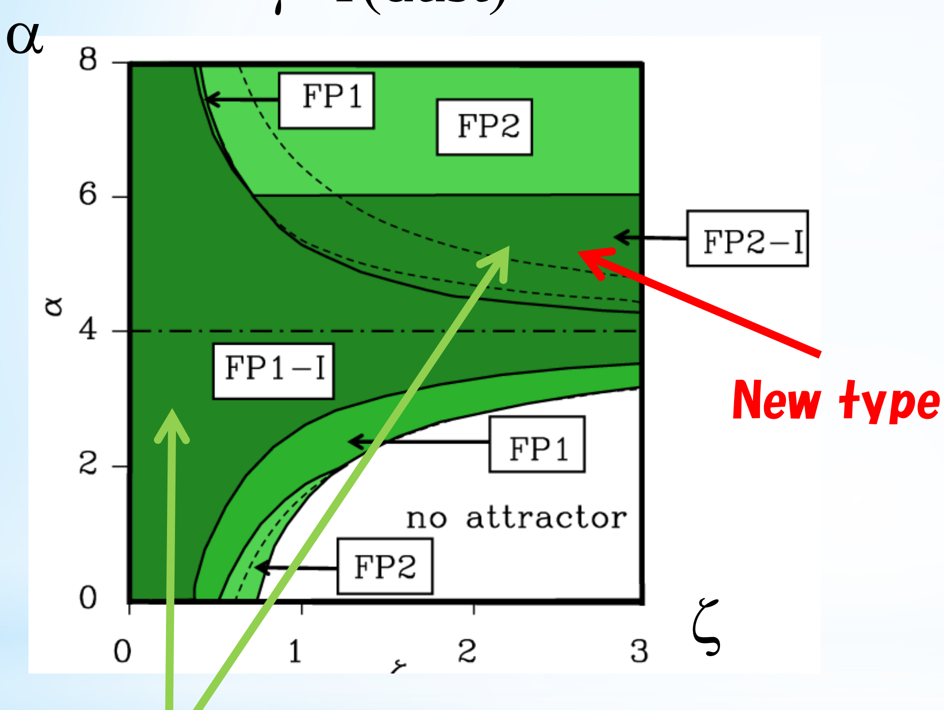
Minkowski in Jordan frame

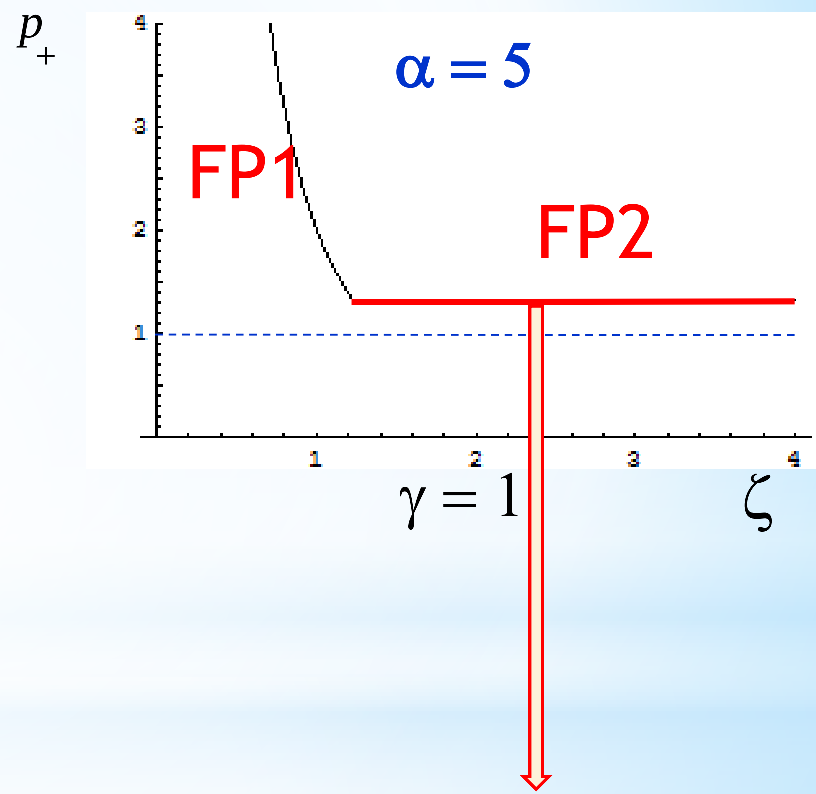


power-law potential $V_0 \to (\kappa\phi)^{\alpha} V_0$

attractor sols. $a \propto t^{p}$

γ=1(dust)





Power-law inflation

acceleration with a steep potential

How about disformal transformation?

Some discussions about
disformal inflation
(or disformal dark energy)

Kaloper(2004),
van de Bruck, Koivisto, Longden (2016)
Zumalacarregui et al (2010)

disformal metric coupled to matter fluid

⟹ acceleration of the Universe

The so-called coincidence problem could be solved

Thank you for your attention