Chiral gravitational waves in single field inflation

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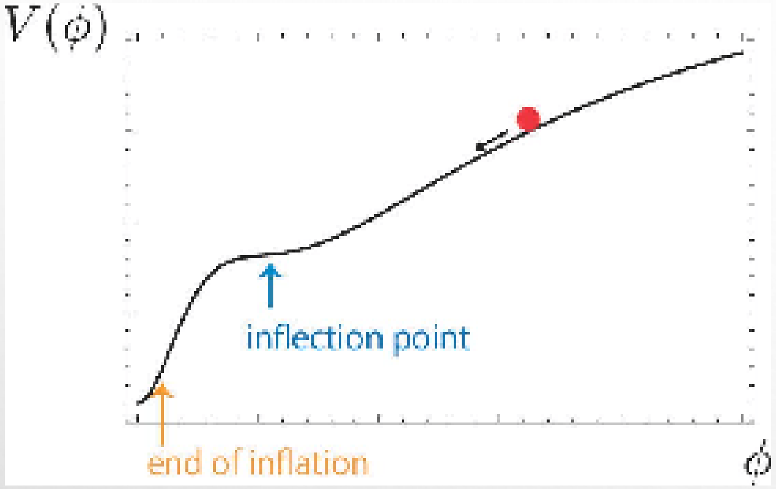
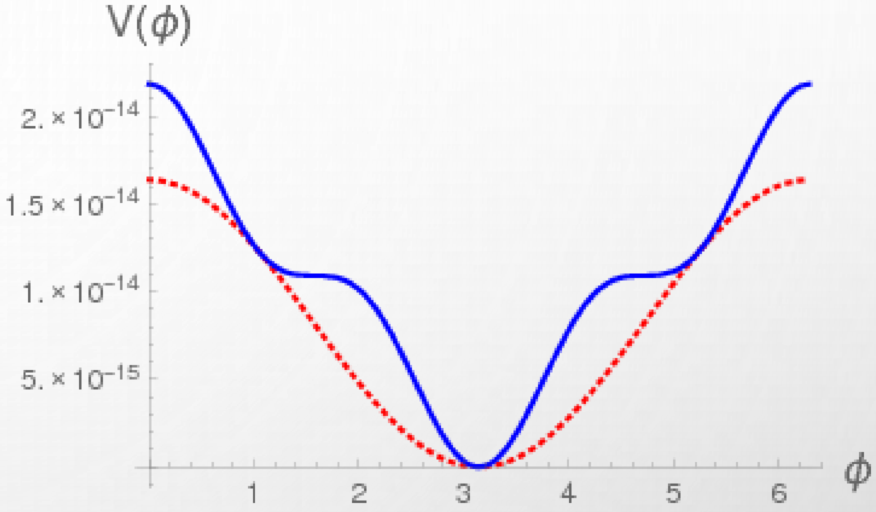
Non-attractor Inflation



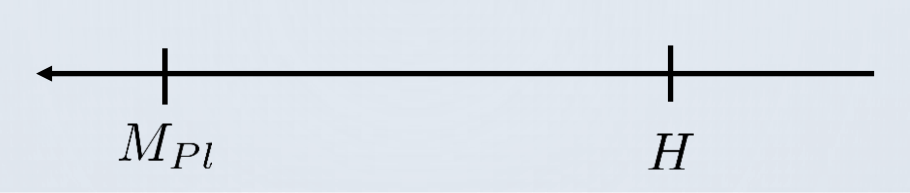
M. Sasaki , T. Suyama, T. Tanaka, S. Yokoyama (2016)



S. Parameswaran, G. Tasinato, I. Zavala (2016)

$$\mathcal{R}(\tau) = \mathcal{C}_1 + \mathcal{C}_2 \int^{\tau} \frac{\mathrm{d}y'}{z^2(y')}$$

Effective Field Theory (EFT) of Inflation



Leading-order

$$S = M_{Pl}^2 \int d^4x \sqrt{-g} \left[\frac{1}{2} R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right.$$
$$\left. + \frac{f1(\phi)}{M_{Pl}^2} (g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi)^2 + \frac{f2(\phi)}{M_{Pl}^2} W^{\mu\nu\rho\sigma} W_{\mu\nu\rho\sigma} + \frac{f3(\phi)}{M_{Pl}^2} \epsilon^{\mu\nu\rho\sigma} W_{\mu\nu}{}^{\kappa\lambda} W_{\rho\sigma\kappa\lambda} \right]$$

Weinberg (2008)

Higher-order corrections

Amplification of Gravitational Waves

Horndeski Action

$$S = \sum_{i=2}^{5} \int \mathrm{d}^4x \sqrt{-g} \mathcal{L}_i(\phi, X),$$

$$X = -\frac{1}{2}\partial_\mu \phi \partial^\mu \phi$$

Horndeski Functions

$$G_2^{(i)} = \rho_i X + \frac{\sqrt{2}}{3} H_0^2 \alpha_i \sqrt{X} - V_i,$$

$$G_3^{(i)} = \frac{\sqrt{2}}{3H_0} \delta_i \sqrt{X},$$

$$G_4^{(i)} = -\frac{\beta_i}{6H_0^2} X,$$

$$G_5^{(i)} = \frac{\sigma_i}{\sqrt{2}H_0^3} \sqrt{X},$$

Schematically

$$(\alpha_i, \beta_i, \delta_i, \rho_i, \sigma_i, V_i) = \begin{cases} \alpha_1, \beta_1, \delta_1, \rho_1, \sigma_1, V_1, & t < t_0, \\ \alpha_2, \beta_2, \delta_2, \rho_2, \sigma_2, V_2, & t > t_0. \end{cases}$$

MM, O. Ozsoy, S. Parameswaran, G. Tasinato, I. Zavala (2019)

Amplification of Gravitational Waves

MM, O. Ozsoy, S. Parameswaran,
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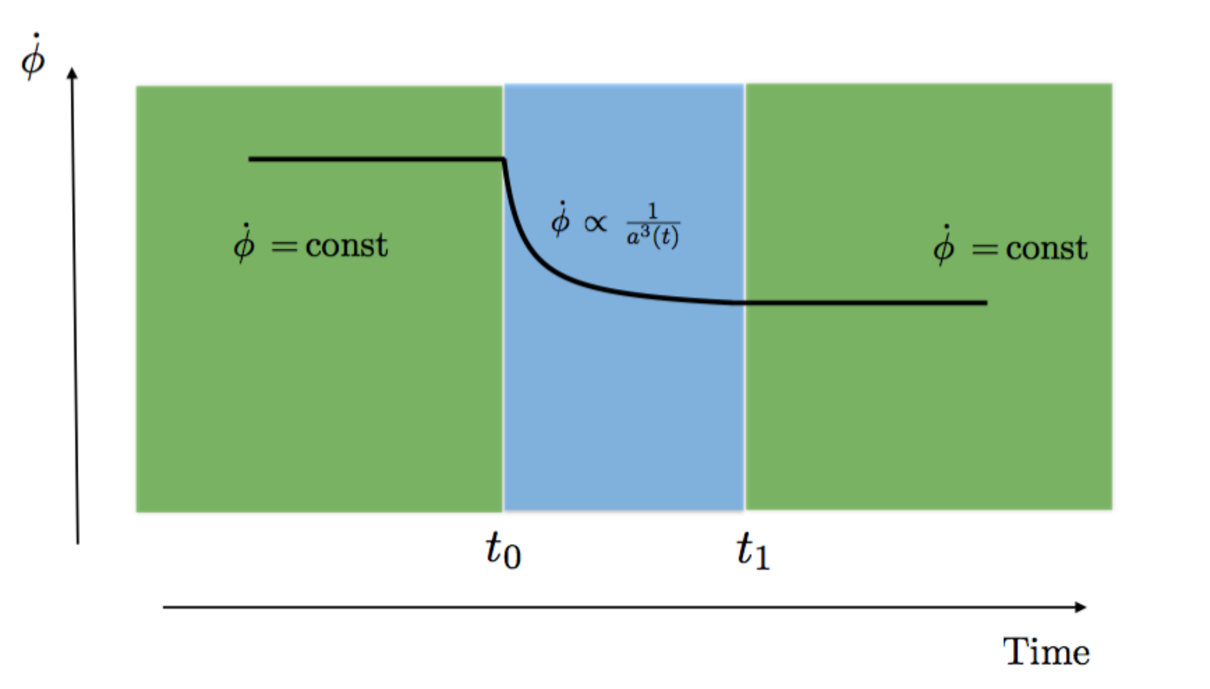


MM, O. Ozsoy, S. Parameswaran,
G. Tasinato, I. Zavala (2019)

$$\dot\phi = -\frac{H_0^2\alpha_1}{3(-\rho_1+\delta_1+\beta_1+\sigma_1)}, \quad t \le t_0$$

$$\dot\phi \simeq -\frac{H_0^2\alpha_1}{3(-\rho_1+\delta_1+\beta_1+\sigma_1)a(t)^3}, \quad t \ge t_0$$

$$\dot\phi = -\frac{H_0^2\alpha_2}{3(-\rho_2+\delta_2+\beta_2+\sigma_2)}, \quad t \ge t_1$$

Tensor Duality

D. Wands (1999), Leech (2000), Leech (2001)

Canonically Normalized Tensor Action

$$S_T^{(2)} = \frac{1}{2}\int \mathrm{d}y\,\mathrm{d}^3x\left[(q'_{ij})^2 - (\nabla q_{ij})^2 + \frac{z''_T}{z_T}q_{ij}^2\right], \quad \text{where} \quad h_{ij} = \frac{q_{ij}}{z_T}$$

Transformation of the effective potential

$$\tilde{z}_T(y) = z_T(y)\left(c_1 + c_2\int^y \frac{\mathrm{d}y'}{z_T^2(y)}\right)$$

⇒

Invariant

$$\frac{\tilde{z}''_T}{\tilde{z}_T} = \frac{z''_T}{z_T}$$

⇒

Unaffected

$$q_{ij}$$

$$\begin{cases} h_{ij} = q_{ij}/z_T \\ \tilde{h}_{ij} = q_{ij}/\tilde{z}_T \end{cases} \Rightarrow \tilde{h}_{ij} = \left(\frac{z_T}{\tilde{z}_T}\right)h_{ij}$$

Tensor Dual

$$P_{\tilde{h}} = \left(\frac{z_T}{\tilde{z}_T}\right)^2 P_h$$

Tensor Powerspectrum

Amplification of Gravitational Waves

MM, O. Ozsoy, S. Parameswaran,
G. Tasinato, I. Zavala (2019)

QuadraticAction for Tensors

$$S_T^{(2)} = \frac{1}{2}\int \mathrm{d}y\, \mathrm{d}^3x\, z_T^2(y)\left[(\partial_y h_{ij})^2 - (\nabla h_{ij})^2\right], \quad z_T^2 = \frac{a^2}{4}\sqrt{\mathcal{F}_T \mathcal{G}_T}$$

Horndeski parameters

$$\mathcal{G}_T = g_{t_i}\frac{\dot\phi^2}{H_0^2} \quad \text{and} \quad \mathcal{F}_T = f_{t_i}\frac{\dot\phi^2}{H_0^2}$$

Effective Potential

$$z_T = -a(t)\frac{\alpha_1 H_0 (f_{t1} g_{t1})^{\frac{1}{4}}}{6(-\rho_1 + \delta_1 + \beta_1 + \sigma_1)}, \quad t < t_0$$

$$\tilde z_T = -\frac{1}{a^2(t)}\frac{\alpha_1 H_0 (f_{t2} g_{t2})^{\frac{1}{4}}}{6(-\rho_1 + \delta_1 + \beta_1 + \sigma_1)}, \quad t > t_0$$

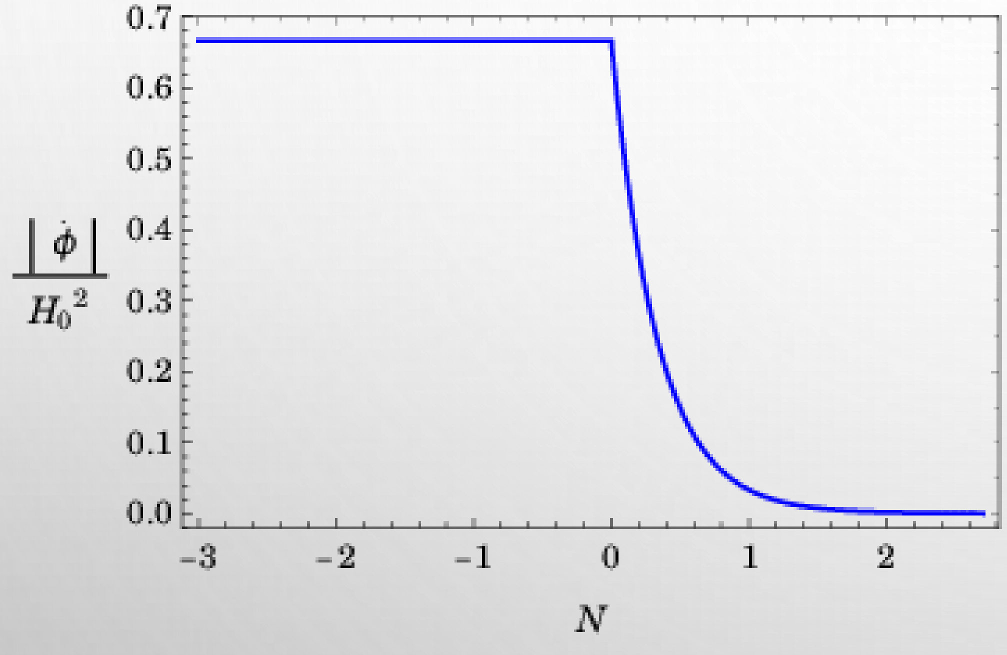
Power spectrum Enhancement

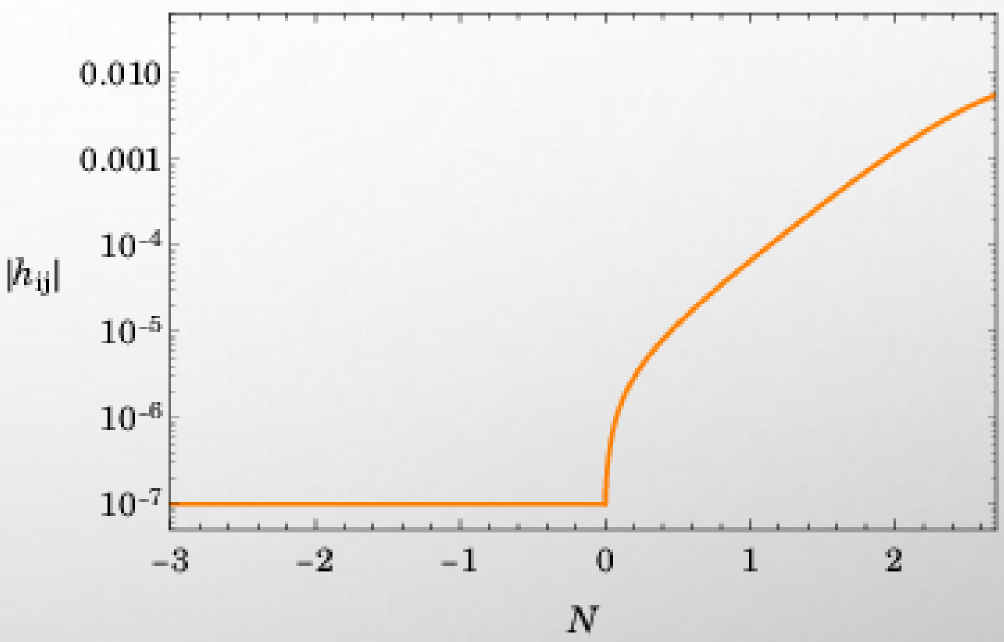
$$\left.\frac{P_{\tilde h}}{P_h}\right|_{t=t_1} = \left.\left(\frac{z_T}{\tilde z_T}\right)^2\right|_{t=t_1} = a(t_1)^6\left(\frac{\beta_1(\beta_1 + 3\sigma_1)}{(\beta_2 + 3\sigma_2)(\beta_2 + 9\sigma_2)}\right)^{\frac{1}{2}}$$

Scale-Invariant!

Amplification of Gravitational Waves

MM, O. Ozsoy, S. Parameswaran,
G. Tasinato, I. Zavala (2019)



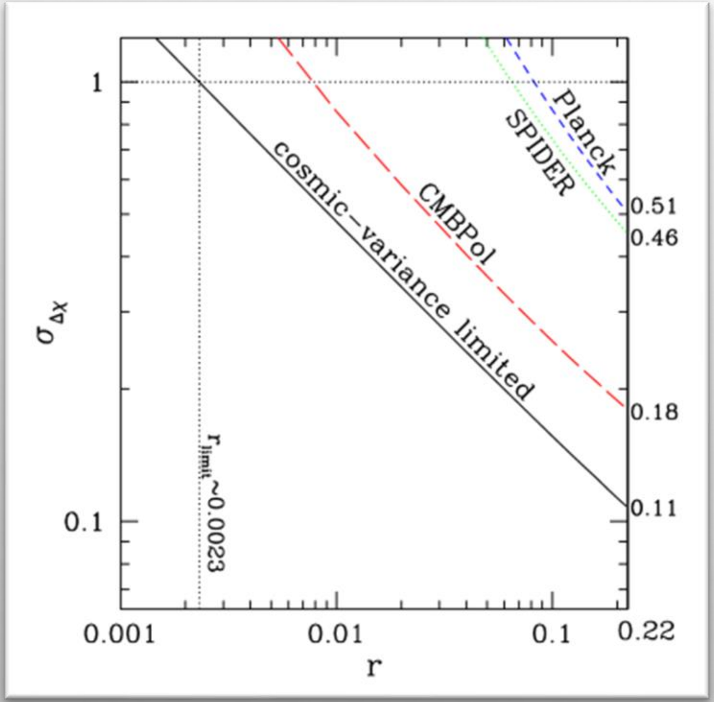


Parity Violation in Gravity

<u>Circular polarization degree</u>

$$\Pi = \frac{\mathcal{P}_h^L - \mathcal{P}_h^R}{\mathcal{P}_h^L + \mathcal{P}_h^R}, \quad \mathcal{P}_h^L(k) \neq \mathcal{P}_h^R(k), \quad -1 \leq \Pi \leq 1$$

Saito, Ichiki Taruya (2007)



Gluscevic, Kamionkowski (2010)

Maximal parity violation $|\Pi| = 1$, at 1σ

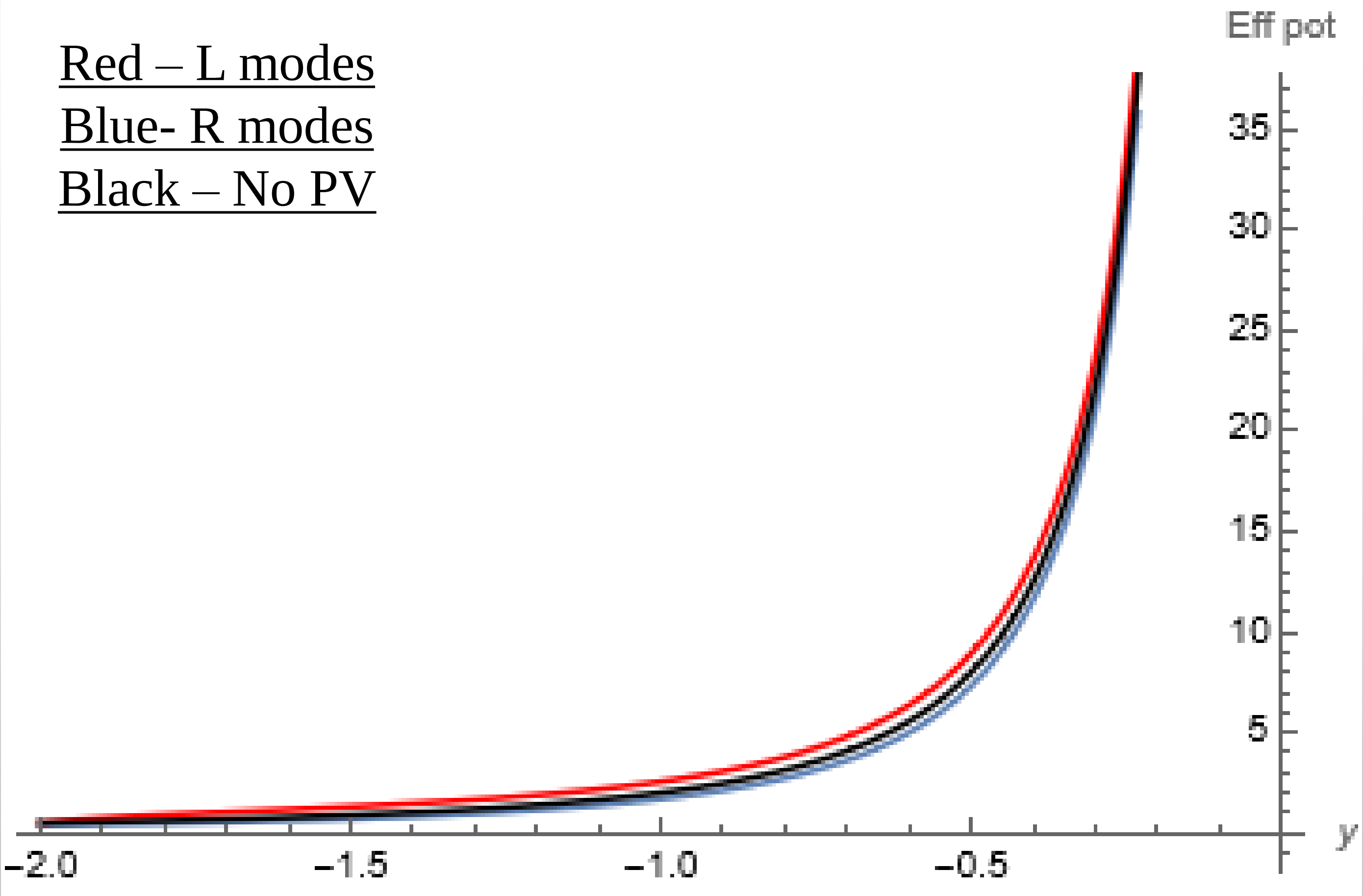
SPIDER $r \simeq 0.064$

PLANCK $r \simeq 0.082$

CMBpol $r \simeq 0.0079$

CV limited $r \simeq 0.0023$

Parity Violation in Gravity



Chern Simons Instability

Action: Einstein-Hilbert + CS

$$S^{(0)} = \frac{M_{Pl}^2}{2} \int \mathrm{d}^4x \left[\sqrt{-g} R + \frac{f_2(\phi)}{\Lambda^2} \epsilon^{\mu\nu\rho\sigma} W_{\mu\nu\kappa\lambda} W^{\kappa\lambda}{}_{\rho\sigma} \right]$$

Alexander, Martin (2004)

Metric perturbation

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

FRW background

$$\mathrm{d}s^2 = a(\eta)^2 [-\mathrm{d}\eta^2 + \mathrm{d}x^2]$$

Amplitudes

$$\mu_{\mathbf{k}}^s(\eta) = \frac{M_{Pl}}{2} h_{\mathbf{k}}^s(\eta) z_{\mathbf{k}}^s(\eta)$$

Evolution equations

$$(\mu_{\mathbf{k}}^s)'' + \left[k^2 - \frac{(z_{\mathbf{k}}^s)''}{z_{\mathbf{k}}^s} \right] \mu_{\mathbf{k}}^s = 0$$

Effective potential

$$z_{\mathbf{k}}^s(\eta) = a\sqrt{1 - \frac{\lambda^s k f_2'}{a^2 \Lambda^2}} = a\sqrt{1 - \frac{\lambda^s k_{ph}}{M_{cs}}}$$

$s = L, R$

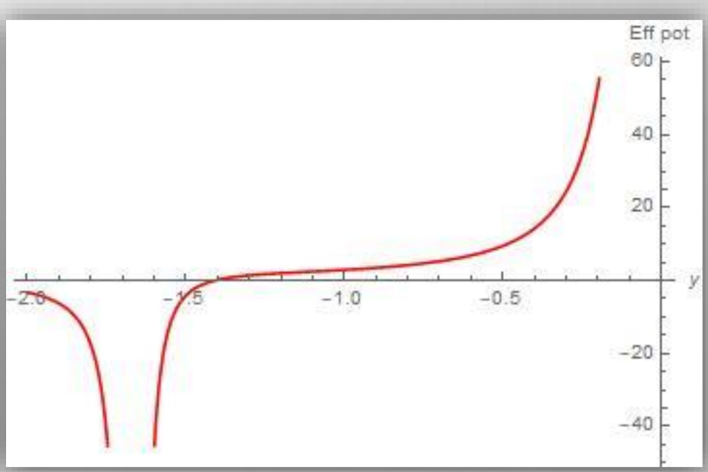
$\lambda^s = \pm 1$

Dyda, Flanagan, Kamionkowski (2012)

CS Instability

$$(z_{\mathbf{k}}^s)^2 = 0$$

$$k_{ph} = M_{cs}$$



EFT of Scalar-Tensor gravity

Extended action:

$$S^{(0)} = \sum_{i=2}^{5} \int \mathrm{d}^4x \sqrt{-g}\mathcal{L}_i(\phi, X) + S^{(0)}_{pp} + S^{(0)}_{pv}$$

$$X = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi$$

Leading-order

Kobayashi, Yamaguchi, Yokoyama (2011)

Higher-order corrections

NLO: Weinberg (2008)

NNLO pv: Chrisostomi, Noui, Charmousis (2018)

NNLO pp: Solomon, Trodden (2018)

Partial extension*:

$$S^{(0)}_{pp} = \frac{M^2_{Pl}}{2}\int \mathrm{d}^4x\sqrt{-g}\left\{\frac{f_1}{\Lambda^2}W^{\mu\nu\rho\sigma}W_{\mu\nu\rho\sigma} + \frac{b_1}{\Lambda^4}W_{\mu\nu\rho\sigma}R^{\nu\sigma}\nabla^\mu\phi\nabla^\rho\phi + \frac{b_2}{\Lambda^4}W_{\mu\nu\rho\sigma}\nabla^\mu\nabla^\rho\phi\nabla^\nu\nabla^\sigma\phi\right\}$$

$$S^{(0)}_{pv} = \frac{M^2_{Pl}}{2}\int \mathrm{d}^4x\,\epsilon^{\mu\nu\rho\sigma}\left\{\frac{f_2}{\Lambda^2}W_{\mu\nu\kappa\lambda}W^{\kappa\lambda}{}_{\rho\sigma} + \frac{d_1}{\Lambda^4}W_{\rho\sigma\kappa\lambda}R^{\lambda}{}_{\nu}\nabla^\kappa\phi\nabla^\mu\phi\right\}$$

*Excluding contributions of the form $\nabla_\alpha W_{\mu\nu\rho\sigma}\nabla^\alpha W^{\mu\nu\rho\sigma}$, W^3.

Parity Violation in Gravity

- **Topological term – switches on when tits coupling is time-dependent**

$$S^{(2)}_{\tilde{W}W} = -\frac{M_{Pl}^2}{8}\int \mathrm{d}^4x \frac{f'(\phi)}{\Lambda^2} 8\epsilon^{ijk0}\left[h'_{qj;i}h'_{qk} - h_{qi;r}h_{qk;jr}\right]$$

Fourier Space

$$h_{ij}(\mathbf{x},\tau) = \frac{1}{(2\pi)^{\frac{3}{2}}}\int \mathrm{d}\mathbf{k} \sum_{s=R,L} p^s_{ij}(\mathbf{k}) h^s(\mathbf{k},\tau) e^{i\mathbf{k}\cdot\mathbf{x}}$$

Tensor Polarizations

$$p^R_{ij} \equiv \frac{1}{\sqrt{2}}(p^+_{ij} + ip^\times_{ij}), \quad \text{and} \quad p^L_{ij} \equiv \frac{1}{\sqrt{2}}(p^+_{ij} - ip^\times_{ij}) = (p^R_{ij})^*$$

Transversality Traceless conditions

$$p^s_{ij}k_j = 0, \quad \text{and} \quad (p^i_i)^s = 0, \quad s = L, R$$

Identity

$$i\frac{k_p}{k}\epsilon^{pjk}p_{ik} = -\lambda^s (p^j_i)^s, \quad \lambda^s = \pm 1$$

Parity Violation in Gravity

EOM

$$(\mu_{\mathbf{k}}^s)'' + \left(k^2 c_T^2(\tau) - \frac{z_s''}{z_s}\right)\mu_{\mathbf{k}}^s = 0$$

Effective Potential

$$\frac{z_s''}{z_s} = \frac{a''}{a} - \mathcal{H}\lambda^s k \frac{\left(\frac{f'}{a^2 G M_{Pl}^2}\right)'}{1 - \lambda^s k\left(\frac{f'}{a^2 G M_{Pl}^2}\right)} - \frac{\lambda^s k}{2}\frac{\left(\frac{f'}{a^2 G M_{Pl}^2}\right)''}{1 - \lambda^s k\left(\frac{f'}{a^2 G M_{Pl}^2}\right)} - \frac{1}{4}(\lambda^s k)^2 \frac{\left[\left(\frac{f'}{a^2 G M_{Pl}^2}\right)'\right]^2}{\left[1 - \lambda^s k\left(\frac{f'}{a^2 G M_{Pl}^2}\right)\right]^2}$$

$$z_s(\tau, \mathbf{k}) \equiv a(\tau)\sqrt{1 - \frac{k\lambda^s f'}{a^2 G M_{Pl}^2}} \qquad G(\tau) = \tilde{g}_{t_2}\frac{(\phi')^2}{H_0^2 a^2} \qquad G(\tau) = \tilde{g}_{t_2}\frac{(\phi')^2}{H_0^2 a^2}$$

$$f'(\phi) = f_0\phi' \simeq -f_0 \frac{M_{Pl}\tilde{\alpha}_1}{3(-\tilde{\rho}_1 + \tilde{\delta}_1 + \tilde{\beta}_1 + \tilde{\sigma}_1)a(t)^2}, \quad t \geq t_0$$

Preliminary Results

First phase – Whittaker solutions

$$\mu_k^s = (-2c_1 k\tau)^{\frac{3}{2}}\sqrt{-\tau}e^{-\frac{\pi\lambda^s F}{4c_1}}U\left(2-\frac{i\lambda^s F}{2c_1}, 4, 2ic_1 k\tau\right), \quad t \leq t_0$$

Second phase – A complete mess of trigonometric functions!

❖ **Difficult to disentangle scale-dependence**

- Instead match the asymptotic expansion of the solutions
- Keep the most dominant term

Possible Constraints

$$a(\tau_1)^2 < \frac{Af_0 g_{t_2} M_{Pl}^3}{k} = \frac{f_0 M_{pl}\tilde{\alpha}_1(\tilde{\beta}_2 + 3\tilde{\sigma}_2)}{18(\tilde{\beta}_1 + \tilde{\delta}_1 - \tilde{\rho}_1 + \tilde{\sigma}_1)}\frac{1}{k}$$

And after some brute force….

$$P_{\tilde{c}\tilde{s}}\big|_{t=t_1} \sim a(\tau_1)^4 P_{cs}, \quad P_{cs} \sim \frac{1}{(c_1)^3}e^{-\frac{\pi\lambda F}{2c_1}}$$

Thank you for your attention!