17-21 April 2017
Cosmological Quests for the Next Decade
KASI, Daejeon

Challenges in perturbation theory calculation of large-scale structure

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Plan of Talk

Perturbation theory of large-scale structure as a precision cosmological tool: limitation and beyond

- UV problem in perturbation theory
- Response function: characterizing nonlinear mode coupling
- Post-collapse PT: new perturbative description beyond shell-crossing in 1D cosmology

Collaborators

F. Bernardeau, S. Colombi (IAP), A. Halle (MPA), I. Hashimoto (YITP), T. Nishimichi (Kavli IPMU), Y. Rasera (Paris Observatory)

Large-scale structure

Matter inhomogeneity over Giga parsec scales

Provide a wealth of cosmological information

Is key observations in post-Planck precision cosmology

Main focuses:

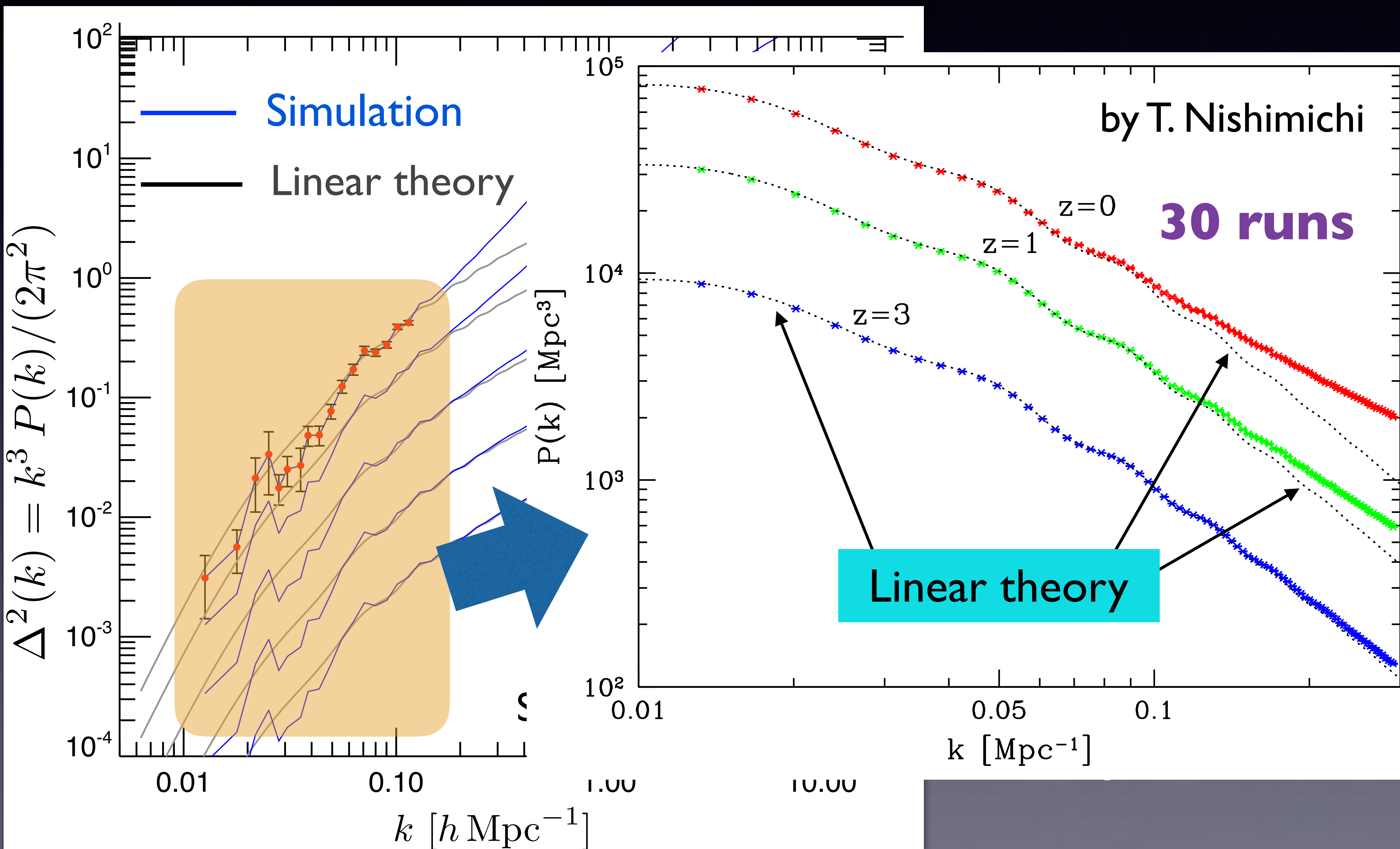
BAO (baryon acoustic oscillations)	Dark energy
RSD (redshift-space distortions)	Test of gravity
Free-streaming damping due to massive-ν	

Need an accurate theoretical description (e.g., for template)

Regime of our interest : $k<0.2$–0.3 h/Mpc at $z\sim0.5\sim1.5$

→ weakly nonlinear regime of gravitational evolution

Power spectrum in simulations



Perturbation theory (PT): reloaded

Single-stream approx. of Vlasov-Poisson system

CDM + baryon → pressureless & irrotational fluid

Basic eqs.

$$\frac{\partial \delta}{\partial t} + \frac{1}{a}\vec{\nabla}\cdot\left[(1+\delta)\vec{v}\right] = 0$$

$$\frac{\partial \vec{v}}{\partial t} + \frac{\dot{a}}{a}\vec{v} + \frac{1}{a}(\vec{v}\cdot\vec{\nabla})\vec{v} = -\frac{1}{a}\vec{\nabla}\Phi$$

$$\frac{1}{a^2}\nabla^2\Phi = 4\pi G\,\overline{\rho}_{\rm m}\,\delta$$

Juszkiewicz ('81), Vishniac ('83), Goroff et al. ('86), Suto & Sasaki ('91), Makino, Sasaki & Suto ('92), Jain & Bertschinger ('94), …

Standard PT $(\delta_1 \ll 1)$

$$\delta = \delta_1 + \delta_2 + \delta_3 + \cdots$$

<u>*Recent progress*</u>

- Improving accuracy by resummation or renormalized PT treatment
- Higher-order calculation & fast PT code (*RegPT*)

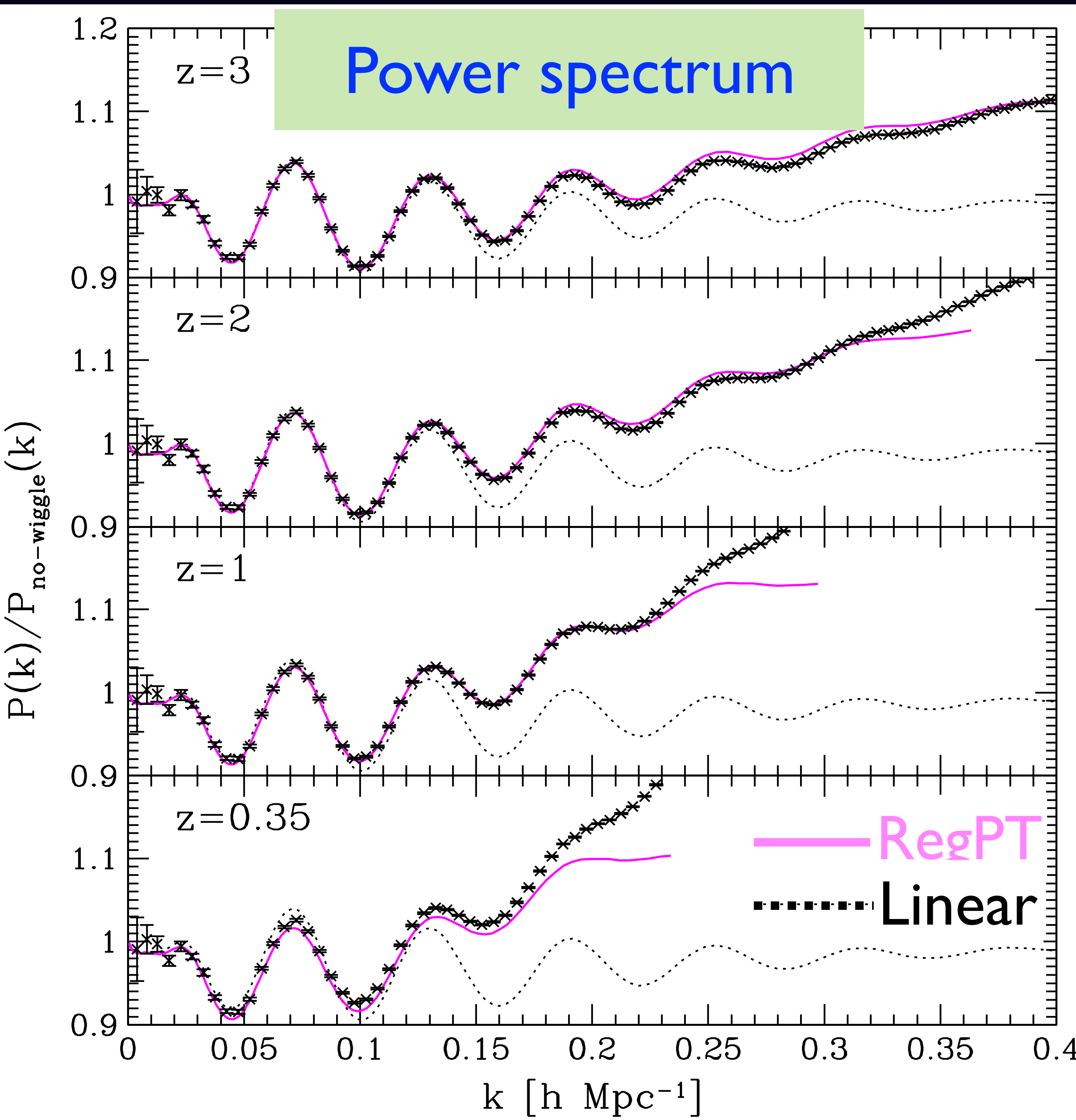
2-loop (next-to-next-to leading order)

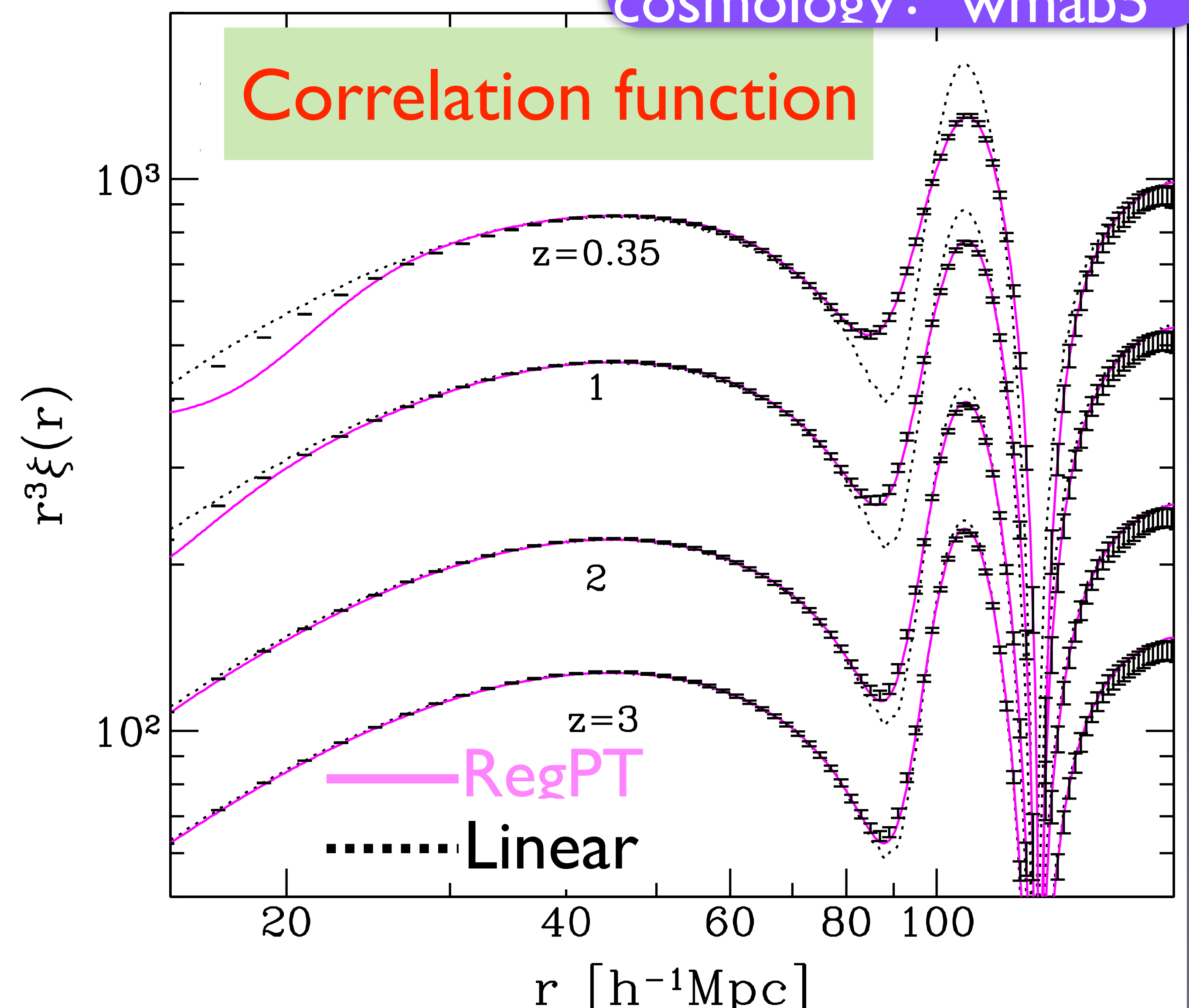
- Incorporating other systematics (massive ν, modified gravity, halo bias,…)

Performance of resummed PT

RegPT fast resummed PT code (http://ascl.net/1404.012)
including 2-loop (*next-to-next-to-leading*) *order*

$L_{\rm box} = 2,048\,h^{-1}\,{\rm Mpc}$
# of particles : $1,024^3$
# of runs : 60
cosmology : WMAP5

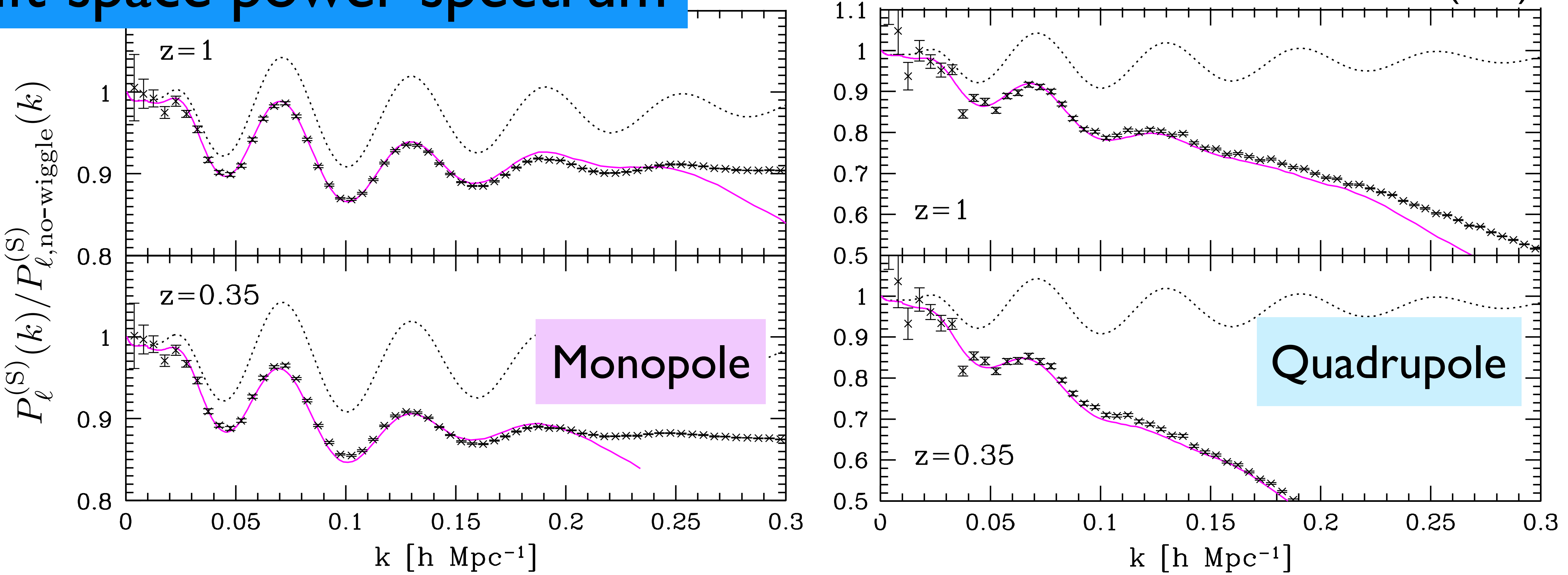




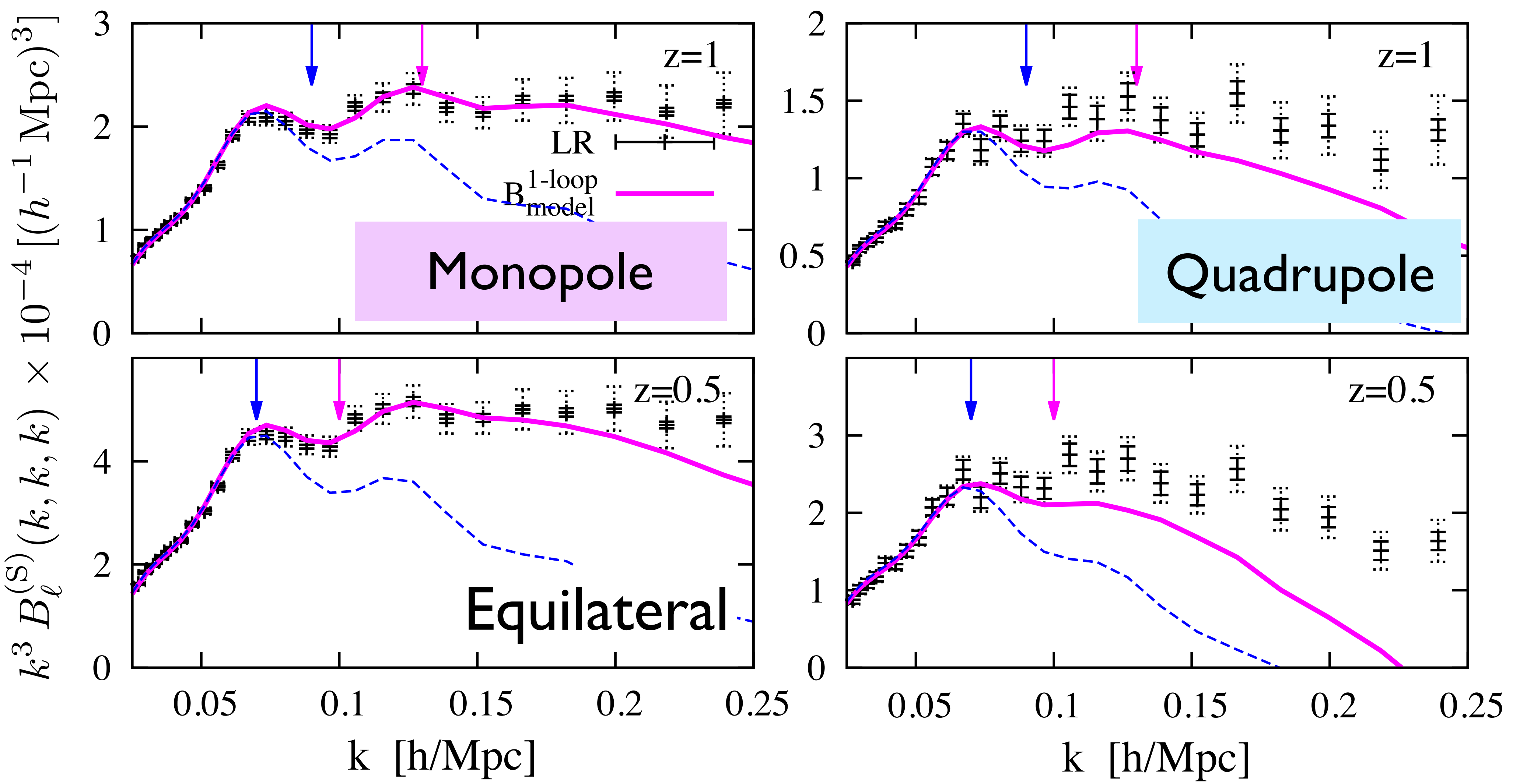
AT, Bernardeau, Nishimichi & Codis ('12)

Redshift-space power spectrum

AT, Nishimichi & Bernardeau ('13)



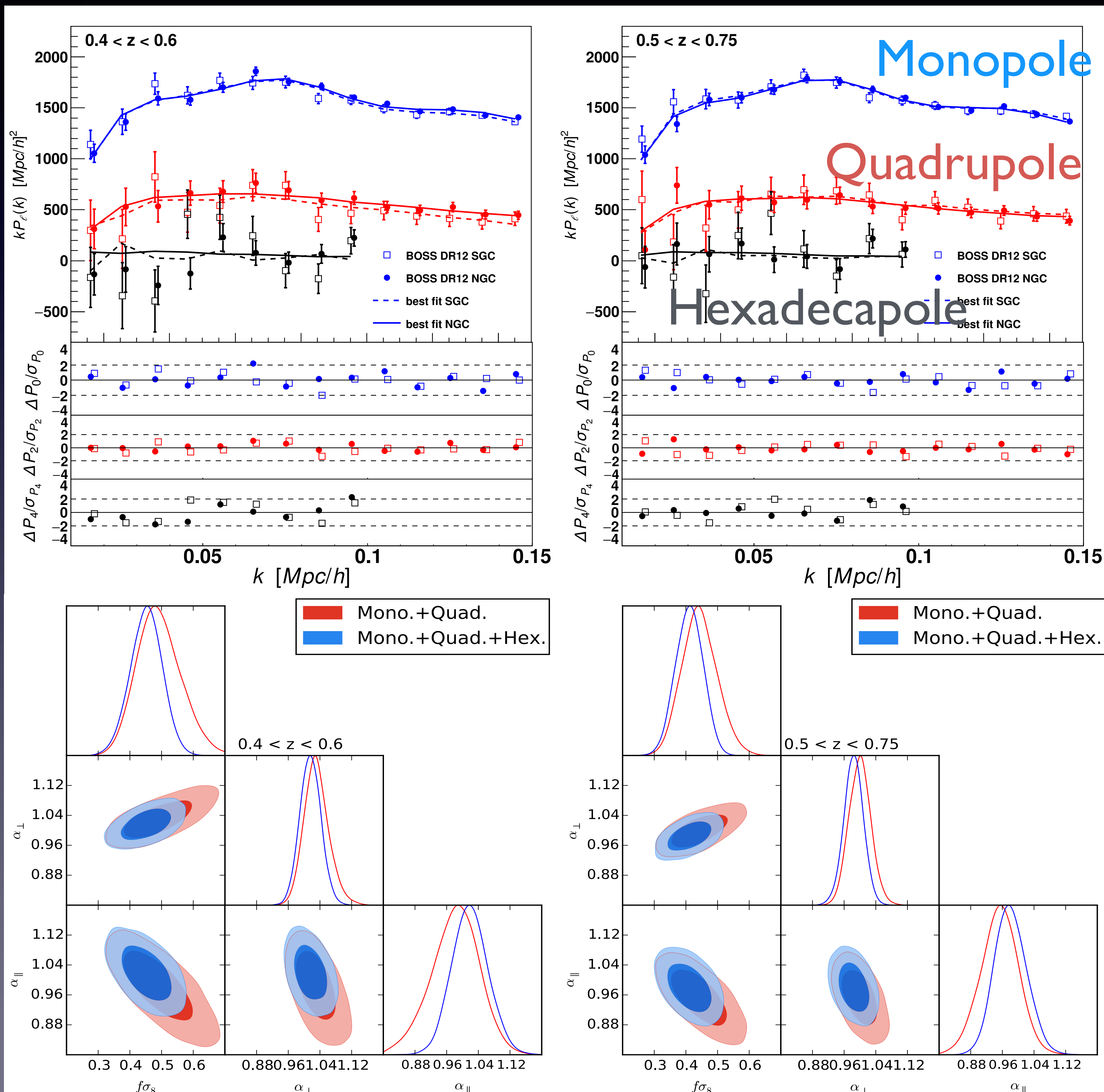
Redshift-space bispectrum



New !!

Hashimoto, Rasera & AT ('17)

Application to BAO/RSD



Beutler et al. ('17)

BOSS DR12

k<0.15 h/Mpc

Constraints on

- geometric distances

  $D_{\rm A}$ & $H^{-1}(z)$

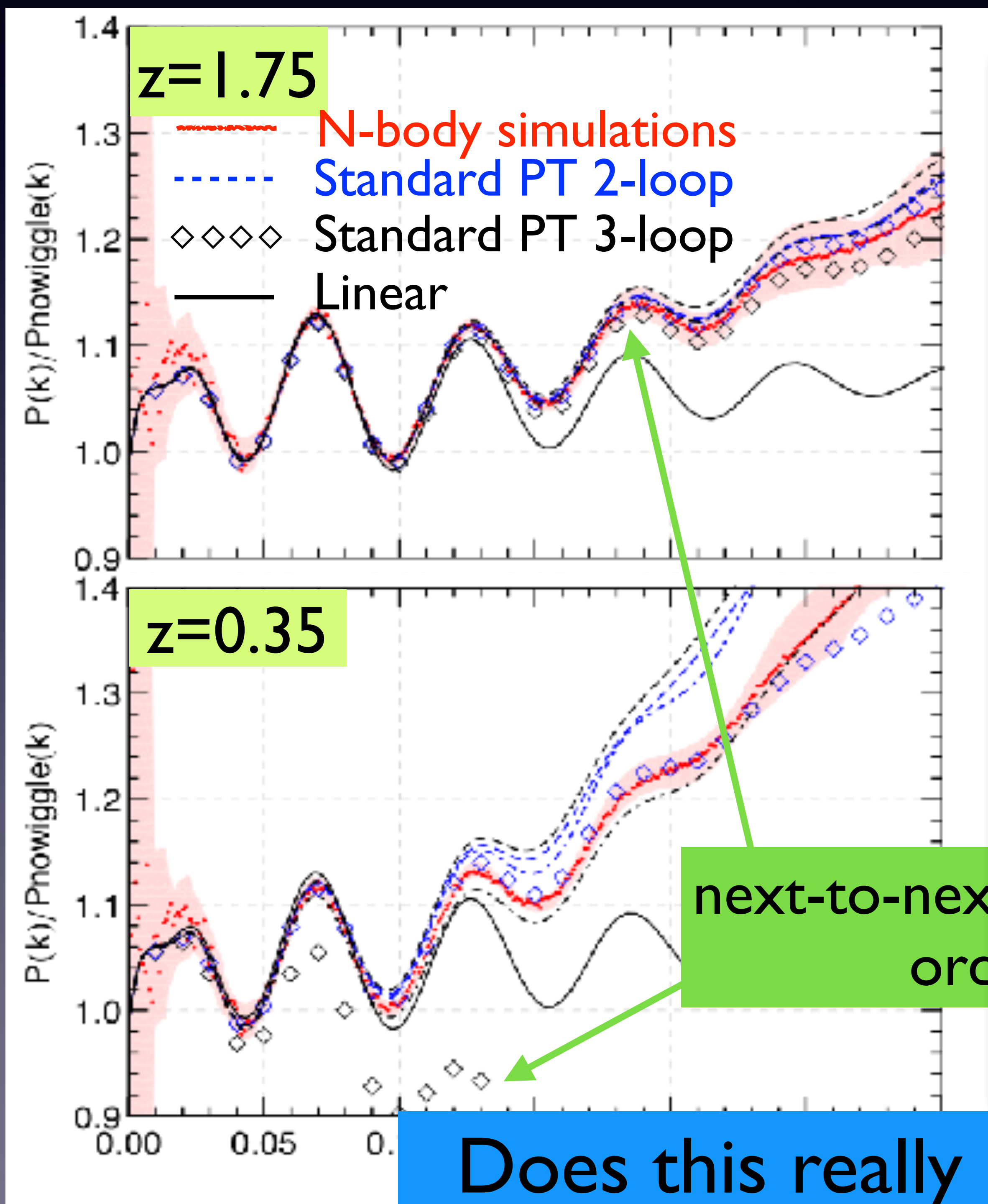
- growth rate

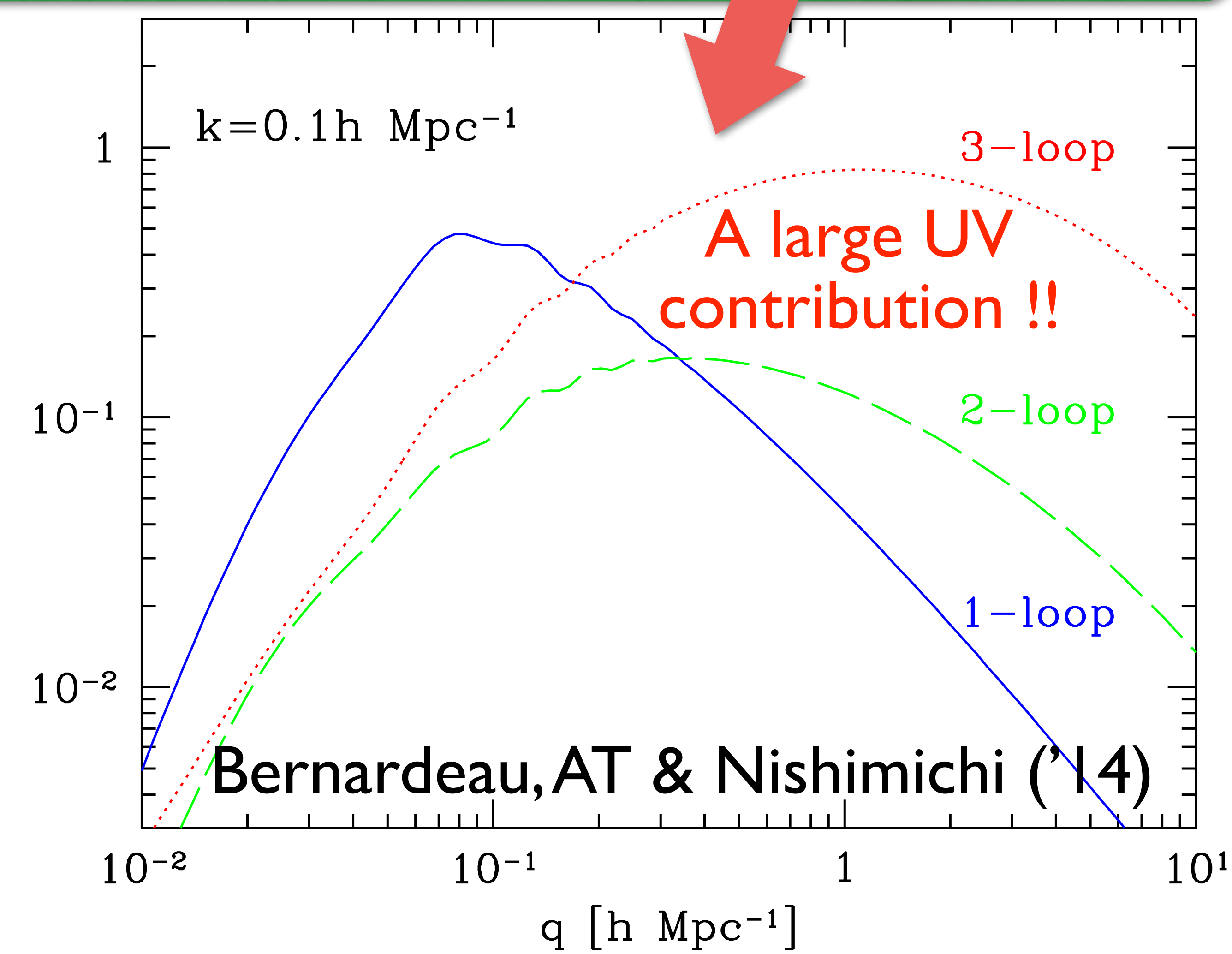
  $f(z)$

3-loop : source of trouble

Further including 3-loop (i.e., next-to-next-to-next-to-leading order), PT calculations start to get worse !!

$$P_{\text{n-loop}}(k) \propto \int d\ln q \; K_{\text{n-loop}}(k,q)\, P_0(q)$$





Does this really happen in real universe ?

Nature of nonlinear mode-coupling

How the small-scale fluctuations affect the evolution of large-scale modes ? (or vice versa)

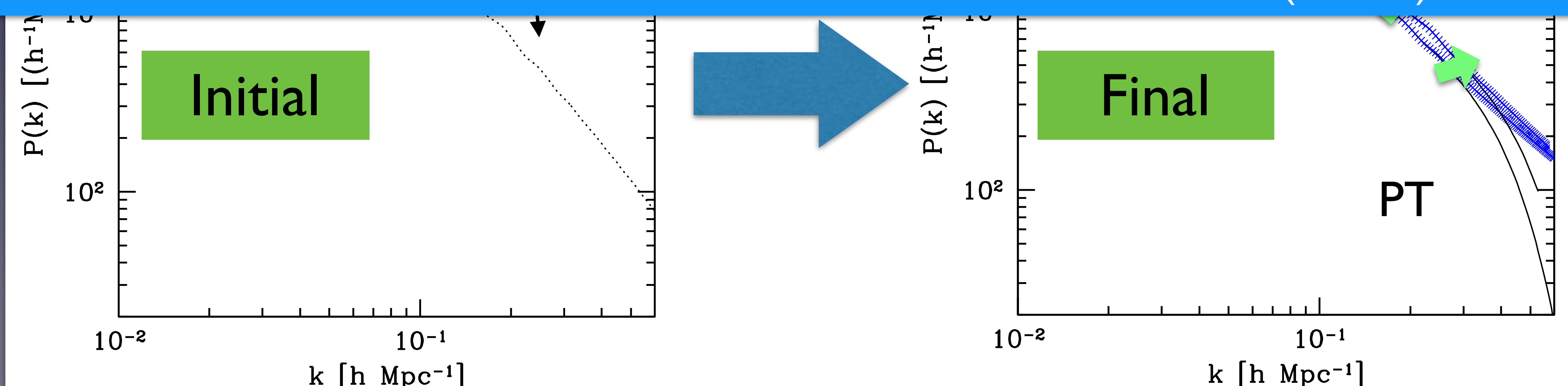
How the small disturbance added in initial power spectrum can contribute to each Fourier mode in final trum ?

Response function

$$\delta P_{\rm nl}(k) = \int d\ln q \, K(k,q) \, \delta P_0(q)$$

Final (nonlinear)

initial (linear)

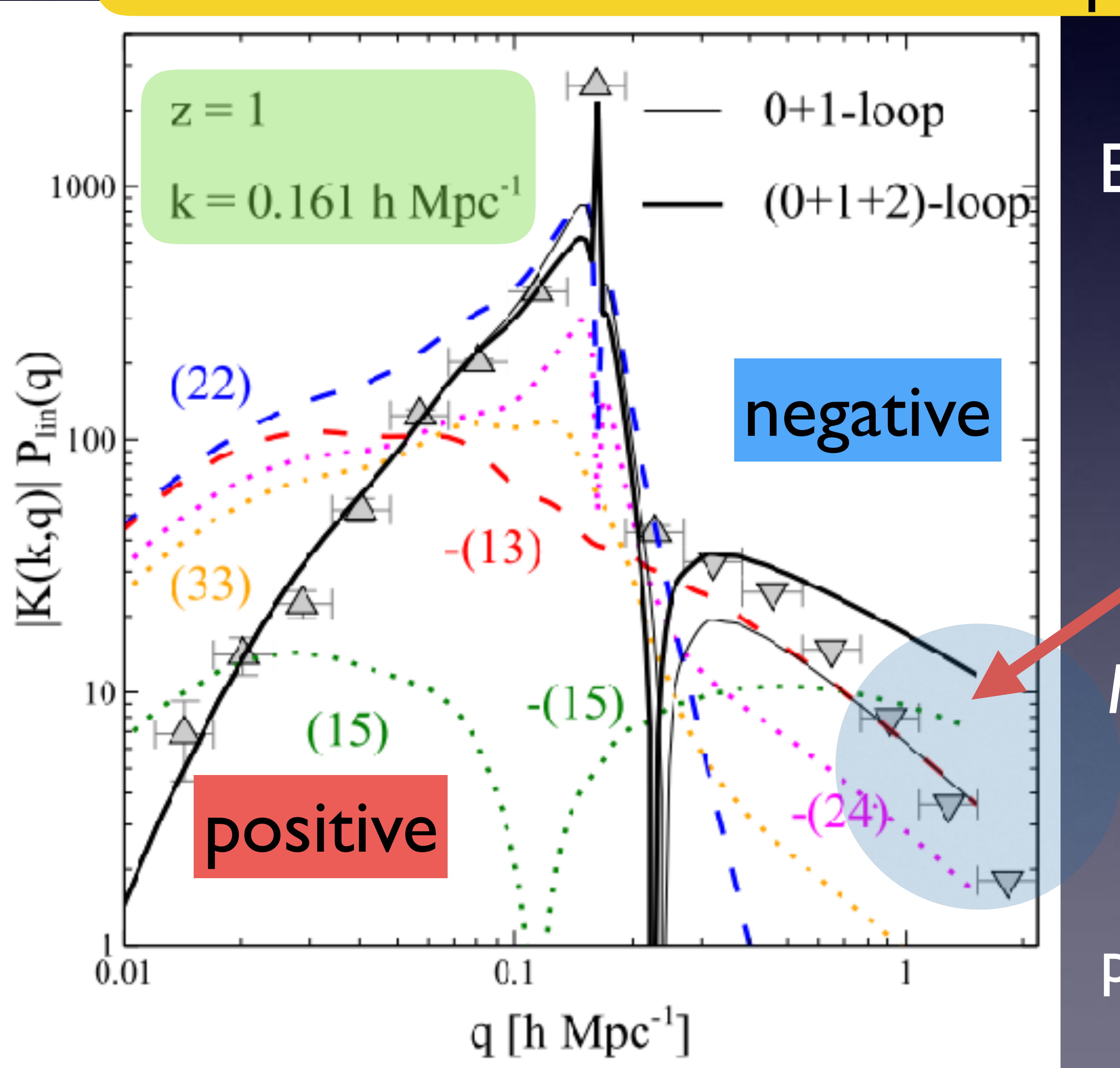


A measurement result

Nishimichi, Bernardeau & AT ('16)

Response of power spectrum at k to a small initial variation at q

$$K(k,q;z) = q\frac{\delta P^{\rm nl}(k;z)}{\delta P^{\rm lin}(q;z)}$$



Even for *low-k* modes,

Standard PT gets a *large UV contribution* *(q-modes)*:

2-loop > 1-loop > N-body

In other words,

low-k mode in simulation is *UV-insensitive*

protected against small-scale uncertainty

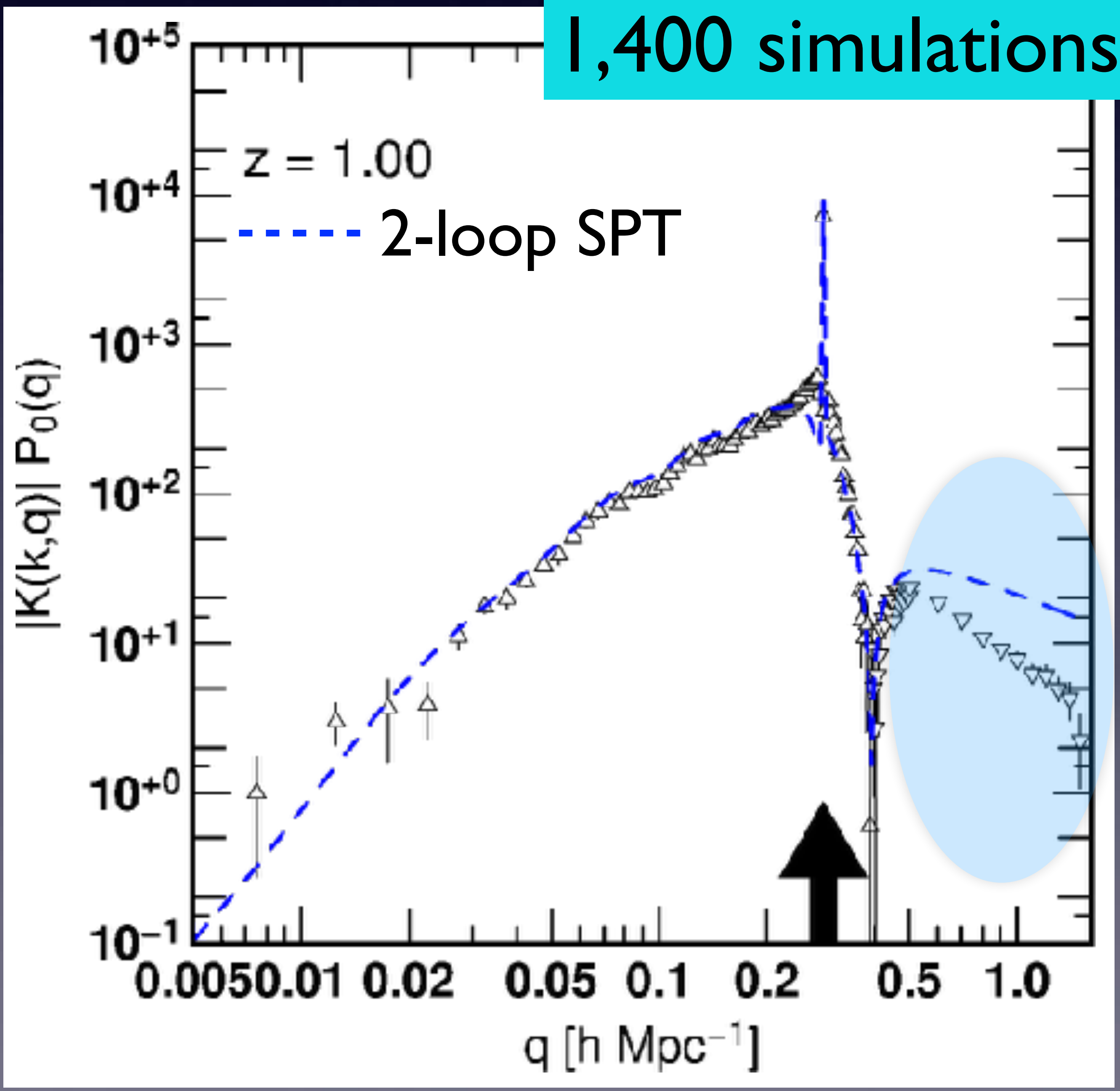
Refined measurement

Nishimichi, Bernardeau & AT ('16 &'17 in prep.)

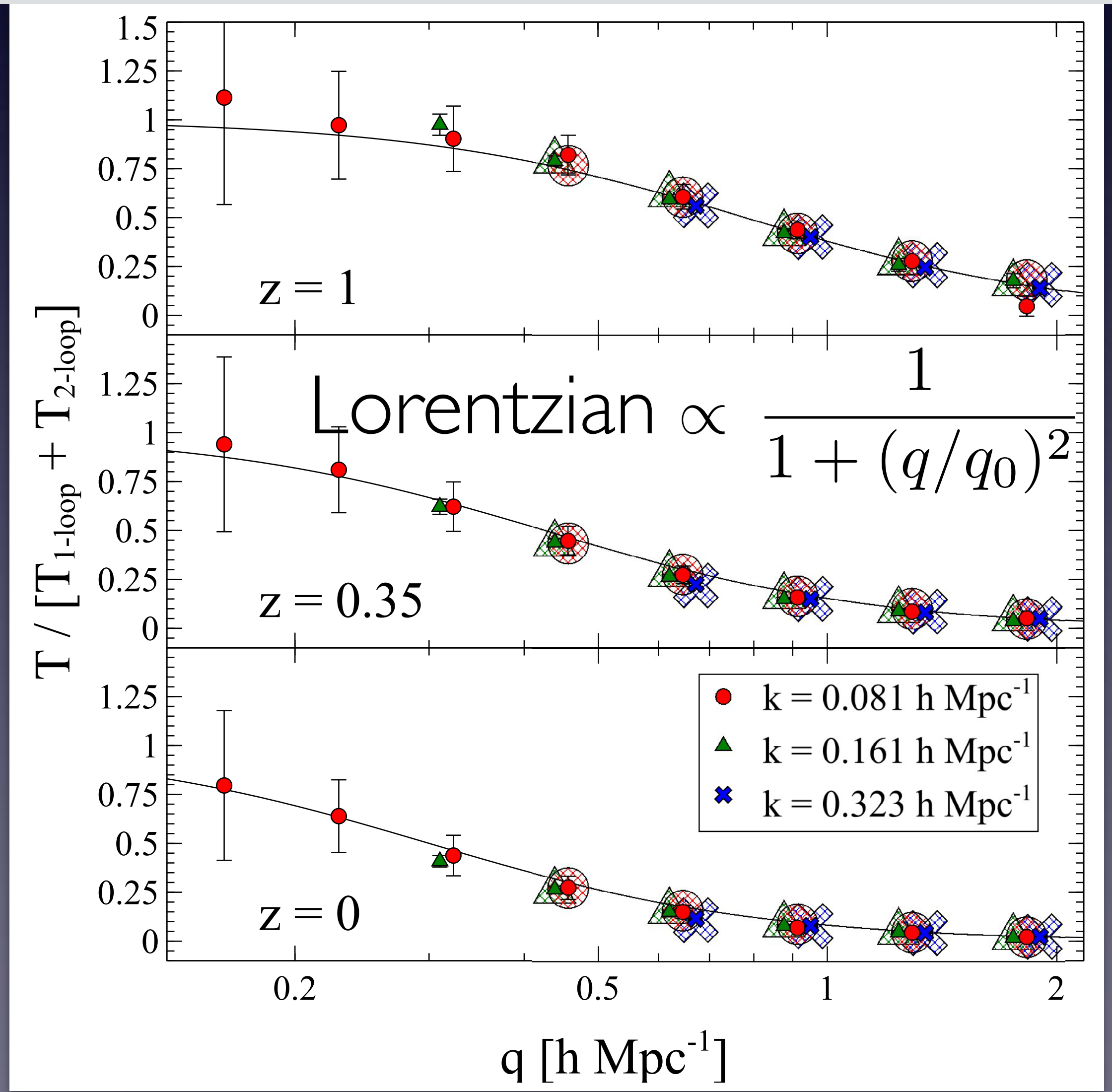
Response of power spectrum at k to a small initial variation at q

$$K(k,q;\,z) = q\frac{\delta P_{\rm nl}(k;\,z)}{\delta P_0(q;\,z)}$$

1,400 simulations



$$T(k,q) = [K(k,q) - K^{\rm lin}(k,q)]/[qP^{\rm lin}(k)]$$



UV suppression is seen at various k

What's wrong ?

Short summary

- Higher-order mode-coupling gets a larger UV contribution

Blas, Garny & Konstandin ('14), Bernardeau, AT & Nishimichi ('14)

However !

- In simulation, actual UV contribution is suppressed

Nishimichi, Bernardeau & AT ('16, '17 in prep.)

Most likely

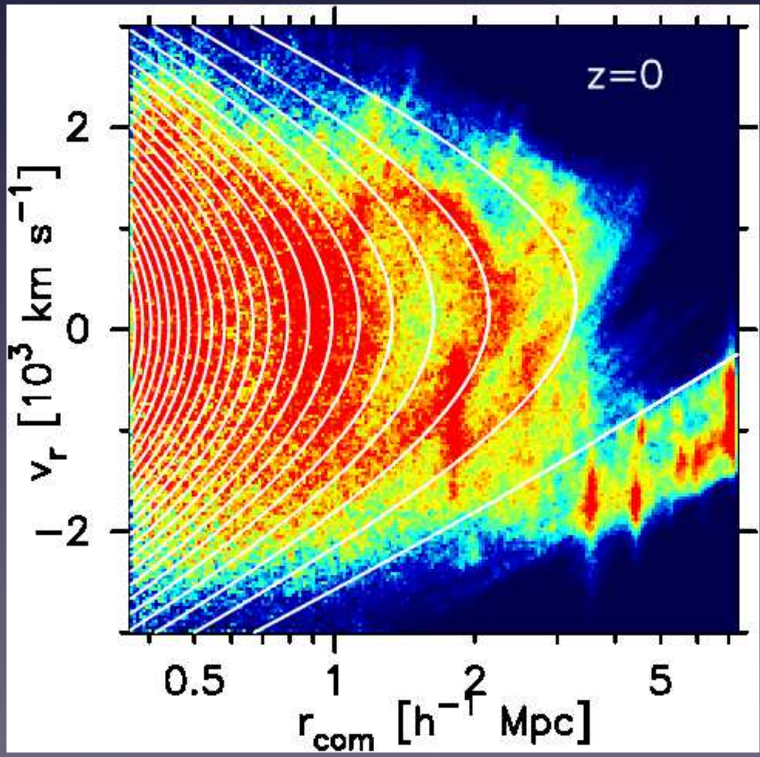
Breakdown of single-stream PT treatment (even at large scales)

What is a role of small-scale dynamics ?

Is there a way to go beyond single-stream PT ?

Multi-stream flows

(formation/merger of halos)

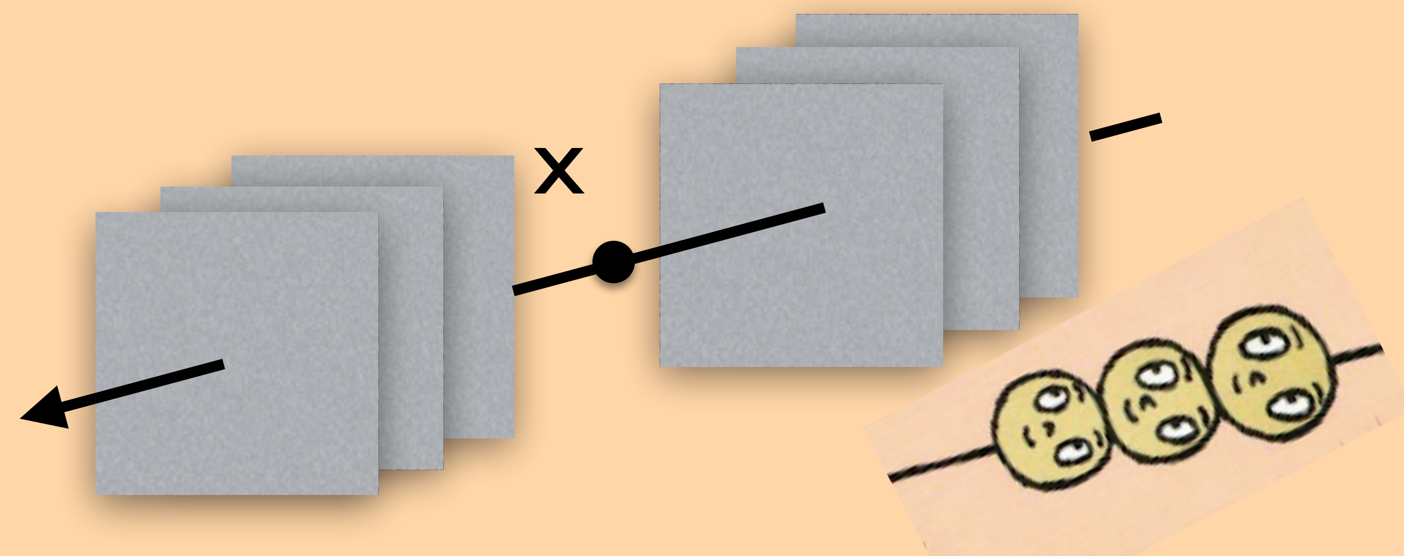


Suto et al. (2016)

1D cosmology

Simplification may help us to understand what's going on

$$\nabla_x^2 \phi(x) = 4\pi\, G\overline{\rho}\, a^2\, \delta(x)$$



Force $\propto$ (# of sheets at RHS) - (# of sheets at LHS)

- Generic features of nonlinear mode-coupling :

  Response function

- Perturbative description beyond shell-crossing: Post-collapse PT

Learn something in simple *1D cosmology*

1D Zel'dovich solution

(Zel'dovich '70)

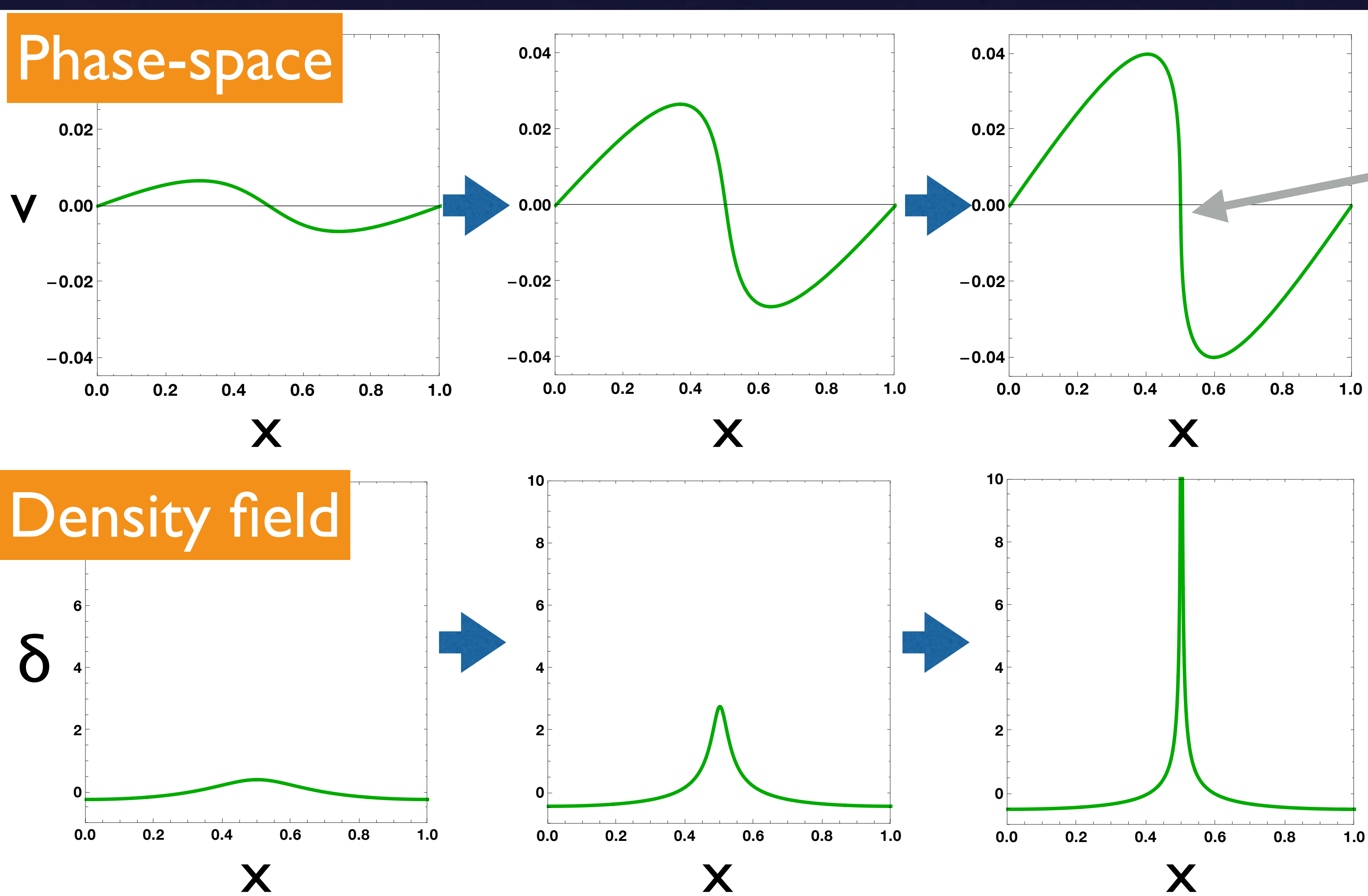
Exact single-stream solution

$$x(q;\tau) = q + \psi(q)\, D_+(\tau)$$

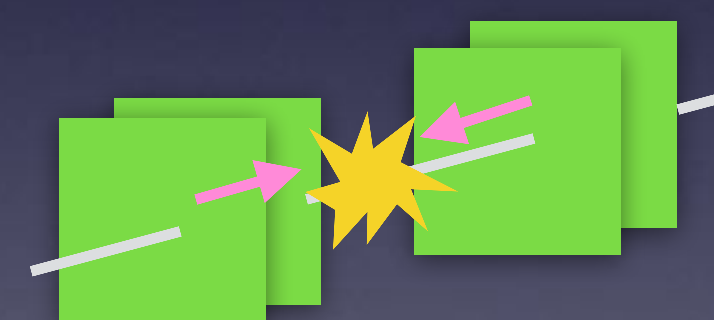
$$\mathrm{v}(q;\tau) = \psi(q)\, \frac{dD_+(\tau)}{d\tau}$$

$D_+(\tau)$: linear growth factor

$\psi(q)$: displacement field

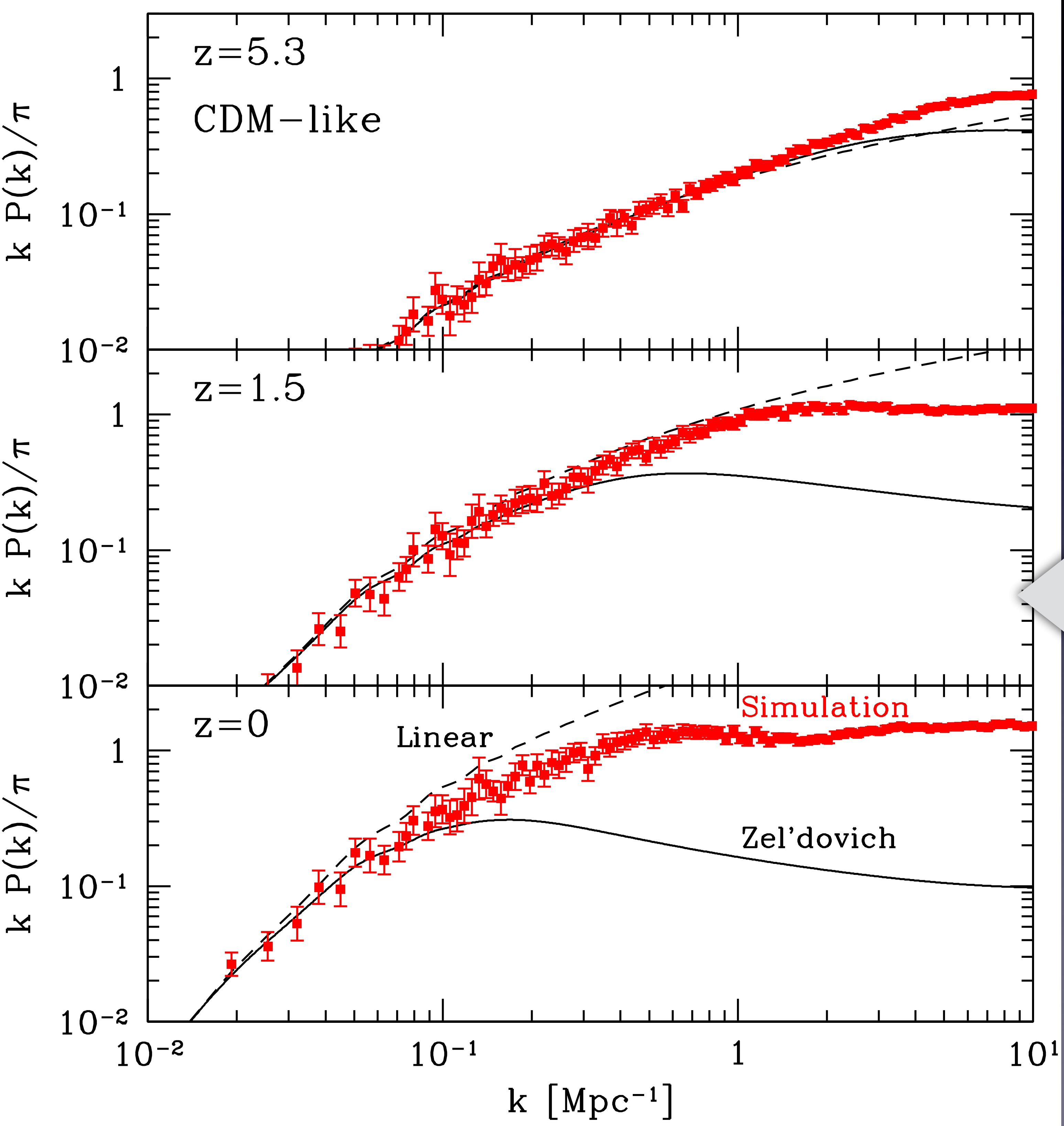


Shell crossing



Solution is exact until shell crossing

Power spectrum in 1D



Initial — Planck ΛCDM

$$P_{\rm 1D}(k) = \frac{k^2}{2\pi} P_{\rm 3D}(k)$$

Dimensionless initial power spectrum is the same as in 3D

manifestation of the limitation of single-stream treatment

$L = 1,000\,\mathrm{Mpc}$

# of particles (sheets) : $200,000$

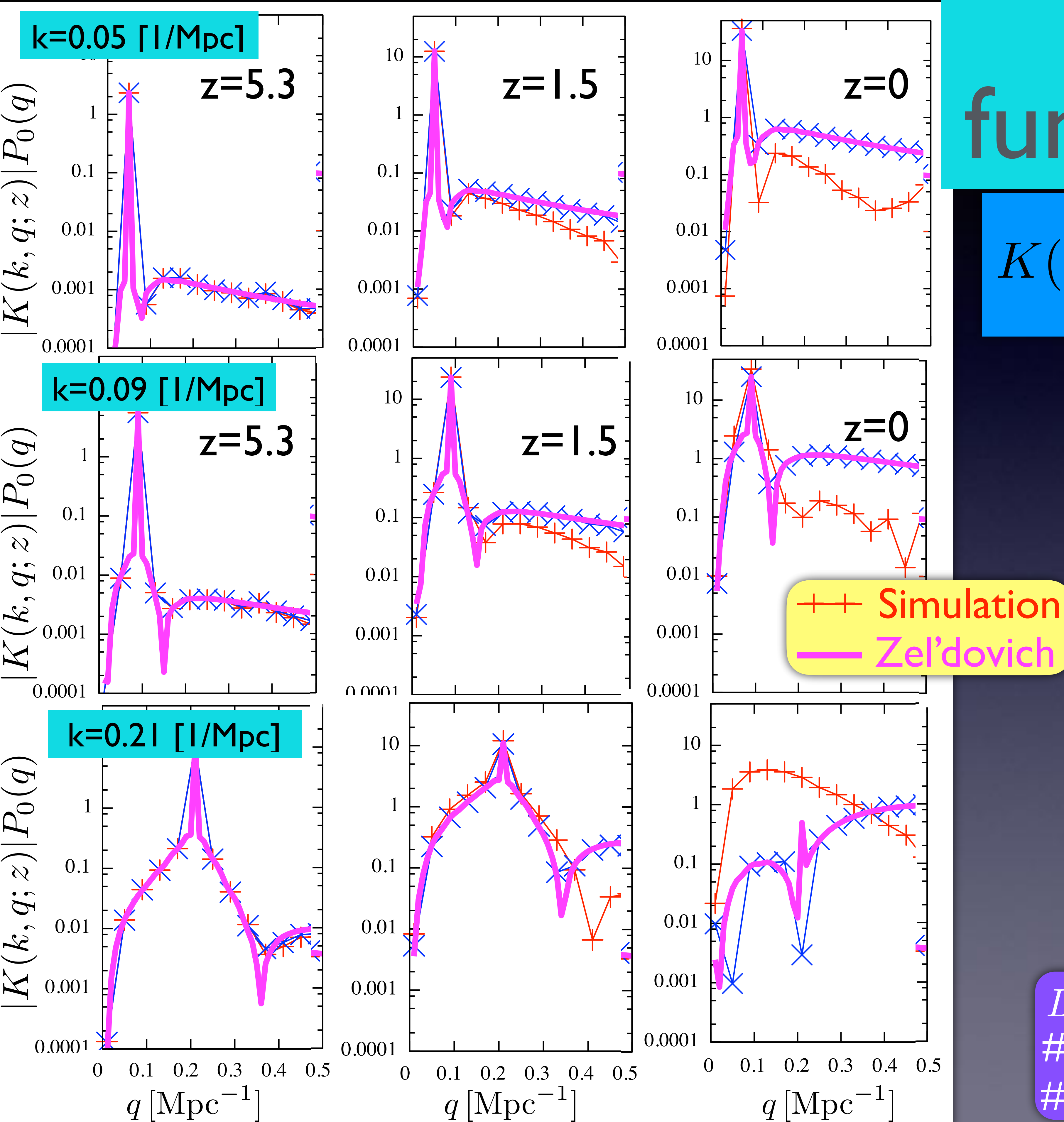
# of runs : 50

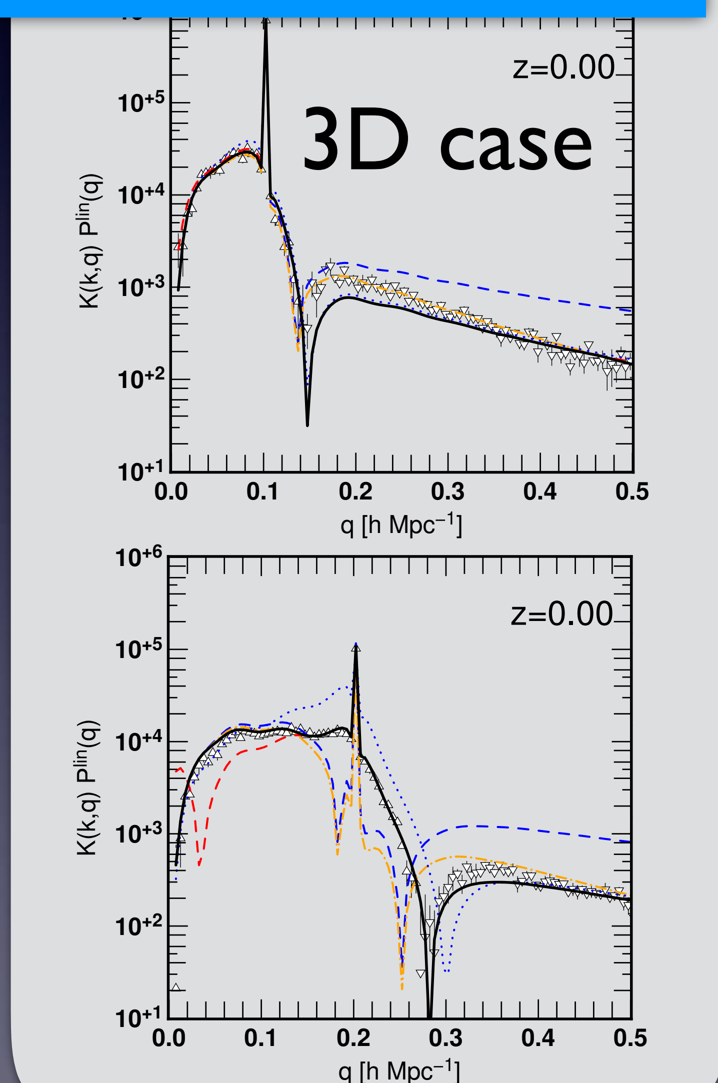
by Vlafroid (PM code)

http://www.vlasix.org/uploads/Main/froid1D.1.5.tar.gz w/ A. Halle, S. Colombi & T. Nishimichi (in progress)

Response function in 1D

$$K(k,q;\,z) = q\,\frac{\delta P_{\rm nl}(k;\,z)}{\delta P_0(q;\,z)}$$

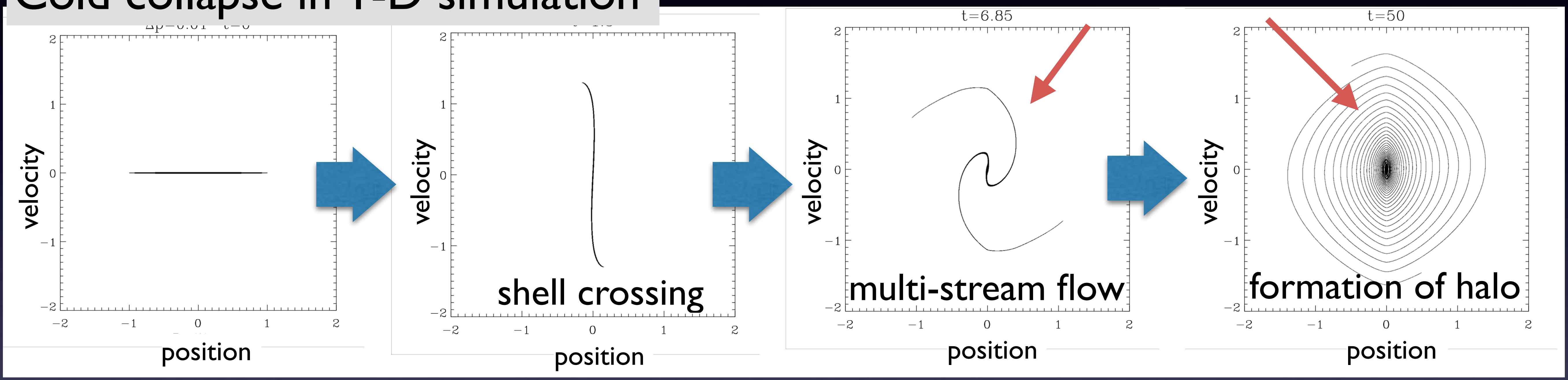




$L = 100,000\mathrm{Mpc}$
\# of particles (sheets) : 10^7
\# of runs : $2,000$ for each q-mode

Post-collapse PT: beyond shell-crossing

AT & Colombi ('17)



Computing back-reaction to the Zel'dovich flow: *Lagrangian*

1. Expand the displacement field around *shell-crossing point*, q_0:

$$x(q;\tau) \simeq A(q_0;\tau) - B(q_0;\tau)(q-q_0) + C(q_0;\tau)(q-q_0)^3$$

2. Compute force $F(x(q;\tau)) = -\nabla_x \Phi(x(q;\tau))$ at multi-stream region

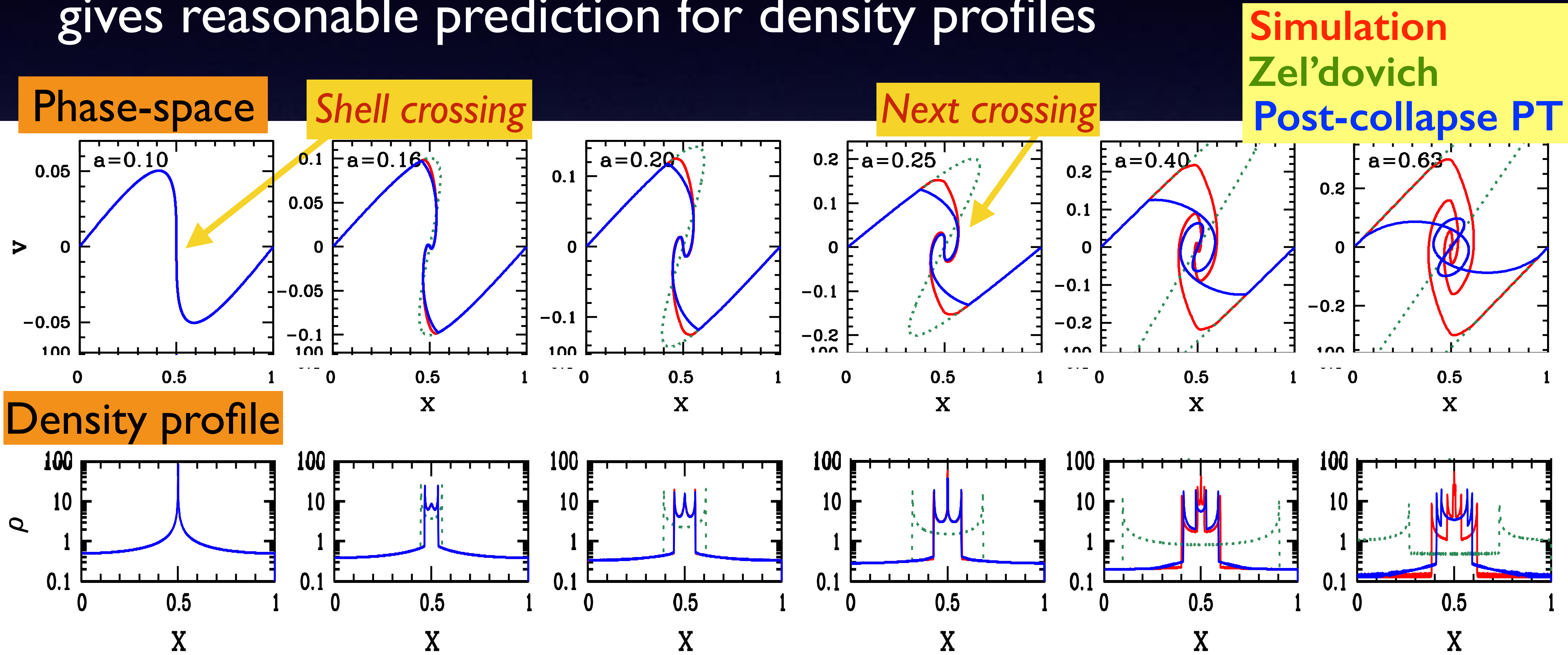
$$\Delta \mathrm{v}(Q;\tau,\,\tau_{\mathrm{q}}) = \int_{\tau_{\mathrm{q}}}^{\tau} d\tau'\,F(x(Q,\tau')), \quad \Delta x(Q;\tau,\,\tau_{\mathrm{q}}) = \int_{\tau_{\mathrm{q}}}^{\tau} d\tau'\,\Delta \mathrm{v}(Q;\tau',\tau_{\mathrm{q}})$$

…… polynomial function of $Q=q-q_0$ *up to 7th order*

Post-collapse PT: single cluster

AT & Colombi ('17)

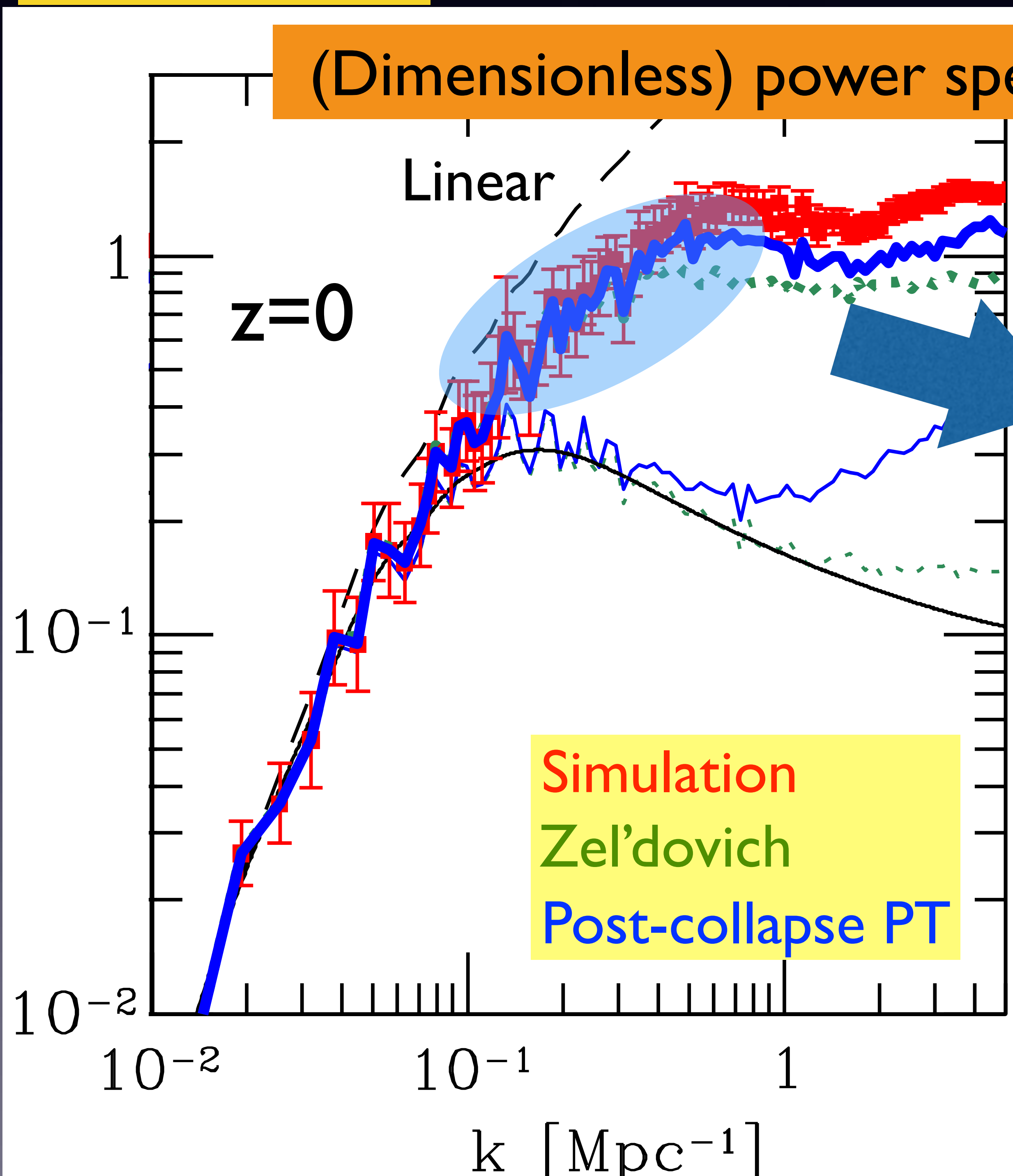
Post-collapse PT basically fails after next shell-crossing, but it still gives reasonable prediction for density profiles



Of course, this does not guarantee the accuracy of power spectrum prediction at small scales (→ next slide)

Post-collapse PT: ΛCDM

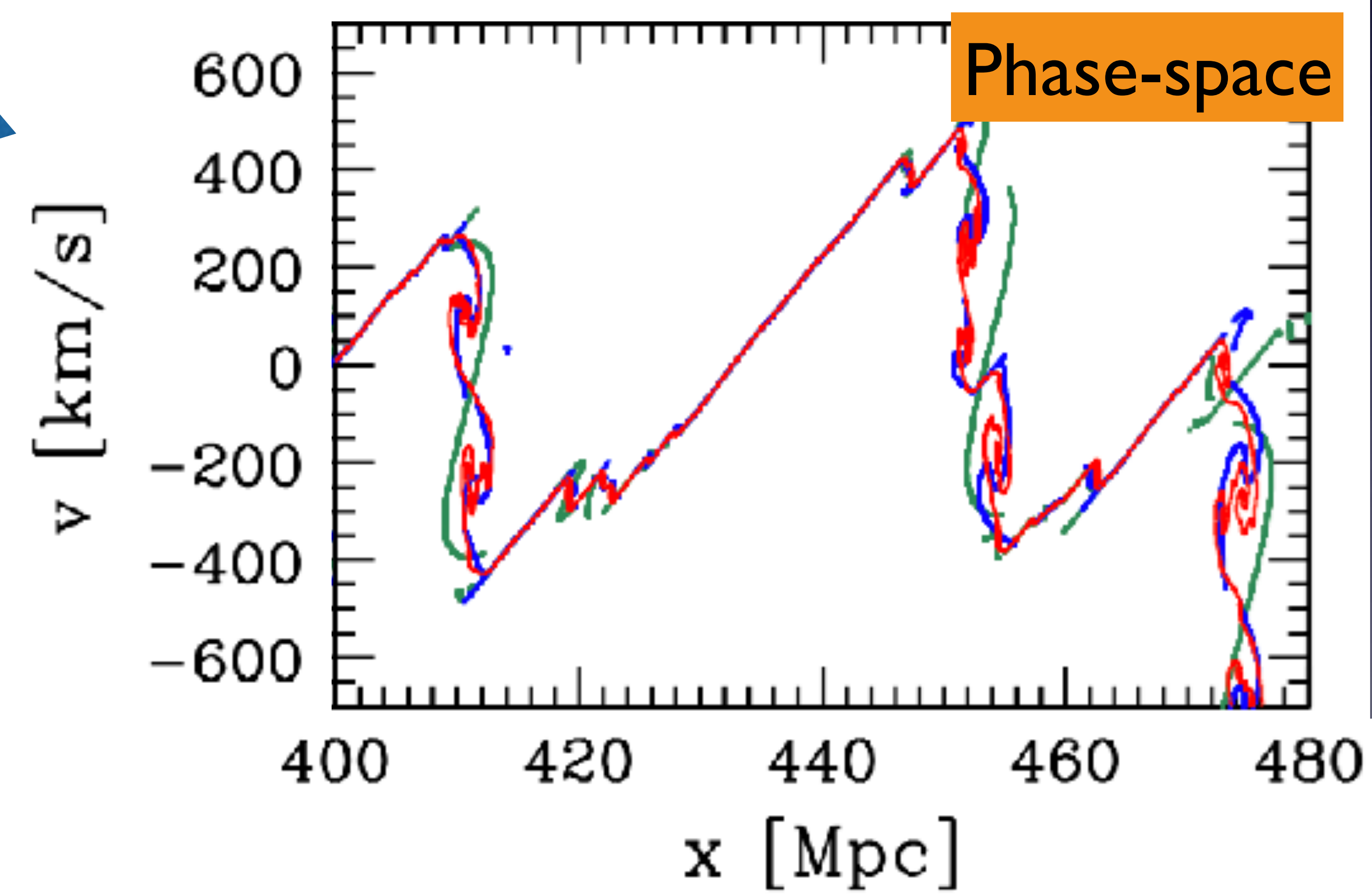
$k\ P(k)/\pi$



Adaptive smoothing

applied to initial density peaks

(with filter scales determined by first-barrier crossing)



AT & Colombi ('17)

Implication to 3D

Combination of the two methods are rather crucial:

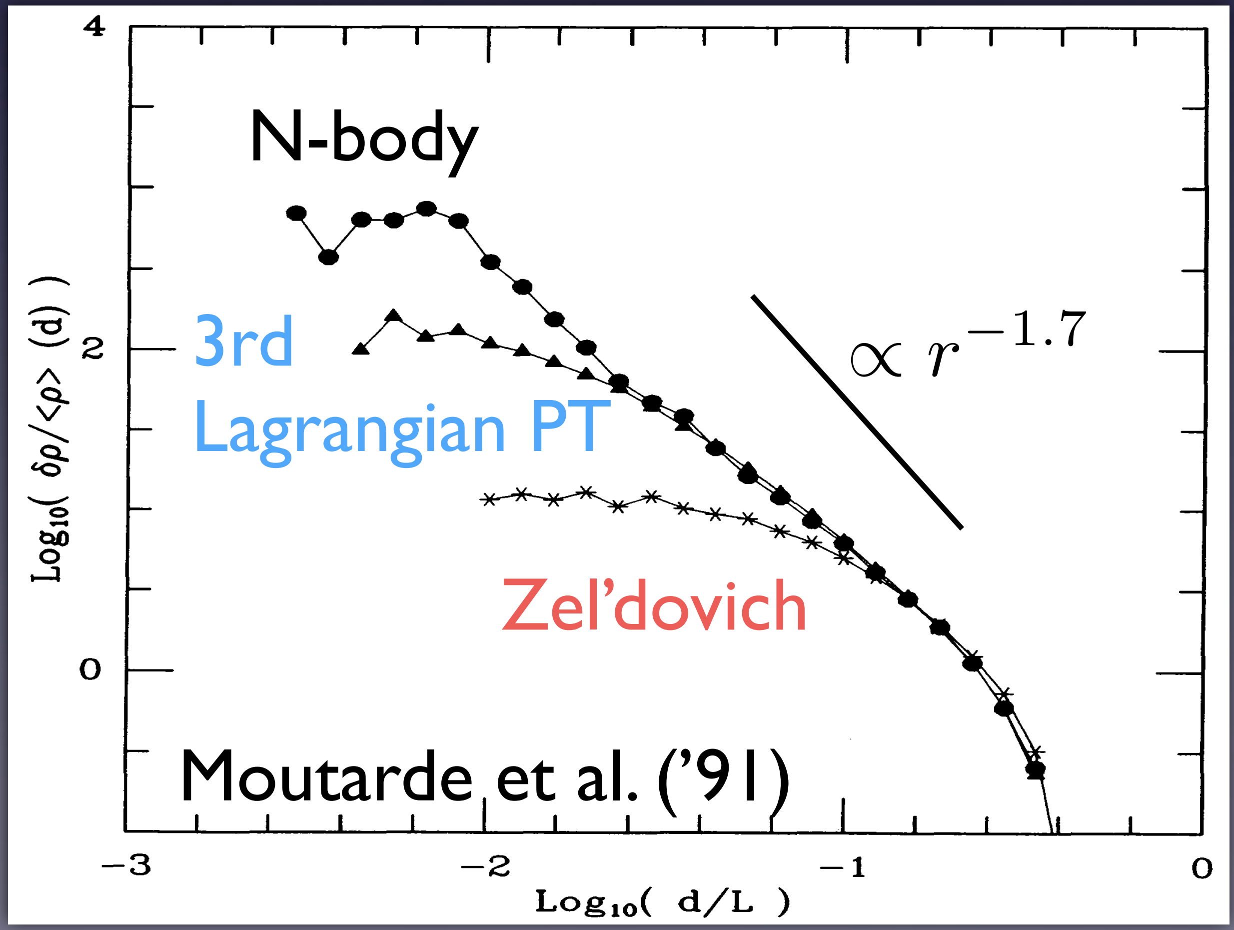
PT scheme beyond shell crossing & *Coarse-graining*

(post-collapse PT) *(adaptive smoothing)*

But, idea & technique are very promising and can be extended to 3D

<u>*Issues to be addressed*</u>

- Accurate pre-collapse description
  - ✓ Zel'dovich approx. is inaccurate
  - ✓ Various topologies of shell crossing
- Tractable analytical calculation of statistical quantities



Summary

Perturbation theory (PT) of large-scale structure has been developed as a precision tool, but it needs to be renovated

- ✓ UV issue in single-stream PT   Do not go to 3-loop !
- ✓ Response function: Characterizing nature of mode coupling
- ✓ Post-collapse PT with adaptive smoothing in 1D:
  Novel scheme beyond shell crossing

Several issues still remain toward a practical application to 3D, and persistent study is needed with a help of 3D Vlasov code

Stay tuned,
Not stick to effective-field-theory approach !