- Planck constraints on inflation in auxiliary vector modified $f(R)$ theory.

- Mehmet Ozkan, Yi Pang, Shinji Tsujikawa

1. Introduction.

* Starobinsky model + auxiliary vector field A_μ = α - attractor model

+ Replacing the Ricci Scalar R

: $R \rightarrow R + A_\mu A^\mu + \beta \nabla_\mu A^\mu$

* extended the general $f(R)$ model.

※ Starobinsky model : $f(R) \simeq R + R^2$

2. Starobinsky model and Its dual description.

* Starobinsky model : $\mathcal{L} = R + \frac{R^2}{6M^2}$ ← additional D.O.F. in G.R.

* Additional D.O.F.

$$\mathcal{L} = F + \frac{F^2}{6M^2} - \varphi (F - R)$$

E.O.M. φ : $F = R$

E.O.M F : $F = 3M^2(\varphi - 1)$

$$\mathcal{L} = R + \frac{R^2}{6M^2} \xleftrightarrow[\text{equivalent}]{} \mathcal{L} = \varphi R - \frac{3}{2} M^2 (\varphi - 1)^2$$

: Brans-Dicke theory ($\omega_{BD} = 0$)

↓ Conformal Tr.

$$\tilde{\mathcal{L}} = \frac{M_p^2}{2} \tilde{R} - \frac{1}{2} \tilde{g}^{\mu\nu} \partial_\mu \phi \, \partial_\nu \phi - V(\phi)$$

where

$$\phi \equiv \sqrt{3/2} \cdot M_p \cdot \ln \varphi$$

$$V \equiv \frac{3}{4} M_p^2 \cdot M^2 \left(1 - e^{-\sqrt{2/3} \cdot \frac{\phi}{M_p}}\right)^2$$

3. Auxiliary Modified Starobinsky Model.

; if) auxiliary Vector field A_μ is Coupled to the Starobinsky model.

then) obtain the "α-attractor model" in the context of supergravity.

* Action. : $\boxed{\mathcal{L} = R + A_\mu A^\mu + \beta \nabla_\mu A^\mu + \frac{1}{6M^2}\left(R + A_\mu A^\mu + \beta \nabla_\mu A^\mu\right)^2}$

$\mathcal{L}$

$$\mathcal{L} = F + \frac{1}{6M^2}F^2 - \varphi\left(F - R - A_\mu A^\mu - \beta \nabla_\mu A^\mu\right)$$

E.O.M. φ : $F = R + A_\mu A^\mu + \beta \nabla_\mu A^\mu$

e.o.m. A_μ : $A_\mu = \frac{1}{2\varphi}\beta \nabla_\mu \varphi$

e.o.m. F : $F = 3M^2(\varphi - 1)$

$\mathcal{L} = \boxed{''''}$

"equivalent"

$$\mathcal{L} = \varphi R - \frac{1}{4\varphi}\beta^2 \nabla_\mu \varphi \nabla^\mu \varphi - \frac{1}{2}M^2(\varphi-1)^2 ;$$

Brans-Dicke theory $\omega_{BD} = \frac{\beta^2}{4}$

↓ Conformal Tr.

$$S = \int d^4x \sqrt{-\tilde{g}}\left[\frac{M_p^2}{2}\tilde{R} - \frac{1}{2}\tilde{g}^{\mu\nu}\nabla_\mu \phi \nabla_\nu \phi - V(\phi)\right]$$

where

$$\alpha \equiv 1 + \frac{\beta^2}{6}, \quad \phi \equiv \frac{\sqrt{\beta^2+6}}{2} M_p \cdot \ln\varphi, \quad V \equiv \frac{3}{4}M_p^2 \cdot M^2\left(1 - e^{-\sqrt{\frac{2}{3\alpha}}\frac{\phi}{M_p}}\right)^2$$

4. Auxiliary Vector Modified $f(R)$ theory and inflationary observables.

; extended $f(R)$ theory $\longrightarrow$ P_s, n_s, r

$\boxed{A.}$ Auxiliary Vector modified $f(R)$

$$\mathcal{L} = f(R) \longrightarrow \mathcal{L} = f(R + A_\mu A^\mu + \beta \nabla_\mu A^\mu)$$

$$\mathcal{L} = f(F) - \varphi\left(F - R - A_\mu A^\mu - \beta \nabla_\mu A^\mu\right)$$

e.o.m φ : $F = R + A_\mu A^\mu + \beta \nabla_\mu A^\mu$

e.o.m A_μ : $\dfrac{1}{2\varphi}\beta \nabla_\mu \varphi = A_\mu$

e.o.m F : $\dfrac{\partial f}{\partial F} = \varphi$

$$\mathcal{L} = \boxed{////} \quad \xleftrightarrow{\text{"equivalent"}} \quad \mathcal{L} = \varphi R - \frac{1}{4\varphi}\beta^2 \nabla_\mu \varphi \nabla^\mu \varphi - \left[\varphi F - f(F)\right]$$

; Brans-Dicke theory ($\omega_{BD} = \beta^2/4$)

$\downarrow$ Conformal Tr.

$$V(\phi) \equiv \frac{M_p^2}{2} e^{-\sqrt{\frac{2}{3\alpha}}\frac{\phi}{M_p}} \left[F - e^{-\sqrt{\frac{2}{3\alpha}}\frac{\phi}{M_p}} f(F)\right]$$

$\boxed{B}$ Observables : P_s, n_s, r

* example

$$S = \int d^4x \cdot \sqrt{-g}\left[f\left(R + A_\mu A^\mu + \beta \nabla_\mu A^\mu \right) \right]$$

① modified Starobinsky

$$f(F) = F + \frac{F^2}{6M^2}$$

② Power - law

$$f(F) = m^{2(1-n)} \cdot F^n$$

; disfavored from data.

③ Generalization of Auxiliary Vector modified Starobinsky model.

$$f(F) = F + m^{2(1-n)} F^n$$